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Topicwise Solved Paper for CDS Mathematics 2020-21

Number System

1. $x^3 + x^2 + 16$ is exactly divisible by x, where x is a positive integer. The number of all such possible values of x is **[CDS-2020-II]**
(a) 3 (b) 4 (c) 5 (d) 6

2. The number of (a, b, c) where a, b, c are positive integers such that abc = 30, is **[CDS-2020-II]**
(a) 30 (b) 27 (c) 9 (d) 8

3. If $I = a^2 + b^2 + c^2$, where a and b are consecutive integers and $c = ab$, then I is **[CDS-2020-II]**
(a) an even number and it is not a square of an integer
(b) an odd number and it is not a square of an integer
(c) square of an even integer
(d) square of an odd integer

4. If the number 23P62971335 is divisible by the smallest odd composite number, then what is the value of P? **[CDS-2020-II]**
(a) 4 (b) 5 (c) 6 (d) 7

5. What is the remainder when the sum $1^5 + 2^5 + 3^5 + 4^5 + 5^5$ is divided by 4? **[CDS-2020-II]**
(a) 0 (b) 1 (c) 2 (d) 3

6. What is the digit in the unit place of 3^{99}? **[CDS-2020-II]**
(a) 1 (b) 3 (c) 7 (d) 9

7. $(x^n - a^n)$ is divisible by $(x - a)$, where $x \neq a$, for every **[CDS-2020-II]**
(a) natural number n
(b) even natural number n only
(c) odd natural number n only
(d) prime number n only

8. If 17^{2020} is divided by 18, then what is the remainder? **[CDS-2020-II]**
(a) 1 (b) 2 (c) 16 (d) 17

9. The sum of all possible products taken two at a time out of the numbers $\pm 1, \pm 2, \pm 3, \pm 4, \pm 5$ is **[CDS-2020-II]**
(a) 0 (b) -30 (c) -55 (d) 55

10. A 60-page book has n lines per page. If the number of lines were reduced by 3 in each page, the number of pages would have to be increased by 10 to given the same writing space. What is the value of n? **[CDS-2020-II]**
(a) 18 (b) 21 (c) 24 (d) 30

11. If $A_n = P_n + 1$, where P_n is the product of the first n prime number, then consider the following statements : **[CDS-2020-II]**
1. A_n is always a composite number.
2. $A_n + 2$ is always an odd number.
3. $A_n + 1$ is always an even number.

Which of the above statements is/are correct?
(a) 1 only (b) 2 only
(c) 3 only (d) 2 and 3 only

12. The sum of the digits of a two digit number is 13 and the difference between the number and that formed by reversing the digits is 27. What is the product of the digits of the number? **[CDS-2020-II]**
(a) 35 (b) 40 (c) 45 (d) 54

13. The sum of the reciprocals of two alternate natural numbers is $\frac{7}{24}$. What is the sum of the numbers? **[CDS-2021-I]**
(a) 12 (b) 13 (c) 14 (d) 16

14. If n is any natural number, then $5^{2n} - 1$ is always divisible by how many natural numbers? **[CDS-2021-I]**
(a) One (b) Four (c) Six (d) Eight

15. If the number 413283P759387 is divisible by 13, then what is the value of P? **[CDS-2021-I]**
(a) 3 (b) 6 (c) 7 (d) 8

16. What is the remainder when $2^{1000000}$ is divided by 7? **[CDS-2021-I]**
(a) 1 (b) 2 (c) 4 (d) 6

17. Consider the pairs of prime numbers (m, n) between 50 and 100 such that $m - n = 6$. How many such pairs are there? **[CDS-2021-I]**
(a) 2 (b) 3 (c) 4 (d) 5

18. What is the remainder when $27^{27} - 15^{27}$ is divided by 6? **[CDS-2021-I]**
(a) 0 (b) 1 (c) 3 (d) 4

19. What is the unit digit in the expansion of 67^{32}? **[CDS-2021-I]**
(a) 1 (b) 3 (c) 7 (d) 9

HCF and LCM

1. LCM of two numbers is 28 times their HCF. The sum of the HCF and the LCM is 1740. If one of these numbers is 240, then what is the other number? **[CDS-2020-II]**
(a) 420 (b) 640 (c) 820 (d) 1040

2. What is the HCF of the polynomials
$x^6 - 3x^4 + 3x^2 - 1$ and $x^3 + 3x^2 + 3x + 1$? **[CDS-2020-II]**
(a) $(x + 1)$ (b) $(x + 1)^2$
(c) $x^2 + 1$ (d) $(x + 1)^3$

3. The HCF and the LCM of two polynomials are $3x + 1$ and $30x^3 + 7x^2 - 10x - 3$ respectively. If one polynomial is $6x^2 + 5x + 1$, then what is the other polynomial? **[CDS-2020-II]**
(a) $15x^2 + 4x + 3$ (b) $15x^2 + 4x - 3$
(c) $15x^2 - 4x + 3$ (d) $15x^2 - 4x - 3$

4. If $(x-k)$ is the HCF of x^2+ax+b and x^2+cx+d, then what is the value of k? **[CDS-2021-I]**

(a) $\frac{d-b}{c-a}$ (b) $\frac{d-b}{a-c}$

(c) $\frac{d+b}{c+a}$ (d) $\frac{d-b}{c+a}$

5. What is the HCF of $x^3-19x+30$ and x^2-5x+6? **[CDS-2021-I]**

(a) $(x+2)(x-3)$ (b) $(x-2)(x+3)$

(c) $(x+2)(x-1)$ (d) $(x-3)(x-2)$

Powers and Roots

1. What is the value of

$$\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+...+\frac{1}{\sqrt{99}+\sqrt{100}}?$$

[CDS-2020-II]

(a) 1 (b) 5 (c) 9 (d) 10

2. If $x^m=\sqrt[14]{x\sqrt{x\sqrt{x}}}$, then what is the value of m? **[CDS-2020-II]**

(a) $\frac{1}{8}$ (b) $\frac{1}{4}$ (c) $\frac{3}{4}$ (d) $\frac{7}{4}$

3. What is the square root of $15-4\sqrt{14}$? **[CDS-2021-I]**

(a) $2\sqrt{2}-\sqrt{7}$ (b) $3\sqrt{2}-\sqrt{7}$

(c) $\sqrt{15}-\sqrt{7}$ (d) $\sqrt{5}-\sqrt{3}$

4. If $p=\frac{\sqrt{3q+2}+\sqrt{3q-2}}{\sqrt{3q+2}-\sqrt{3q-2}}$

then what is the value of $p^2-3pq+2$? **[CDS-2021-I]**

(a) 0 (b) 1 (c) 2 (d) 3

5. What is the value of x, if

$$\frac{b+\sqrt{b^2-2bx}}{b-\sqrt{b^2-2bx}}=a?$$

[CDS-2021-I]

(a) $\frac{ab}{(a+b)}$ (b) $\frac{2ab}{(a+1)}$

(c) $\frac{2ab}{(a+1)^2}$ (d) $\frac{ab}{(a+b)^2}$

Percentage

1. The number of items in a booklet is N. In the first year there is an increase of x% in this number and in the subsequent year there is a decrease of x%. At the end of the two years, what will be the number of items in the booklet? **[CDS-2020-II]**

(a) Less than N
(b) Equal to N
(c) More than N
(d) It depends on the value of N

2. A trader gives successive discounts of 20%, 10% and 5% respectively. What is the overall discount? **[CDS-2021-I]**

(a) 30% (b) 31.6% (c) 32.8% (d) 35%

3. X sells his goods 25% cheaper than Y and 25% dearer than Z. How much percentage is Z's goods cheaper than Y? **[CDS-2021-I]**

(a) $\frac{100}{3}$% (b) 40%

(c) 50% (d) $\frac{200}{3}$%

Ratio and Proportion

1. Three persons start a business with capitals in the ratio $\frac{1}{3}:\frac{1}{4}:\frac{1}{5}$. The first person withdraws half his capital after 4 months. What is the share of profit if the business fetches an annual profit of ₹96,800? **[CDS-2020-II]**

(a) ₹32,000 (b) ₹34,500

(c) ₹36,000 (d) ₹36,800

2. In a mixture of 80 litres of a liquid and water, 25% of the mixutre is the liquid. How much water should be added to the mixture so that the liquid becomes 20% of the mixture? **[CDS-2021-I]**

(a) 15 litres (b) 20 litres

(c) 24 litres (d) 25 litres

3. The incomes of A, B and C are in the ratio 7 : 9 : 12 and their expenditures are in the ratio 8 : 9 : 15. If A's saving is one-fourth of his income, then the ratio of savings of A, B and C is **[CDS-2021-I]**

(a) 56 : 99 : 69 (b) 99 : 56 : 69

(c) 69 : 56 : 99 (d) 99 : 69 : 56

Average

1. Ena was born 4 years after her parents marriage. Her mother is 3 years younger than her father and 24 years older than Ena, who is 13 years old. At what age did Ena's father get married? **[CDS-2020-II]**

(a) 25 years (b) 24 years

(c) 23 years (d) 22 years

2. Mahesh is 60 years old. Ram is 5 years younger to Mahesh and 4 years elder to Raju. Babu is younger brother of Raju and he is 6 years younger. What is the age difference between Mahesh and Babu? **[CDS-2020-II]**

(a) 18 years (b) 15 years

(c) 13 years (d) 11 years

3. The marks obtained by 5 students are 21, 27, 19, 26, 32. Later on 5 grace marks are added to each student. What are the average marks of the revised marks of the students? **[CDS-2021-I]**

(a) 26 (b) 30 (c) 31 (d) 32

4. Let the average score of a class of boys and girls in an examination be p. The ratio of boys and girls in the class is 3 : 1. If the average score of the boys is $(p+1)$, then what is the average score of the girls? **[CDS-2021-I]**

(a) $(p-1)$ (b) $(p-2)$

(c) $(p-3)$ (d) p

Simple and Compound Interest

1. The simple interest on a certain sum is one-fourth of the sum. If the number of years and the rate of annual interest are numerically equal, then the number of years is **[CDS-2020-II]**
(a) 2.5 (b) 3 (c) 3.5 (d) 5

2. A sum of money was invested at simple interest at a certain rate for 5 years. Had it been invested at a 5% higher rate, it would have fetched ₹500 more. What was the principal amount? **[CDS-2021-I]**
(a) ₹2,000 (b) ₹1,800
(c) ₹1,600 (d) ₹1,200

3. The difference between the compound interest (compounded annually) and the simple interest on a certain sum of money at 12% per annum for 2 years is ₹72. What is the principal amount? **[CDS-2021-I]**
(a) ₹6,500 (b) ₹6,000
(c) ₹5,500 (d) ₹5,000

Profit and Loss

1. A shopkeeper sells his articles at their cost price but uses a faulty balance which reads 1000 gm for 800 gm. What is the actual profit percentage? **[CDS-2020-II]**
(a) 20% (b) 25% (c) 30% (d) 40%

2. If a television set is sold at ₹x, a loss of 28% would be incurred. If it is sold at ₹y, a profit of 12% would be incurred. What is the ratio of y to x? **[CDS-2020-II]**
(a) 41 : 9 (b) 31 : 9
(c) 23 : 9 (d) 14 : 9

3. A person sold an article for ₹ 75 which cost him ₹ x. He finds that he realised x% profit on his outlay. What is x equal to ? **[CDS-2020-II]**
(a) 20% (b) 25% (c) 50% (d) 100%

4. The cost price of 100 mangoes is equal to the selling price of 80 mangoes. What is the profit percentage? **[CDS-2021-I]**
(a) 16% (b) 20% (c) 24% (d) 25%

Time , Speed and Distance

1. A train of length 110 m is moving at a uniform speed of 132 km/hr. The time required to cross a bridge of length 165 m is **[CDS-2020-II]**
(a) 6.5 seconds (b) 7 seconds
(c) 7.5 seconds (d) 8.5 seconds

2. By increasing the speed of his car by 15 km/hr, a person covers a distance of 300 km by taking an hour less than before. What was the original speed of the car? **[CDS-2020-II]**
(a) 45 km/hr (b) 50 km/hr
(c) 60 km/hr (d) 75 km/hr

3. *X, Y* and *Z* travel from the same place with uniform speeds 4 km/hr, 5 km/hr and 6 km/hr. respectively. *Y* starts 2 hours after *X*. How long after *Y* must *Z* start in order that they overtake X at the same instant? **[CDS-2020-II]**
(a) $\frac{3}{2}$ hours (b) $\frac{4}{3}$ hours
(c) $\frac{9}{8}$ hours (d) $\frac{11}{8}$ horus

4. A car did a journey in t hours. Had the average speed been x kmph greater, the journey would have taken y hours less. How long was the journey? **[CDS-2020-II]**
(a) $x(t-y)ty$ (b) $x(t-y)ty^{-1}$
(c) $x(t-y)ty^{-2}$ (d) $x(t+y)ty$

5. A train travels 600 km in 5 hours and the next 900 km in 10 hours. What is the average speed of the train? **[CDS-2021-I]**
(a) 80 km/hr. (b) 90 km/hr.
(c) 100 km/hr. (d) 120 km/hr.

6. Walking at $\frac{4}{5}$th of his usual speed, a man is 12 minutes late for his office. What is the usual time taken by him to cover that distance? **[CDS-2021-I]**
(a) 48 minutes (b) 50 minutes
(c) 54 minutes (d) 60 minutes

7. A train 200 m long passes a platform 100 m long in 10 seconds. What is the speed of the train? **[CDS-2021-I]**
(a) 40 m/s (b) 30 m/s
(c) 25 m/s (d) 20 m/s

Time & Work

1. If x men working x hours per day can do x units of work in x day, then y men working y hours per day in y days would be able to do k units of work. What is the value of k? **[CDS-2020-II]**
(a) x^2y^{-3} (b) x^3y^{-2}
(c) y^2x^{-3} (d) y^3x^{-2}

2. If 20 persons can clean 20 floors in 20 days, then in how many days can 16 persons clean 16 floors? **[CDS-2021-I]**
(a) 25 days (b) 24 days
(c) 20 days (d) 16 days

3. Let the work done by $(x-1)$ men in $(x+1)$ days be y. Let the work done by $(x+2)$ men in $(x-1)$ days be z. If $y : z = 9 : 10$, then what is the value of x? **[CDS-2021-I]**
(a) 8 (b) 9 (c) 10 (d) 12

Logarithm

1. If $6^{3-4x}4^{x+5} = 8$ (Given $\log_{10}2 = 0.301$ and $\log_{10}3 = 0.477$), then which one of the following is correct? **[CDS-2020-II]**
(a) $0 < x < 1$ (b) $1 < x < 2$
(c) $2 < x < 3$ (d) $3 < x < 4$

2. The Euclidean algorithm is used to calculate the **[CDS-2020-II]**
(a) square root of an integer
(b) cube root of an integer
(c) square of an integer
(d) HCF of two integer

3. What is $\log_{10} 31.25$ equal to? **[CDS-2021-I]**
(a) $3 - 5\log_{10} 2$ (b) $3 - 2\log_{10} 2$
(c) $5 - 5\log_{10} 2$ (d) $5 - 3\log_{10} 2$

4. If $5^{x-3} = 8$, then what is x equal to? **[CDS-2021-I]**
(a) $\frac{3}{1-\log_{10} 2}$ (b) $\frac{3}{1+\log_{10} 2}$
(c) $\frac{2}{1-\log_{10} 2}$ (d) $\frac{5}{1-\log_{10} 2}$

5. What is the least value of $3\sin^2\theta + 4\cos^2\theta$? **[CDS-2021-I]**
(a) 5 (b) 4 (c) 3 (d) 2

6. If $\sin\theta\cos\theta = k$, where $0 \le \theta \le \frac{\pi}{2}$, then which one of the following is correct? **[CDS-2021-I]**
(a) $0 \le k \le 1$ (b) $0 \le k \le 0.5$ only
(c) $0.5 \le k \le 1$ only (d) $0 < k < 1$

7. If $p = \sin^2\theta + \cos^4\theta$ for $0 \le \theta \le \frac{\pi}{2}$, then consider the following statements :
1. p can be less than $\frac{3}{4}$
2. p can be more than 1.

Which of the above statements is/are correct? **[CDS-2021-I]**
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

8. What is the ratio of greatest to the smallest value of $2 - 2\sin x - \sin^2 x$, $0 \le x \le \frac{\pi}{2}$? **[CDS-2021-I]**
(a) –3 (b) –1 (c) 1 (d) 3

Basic Operation and Factorization

1. The number of different solutions of the equation $x + y + z = 12$, where each of x, y and z is a positive integer, is **[CDS-2020-II]**
(a) 53 (b) 54 (c) 55 (d) 56

2. What is the value of **[CDS-2020-II]**
$$\frac{a^2+ac}{a^2c-c^3} - \frac{a^2-c^2}{a^2c+2ac^2+c^3} - \frac{2c}{a^2-c^2} + \frac{3}{a+c}?$$
(a) 0 (b) 1
(c) $\frac{ac}{a^2+c^2}$ (d) $\frac{6}{a+c}$

3. If $\frac{x}{b+c} = \frac{y}{c+a} = \frac{z}{b-a}$, then which one of the following is correct? **[CDS-2020-II]**
(a) $x+y+z=0$ (b) $x-y-z=0$
(c) $x+y-z=0$ (d) $x+2y+3z=0$

4. $1 - x - x^n + x^{n+1}$, where n is a natural number, is divisible by **[CDS-2020-II]**
(a) $(1+x)^2$ (b) $(1-x)^2$
(c) $1-2x-x^2$ (d) $1+2x-x^2$

5. When a ball is allowed to fall, the time it takes to fall any distance varies as the square root of the distance and it takes 4 seconds to fall 78.40 m. How long would it take to fall 122.50 m? **[CDS-2020-II]**
(a) 5 seconds (b) 5.5 seconds
(c) 6 seconds (d) 6.5 seconds

6. For how many real values of k is $6kx^2 + 12kx - 24x + 16$ a perfect square for every integer x? **[CDS-2021-I]**
(a) Zero (b) One
(c) Two (d) Four

7. If $x + \frac{1}{x} = \frac{5}{2}$, then what is $x^4 - \frac{1}{x^4}$ equal to? **[CDS-2021-I]**
(a) $\frac{195}{16}$ (b) $\frac{255}{16}$ (c) $\frac{625}{16}$ (d) 0

8. $4x^3 + 12x^2 - x - 3$ is divisible by **[CDS-2021-I]**
(a) $(2x+1)$ only
(b) $(2x-1)$ only
(c) Both $(2x+1)$ and $(2x-1)$
(d) Neither $(2x+1)$ nor $(2x-1)$

9. Which one of the following fractions will have minimum change in its value if 3 is added to both the numerator and the denominator of all the fractions? **[CDS-2021-I]**
(a) $\frac{2}{3}$ (b) $\frac{3}{4}$ (c) $\frac{4}{5}$ (d) $\frac{5}{6}$

10. If $\frac{1}{1\times2} + \frac{1}{2\times3} + \frac{1}{3\times4} + \ldots + \frac{1}{n(n+1)} = \frac{99}{100}$ then what is the value of n? **[CDS-2021-I]**
(a) 98 (b) 99 (c) 100 (d) 101

11. The expression $\frac{(x^3-1)(x^2-9x+14)}{(x^2+x+1)(x^2-8x+7)}$ simplifies to **[CDS-2021-I]**
(a) $(x-1)$ (b) $(x-2)$
(c) $(x-7)$ (d) $(x+2)$

12. What should be added to $\frac{1}{(x-2)(x-4)}$ to get $\frac{2x-5}{(x^2-5x+6)(x-4)}$? **[CDS-2021-I]**
(a) $\frac{1}{(x^2-7x+12)}$ (b) $\frac{1}{(x^2+7x+12)}$
(c) $\frac{1}{(x^2-7x-12)}$ (d) $\frac{1}{(x^2+7x-12)}$

13. If $\frac{x}{a} + \frac{y}{b} = a + b$ and $\frac{x}{a^2} + \frac{y}{b^2} = 2$, then what is $\frac{x}{a^2} - \frac{y}{b^2}$ equal to? **[CDS-2021-I]**
(a) –2 (b) –1 (c) 0 (d) 1

14. Consider the following statements:
    1. If x is directly proportional to z and y is directly proportional to z, then $(x^2 - y^2)$ is directly proportional to z^2.
    2. If x is inversely proportional to z and y is inversely proportional to z, then (xy) is inversely proportional to z^2.

    Which of the above statement is/are correct? **[CDS-2021-I]**

    (a) 1 only (b) 2 only
    (c) Both 1 and 2 (d) Neither 1 nor 2

15. What is $\frac{8x}{1-x^4} - \frac{4x}{x^2+1} + \frac{x+1}{x-1} - \frac{x-1}{x+1}$ equal to ? **[CDS-2021-I]**

    (a) 0 (b) 1 (c) 2 (d) 4

16. For what integral value of x is

    $$\frac{12}{7 - \frac{6}{7 - \frac{3}{5-x}}} = x?$$ **[CDS-2021-I]**

    (a) 4 (b) 3 (c) 2 (d) 1

17. If $x(x-1)(x-2)(x-3) + 1 = k^2$, then which one of the following is a possible expression for k? **[CDS-2021-I]**

    (a) $x^2 - 3x + 1$ (b) $x^2 - 3x - 1$
    (c) $x^2 + 3x - 1$ (d) $x^2 - 2x - 1$

18. What is $\frac{1}{bc(a-b)(b-c)} + \frac{1}{ca(b-c)(b-a)} + \frac{1}{ab(c-a)(c-b)}$ equal to ? **[CDS-2021-I]**

    (a) a + b + c (b) 3
    (c) ab + bc + ca (d) 0

19. How many terms are there in the following product?

    $$(a_1 + a_2 + a_3)(b_1 + b_2 + b_3 + b_4)(c_1 + c_2 + c_3 + c_4 + c_5)$$ **[CDS-2021-I]**

    (a) 15 (b) 30 (c) 45 (d) 60

20. If a + b + c = 0, then which of the following are correct?
    1. $a^3 + b^3 + c^3 = 3abc$
    2. $a^2 + b^2 + c^2 = -2(ab + bc + ca)$
    3. $a^3 + b^3 + c^3 = -3ab(a + b)$

    Select the correct answer using the code given below. **[CDS-2021-I]**

    (a) 1 and 2 only (b) 2 and 3 only
    (c) 1 and 3 only (d) 1, 2 and 3

Linear Equations in One and Two Variables

1. If $ab + xy - xb = 0$ and $bc + yz - cy = 0$, then what is $\frac{x}{a} + \frac{c}{z}$ equal to ? **[CDS-2020-II]**

    (a) $\frac{y}{b}$ (b) $\frac{b}{y}$ (c) 1 (d) 0

2. If $(p+2)(2q-1) = 2pq - 10$ and $(p-2)(2q-1) = 2pq - 10$, then what is pq equal to ? **[CDS-2020-II]**

    (a) – 10 (b) – 5 (c) 5 (d) 10

Quadratic Equations

1. If the roots of the quadratic equation $x^2 - 4x - \log_{10} N = 0$ are real, then what is the minimum value of N? **[CDS-2020-II]**

    (a) 1 (b) $\frac{1}{10}$ (c) $\frac{1}{100}$ (d) $\frac{1}{10000}$

2. What is the square root of $4x^4 + 8x^3 - 4x + 1$? **[CDS-2020-II]**

    (a) $2x^2 - 2x - 1$ (b) $2x^2 - x - 1$
    (c) $2x^2 + 2x + 1$ (d) $2x^2 + 2x - 1$

3. If the equation $x^2 + y^2 - 2xy\sin^2\theta = 0$ contains real solution for x and y, then **[CDS-2021-I]**

    (a) $x = y$ (b) $x = -y$
    (c) $x = 2y$ (d) $2x = y$

4. If the equation $4x^2 - 2kx + 3k = 0$ has equal roots, then what are the values of k? **[CDS-2021-I]**

    (a) 4, 12 (b) 4, 8 (c) 0, 12 (d) 0, 8

5. If the sum as well as the product of the roots of the equation $px^2 - 6x + q = 0$ is 6, then what is $(p + q)$ equal to ? **[CDS-2021-I]**

    (a) 8 (b) 7 (c) 6 (d) 5

Set Theory

1. Let $d(n)$ denote the number of positive divisors of a positive integer n. Which of the following are correct?
    1. $d(5) = d(11)$
    2. $d(5).d(11) = d(55)$
    3. $d(5) + d(11) = d(16)$

    Select the correct answer using the code given below : **[CDS-2020-II]**

    (a) 1 and 3 only (b) 1 and 2 only
    (c) 2 and 3 only (d) 1, 2 and 3

2. If x varies as y, then which of the following is/are correct?
    1. $x^2 + y^2 +$ varies as $x^2 - y^2$
    2. $\frac{x}{y^2}$ varies inversely as y
    3. $\sqrt[2]{x^2 y}$ varies as $\sqrt[2n]{x^4 y^2}$

    Select the correct answer usign the code given below : **[CDS-2020-II]**

    (a) 1 and 2 only (b) 2 and 3 only
    (c) 3 only (d) 1, 2 and 3

3. How many pairs of (x, y) can be chosen from the set {2, 3, 6, 8, 9} such that $\frac{x}{y} + \frac{y}{x} = 2$, where $x \neq y$? **[CDS-2021-I]**

    (a) Zero (b) One
    (c) Two (d) Three

Trigonometric Ratios & Identities

1. If cosec θ – sin θ = m and sec θ – cos θ = n, then what is $m^{\frac{4}{3}}n^{\frac{2}{3}} + m^{\frac{2}{3}}n^{\frac{4}{3}}$ equal to ? **[CDS-2020-II]**
(a) 0 (b) 1
(c) mn (d) m^2n^2

2. If cos θ + sec θ = k, then what is the value of $\sin^2\theta - \tan^2\theta$? **[CDS-2020-II]**
(a) $4 - k$ (b) $4 - k^2$
(c) $k^2 - 4$ (d) $k^2 + 2$

3. Consider the following statements :
1. $\sin\theta = x + \frac{1}{x}$ is possible for some real value of x.
2. $\cos\theta = x + \frac{1}{x}$ is possible for some real value of x.

Which of the above statements is/are correct? **[CDS-2020-II]**
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

4. Consider the following statemetns :
1. The equation $2\sin^2\theta - \cos\theta + 4 = 0$ is possible for all θ.
2. tan θ + cot θ cannot be less than 2, where $0 < \theta < \frac{\pi}{2}$.

Which of the above statements is/are correct? **[CDS-2020-II]**
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

5. What is the maximum value of 3 sin θ – 4 ? **[CDS-2020-II]**
(a) –4 (b) – 1 (c) 0 (d) 1

6. If $\sin\theta + \cos\theta = \sqrt{2}$, then what is $\sin^6\theta + \cos^6\theta + 6\sin^2\theta\cos^2\theta$ equal to ? **[CDS-2020-II]**
(a) $\frac{1}{4}$ (b) $\frac{3}{4}$ (c) 1 (d) $\frac{7}{4}$

7. What is the least value of $9\sin^2\theta + 16\cos^2\theta$? **[CDS-2020-II]**
(a) 0 (b) 9 (c) 16 (d) 25

8. If cos 47° + sin 47° = k, then what is the value of $\cos^2 47° - \sin^2 47°$? **[CDS-2020-II]**
(a) $k\sqrt{2-k^2}$ (b) $-k\sqrt{2-k^2}$
(c) $k\sqrt{1-k^2}$ (d) $-k\sqrt{1-k^2}$

9. If cosec θ – sin θ = p^3 and sec θ – cos θ = q^3 then what is the value of tan θ ? **[CDS-2020-II]**
(a) $\frac{p}{q}$ (b) $\frac{q}{p}$
(c) pq (d) p^2q^2

10. If 0 ≤ α, β ≤ 90° such that cos (α – β) = 1, then what is sin α – sin β + cos α – cos β equal to ? **[CDS-2020-II]**
(a) – 1 (b) 0 (c) 1 (d) 2

11. Consider the following statements :
1. The value of cos 61° + sin 29° cannot exceed 1.
2. The value of tan 23° – cot 67° is less than 0.

Which of the above statements is/are correct? **[CDS-2020-II]**
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

12. Consider the following inequalities :
1. sin1° < cos 57° 2. cos 60° > sin 57°

Which of the above is/are correct? **[CDS-2021-I]**
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

13. If p = sec θ – tan θ and q = cosecθ + cotθ, then what is $p + q(p - 1)$ equal to ? **[CDS-2021-I]**
(a) –1 (b) 0 (c) 1 (d) 2

14. If cosecθ – cotθ = m, then what is cosecθ equal to ? **[CDS-2021-I]**
(a) $m + \frac{1}{m}$ (b) $m - \frac{1}{m}$
(c) $\frac{m}{2} + \frac{2}{m}$ (d) $\frac{m}{2} + \frac{1}{2m}$

15. Let ABC be a triangle right angled at C, then what is tan A + tan B equal to ? **[CDS-2021-I]**
(a) $\frac{a}{bc}$ (b) $\frac{a^2}{bc}$ (c) $\frac{b^2}{ca}$ (d) $\frac{c^2}{ab}$

16. Let cosα + cosβ = 2 and sinα + sinβ = 0, where 0 ≤ α ≤ 90°, 0 ≤ β ≤ 90°. What is the value of cos2α – cos 2β? **[CDS-2021-I]**
(a) 0
(b) 1
(c) 2
(d) Cannot be determined due to insufficient data

17. If secθ + cosθ = $\frac{5}{2}$, where 0 ≤ θ ≤ 90°, then what is the value of $\sin^2\theta$? **[CDS-2021-I]**
(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) 1

18. What is (1 + cotθ – cosecθ) (1 + tanθ + secθ) equal to ? **[CDS-2021-I]**
(a) 4 (b) 3 (c) 2 (d) 1

19. If 6 + 8 tanθ = secθ and 8 – 6 tanθ = k sec θ, then what is the value of k^2? **[CDS-2021-I]**
(a) 11 (b) 22 (c) 77 (d) 99

Height and Distance

1. A pole on the ground leans at 60° with the vertical. At a point x metre away from the base of the pole on the ground, two halves of the pole subtend the same angle. If the pole and the point are in the same vertical plane, then what is the length of the pole? **[CDS-2021-I]**
(a) $\sqrt{2}x$ metre (b) $\sqrt{3}x$ metre
(c) $2x$ metre (d) $2\sqrt{2}x$ metre

2. A vertical tower standing at the corner of a rectangular field subtends angles of 60° and 45° at the two nearer corners. If θ is the angle that the tower subtends at the farthest corner, then what is $\cot\theta$ equal to? **[CDS-2021-I]**

(a) $\frac{1}{2}$ (b) 2 (c) $\frac{2}{\sqrt{3}}$ (d) $\frac{4}{\sqrt{3}}$

Area and Perimeter

1. The two sides of a triangle are 40 cm and 41 cm. If the perimeter of the triangle is 90 cm, what is its area ? **[CDS-2020-II]**

(a) 90 cm^2 (b) 135 cm^2
(c) 150 cm^2 (d) 180 cm^2

2. The diagonals of a rhombus differ by 2 units and its perimeter exceeds the sum of the diagonals by 6 units. What is the area of the rhombus? **[CDS-2020-II]**

(a) 48 square units (b) 36 square units
(c) 24 square units (d) 12 square units

3. What is the area of a right-angled triangle, if the radius of the circumcircle is 5 cm and altitude drawn to the hypotenuse is 4 cm? **[CDS-2020-II]**

(a) 20 cm^2 (b) 18 cm^2
(c) 16 cm^2 (d) 10 cm^2

4. The length of a rectangle is increased by 10% and breadth is decreased by 10%. Then the area of the new rectangle is **[CDS-2020-II]**

(a) neither increased nor decreased
(b) increased by 1%
(c) decreased by 1%
(d) decreased by 10%

5. A bicycle wheel makes 5000 revolutions in moving 11 km. What is the radius of the wheel? (Assume $\pi = \frac{22}{7}$) **[CDS-2020-II]**

(a) 17.5 cm (b) 35 cm
(c) 70 cm (d) 140 cm

6. If the perimeter of a circle and a square are equal, then what is the ratio of the area of the circle to that of the square? **[CDS-2020-II]**

(a) $1:\pi$ (b) $2:\pi$ (c) $3:\pi$ (d) $4:\pi$

7. A triangle and a parallelogram have equal areas and equal bases. If the altitude of the triangle is k times the altitude of the parallelogram, then what is the value of k? **[CDS-2020-II]**

(a) 4 (b) 2 (c) 1 (d) $\frac{1}{2}$

8. Areas of the squares are in the ratio $m^2 : n^4$. What is the ratio of their perimeters? **[CDS-2020-II]**

(a) $m : n$ (b) $n : m$
(c) $m : n^2$ (d) $m^2 : n$

9. Four circular coins of equal radius are placed with their centres coinciding with four vertices of a square. Each coin touches two other coins. If the uncovered area of the squrea is 42 cm^2, then what is the radius of each coin? (Assume $\pi = \frac{22}{7}$) **[CDS-2020-II]**

(a) 5 cm (b) 7 cm (c) 10 cm (d) 14 cm

10. The sides of a right-angled triangle are in the ratio $x : (x-1) : (x-18)$. What is the perimeter of the triangle? **[CDS-2021-I]**

(a) 28 units (b) 42 units
(c) 56 units (d) 84 units

11. If the perimeter of a right-angled triangle is 30 cm and the hypotenuse is 13 cm, then what is the area of the triangle? **[CDS-2021-I]**

(a) 24 cm^2 (b) 27 cm^2
(c) 30 cm^2 (d) 36 cm^2

12. A wire is in the form of a circle of radius 70 cm. If it is bent in the form of a rhombus, then what is its side length? (Take $\pi = \frac{22}{7}$) **[CDS-2021-I]**

(a) 55 cm (b) 75 cm
(c) 95 cm (d) 110 cm

13. If the perimeter of a semicircular park is 360 m, then what is its area? (Take $\pi = \frac{22}{7}$) **[CDS-2021-I]**

(a) 3850 m^2 (b) 7700 m^2
(c) 11550 m^2 (d) 15400 m^2

Volume and Surface Area

1. A river 3 m deep and 40 m wide is flowing at the rate of 2 km/hr and falls into the sea. What is the amount of water in litres that will fall into the sea from this river in a minute? **[CDS-2020-II]**

(a) 40,00,000 litres (b) 4,00,000 litres
(c) 40,000 litres (d) 4,000 litres

2. If H, C and V are respectively the height, curved surface area and volume of a cone, then what is $3\pi VH^3 + 9V^2$ equal to ? **[CDS-2020-II]**

(a) C^2H^2 (b) $2\,C^2H^2$
(c) $5\,C^2H^2$ (d) $7\,C^2H^2$

3. The surface areas of two spheres are in the ratio 1 : 4. What is the ratio of their volumes ? **[CDS-2020-II]**

(a) 1 : 16 (b) 1 : 12
(c) 1 : 10 (d) 1 : 8

4. The length, breadth and height of a brick are 20 cm, 15 cm and 10 cm respectively. The number of bricks required to construct a wall with dimensions 45 m length, 0.15 m breadth and 3 m height is **[CDS-2020-II]**

(a) 12450 (b) 11250
(c) 6750 (d) None of the above

5. The volumes of two cones are in the ratio 1 : 4 and their diameters are in the ratio 4 : 5. What is the ratio of their heights? **[CDS-2020-II]**

(a) 25 : 64 (b) 16 : 25
(c) 9 : 16 (d) 5 : 9

6. The radii of the flat circular faces of a bucket are x and $2x$. If the height of the bucket is $3x$. What is the capacity of the bucket ? (Assume $\pi = \frac{22}{7}$) **[CDS-2020-II]**
   (a) $11x^3$ (b) $22x^3$
   (c) $44x^3$ (d) $55x^3$

7. If p, q, r, s and t represent length, breadth, height, surface area and volume of a cuboid respectively, then what is $\frac{1}{p}+\frac{1}{q}+\frac{1}{r}$ equal to ? **[CDS-2020-II]**
   (a) $\frac{s}{t}$ (b) $\frac{2t}{s}$ (c) $\frac{s}{2t}$ (d) $\frac{2s}{t}$

8. A cloth of 3 m width is used to make a conical tent 12 m in diameter with a slant height of 7 m. What is the length of the cloth ? (Take $\pi = \frac{22}{7}$) **[CDS-2021-I]**
   (a) 21 m (b) 28 m (c) 44 m (d) 66 m

9. A sphere of diameter 6 cm is dropped into a cylindrical vessel partly filled with water. The radius of the vessel is 6 cm. If the sphere is completely submerged in water, then by how much will the surface level of water be raisded? **[CDS-2021-I]**
   (a) 0.5 cm (b) 1 cm (c) 1.5 cm (d) 2 cm

10. A sector is cut from a circle of radius 21 cm. If the length of the arc of the sector is 55 cm, then what is the area of the sector? **[CDS-2021-I]**
    (a) 577.5 cm^2 (b) 612.5 cm^2
    (c) 705.5 cm^2 (d) 725.5 cm^2

11. The surface area of a cube is equal to that of a sphere. If x is the volume of the cube and y is the volume of the sphere, then what is $x^2 : y^2$ equal to : **[CDS-2021-I]**
    (a) $\pi : 6$ (b) $6 : \pi$
    (c) $\pi : 3$ (d) $3 : \pi$

12. A cone and a hemisphere have equal bases and equal volumes. What is the ratio of the height of the cone to the radius of the hemisphere ? **[CDS-2021-I]**
    (a) 1 : 1 (b) 2 : 1
    (c) 3 : 2 (d) 4 : 3

13. A solid sphere of diameter 60 mm is melted to stretch into a wire of length 144 cm. What is the diameter of the wire ? **[CDS-2021-I]**
    (a) 0.5 cm (b) 1 cm
    (c) 1.5 cm (d) 2 cm

14. The ratio of the radius of base to the height of a cylinder is 2 : 3. If the volume of the cylinder is 1617 cm^3, then what is the curved surface area of the cylinder? (Take $\pi = \frac{22}{7}$) **[CDS-2021-I]**
    (a) 242 cm^2 (b) 385 cm^2
    (c) 462 cm^2 (d) 770 cm^2

15. The difference between the outside and the inside surface area of a cylindrical pipe 14 cm long is 44 cm^2. The pipe is made of 99 cm^3 of metal. If R is the outer radius and r is the inner radius of the pipe, then what is $(R + r)$ equal to? (Take $\pi = \frac{22}{7}$) **[CDS-2021-I]**
    (a) 9 cm (b) 7.5 cm
    (c) 6 cm (d) 4.5 cm

16. A metal solid cube of edge 24 cm is melted and made into three small cubes. If the edges of two small cubes are 12 cm and 16 cm, then what is the surface area of the third small cube? **[CDS-2021-I]**
    (a) 1200 cm^2 (b) 1800 cm^2
    (c) 2400 cm^2 (d) 3600 cm^2

17. A conical vessel whose internal radius is 5 cm and height 24 cm is full of water. The water is emptied into a cylindrical vessel with internal radius 10 cm. What is the height of which the water rises? **[CDS-2021-I]**
    (a) 1 cm (b) 2 cm
    (c) 3 cm (d) 4 cm

18. A metal solid cube of side 22 cm is melted to make a cone of height 21 cm. What is the radius of the base of the cone? (Take $\pi = \frac{22}{7}$) **[CDS-2021-I]**
    (a) 11 cm (b) 16.5 cm (c) 22 cm (d) 27.5 cm

19. A cone of height 24 cm has a curved surface area 550 cm^2. What is the ratio of its radius to slant height? (Take $\pi = \frac{22}{7}$) **[CDS-2021-I]**
    (a) $\frac{5}{12}$ (b) $\frac{5}{13}$ (c) $\frac{7}{25}$ (d) $\frac{7}{27}$

20. A rectangular paper is 44 cm long and 22 cm wide. Let x be the volume of the largest cylinder formed by rolling the paper along its length and y be the volume of the largest cylinder formed by rolling the paper along its width. What is the ratio of x to y? (Take $\pi = \frac{22}{7}$) **[CDS-2021-I]**
    (a) 1 : 1 (b) 2 : 1 (c) 1 : 2 (d) 3 : 2

21. A hollow sphereical shell is made up of a metal of density 3 g/cm^3. If the internal and external radii are 5 cm and 6 cm respectively, then what is the mass of the shell? (Take $\pi = \frac{22}{7}$) **[CDS-2021-I]**
    (a) 1144 g (b) 1024 g
    (c) 840 g (d) 570 g

Lines and Angles

1. The difference between two angles is 15° and the sum of the angles in radius is $\frac{5\pi}{12}$. The bigger angle is k times the smaller angle. What is k equal to? **[CDS-2020-II]**
   (a) $\frac{4}{3}$ (b) $\frac{3}{2}$ (c) $\frac{6}{5}$ (d) $\frac{7}{6}$

Triangles and its Properties

1. In a ΔABC, $AC = 12$ cm, $AB = 16$ cm and AD is the bisector of $\angle A$. If $BD = 4$ cm, then what is DC equal to ? **[CDS-2020-II]**
   (a) 2 cm (b) 3 cm
   (c) 4 cm (d) 5 cm
2. ABC is an equilateral triangle. The side BC is trisected at D such that $BC = 3\ BD$. What is the ratio of AD^2 to AB^2 ? **[CDS-2020-II]**
   (a) 7 : 9 (b) 1 : 3
   (c) 5 : 7 (d) 1 : 2
3. In a triangle, values of all the angles are integers (in degree measure). Which one of the following cannot be the proportion of their measures ? **[CDS-2020-II]**
   (a) 1 : 2 : 3 (b) 3 : 4 : 5
   (c) 5 : 6 : 7 (d) 6 : 7 : 8
4. In a triangle ABC, if $2\ \angle A = 3\ \angle B = 6\ \angle C$, then what is $\angle A + \angle C$ equal to? **[CDS-2020-II]**
   (a) 90° (b) 120°
   (c) 135° (d) 150°
5. The lengths of the sides of a right-angled triangle are consecutive even integers (in cm). What is the product of these integers? **[CDS-2020-II]**
   (a) 60 (b) 120 (c) 360 (d) 480
6. The sum of the squares of sides of a right-angled triangle is 8,450 square units. What is the length of its hypotenuse? **[CDS-2020-II]**
   (a) 50 units (b) 55 units
   (c) 60 units (d) 65 units
7. AD is the median of the triangle ABC. If P is any point on AD, then which one of the following is correct? **[CDS-2020-II]**
   (a) Area of triangle PAB is greater than the area of triangle PAC
   (b) Area of triangle PAB is equal to area of triangle PAC
   (c) Area of triangle PAB is one-fourth of the area of triangle PAC
   (d) Area of triangle PAB is half of the area of triangle PAC
8. ABC is a triangle right-angled at C. Let P be any point on AC and Q be any point on BC. Which of the following statements is/are correct?
   1. $AQ^2 + BP^2 = AB^2 + PQ^2$
   2. $AB = 2PQ$

   Select the correct answer using the code given below : **[CDS-2020-II]**
   (a) 1 only
   (b) 2 only
   (c) Both 1 and 2
   (d) Neither 1 nor 2
9. ABC is a triangle right angled at B. Let M and N be two points on AB such that $AM = MN = NB$. Let P and Q be two points on AC such that PM is parallel to QN and QN is parallel to CB. If $BC = 12$ cm, then what is $(PM + QN)$ equal to? **[CDS-2021-I]**
   (a) 10 cm (b) 11 cm (c) 12 cm (d) 13 cm
10. An equilateral triangle ABC and a scalene triangle DBC are inscribed in a circle on same side of the arc. What is $\angle BDC$ equal to? **[CDS-2021-I]**
    (a) 30° (b) 45° (c) 60° (d) 90°
11. ABC is a triangle right angled at C. Let p be the length of the perpendicular drawn from C on AB. If $BC = 6$ cm and $CA = 8$ cm, then what is the value of p? **[CDS-2021-I]**
    (a) 5.4 cm (b) 5 cm
    (c) 4.8 cm (d) 4.2 cm
12. ΔABC is similar to ΔDEF. The perimeters of ΔABC and ΔDEF are 40 cm and 30 cm respectively. What is the ratio of $(BC + CA)$ to $(EF + FD)$ equal to ? **[CDS-2021-I]**
    (a) 5 : 4 (b) 4 : 3
    (c) 3 : 2 (d) 2 : 1
13. Two isosceles triangles have equal vertical angles and their areas are in the ratio 4.84 : 5.29. What is the ratio of their corresponding heights? **[CDS-2021-I]**
    (a) 11 : 23 (b) 23 : 25
    (c) 22 : 23 (d) 484 : 529
14. ABC is a triangle right angled at A and AD is perpendicular to BC. If $BD = 8$ cm and $DC = 12.5$ cm, then what is AD equal to ? **[CDS-2021-I]**
    (a) 7.5 cm (b) 8.5 cm
    (c) 9 cm (d) 10 cm

Quadrilateral and Polygon

1. What is the magnitude (in radian) of the interior angle of a regular pentagon? **[CDS-2020-II]**
   (a) $\frac{\pi}{5}$ (b) $\frac{2\pi}{5}$ (c) $\frac{3\pi}{5}$ (d) $\frac{4\pi}{5}$
2. In a quadrilateral $ABCD$, $\angle B = 90°$ and $AB^2 + BC^2 + CD^2 - AD^2 = 0$, then what is $\angle ACD$ equal to ? **[CDS-2020-II]**
   (a) 30° (b) 60° (c) 90° (d) 120°
3. $ABCD$ is a cyclic quadrilateral. The bisectors of the angles A, B, C and D cut the circle at P, Q, R and S respectively. What is $\angle PQR + \angle RSP$ equal to ? **[CDS-2020-II]**
   (a) 90° (b) 135° (c) 180° (d) 270°
4. Consider the following statements :
   1. The diagonals of a trapezium divide each other proportionally.
   2. Any line drawn parallel to the parallel sides of a trapezium divides the non-parallel sides proportionally.

   Which of the above statements is/are correct? **[CDS-2020-II]**
   (a) 1 only (b) 2 only
   (c) Both 1 and 2 (d) Neither 1 nor 2
5. If the sum of all interior angles of a regular polygon is twice the sum of all its exterior angles, then the polygon is **[CDS-2020-II]**
   (a) Hexagon (b) Octagon
   (c) Nonagon (d) Decagon

6. $ABCD$ is a trapezium in which AB is parallel to DC and $2AB = 3DC$. The diagonals AC and BD intersect at O. What is the ratio of the area of ΔAOB to that of ΔDOC? **[CDS-2021-I]**
(a) 2 : 1 (b) 3 : 2
(c) 4 : 1 (d) 9 : 4

7. A circle touches all the four sides of a quadrilateral $ABCD$. If $AB = 9$ cm, $BC = 8$ cm and $CD = 12$ cm, then what is DA equal to ? **[CDS-2021-I]**
(a) 14 cm (b) 13 cm (c) 12 cm (d) 11 cm

8. In a trapezium $ABCD$, AB is parallel to DC. The diagonals AC and BD intersect at P. If $AP : PC = 4 : (4x - 4)$ and $BP : PD = (2x - 1) : (2x + 4)$, then what is the value of x? **[CDS-2021-I]**
(a) 4 (b) 3 (c) $\frac{3}{2}$ (d) 2

Circle

1. If radius of a sphere is rational, then which of the following is/are correct?
   1. Its surface area is rational
   2. Its volume is rational

   Select the correct answer using the code given below : **[CDS-2020-II]**
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

2. ABC is a triangle inscribed in a semicircle of diameter AB. What is $\cos(A + B) + \sin(A + B)$ equal to ? **[CDS-2020-II]**
(a) 0 (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) 1

3. A road curve is to be laid out of a circle. What radius should be used if the track is to change direction by 42° in distance of 44 m? (Assume $\pi = \frac{22}{7}$) **[CDS-2020-II]**
(a) 60 m (b) 66 m (c) 75 m (d) 80 m

4. How many solid lead balls each of diameter 2 mm can be made from a solid lead ball of radius 8 cm? **[CDS-2020-II]**
(a) 512 (b) 1024 (c) 256000 (d) 512000

5. A circle is inscribed in a triangle ABC. It touches the sides AB and AC at M and N respectively. If O is the centre of the circle and $\angle A = 70°$, then what is $\angle MON$ equal to ? **[CDS-2020-II]**
(a) 90° (b) 100° (c) 110° (d) 120°

6. What is the area of segment of a circle of radius r subtending an angle θ at the centre ? **[CDS-2020-II]**
(a) $\frac{1}{2}r^2\theta$
(b) $\frac{1}{2}r^2\left(\theta - 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)$
(c) $\frac{1}{2}r^2\left(\theta - \sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)$
(d) $\frac{1}{2}r^2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

7. AB and CD are the diameters of a circle which intersect at P. Join AC, CB, BD and DA. If $\angle PAD = 60°$, then what is $\angle BPD$ equal to? **[CDS-2021-I]**
(a) 30° (b) 60° (c) 90° (d) 120°

8. The sides of a triangle ABC are 4 cm, 6 cm and 8 cm. With the vertices of the triangle as centres, three circles are drawn each touching the other two externally. What is the sum of the radii of the three circles? **[CDS-2021-I]**
(a) 6 cm (b) 7 cm (c) 9 cm (d) 10 cm

9. Let PAB be a secant to a circle intersecting the circle at A and B. Let PT be the tangent segment. If $PA = 9$ cm and PT = 12 cm, then what is AB equal to? **[CDS-2021-I]**
(a) 5 cm (b) 6 cm (c) 7 cm (d) 9 cm

Statistics

1. Fifteen candidates appeared in an examination. The marks of the candidates who passed in the examination are 9, 6, 7,8, 8, 9, 6, 5, 4 and 7. What is the median of marks of all the fifteen candidates? **[CDS-2020-II]**
(a) 6 (b) 6.5 (c) 7 (d) 7.5

2. If the yield (in gm) of barley from 7 plots of size one square yard each, were found to be 180, 191, 175, 111, 154, 141 and 176, then what is the median yield? **[CDS-2020-II]**
(a) 111 gm (b) 154 gm
(c) 175 gm (d) 176 gm

3. Which one of the following measures of central tendency will be used to determine the average size of the shoe sold in the shop? **[CDS-2020-II]**
(a) Arithmetic mean (b) Geometric mean
(c) Median (d) Mode

4. When the class intervals have equal width, the heigth of a rectangle in a histogram represents **[CDS-2020-II]**
(a) Width of the class
(b) Lower class limit
(c) Upper class limit
(d) Frequency of the class

5. The ages of 7 family members are 2, 5, 12, 18, 38, 40 and 60 years respectively. After 5 years a new member aged x years is added. If the mean age of the family now goes up by 1.5 years, then what is the value of x? **[CDS-2020-II]**
(a) 1 (b) 2 (c) 3 (d) 4

6. The mean weight of 100 students in a class is 46 kg. The mean weight ofboys is 50 kg and that of girls is 40 kg. The number of boys exceeds the number of girls by **[CDS-2020-II]**
(a) 10 (b) 15 (c) 20 (d) 25

7. What is the algebraic sum of the deviations from the mean of a set of values 25, 65, 73, 75, 83, 76, 17, 15, 7, 14 ? **[CDS-2020-II]**
(a) -1 (b) 0 (c) 1 (d) 2

8. The mean of five observations $x, x+2, x+4, x+6, x+8$ is m. What is the mean of the first five observations? **[CDS-2020-II]**
(a) m (b) $m-1$ (c) $m-2$ (d) $m-3$

9. What is the median of 2, 4, 6, ..., 100? **[CDS-2020-II]**
(a) 48 (b) 49 (c) 50 (d) 51

10. The harmonic mean and the geometric mean of two numbers are 10 and 12 respectively. What is their arithmetic mean? **[CDS-2020-II]**
(a) $\frac{25}{3}$ (b) $\sqrt{120}$ (c) 11 (d) 14.4

11. Consider the following data with regard to production of cars (in lakhs) :

	Year 2015	Year 2016
Country A	35	38
Country B	45	47
Country C	88	93
Country D	75	79
Country E	58	60.9

In which of the countries, the production of cars has increased by more than or equal to 5% in 2016 over 2015? **[CDS-2021-I]**
(a) B and E (b) A, C and D only
(c) A, C, D and E (d) A, D and E only

12. The following table shows the marks of 90 students in a test of 80 marks :

Marks	Number of students
1-10	5
11-20	8
21-30	10
31-40	13
41-50	18
51-60	17
61-70	12
71-80	7

The percentage of students who have obtained less than or equal to 50% marks is **[CDS-2021-I]**
(a) 30% (b) 40% (c) 45% (d) 60%

13. What is the median of the following data :
2, 3, –1, 2, 6, 8, 9 **[CDS-2021-I]**
(a) 2 (b) 3 (c) 4 (d) 5

14. What is the arithmetic mean of the first ten composite numbers? **[CDS-2021-I]**
(a) 8.5 (b) 9.5 (c) 10.2 (d) 11.2

15. Let p be the mean of m observations and q be the mean of n observations, where $p \leq q$. If the combined mean of $(m+n)$ observations is c, then which one of the following is correct? **[CDS-2021-I]**
(a) $c \leq p$ (b) $c \geq p$
(c) $p \leq c \leq q$ (d) $q \leq c \leq p$

Directions: For the next four (4) items, consider the following data with regard to different types (I, II, III, IV, V) of multivitamin tablets produced in a company (in lakhs):

Year	I	II	III	IV	V
2000	160	80	70	90	75
2001	200	150	85	160	100
2002	135	35	44	95	85
2003	240	95	120	80	120
2004	180	110	85	95	115
2005	210	150	100	92	110

16. Which product is produced least over the years 2000-2005? **[CDS-2021-I]**
(a) Type II (b) Type III
(c) Type IV (d) Type V

17. In which one of the following pairs of years, the difference in total number of tablets produced between them is minimum? **[CDS-2021-I]**
(a) (2003, 2005) (b) (2001, 2005)
(c) (2003, 2004) (d) (2000, 2002)

18. The ratio of percentage drop in total production in 2004 compared to 2001 to that in 2000 compared to 2001, is **[CDS-2021-I]**
(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{5}$

19. In which year, the production of Type I is more than the sum of the production of Type III and Type IV? **[CDS-2021-I]**
(a) 2001 (b) 2002 (c) 2003 (d) 2004

HINTS & EXPLANATIONS

Number System

1. (c) $f(x) = x^3 + x^2 + 16$ is exactly divisible by x.

$$\therefore I = \frac{f(x)}{x} = \frac{x^3 + x^2 + 16}{x} = x^2 + x + \frac{16}{x}$$

For I to be integer,
$x = 1, 2, 4, 8$ and 16.
Hence, $n(x) = 5$.

2. (b) Factor of $30 = 1, 2, 3, 5, 6, 10, 15$ and 30

Given that $a \cdot b \cdot c = 30$

$\therefore (a, b, c) = (1,2,15), (1,3,10), (1,5,6), (2,3,5)$

Here product of each triplets = 30
i.e. $1 \times 2 \times 15 = 1 \times 15 \times 2 = 2 \times 15 \times 1 = 30$
So, each triplet can be arrange into 6 ways.
Number of $(a, b, c) = 6 \times 4 = 24$.
Now, for $(a, b, c) = (1, 1, 30)$
We get $1 \times 1 \times 30 = 1 \times 30 \times 1 = 30 \times 1 \times 1 = 30$
i.e. Number of $(a, b, c) = 3$.
Hence, total number of ways $= 24 + 3 = 27$

3. (d) We know, square of even number is always even and square of odd number is always odd.
So, when a = even, $(a)^2$ = even
and b = odd, $(b)^2$ = odd.
Again, product of even and odd number is always even.
So, $c = a \,.\, b$ = even.
$\therefore c^2$ = even.

Now, Sum $I = \underset{\text{(even)}}{a^2} + \underset{\text{(odd)}}{b^2} + \underset{\text{(even)}}{c^2}$ = odd number

Hence, I = square of odd integer.

4. (a) Smallest odd composite number = 9
Given number 23P62971335 is divisible by 9 when, sum of its digits is divisible by 9.
i.e. $(2 + 3 + P + 6 + 2 + 9 + 7 + 1 + 3 + 3 + 5) = 41 + P$.
For $P = 4$, $41 + 4 = 45$ is divisible by 9.

5. (b) Here periodicity of 2 and 3 are '4' and periodicity of 4 is 2.
So,

Number	Unit digit	Last 2 digit
1^5	1	01
2^5	2	32
3^5	3	43
4^5	4	24
5^5	5	25
		125

Now number '....125' when divided by 4 gives remainder '1'.

6. (c) Periodicity of 3 is 4.
Now, $99 = 4 \times 24 + 3$

$\therefore (3)^{99} = (3)^{4 \times 24 + 3} = (3)^3 = ...7$

$\therefore$ Unit digit = 7.

7. (a) $(x^n - a^n)$ is divisible by $(x - a)$ for all natural values of n.

8. (a) $(17)^{2020} = (18 - 1)^{2020}$

$$= {}^{2020}C_0 (18)^{2020} - {}^{2020}C_1 (18)^{2019} \ldots\ldots - {}^{2020}C_{2019} (18) + {}^{2020}C_{2020}$$

$$= 18\{{}^{2020}C_0 (18)^{2019} - {}^{2020}C_1 (18)^{2018} \ldots - {}^{2020}C_{2019}\}$$

Hence, on dividing $(17)^{2020}$ by 18, we get remainder = 1.

9. (c) Product

$= 5 \times (-4) + (4)(-3) + (-2) \times (3) + (2)(-1) + (-5) \times (3)$
$= -55$.

10. (b) Here, $60 \times n = (60 + 10)(n - 3)$

$\Rightarrow 60n = 70n - 210$

$\Rightarrow n = \frac{210}{10} = 21$.

11. (d) $A_2 = P_2 + 1 = 2 \times 3 + 1 = 7$

$A_3 = P_3 + 1 = 2 \times 3 \times 5 + 1 = 31$

$A_4 = P_4 + 1 = 2 \times 3 \times 5 \times 7 + 1 = 211$.

Now, A_2, A_3 and A_4 are odd prime numbers.
So,
(1) A_n is always a composite number (Incorrect).
(2) A_{n+1} is always even number (Correct)
(3) A_{n+2} is always odd number (Correct)

12. (b) Let two digits are x and y.
then,

$x + y = 13$ ...(i)

and $(10x + y) - (10y + x) = 27$

$(x - y) = 3$ ...(ii)

From (i) and (ii), we get $x = 8$ and $y = 5$
Product $xy = 8 \times 5 = 40$.

13. (d) $m = \text{cosec}\,\theta - \cot\theta$
As, $\text{cosec}^2\theta - \cot^2\theta = 1$

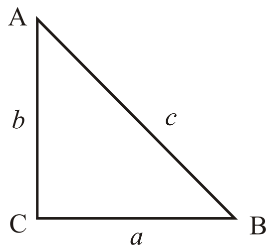
$\therefore \quad \text{cosec}\,\theta + \cot\theta = \frac{1}{m}$

then 2 cosec θ = $m+\frac{1}{m}$

$\therefore$ cosec θ = $\frac{m}{2}+\frac{1}{2m}$

14. (d) An right angle triangle, $(AC)^2+(BC)^2=(AB)^2$

$\Rightarrow (b)^2+(a)^2=c^2$



Now, $\tan A+\tan B=\frac{a}{b}+\frac{b}{a}=\frac{a^2+b^2}{ab}$

$=\frac{c^2}{ab}$

15. (c) Percent increase in production of cars in 2016 over 2015 in

Country A = $\left(\frac{38-35}{35}\right)\times 100=8.57\%$

Country B = $\left(\frac{47-45}{45}\right)\times 100=4.44\%$

Country C = $\left(\frac{93-88}{88}\right)\times 100=5.68\%$

Country D = $\left(\frac{79-75}{75}\right)\times 100=5.33\%$

Country E = $\left(\frac{60.9-58}{58}\right)\times 100=5\%$

Hence, in country A, C, D and E, Production of cars has increased by $\geq 5\%$

16. (b) Total number of students = 5 + 8 + 10 + 13 + 18 + 17 + 12 + 7 = 90

Number of Students, obtained $\leq 50\%$

= 5 + 8 + 10 + 13 = 36

$\therefore$ Required percent = $\frac{36}{103}\times 100=40\%$

17. (d) First ten composite numbers are 4, 6, 8, 9, 10, 12, 14, 15, 16 and 18

Mean = $\frac{1}{10}[4+6+8+9+10+12+14+15+16+18]$

= 11.2

18. (c) Sum of m observation = $p.m$
and, sum of n observation = $q.n$
combined mean of $(m+n)$ observation = $c.(m+n)$.
According to the question,
$c(m+n)=p.m+q.n$
For $p=q$, $c=p$
For $p<q$, $c\leq q$
Hence, $p\leq c\leq q$.

19. (c) Percentage drop in production in 2004 compare to 2001

$=\left(\frac{695-585}{695}\right)\times 100=15.83\%$

Percentage drop in production in 2000 compare to 2001

$=\left(\frac{695-475}{695}\right)\times 100=31.65\%$

Required ratio = $\frac{15.83}{31.65}=\frac{1}{2}$

HCF and LCM

1. (a) LCM + HCF = 1740
and LCM = 28 (HCF)

$\therefore$ 29HCF = 1740 $\Rightarrow$ HCF = $\frac{1740}{29}$

Now, LCM × HCF = 240 × Other number

28 · HCF × HCF = 240 × Other number

$\Rightarrow 28\times\left(\frac{1740}{29}\right)^2=240\times$ Other number

$\therefore$ Other number $=28\times\left(\frac{1740}{29}\right)^2\times\frac{1}{240}=420.$

2. (d)

$f(x)=x^6-3x^4+3x^2-1=(x^2-1)^3=(x+1)^3(x-1)^3$

Again, $g(x)=x^3+3x^2+3x+1$

$=x^2(x+1)+2x(x+1)+(x+1)$

$=(x+1)(x^2+2x+1)=(x+1)^3$

$\therefore$ H.C.F. = $(x+1)^3$.

3. (d) One polynomial × other polynomial = H.C.F. × L.C.M.

$(6x^2+5x+1)$ × Other polynomial

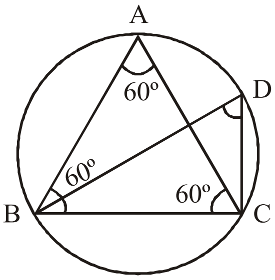
$=(3x+1)(30x^3+7x^2-10x-3)$

$\therefore$ Other polynomial

$=\frac{(3x+1)(3x+1)(2x+1)(5x-3)}{(3x+1)(2x+1)}$

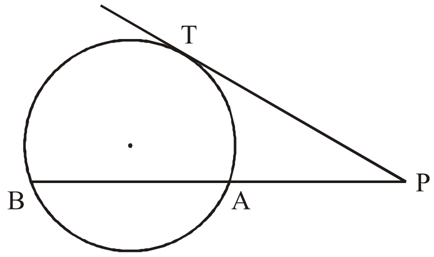
$=(3x+1)(5x-3)=15x^2-4x-3.$

4. (c) As $\angle BAC$ and $\angle BDC$ are same sides of arc BC



$\therefore \quad \angle BAC = \angle BDC = 60^\circ$

5. (c) From tangent properties,



$(PT)^2 = PA.PB$

$(12)^2 = 9.(PA + AB)$

$\therefore \quad (PB + 9) = \dfrac{144}{9} = 16$

Hence, PB = 16 – 9 = 7 cm.

Powers and Roots

1. (c) $\dfrac{1}{1+\sqrt{2}} + \dfrac{1}{\sqrt{2}+\sqrt{3}} + \dfrac{1}{\sqrt{3}+\sqrt{4}} + \ldots.. + \dfrac{1}{\sqrt{99}+\sqrt{100}}$

$$= \frac{\sqrt{2}-1}{2-1} + \frac{\sqrt{3}-\sqrt{2}}{3-2} + \frac{\sqrt{4}-\sqrt{3}}{4-3} + \ldots. + \frac{\sqrt{100}-\sqrt{99}}{100-99}$$

$$= \sqrt{2} - 1 + \sqrt{3} - \sqrt{2} + \sqrt{4} - \sqrt{3} + \ldots. + \sqrt{100} - \sqrt{99}$$

$$= \sqrt{100} - 1 = 10 - 1 = 9.$$

2. (a) $x^m = \sqrt[14]{x\sqrt{x\sqrt{x}}} = \sqrt[14]{x \cdot \sqrt{x^{1+\frac{1}{2}}}} = \sqrt[14]{x^{1+\frac{3}{4}}}$

$$= x^{\frac{7}{4}\times\frac{1}{14}} = x^{\frac{1}{8}}$$

$$\therefore m = \frac{1}{8}$$

3. (a) $p + q(p - 1) = (\sec\theta - \tan\theta) + (\text{cosec}\,\theta + \cot\theta)$
$(\sec\theta - \tan\theta - 1)$

$$= \left(\frac{1-\sin\theta}{\cos\theta}\right) + \left(\frac{1+\cos\theta}{\sin\theta}\right)\left(\frac{1-\sin\theta-\cos\theta}{\cos\theta}\right)$$

$$= \frac{(1-\sin\theta).\sin\theta + (1-\cos^2\theta) - \sin\theta.(1+\cos\theta)}{\sin\theta.\cos\theta}$$

$$= \frac{\sin\theta - \sin^2\theta + 1 - \cos^2\theta - \sin\theta - \sin\theta.\cos\theta}{\sin\theta.\cos\theta}$$

$= -1$

4. (a) Total number of tablet produced in
Year 2000 = 160 + 80 + 70 + 90 + 75 = 475
Year 2001 = 200 + 150 + 85 + 160 + 100 = 695
Year 2002 = 135 + 35 + 44 + 95 + 85 = 394
Year 2003 = 240 + 95 + 120 + 80 + 120 = 655
Year 2004 = 180 + 110 + 85 + 95 + 115 = 585
Year 2005 = 210 + 150 + 100 + 92 + 110 = 662
Now, difference in production in two years
Year: 2003 – 2005 = 662 – 655 = 7 {Minimum}
Year: 2001 – 2005 = 695 – 662 = 33
Year: 2003 – 2004 = 655 – 585 = 70
Year: 2000 – 2002 = 475 – 394 = 81

5. (c) In 2003, production of type I = 240.
Sum of production of type III & IV = 120 + 80 = 200
Hence, production of type I is more then sum of production of type III & IV.

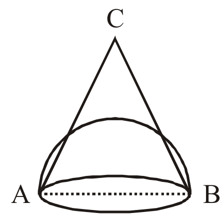
Percentage

1. (a) After first year, number of items $= N \times \left(\dfrac{x}{100} + 1\right)$

After second year, number of items

$$= N\left(\frac{x}{100}+1\right)\left(1-\frac{x}{100}\right) = N(1-0.001x^2) < N$$

2. (b) Let AB (= $2r$) is the diameter of the base of cone & hemisphere.



According to the question

Volume of cone = Volume of hemisphere

$$\frac{1}{3} \times \pi.r^2 \times h = \frac{2}{3}\pi.r^3$$

Where h = Height of the cone.

$$\therefore \quad \frac{h}{r} = \frac{2}{1} \qquad \Rightarrow \quad 2:1$$

3. (c) Volume of Cube = Volume of the cone

$$(23)^3 = \frac{1}{3} \times \pi \times (r)^2 \times 21$$

$\therefore \quad r = 22$ cm

Ratio and Proportion

1. (a) L.C.M. of 3, 4 and 5 = 60 unit
Now, Ratio of investment (initially)
$$= \frac{60}{3} : \frac{60}{4} : \frac{60}{5} = 20:15:12$$
Now, $(20 \times 4 + 10 \times 8) : 15 \times 12 : 12 \times 12$
$\Rightarrow 160:180:144 \Rightarrow 40:45:36.$
$\therefore$ Profit share of A
$$= \frac{40}{(40+45+36)} \times 96800 = 32000.$$

2. (c) Curved surface area of the cone = $\pi.r.l$
$$550 = \frac{22}{7} \times rl$$
$\therefore \quad rl = 175 \qquad \Rightarrow \quad = \frac{175}{l}$
Now, slant height : $(l)^2 = h^2 + r^2$
$$l^2 = (24)^2 + \left(\frac{175}{l}\right)^2$$
$$l^4 - 576l^2 - 30625 = 0$$
On solving, $l = 25$ and $r = 7$
Ratio : $\frac{r}{l} = \frac{7}{25}$

3. (c) As vertical angles of two triangle are same, then two triangles are similar.
Ratio of areas of two triangles = Ratio of square of the sides of two triagnle.
$$\therefore \quad \frac{\text{Height}(h_1)}{\text{Height}(h_2)} = \sqrt{\frac{4.84}{5.29}} = \frac{22}{23}$$

Average

1. (c) Mother's age = Ena's age + 24
= 13 + 24 = 37 years
Father's age = 37 + 3 = 40 years
Father's age at the time of his marriage
= 40 – 13 – 4 = 23 years.

2. (b) Mahesh's age = 60 years
Ram's age = 60 – 5 = 55 years
and Raju's age = 55 – 4 = 51 years
Babu's age = 51 – 6 = 45 years
$\therefore$ Mahesh's age – Babu's age = 60 – 45 = 15 years.

3. (d) $(a_1 + a_2 + a_3)(b_1 + b_2 + b_3 + b_4)(c_1 + c_2 + c_3 + c_4 + c_5)$
Total no. of terms = 3 × 4 × 5 = 60.

4. (b) As ΔABC and ΔDEF are similar,
$$\therefore \quad \frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta DEF} = \frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{DF}$$
So, $\frac{40}{30} = \frac{BC+CA}{EF+DF} = \frac{4}{3}.$

Simple and Compound Interest

1. (d) Here, S.I. $= \frac{\text{Sum}}{4}$ and rate of interest (r) = time (t)
Now,
$$\text{S.I.} = \frac{\text{Sum}}{4} = \frac{\text{Sum} \times \text{rate} \times \text{time}}{100}$$
$$\Rightarrow \frac{100}{4} = \text{time} \times \text{time}$$
$$\therefore \text{ time} = \sqrt{\frac{100}{4}} = 5 \text{ years}$$

2. (b) Length of wire = 144 cm = 1440 mm.
Volume of sphere = Volume of wire.
$$\frac{4}{3}\pi\left(\frac{60}{2}\right)^3 = \pi \cdot (r)^2 .(1440)$$
$$r^2 = \frac{4 \times 30 \times 30}{144}$$
$$\Rightarrow \quad r = \sqrt{\frac{4 \times 30 \times 30}{144}} = \frac{2 \times 30}{12} = 5 \text{ mm}$$
$\therefore$ Diameter = $2r = 2 \times 5 = 10$ mm = 1 cm.

3. (c) Volume of the cylinder = $\pi r^2 \cdot h$
$1617 = \pi.(2k)^2.3k$
$$\therefore \quad k = \sqrt[3]{\frac{1617 \times 7}{22 \times 12}}$$
$$= \sqrt[3]{\frac{7 \times 7 \times 7 \times 3 \times 11}{2 \times 2 \times 2 \times 11 \times 3}} = 3.5$$
$\therefore \quad r = 2k = 7$cm, $h = 3k = 10.5$
Curved Surface Area = $2\pi.r.h$
$$= 2 \times \frac{22}{7} \times 7 \times 10.5 = 462 \text{ cm}^2$$

Profit and Loss

1. (b) Profit % $= \left(\frac{1000 - 800}{800}\right) \times 100 = 25\%.$

2. (d) Cost price of the T.V.
$$= x \times \frac{100}{(100-28)} = y \times \frac{100}{(100+12)}$$
$$\therefore \frac{x}{72} = \frac{y}{112}$$
$$\therefore \frac{y}{x} = \frac{112}{72} = \frac{14}{9}.$$

3. (c) $x \times \left(1+\frac{x}{100}\right) = 75$

$\Rightarrow x + 0.01x^2 = 75 \Rightarrow 0.01x^2 + x - 75 = 0$

$\therefore x = 50\%.$

4. (b) Volume of water in conical vessel = Volume of water in cylindrical vessel

$\Rightarrow \frac{1}{3}\pi(r_1)^2 \times h_1 = \pi(r_2)^2.h_2$

$\Rightarrow \frac{1}{3} \times (5)^2 \times 24 = (10)^2 \times h_2$

$\therefore \quad h_2 = \frac{5 \times 5 \times 8}{10 \times 10} = 2$ cm

Time , Speed and Distance

1. (c) Speed of the train = 132 km/hr $= 132 \times \frac{5}{18}$

$\therefore$ Time required to cross the bridge

$= \frac{(110+165)\times 18}{132 \times 5} = \frac{275 \times 18}{132 \times 5} = 7.5$ sec.

2. (c) $\frac{300}{s} - \frac{300}{(s+15)} = 1$

(where s = original speed of the car)

$\therefore 300 \times 15 = s(s+15) \Rightarrow s^2 + 15s - 4500 = 0$

$\Rightarrow (s+75)(s-60) = 0$

$\therefore s = 60$ km/hr.

(Here, $s = -75$ km/hr, neglecting -ve values)

3. (b) Let all three meet at time t hours when z start and z started after K hours.

Then,

Distance traveled by x = Distance traveled by y = Distance traveled by z.

$\therefore 4(2+K+t) = 5(K+t) = 6 \cdot t$

$\Rightarrow 6t = 5(K+t) \Rightarrow t = 5K$ ...(i)

Again, $4(2+K+t) = 5(K+t)$

$\Rightarrow K + t = 8$ ...(ii)

From (i) and (ii),

$K = \frac{8}{6} = \frac{4}{3}$ hours

4. (b) Let the distance = d km

Then, speed $(s) = d \cdot t$

Now, $\frac{d}{(t-y)} - \frac{d}{t} = x$

$\Rightarrow dy = x \cdot t(t-y)$

$\therefore d = x \cdot t(t-y) \cdot y^{-1}$

5. (d) Difference in surface Area = $2\pi.h(R-r) = 44$

$\Rightarrow \quad 2 \times \frac{22}{7} \times 14(R-r) = 44$

$\therefore \quad R - r = 0.5$ . . . (i)

Again, metal used = $\pi(R^2 - r^2).h$

$99 = \frac{22}{7}(R^2 - r^2).14$

$R^2 - r^2 = 2.25$ . . . (ii)

From (i) and (ii), we get

$(R+r) = \frac{2.25}{0.5} = 4.5$ cm.

6. (c) Sum of volumes of three small cube = Volume of big cube.

$\frac{4}{3}\pi((12)^3 + (16)^3 + r^3) = \frac{4}{3}\pi(24)^3$

$\therefore \quad r^3 = (24)^3 - [(12)^3 + (16)^3]$

$= (24)^3 - [(16+12)\{(16)^2 + (12)^2 - 16 \times 12\}]$

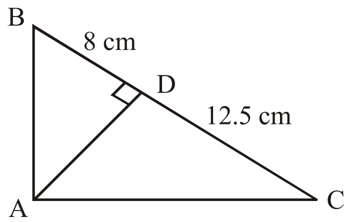
$= (24)^3 - [28\{256 + 144 - 192\}]$

$\therefore \quad r^3 = 8000 \Rightarrow \quad r = 20$

Surface Area of third Cube

$= 6(r)^2 = 6 \times (20)^2 = 2400$ cm^2

7. (d)



From triangle properties.

$\frac{BD}{AD} = \frac{AD}{CD}$

$\therefore \quad AD = \sqrt{BD \times CD} = \sqrt{8 \times 12.5}$

$= 10$ cm.

Time & Work

1. (d) Here from $\frac{M_1 \cdot D_1 \times H_1}{W_1} = \frac{M_2 D_2 H_2}{W_2}$

$\therefore \frac{X \cdot X \cdot X}{X} = \frac{Y \cdot Y \cdot Y}{K}$

$\therefore K = Y^3 X^{-2}$

2. (b) When paper is folded along its width.

Let circumference of the base = width of the paper

$\therefore \quad 2\pi.r = 22$

$\therefore \quad r = \frac{7}{2}$ cm

$$\text{Volume of Cylinder (Y)} = \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 44$$

$$\text{Similarly, Volume (X)} = \frac{22}{7} \times (7)^2 \times 22$$

$$\text{Ratio}\left(\frac{X}{Y}\right) = \frac{\frac{22}{7} \times (7)^2 \times 22}{\frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 44} = 2:1$$

3. (a) Volume of material used in the shell

$$= \frac{4}{3}\pi(6^3 - 5^3) = \frac{4}{3}\pi \times 91$$

$$\therefore \text{ Mass of the shell} = \frac{4}{3} \times \frac{22}{7} \times 91 \times 3 = 1144 \text{ g}$$

Logarithm

1. (b) $6^{3-4x} \cdot 4^{x+5} = 8$

Taking log on both sides

$\log(6^{3-4x} \cdot 4^{x+5}) = \log 8$

$\Rightarrow \log_{10} 6^{(3-4x)} + \log_{10} 4^{(x+5)} = \log_{10} 2^3$

$\Rightarrow (3-4x)\log_{10} 6 + (x+5)\log_{10} 4 = 3\log 2$

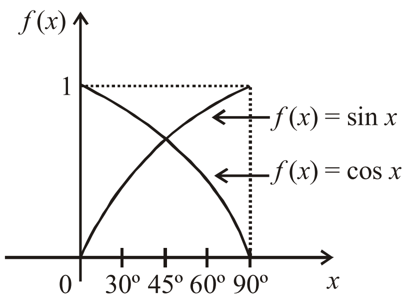
$\Rightarrow (3-4x)\{\log_{10} 2 + \log_{10} 3\} + 2(x+5) \cdot \log_{10} 2 = 3\log 2$

$\Rightarrow (3-4x)\{0.301 + 0.477\} + 2(x+5)(0.301) = 3(0.301)$

On solving we get $1 < x < 2$.

2. (d) Euclidean algorithm is used to calculate HCF of two integers.

3. (a) From graph :



Thus, for $0 < x < 45^\circ$, $\cos x > \sin x$

and for, $45^\circ < x < 90^\circ$, $\sin x > \cos x$

Now, (1) $\sin 1^\circ < \cos 57^\circ \quad \Rightarrow \sin(1^\circ) < \cos(90^\circ - 33^\circ)$

$\sin 1^\circ < \sin 33^\circ$ {True}

4. (a) $\cos\alpha + \cos\beta = 2$

We know that, for $0 \le \alpha, \beta \le 90^\circ$

$0 \le \cos\alpha, \cos\beta \le 1$

$\therefore \cos\alpha = \cos\beta = 1. \Rightarrow \alpha = \beta = 0$

Now, $\cos 2\alpha - \cos 2\beta = \cos 2\alpha - \cos 2\alpha = 0$

5. (c) $\text{Sec}\,\theta + \cos\theta = \frac{5}{2}$

$\frac{1+\cos^2\theta}{\cos\theta} = \frac{5}{2} \quad \Rightarrow 2\cos^2\theta - 5\cos\theta + 2 = 0$

$(2\cos\theta - 1)(\cos\theta - 2) = 0$

$\therefore$ As, $\cos\theta \ne 2, \quad \therefore \cos\theta = \frac{1}{2}$

$\sin^2\theta = 1 - \cos^2\theta = 1 - \left(\frac{1}{2}\right)^2 = \frac{3}{4}$

6. (c) $(1 + \cot\theta - \text{cosec}\,\theta)(1 + \tan\theta + \sec\theta)$

$$\Rightarrow \frac{(\sin\theta + \cos\theta - 1)(\cos\theta + \sin\theta + 1)}{\sin\theta.\cos\theta}$$

$$= \frac{(\sin\theta + \cos\theta)^2 - 1}{\sin\theta.\cos\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta + 2\sin\theta.\cos\theta - 1}{\sin\theta.\cos\theta} = \frac{1 + 2\sin\theta.\cos\theta - 1}{\sin\theta.\cos\theta}$$

$= 2.$

7. (d) $6 + 8\tan\theta = \sec\theta$ ...(i)

and $8 - 6\tan\theta = k\sec\theta$ ...(ii)

On, squaring and adding equation (i) and (ii), we get

$36 + 64\tan^2\theta + 64 + 36\tan^2\theta = (1 + k^2).\sec^2\theta$

$100(1 + \tan^2\theta) = (1 + k^2)(1 + \tan^2\theta)$

$\therefore (1 + k^2) = 100 \quad \Rightarrow k^2 = 99$

8. (b)

Basic Operation and Factorization

1. (c) Given equation : $x + y + z = 12$.

As, x, y, z are positive integer.

$\therefore 1 \le x, y, z \le 10$.

Possible triplet (x, y, z) : (1, 1, 10), (1, 2, 9), (1, 3, 8), (1, 4, 7), (1, 5, 6), (1, 6, 5), (1, 7, 4), (1, 8, 3), (1, 9, 2), (1, 10, 1)

Total number of sets = 10

Similarly for, (2, 1, 9), (2, 2, 8), ... (2, 9, 1), ...

Total number of sets = 9.

For triplet (x, y, z) = (10, 1, 1), ...

Total number of sets = 1.

Hence, total number of solution sets

= 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 55.

2. (d) $\frac{a^2 + ac}{a^2c - c^3} - \frac{a^2 - c^2}{a^2c + 2ac^2 + c^3} - \frac{2c}{a^2 - c^2} + \frac{3}{a+c}$

$$= \frac{a(a+ac)}{c(a^2 - c^2)} - \frac{(a^2 - c^2)}{c(a+c)^2} - \frac{2c}{a^2 - c^2} + \frac{3(a-c)}{a^2 - c^2}$$

$$= \frac{a(a+c)}{c(a^2-c^2)} - \frac{(a-c)^2}{c(a^2-c^2)} + \frac{(3a-5c)c}{c(a^2-c^2)}$$

$$= \frac{a^2+ac-a^2-c^2+2ac+3ac-5c^2}{c(a^2-c^2)}$$

$$= \frac{6ac-6c^2}{c(a^2-c^2)} = \frac{6}{(a+c)}.$$

3. (b) $\frac{x}{b+c} = \frac{y}{c+a} = \frac{z}{b-a} = K$ (Let)

$\therefore x = K(b+c),\ y = K(c+a),\ z = K(b-a)$

Now, $x - y - z = K(b+c) - K(c+a) - K(b-a) = 0.$

4. (b) $f(x) = 1 - x - x^n + x^{n+1}$

$f(x) = (1-x) - x^n(1-x)$

$= (1-x)(1-x^n) = (1-x)(1-x)(1+x^{n-1}...)$

$\therefore f(x)$ is divisible by $(1-x)^2$.

5. (a) Here time to fall $(t)^2 \alpha$ distance (d)

$t^2 = K \cdot d$

$\Rightarrow (4)^2 = K \cdot (78.4) \ \Rightarrow K = \frac{16}{78.4}$

Now, $t = \sqrt{\frac{16}{78.4} \times 122.5} = \frac{4 \times 35}{28} = 5$ sec.

6. (c) Radius of the tent $= \frac{12}{2} = 6$ m

lateral surface Area of the tent = Area of the cloth

$\pi.r.l$ = (length of the cloth) × 3

$\frac{22}{7} \times 6 \times 7 = (l) \times 3$

$\therefore$ length of the cloth $(l) = 44$ m.

7. (b) As the sphere is completely submerged in water, then

Surface Area of the sphere = Volume of the water displaced in the cylinder.

$\therefore\ 4\pi(r_s)^2 = \pi(r_c)^2 \times h$

$4\pi(3)^2 = \pi(6)^2 \times h$

$\therefore\ h = 1$ cm.

8. (b) Perimeter of the semicircular park

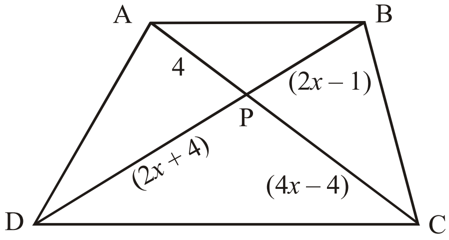
$= \pi r + 2r = 360$

$\therefore\ \left(\frac{22}{7} + 2\right) r = 360$

$\Rightarrow r = 70$ m

Area of the park $= \frac{1}{2} \times \frac{22}{7} \times (70)^2 = 7700$ m^2.

9. (b) As diagonals of the trapezium divides each other in equal ratio



$\therefore\ \frac{4}{4x-4} = \frac{2x-1}{2x+4}$

$2x + 4 = (2x-1)(x-1)$

$2x^2 - 5x - 3 = 0$

$\Rightarrow (x-3)(2x+1) = 0$

Hence, $x = 3$.

10. (a) Let side of the cube is p and radius of the sphere is r. Then, According to the question,

$6p^2 = 4\pi(r)^2$

$\frac{p}{r} = \left(\frac{4\pi}{6}\right)^{\frac{1}{2}}$

Now, $\frac{\text{Volume of Cube}}{\text{Volume of Sphere}} = \frac{x}{y} = \frac{p^3}{\frac{4}{3}\pi r^3}$

$= \left(\frac{4\pi}{6}\right)^{\frac{3}{2}} \times \frac{3}{4\pi} = \frac{\sqrt{4\pi}}{2(6)^{\frac{1}{2}}}$

$\therefore\ \frac{x^2}{y^2} = \frac{4\pi}{4.6} = \frac{\pi}{6}$

11. (c) Let ΔABC is a right angle triangle with right angled at B and side

$AB : BC : AC = (x-1) : (x-18) : x$

$\therefore\ (AB)^2 + (BC)^2 = (AC)^2$

$\Rightarrow (x-1)^2 + (x-18)^2 = x^2$

$x^2 - 38x + 325 = 0$

$(x-25)(x-13) = 0$

$\Rightarrow x = 25$

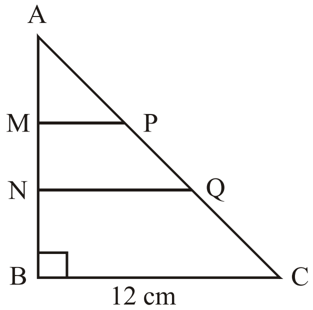
Now, side $AB = (x-1) = 24$

$BC = (x-18) = (25-18) = 7$

$AC = x = 25$

Perimeter of the triangle ABC = 25 + 24 + 7 = 56 units

12. (c) Here ΔAMP, ΔANQ and ΔABC are similar triangle.



As, AM = MN = NB = x (Let)
From ΔAPM and ΔANQ.

Then, $\frac{PM}{QN} = \frac{AM}{AN}$

Now, $\frac{QN + PM}{PM} = \frac{AN + AM}{AM}$

$\frac{QN + PM}{PM} = \frac{2x + x}{x} = \frac{3}{1}$ . . . (i)

Again, from ΔAMP and ΔABC

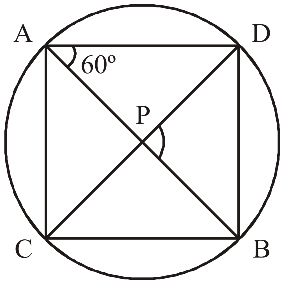
$\frac{BC}{PM} = \frac{AB}{AM}$

$\frac{12}{PM} = \frac{3x}{x}$ $\Rightarrow$ PM = 4

Putting in equation (i), we get

QN + PM = 4 × 3 = 12 cm.

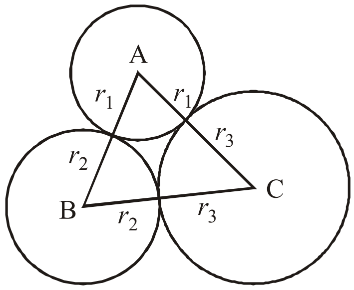
13. (d) As two diameters AB and CD intersects at point P. So, point P is the centre of the circle and BCAD is a cyclic quadrilateral.



So, $\angle BPD = 2 \times (\angle BAD)$ {As angle made by the arc at centre is two times the angel made at circumference}

$= 2 \times 60^\circ$
$= 120^\circ$

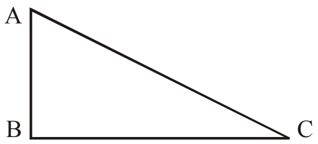
14. (c)



As shown in the fig. three circles of radius r_1, r_2 and r_3 are drawn touches each other externally.
Sum of three sides = AB + BC + AC = 4 + 6 + 8
= 18 cm

$\therefore$ $(r_1 + r_2) + (r_2 + r_3) + (r_3 + r_1) = 18$
$2(r_1 + r_2 + r_3) = 18$
$\therefore$ $r_1 + r_2 + r_3 = 9$ cm.

15. (c)

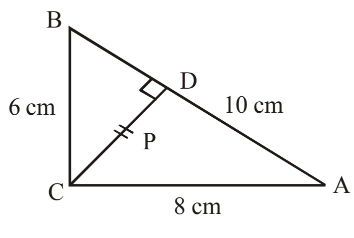


Let ΔABC is a right angled triangle, with hypotenuse AC = 13 cm
Perimeter of the triangle = 30cm
$\therefore$ AB + BC = 30 – 13 = 17 cm
So, AB = 17 – BC
In right, angle triangle, $(AB)^2 + (BC)^2 = (AC)^2$
$(17 - BC)^2 + (BC)^2 = (13)^2$
$289 + 2(BC)^2 - 34 \cdot BC = 169$
$(BC)^2 - 17(BC) + 60 = 0$
$(BC - 5)(BC - 12) = 0$
$\therefore$ BC = 5 or 12
and AB = 17 – (5 or 12) = 12 or 5.

Area of the triangle $\Delta ABC = \frac{1}{2} \times AB \times BC$

$= \frac{1}{2} \times 12 \times 5 = 30\ cm^2$

16. (c) $AB = \sqrt{(6)^2 + (8)^2} = 10$ cm

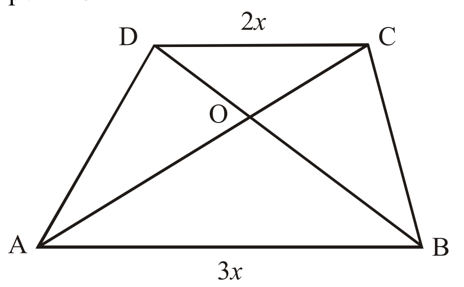


From properties of right angle triangle,

$CD = \frac{AC \times BC}{AB}$

$= \frac{8 \times 6}{10} = 4.8$ cm.

17. (d) Let the diagonals AC and BD intersect each other at point 'O'



Then, $\frac{AO}{OC} = \frac{BO}{OD} = k$ (says)

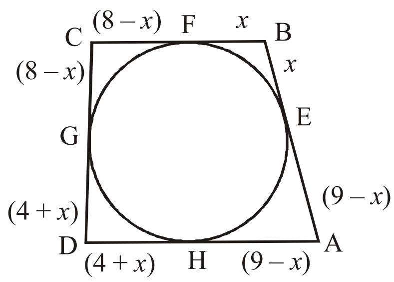
$\Rightarrow$ AO = k(OC) and BO = k(OD)

also, $\angle AOB = \angle COD$

$\therefore$ $\Delta AOB \sim \Delta COD$

So, $\frac{\text{Area of } \Delta AOB}{\text{Area of } \Delta COD} = \frac{(AB)^2}{(CD)^2} = \left(\frac{3x}{2x}\right)^2 = \frac{9}{4}$

18. (b)



$\therefore$ Side DA = AH + DH = $9 - x + 4 - x = 13$ cm.

19. (b) Average marks = $\frac{21+27+19+26+32}{5} = 25$

When 5 marks is added to each students marks then, New Average marks = 25 + 5 = 30.

20. (b) Production of Type-I = 160 + 200 + 135 + 240 + 180 + 210 = 1125

Similarly,

Production of Type-II = 80 + 150 + . . . + 150 = 620

Production of Type-III = 70 + 85 + . . . + 100 = 504

Production of Type-IV = 90 + 160 + . . . + 92 = 612

Production of Type-V = 75 + 100 + . . . + 110 = 605

Hence, production of type-III is least over the year 2000 - 2005.

Linear Equations in One and Two Variables

1. (c) $ab + xy - xb = 0 \Rightarrow b + \frac{x}{a}(y - b) = 0$

$$\Rightarrow \frac{x}{a} = \frac{b}{b-y}$$

and $bc + yz - cy = 0 \Rightarrow y + \frac{c}{z}(b - y) = 0$

$$\Rightarrow \frac{c}{z} = \frac{y}{y-b}$$

Now, $\frac{x}{a} + \frac{c}{z} = \frac{b}{b-y} + \frac{y}{y-b} = \frac{b-y}{b-y} = 1.$

2. (c) $(p+2)(2q-1) = 2pq - 10$

$\Rightarrow 4q - p - 2 = -10 \Rightarrow p - 4q = 8$ ...(i)

$(p-2)(2q-1) = 2pq - 10$

$\Rightarrow p + 4q = 12$ ...(ii)

From (i) and (ii), $p = 10$ and $q = \frac{1}{2}$

Now, $p \cdot q = 10 \times \frac{1}{2} = 5.$

Quadratic Equations

1. (d) For real root $D \geq 0$

$(-4)^2 - 4 \cdot (1) \cdot (-\log_{10} N) \geq 0$

$16 + 4\log_{10} N \geq 0$

$\Rightarrow (4)\{4 + \log_{10} N\} \geq 0$

$\therefore \log_{10} N \geq -4$

or $N \geq 10^{-4} \Rightarrow N \geq \frac{1}{10000}$

$N_{(\text{Minimum})} = \frac{1}{10000}$

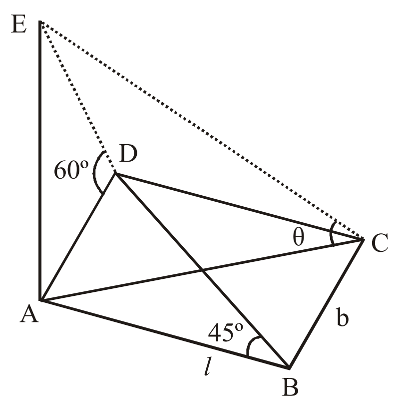
2. (d) $f(x) = 4x^4 + 8x^3 - 4x + 1$

Now,

$(2x^2 + 2x - 1)^2 = 4x^4 + 4x^2 + 1 + 8x^3 - 4x - 4x^2$

$= 4x^4 + 8x^3 - 4x + 1$

3. (c) Let ABCD is a rectangle of Length l and vertical tower AE is standing at the corner A.



Then, $\angle ABE = 45^\circ$ and $\angle ADE = 60^\circ$ and $\angle ACE = \theta$

From ΔABE, AE = AB. tan 45° = AB = l

From ΔADE, AD = AE. cot60° = $\frac{l}{\sqrt{3}}$

Now, Diagonal AC = $\sqrt{(AB)^2 + (AD)^2}$

$$= \sqrt{(l)^2 + \left(\frac{l}{\sqrt{3}}\right)^2} = \frac{2l}{\sqrt{3}}$$

Again from ΔACE, $\cot\theta = \frac{AC}{AE} = \frac{\frac{2l}{\sqrt{3}}}{l} = \frac{2}{\sqrt{3}}$

4. (a) As length ($2\pi r$) subtends angle 360° at the centre.

So, Angle subtends by 55 = $\frac{360^\circ \times 55}{2\pi \times 21} \approx 150^\circ$

Now, Area of the sector of length 55 cm

$$= \frac{150^\circ}{360^\circ} \times \pi (21)^2 = 577.5 \text{ cm}^2$$

5. (d) Side length of the rhombus = $\frac{\text{Perimeter of the circle}}{4}$

$$= \frac{2 \times \frac{22}{7} \times 70}{4} = 110 \text{ cm}$$

Set Theory

1. (b)

(1) d (5) = divisor of 5 i.e. 1, 5

and d(11) = Divisor of 11 i.e. 1, 11

So, d(5) = d(11)

$\Rightarrow$ Number of divisor of 5 = Number of divisor of 11

(2) d(5) . d(11) = 2 × 2 = 4

d(55) = Divisor of 55 = 1, 5, 11 and 55

Number of divisor of 55 = 4

$\therefore$ d(5) . d(11) = d(55).

(3) $d(5) + d(11) = 2 + 2 = 4$

$d(16) = 1, 2, 4, 8, 16$

$\therefore$ Number of divisor of 16 = 5

Hence, $d(5) + d(11) \neq d(16)$.

Thus, (1) and (2) are correct but (3) is not correct.

2. (b) $x \,\alpha\, y \Rightarrow x = K \cdot y$, where K = constant.

(1) Now, $x^2 + y^2 \alpha x^2 - y^2$

$$\Rightarrow (Ky)^2 + y^2 = z\left\{(Ky)^2 - y^2\right\}$$

$$\Rightarrow (K^2+1)y^2 = (K^2-1)y^2.z$$

$$\therefore z = \frac{K^2+1}{(K^2-1)} \qquad \text{(Not true)}$$

(2) $\frac{x}{y^2} \alpha \frac{1}{y}$

$$\Rightarrow \frac{Ky}{y^2} \alpha \frac{1}{y} \Rightarrow \frac{K}{y} \alpha \frac{1}{y} \qquad \text{(True)}$$

(3) $\sqrt[n]{x^2 y} \alpha \sqrt[2n]{x^4 y^2}$

$$\Rightarrow (x^2 y)^{\frac{1}{n}} \alpha (x^4 y^2)^{\frac{1}{2n}}$$

$$\Rightarrow (x^2 y)^{\frac{1}{n}} \alpha (x^2 y)^{\frac{1}{n}}$$

3. (b) Arranging data in increasing order:

–1, 2, 2, 3, 6, 8, 9.

Number of median data = $\left(\frac{7+1}{2}\right)^{\text{th}}$ term = 4th term

Hence, median = 3

Trigonometric Ratios & Identities

1. (b) $m = \text{cosec}\theta - \sin\theta = \frac{1-\sin^2\theta}{\sin\theta} = \frac{\cos^2\theta}{\sin\theta}$

$$n = \sec\theta - \cos\theta = \frac{1-\cos^2\theta}{\cos\theta} = \frac{\sin^2\theta}{\cos\theta}$$

Now, $m^{\frac{4}{3}} \cdot n^{\frac{2}{3}} + m^{\frac{2}{3}} \cdot n^{\frac{4}{3}} = (m.n)^{\frac{2}{3}} \{m^{\frac{2}{3}} + n^{\frac{2}{3}}\}$

$$= \left(\frac{\cos^2\theta}{\sin\theta} \cdot \frac{\sin^2\theta}{\cos\theta}\right)^{\frac{2}{3}} \cdot \left\{\left(\frac{\cos^2\theta}{\sin\theta}\right)^{\frac{2}{3}} + \left(\frac{\sin^2\theta}{\cos\theta}\right)^{\frac{2}{3}}\right\}$$

$$= (\sin\theta \cdot \cos\theta)^{\frac{2}{3}} \cdot \left\{\frac{\cos^2\theta + \sin^2\theta}{(\sin\theta \cdot \cos\theta)^{\frac{2}{3}}}\right\}$$

$= 1$

2. (b) $\cos\theta + \sec\theta = K$

Squaring on both sides,

$$(\cos\theta + \sec\theta)^2 = K^2$$

$$\Rightarrow \cos^2\theta + \sec^2\theta + 2 \cdot \cos\theta \cdot \sec\theta = K^2$$

$$\Rightarrow 1 - \sin^2\theta + 1 + \tan^2\theta + 2 = K^2$$

$$\Rightarrow \sin^2\theta - \tan^2\theta = 4 - K^2$$

3. (d) Minimum values of $x+\frac{1}{x}=2$

But, $-1 \le (\sin\theta, \cos\theta) \le 1$

Hence, both (1) and (2) are not correct.

4. (b)

(1) $2\sin^2\theta - \cos\theta + 4 = 0$

$\Rightarrow 2(1-\cos^2\theta) - \cos\theta + 4 = 0$

$\Rightarrow 2\cos^2\theta + \cos\theta - 6 = 0$

$\Rightarrow (2\cos\theta - 3)(\cos\theta + 2) = 0$

$\therefore \cos\theta = \frac{3}{2}$ and –2.

But $-1 \le \cos\theta \le 1$

$\because$ (1) is not correct.

(2) $\tan\theta + \cot\theta = \tan\theta + \frac{1}{\tan\theta}$

As minimum value of $x+\frac{1}{x}=2$

$\therefore \tan\theta + \frac{1}{\tan\theta} \ge 2$

Hence, (2) is correct.

5. (b) $y = 3\sin\theta - 4$

$y_{\max} = 3(1) - 4 = -1$

$\{\because -1 \le \sin\theta \le 1\}$

6. (d) $\sin\theta + \cos\theta = \sqrt{2}$

Squaring on both sides,

$\sin^2\theta + \cos^2\theta + 2\sin\theta\cdot\cos\theta = 2$

$\Rightarrow \sin 2\theta = 1 \;\Rightarrow \sin 2\theta = \sin 90°$

$\therefore \theta = 45°$.

Now,

$\sin^6\theta + \cos^6\theta + 6\sin^2\theta\cdot\cos^2\theta$

$= (\sin^2\theta)^3 + (\cos^2\theta)^3 + 3\cdot\sin^2\theta\cdot\cos^2\theta$

$(\sin^2\theta + \cos^2\theta) + 3\sin^2\theta * \cos^2\theta$

$= (\sin^2\theta + \cos^2\theta)^3 + \frac{3}{4}(2\sin\theta\cdot\cos\theta)^2$

$= 1 + \frac{3}{4}\sin 2\theta = 1 + \frac{3}{4} = \frac{7}{4}$.

7. (b) $y = 9\sin^2\theta + 16\cos^2\theta$

$= 9(\sin^2\theta + \cos^2\theta) + 7\cos^2\theta$

$= 9 + 7(0) = 9$.

8. (a) $\cos 47° + \sin 47° = K$

Squaring on both sides,

$\cos^2 47° + \sin^2 47° + 2\sin 47°\cdot\cos 47° = K^2$

$\Rightarrow 2\sin 47°\cdot\cos 47° = K^2 - 1$

$\Rightarrow \sin(2\times 47°) = K^2 - 1$

$\Rightarrow \sin^2(2\times 47°) = (K^2-1)^2$

$\Rightarrow 1 - \sin^2(2\times 47°) = 1 - (K^2-1)^2$

$\Rightarrow \cos^2(2\times 47°) = 2K^2 - K^4$

$\Rightarrow \cos(2\times 47°) = K\sqrt{2-K^2}$

$\therefore \cos^2(47°) - \sin^2(47°) = K\sqrt{2-K^2}$

9. (b) $\text{cosec}\theta - \sin\theta = p^3$

$\Rightarrow \frac{1-\sin^2\theta}{\sin\theta} = p^3 \Rightarrow \frac{\cos^2\theta}{\sin\theta} = p^3$

and, $\sec\theta - \cos\theta = q^3$

$\Rightarrow \frac{1-\cos^2\theta}{\cos\theta} = q^3 \Rightarrow \frac{\sin^2\theta}{\cos\theta} = q^3$

Now, $\frac{q^3}{p^3} = \frac{\sin^3\theta}{\cos^3\theta} \Rightarrow \tan\theta = \frac{q}{p}$.

10. (b) $\cos(\alpha-\beta) = 1$

$\Rightarrow \cos(\alpha-\beta) = \cos 0°$

$\therefore \alpha - \beta = 0 \Rightarrow \alpha = \beta$.

Now, $\sin\alpha - \sin\beta + \cos\alpha - \cos\beta$

$= \sin\alpha - \sin\alpha + \cos\alpha - \cos\alpha = 0$.

11. (a)

(1) $y = \cos 61° + \sin 29°$

$\Rightarrow y = \cos(90° - 29°) + \sin 29°$

$\Rightarrow \sin 29° + \sin 29°$

$\Rightarrow y = 2\sin 29°$.

$\therefore y_{\max} = 2\sin 29° < 2\sin 30°$

$= 2\sin 29° < 2\times\frac{1}{2} \Rightarrow y < 1$.

(2) $\tan 23° - \cot 67°$

$= \tan 23° - \cot(90° - 23°)$

$= \tan 23° - \tan 23° = 0$.

12. (a) $\log_{10}(31.25) = \log_{10}\left(\frac{3125}{100}\right)$

$= \log_{10}(3125) - \log_{10}(100)$
$= \log_{10} 5^5 - 2$
$= 5\log_{10} 5 - 2$

$= 5\log_{10}\left(\frac{10}{2}\right) - 2$

$= 5 - 5\log_{10} 2 - 2 = 3 - 5\log_{10} 2$

13. (a) $15 - 4\sqrt{14} = 8 + 7 - 4\sqrt{14}$

$= (2\sqrt{2})^2 + (\sqrt{7})^2 - 2.2\sqrt{2}.\sqrt{7}$

$= (2\sqrt{2} - \sqrt{7})^2$

Hence, Square root of $15 - 4\sqrt{14} = (2\sqrt{2} - \sqrt{7})$

14. (c) Let two natural numbers are n and $(n + 2)$
According to the question,

$\frac{1}{n} + \frac{1}{n+2} = \frac{7}{24}$

$\frac{2(n+1)}{n(n+2)} = \frac{7}{24}$

For $n = 6$, $\frac{2(6+1)}{6 \times 8} = \frac{7}{24}$

Thus, two numbers are $n = 6$ and $(n + 2) = 8$
Sum of these numbers $= 6 + 8 = 14$

15. (d) For $n = 1$, $5^2 - 1 = 24$
24 is divisible by 1, 2, 3, 4, 6, 8, 12 and 24.
For $n = 2$, $5^4 - 1 = 624$
624 is divisible by all numbers by which 24 is completely divisible.
Similarly, for all natural values of n,
$5^{2n} - 1$, is always divisible by 8 natural numbers

16. (a) $5^{x-3} = 8$
Taking log on both sides, we get
$(x - 3) \log 5 = \log 8$
$(x - 3) = \log_5 8 = \log_5 2^3$
$(x - 3) = 3\log_5 2$

$\frac{x}{3} - 1 = \frac{\log 2}{\log 5}$

$\frac{x}{3} = \frac{\log 2}{\log 5} + 1$

$\frac{x}{3} = \frac{\log 2 + \log 5}{\log 5} = \frac{\log(2 \times 5)}{\log 5} = \frac{\log 10}{\log 5}$

$x = \frac{3}{\log_{10} 5} = \frac{3}{\log_{10}\left(\frac{10}{2}\right)} = \frac{3}{\log_{10} 10 - \log_{10} 2}$

$= \frac{3}{1 - \log_{10} 2}$

17. (c) $y = 3\sin^2\theta + 4\cos^2\theta$
$y = 3(\sin^2\theta + \cos^2\theta) + \cos^2\theta$
$= 3 + \cos^2\theta$
Least value of $\cos^2\theta = 0$
$\therefore\ y_{\min} = 3 + 0 = 3$

18. (b) $k = \sin\theta.\cos\theta$

$= \frac{1}{2}(2\sin\theta.\cos\theta)$

$k = \frac{1}{2}(\sin 2\theta)$

For $0 \le \theta \le \frac{\pi}{2}$, $0 \le \sin 2\theta \le 1$

$\therefore\ k = [0, 0.5]$

19. (d) $p = \sin^2\theta + \cos^4\theta$
$= 1 - \cos^2\theta + \cos^4\theta$
$= (1 - \cos^2\theta)^2 + \cos^2\theta$
$\therefore\ p = (\sin^2\theta)^2 + \cos^2\theta = \cos^2\theta + \sin^4\theta$
Now, $2p = \sin^2\theta + \cos^4\theta + \cos^2\theta + \sin^4\theta$
$= (\sin^2\theta + \cos^2\theta) + (\sin^4\theta + \cos^4\theta)$
$= 1 + (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta.\cos^2\theta$

$= 2 - \frac{1}{2}(\sin 2\theta)^2$

$p = 1 - \frac{1}{4}(\sin 2\theta)^2$

As $\sin 2\theta \in [0, 1]$ in $0 \le \theta \le \frac{\pi}{2}$

$\therefore\ 0 < p \le 1$ Hence, Both (1) and (2) are wrong.

Height and Distance

1. (*) Let $y = 2 - 2\sin x - \sin^2 x$
$= 3 - (1 + 2\sin x + \sin^2 x)$
$= 3 - (1 + \sin x)^2$

In $0 \le x \le \frac{\pi}{2}$, $0 \le \sin x \le 1$

Now, $y_{\min} = 3 - (1 + 1)^2 = -1$
$y_{\max} = 3 - (1 + 0)^2 = 2$

$\therefore\ \frac{y_{\max}}{y_{\min}} = \frac{2}{-1} = -2$

2. (a) $x^2 + y^2 - 2xy\sin^2\theta = 0$
For $\sin^2\theta = 1$,
$x^2 + y^2 - 2xy = 0 \quad \Rightarrow \quad (x - y)^2 = 0$
$\therefore\ x = y$

Area and Perimeter

1. (d) Here side $a = 40$, $b = 41$, perimeter $s = 90$

$\therefore$ Side $c = 90 - 40 - 41 = 9$ cm.

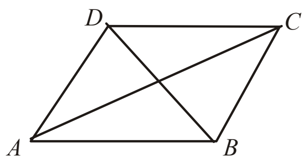
As $(40)^2 + (9)^2 = (41)^2$

$\Rightarrow (a)^2 + (c)^2 = (b)^2$

$\therefore$ Triangle of sides a, b, c are right angle triangle and

Area (A) $= \frac{1}{2} \times a \times c = \frac{1}{2} \times 40 \times 9 = 180 \text{ cm}^2$.

2. (c) Let $ABCD$ is a rhombus with diagonal $BD = 2x$ and $AC = (2x + 2)$.



ATQ,

$4 \times AB = AC + BD + 6$

$$\Rightarrow 4 \times \sqrt{\left(\frac{2x+2}{2}\right)^2 + \left(\frac{2x}{2}\right)^2} = (2x+2) + 2x + 6$$

$$\Rightarrow 4\sqrt{(x+1)^2 + x^2} = 4(x+2)$$

On squaring both sides, we get

$$(x+1)^2 + x^2 = (x+2)^2$$

$$\Rightarrow 2x^2 + 2x + 1 = x^2 + 4x + 4$$

$$\Rightarrow x^2 - 2x - 3 = 0$$

$\therefore x = 3$

Diagonal $BD = 2x = 6$ cm, $AC = 6 + 2 = 8$ cm.

Area of rhombus $= \frac{1}{2} \times 8 \times 6 = 24 \text{ cm}^2$

3. (a) Hypotenouse of a right angle triangle
= Diameter of the circumcircle
$= 2 \times 5 = 10$ cm.
Altitude = 4 cm (given)

$\therefore$ Area of the triangle $= \frac{1}{2} \times 10 \times 4 = 20 \text{ cm}^2$.

4. (c) Here, $l' = l + \frac{10l}{100} = 1.1l$

$$b' = b - \frac{10b}{100} = 0.9b.$$

Area $A = l \cdot b$ and $A' = l' \cdot b' = 1.1l \times 0.9b = 0.99lb$

So, $\left(\frac{A - A'}{A}\right) \times 100 = \left(\frac{lb - 0.99lb}{lb}\right) \times 100$

= 1% decreases.

5. (b) Let radius of the wheel = r cm

Then, $5000 \times 2 \times \frac{22}{7} \times r = 11 \times 100000$

$$\Rightarrow r = \frac{11 \times 7 \times 100}{22 \times 2 \times 5} = 35 \text{ cm.}$$

6. (d) Perimeter of circle = Perimeter of square

$$2\pi r = 4s \Rightarrow s = \frac{\pi r}{2}$$

Now, $\frac{\text{Area of circle}}{\text{Area of square}} = \frac{\pi r^2}{s^2} = \frac{\pi r^2}{\left(\frac{\pi r}{2}\right)^2} = \frac{4}{\pi}$

7. (b) Base of triangle = Base of parallelogram = b.
Height of parallelogram = h
$\therefore$ Height of triangle = $k.h$
ATQ,
Area of Triangle = Area of parallelogram

$$\Rightarrow \frac{1}{2} \times kh \times b = h \times b$$

$\therefore k = 2$.

8. (c) Area of square, $A \propto (\text{Side})^2$

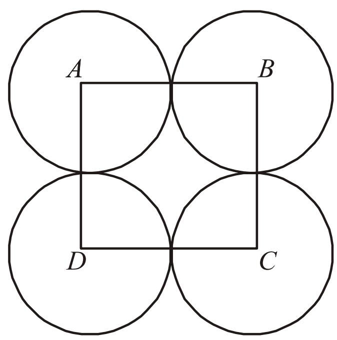
$$\therefore \frac{A_1}{A_2} = \left(\frac{S_1}{S_2}\right)^2 \Rightarrow \frac{m^2}{n^4} = \left(\frac{s_1}{s_2}\right)^2$$

$$\therefore \frac{s_1}{s_2} = \frac{m}{n^2}$$

As perimeter $(p) \propto$ Side (s)

$$\therefore \frac{p_1}{p_2} = \frac{m}{n^2}$$

9. (b) Let $ABCD$ is a square, such that $AB = 2r$.



Then, radius of each circle $= \frac{2r}{2} = r$.

Now, by symmetry

Uncovered Area

= Area of (Square – Circle)

$= (2r)^2 - \pi \cdot r^2$

$\therefore (4-\pi)r^2 = 42$

$\Rightarrow \left(4-\frac{22}{7}\right)\cdot r^2 = 42$

$\therefore r = \sqrt{\frac{42\times 7}{6}} = 7$ cm.

10. (b) $\frac{(x^3-1)(x^2-9x+14)}{(x^2+x+1)(x^2-8x+7)}$

$= \frac{(x-1)(x^2+x+1)(x^2-7x-2x+14)}{(x^2+x+1)(x^2-7x-x+7)}$

$= \frac{(x-1)(x^2+x+1)[x(x-7)-2(x-7)]}{(x^2+x+1)[x(x-7)-1(x-7)]}$

$= \frac{(x-1)(x^2+x+1)(x-7)(x-2)}{(x^2+x+1)(x-7)(x-1)}$

$= (x-2)$

11. (a) $\frac{8x}{1-x^4} - \frac{4x}{x^2+1} + \frac{x+1}{x-1} - \frac{x-1}{x+1}$

$= \frac{8x}{1-x^4} - \frac{4x(1-x^2)}{1-x^4} + \frac{(x+1)^2-(x-1)^2}{x^2-1}$

$= \frac{4x+4x^3}{1-x^4} - \frac{4x}{1-x^2}$

$= \frac{4x+4x^3-4x(1+x^2)}{1-x^4}$

$= 0$

12. (b) Sum of the roots $\alpha+\beta = \frac{6}{p} - 6$ (Given)

$\Rightarrow p = 1$

Product of the roots $\alpha.\beta = \frac{q}{p} = 6$ (Given)

$\Rightarrow q = 6$

$(p+q) = 1+6 = 7$

13. (c) $f(x) = 4x^3 + 12x^2 - x - 3$

$= 4x^2(x+3) - 1(x+3)$

$= (x+3)(4x^2-1)$

$= (x+3)(2x+1)(2x-1)$

Hence, $f(x)$ is divisible by both $(2x+1)$ and $(2x-1)$

Volume and Surface Area

1. (a) Rate of flow = 2 km/hr $= \frac{2000}{60}$ m/min.

Amount of water flow into the sea in a minute

$= 3\times 40\times \frac{2000}{60} = 4000\,\text{m}^3$

Now, 1 m^3 = 1000 litres

$\therefore$ 4000 m^3 = 40,00,000 litres

2. (a) Let radius of the cone is r.

Then $C^2 = \pi^2 r^2 l^2 = \pi^2 r^2 (r^2 + H^2)$ and

$V = \frac{1}{3}\pi r^2 \cdot H$

$\therefore 3\pi \cdot V \cdot H^3 + 9V^2$

$= 3\pi\left(\frac{1}{3}\pi r^2 H\right)\cdot H^3 + 9\cdot\left(\frac{1}{3}\pi r^2 H\right)^2$

$= \pi^2 r^2 H^4 + \pi^2 r^4 H^2$

$= \pi^2 \cdot r^2 H^2 \{H^2 + r^2\} = \pi^2 \cdot r^2 H^2 \cdot l^2 = C^2 \cdot H^2$.

3. (d) Radius $(r)^2 \propto$ Surface area (s)

$\therefore \frac{r_1}{r_2} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

$\therefore$ Volume, $\frac{v_1}{v_2} = \frac{(r_1)^3}{(r_2)^3} = \frac{1}{(2)^3} = \frac{1}{8}$.

4. (c) Number of bricks required

$= \frac{4500\times 15\times 300}{20\times 15\times 10} = 6750.$

5. (a) For cones,

$\frac{v_1}{v_2} = \frac{(d_1)^2 \cdot h_1}{(d_2)^2 \cdot h_2} \Rightarrow \frac{1}{4} = \frac{(4)^2\times h_1}{(5)^2\times h_2}$

$\therefore \frac{h_1}{h_2} = \frac{25}{64}$.

6. (b) Capacity of the bucket

$= \frac{1}{3}\pi \cdot h(R^2 + r\cdot R + r^2)$

$= \frac{1}{3}\times\frac{22}{7}\times 3x((2x)^2 + 2x\cdot x + x^2)$

$= \frac{22}{7}x(7x^2) = 22x^3$.

7. (c) Side of cuboid are p, q and r, then

Surface area, $s = 2(pq + qr + pr)$

Volume, $t = p \cdot q \cdot r$

Now, $\frac{1}{p}+\frac{1}{q}+\frac{1}{r}=\frac{pq+qr+rp}{pqr}=\frac{s}{2t}$

8. (b) Given equation, $6kx^2 + 12(k-2)x + 16 = 0$
Given equation is perfect square, when D = 0
$\{12(k-2)\}^2 - 4.6x.16=0$
$3(k-2)^2 - 8k = 0$
$3k^2 - 20k + 12 = 0$
$(3k-2)(k-6)=0$

$k = \frac{2}{3}$ and 6

Hence, number of real values of $k = 2$

9. (b) $x+\frac{1}{x}=\frac{5}{2} \Rightarrow \frac{x^2+1}{x}=\frac{4+1}{2}=\frac{(2)^2+1}{2}$

$\therefore \quad x = 2$

Now, $x^4-\frac{1}{x^4}=(2)^4-\frac{1}{(2)^4}=\frac{256-1}{16}$

$=\frac{255}{16}$

10. (c) Given equation: $4x^2 - 2kx + 3k = 0$
For equal roots, D = 0
$(-2k^2)^2 - 4.4.3k = 0$
$4.k(k - 1^2) = 0$
$k = 0$ or 12

11. (b) $\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+...+\frac{1}{n(n+1)}=\frac{99}{100}$

$\frac{2-1}{1\times 2}+\frac{3-2}{2\times 3}+\frac{4-3}{3\times 4}+...+\frac{(n+1)-n}{n(n+1)}=\frac{99}{100}$

$1-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+...+\frac{1}{n}-\frac{1}{n+1}=\frac{99}{100}$

$1-\frac{1}{n+1}=\frac{99}{100}$

$\therefore \quad \frac{n}{n+1}=\frac{99}{100}$

Hence, $n = 99$

12. (b) Price after discount

$=100\times\left(\frac{100-20}{100}\right)\times\left(\frac{100-10}{100}\right)\times\left(\frac{100-5}{100}\right)$

$= 68.4\%$

Hence, overall discount = 100 – 68.4 = 31.6%

13. (a) Let the sum invested was ₹ p

then, $\frac{p.(r+5).5}{100}-\frac{p.r.5}{100}=500$

{Where r = rate of interest}

$\therefore \quad \frac{p\times 5\times 5}{100}=500 \quad \Rightarrow \quad p = ₹\,2{,}000$

14. (d) Let principal amount is ₹ p

then, $p\left[\left(1+\frac{12}{100}\right)^2-1\right]-\frac{p\times 12\times 2}{100}=72$

$\{p(1.12)^2 - 1\} - 0.24p = 72$
or, $0.2544p - 0.24p = 72$
$0.0144p = 72$

$\therefore \quad p=\frac{72}{0.0144}=₹\,5000$

Short Trick :

Difference between C.I. and S.I. after 2 years $= p\left(\frac{r}{100}\right)^2$

According to the question,

$p\left(\frac{12}{100}\right)^2=72$

$p(0.0144) = 72$

$\therefore \quad p=\frac{72}{0.0144}=₹\,5000$

15. (c) Average Speed $=\frac{\text{Total distance covered}}{\text{total time taken}}$

$=\frac{600+900}{5+10}=\frac{1500}{15}$

$= 100$ km/hr.

16. (a) Let man is walking with speed s meter/min and his usual time of travel is t min

then, $s.t=\frac{4}{5}s(t+12)$

$5t = 4t + 48 \quad \Rightarrow \quad t = 48$ min

17. (d) cost price of 100 mangoes = selling price of 80 mangoes
$\therefore$ Profit = selling price of 100 – 80 = 20 mango

$\therefore \quad$ profit % $=\frac{20}{80}\times 100 = 25\%$

18. (b) Let Y sell the goods at ₹ 100.

Then, X sell the goods at $100\times\frac{(100-25)}{100}=₹\,75$

and Z sell the goods at $75\times\frac{100}{(100+25)}=₹\,60$

$\therefore \quad$ Z's good cheaper than Y $=\frac{100-60}{100}\times 100 = 40\%$

19. (b) In 80 litres, liquid $= 80 \times \frac{25}{100} = 20$ litres

Water $= 80 - 20 = 60$ litres

Volume of new mixture $= 20 \times \frac{100}{20} = 100$

Hence, Amount of water to be added $= 100 - (60 + 20)$
$= 20$ litres

20. (c) As 20 persons can clean 20 floors in 20 days
So, 1 person can clean 1 floor in 20 days
$\therefore$ 16 persons can clean 16 floors in 20 days.

21. (a) From $\frac{M_1.D_1}{W_1} = \frac{M_2.D_2}{W_2}$

$$\therefore \frac{(x-1)(x+1)}{(x+2)(x-1)} = \frac{y}{z} = \frac{9}{10}$$

$$\frac{(x+1)}{(x+2)} = \frac{9}{10} \quad \Rightarrow x = 8$$

Lines and Angles

1. (b) Difference, $A - B = 15°$

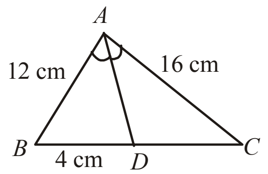
Sum, $A + B = \frac{5\pi}{12} = 75°$.

$\therefore A = 45°$ and $B = 30°$.

$\therefore \frac{A}{B} = \frac{45°}{30°} = \frac{3}{2}$.

Triangles and its Properties

1. (b)



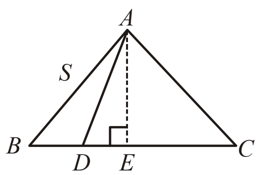
As AD bisect $\angle BAC$

$$\therefore \frac{CD}{BD} = \frac{AB}{AC}$$

$$\Rightarrow \frac{CD}{4} = \frac{12}{16}$$

$$\therefore CD = \frac{12}{16} \times 4 = 3 \text{ cm}.$$

2. (a) Let ΔABC is an equilateral triangle of side S and $AE \perp BC$.



Then, $BD = \frac{S}{3}$ and $BE = \frac{S}{2}$

$$DE = BE - BD = \frac{S}{2} - \frac{S}{3} = \frac{S}{6}$$

From ΔABE,

$$\frac{AE}{AB} = \sin 60°$$

$$\Rightarrow AE = \frac{S\sqrt{3}}{2}$$

Now, $AD^2 = (AE)^2 + (DE)^2$

$$= \left(\frac{S\sqrt{3}}{2}\right)^2 + \left(\frac{S}{6}\right)^2 = \frac{7}{9}S^2.$$

Now, $\frac{AD^2}{AB^2} = \frac{\frac{7}{9}S^2}{S^2} = \frac{7}{9}$.

3. (d) Sum of angles of a triangle $= 180°$

$x + 2x + 3x = 180°$

$\Rightarrow 6x = 180° \Rightarrow x = 30°$.

That gives integer angles.

But for $6x + 7x + 8x = 180$

$\Rightarrow 21x = 180° \Rightarrow x = 8.57°$.

So, 6 : 7 : 8 does not gives angles value in integer.

4. (b) $2\angle A = 3\angle B = 6\angle C = K$

$\therefore \angle A = \frac{K}{2}, \angle B = \frac{K}{3}, \angle C = \frac{K}{6}$

Sum of angles $= 180°$

$$\frac{K}{2} + \frac{K}{3} + \frac{K}{6} = 180° \Rightarrow K = 180°$$

$$\therefore \angle A + \angle C = \frac{K}{2} + \frac{K}{6} = \frac{4K}{6} = \frac{4}{6} \times 180° = 120°$$

5. (d) Let length of right angled triangles are $(x - 2)$, x and $(x + 2)$.
Then,

$(x-2)^2 + x^2 = (x+2)^2$

$\Rightarrow 2x^2 - 4x + 4 = x^2 + 4x + 4$

$\Rightarrow x^2 - 8x = 0$

$\Rightarrow x = 8$

Three sides are $(8 - 2) = 6$, 8 and $(8 + 2) = 10$.

Products $= 6 \times 8 \times 10 = 480$.

6. (d) Let a, b and c are three sides of right angle triangle, such that

$a^2 + b^2 = c^2$

Again, from question, $a^2+b^2+c^2=8450$

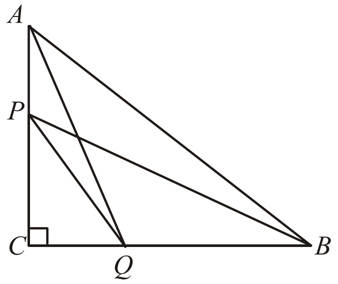
Then, from two equations,

$c^2+c^2=8450$

$\Rightarrow c=\sqrt{4225}=65$ units.

7. (b) As median divide the sides into two equal parts.
So, area of ΔAPB = Area of ΔAPC.

8. (a) ΔACB and ΔPCQ are right triangle.



$\therefore AB^2=AC^2+BC^2$ and $PQ^2=PC^2+CQ^2$

Now, ΔACQ and ΔBCP is a right angle triangle.

$AQ^2=AC^2+CQ^2$ ...(I)

and $BP^2=BC^2+PC^2$ ...(II)

$AQ^2+BP^2=AC^2+BC^2+CQ^2+PC^2$

$=AB^2+PQ^2$

As point P and Q are not mid point of side AC and BC.

$\therefore AB \neq 2PQ$.

9. (a) Let A should be add,

$$\frac{1}{(x-2)(x-4)}+A=\frac{2x-5}{(x^2-5x+6)(x-4)}$$

$$A=\frac{2x-5}{(x^2-5x+6)(x-4)}-\frac{1}{(x-2)(x-4)}$$

$$=\frac{2x-5}{(x^2-3x-2x+6)(x-4)}-\frac{1}{(x-2)(x-4)}$$

$$=\frac{2x-5}{(x-3)(x-2)(x-4)}-\frac{1}{(x-2)(x-4)}$$

$$=\frac{2x-5-(x-3)}{(x-3)(x-2)(x-4)}=\frac{x-2}{(x-3)(x-2)(x-4)}$$

$$=\frac{1}{(x-3)(x-4)}=\frac{1}{x^2-7x+12}$$

10. (b) $x^2+ax+b=0$

$x^2+cx+d=0$

if $(x-k)$ is the factor of Eq^n

Then, putting $x=k$ in the eq^n

$k^2+ak+b=0$ ...(1)

$k^2+ck+d=0$ ...(2)

eq^n (1) – eq^n(2)

$k(a-c)+b-d=0$

$k(a-c)=(d-b)$

$$k=\frac{(d-b)}{(a-c)}$$

11. (c) $$\frac{12}{7-\frac{6}{7-\frac{3}{5-x}}}=x$$

$$x=\frac{12}{7-\frac{6}{\frac{32-7x}{5-x}}}=\frac{12}{7-\frac{30-6x}{32-7x}}$$

$$=\frac{12}{\frac{194-43x}{32-7x}}=\frac{12(32-7x)}{192-42x}$$

$$=\frac{12(32-7x)}{6(32-7x)}=\frac{12}{6}=2$$

$\therefore\ x=2$

12. (c) Let number of boys and girls in the Class is $3x$ and x respectively

Total marks of the class $=p.(3x+x)=4xp$

Again total marks of the boys $=3x.(p+1)$

$\therefore$ Total marks of the girls $=4xp-3x(p+1)$

$=x.p-3x=(p-3)x$

Hence, Average marks of the girls$=\dfrac{(p-3)x}{x}=(p-3)$

13. (a) Let income of the A, B and C are $7x$, $9x$ and $12x$ and their expenditure are $8y$, $9y$ and $15y$ respectively.

then, A's Saving $=7x-8y$

B's Saving $=9x-9y$

C's Saving $=12x-15y$

According to the question,

$$7x-8y=\frac{7x}{4}$$

$$\frac{21x}{4}=8y\Rightarrow\ y=\frac{21}{32}x$$

Now, A's Ratio of saving of A, B and c

$$=\frac{7x}{4}:9\left(x-\frac{21}{32}x\right):\left(12x-15.\frac{21}{32}x\right)$$

$=56:99:69$

14. (b) Speed of the train $=\dfrac{(200+100)}{10}=30$ m/sec

Quadrilateral and Polygon

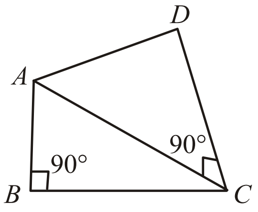
1. (c) Interior angle of polygon of n sides

$$= \frac{(n-2)\times\pi}{n}$$

Here, $n = 5$.

$\therefore$ Interior angle $= \frac{(5-2)\pi}{5} = \frac{3\pi}{5}$.

2. (c)



$\angle B = 90°$

From ΔABC, $AC^2 = AB^2 + BC^2$

Given that

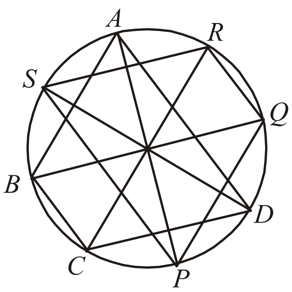
$AB^2 + BC^2 + CD^2 - AD^2 = 0$

$\Rightarrow AC^2 + CD^2 - AD^2 = 0$

$\Rightarrow AC^2 + CD^2 = AD^2$

$\therefore$ ΔACD, is a right angle triangle and $\angle ACD = 90°$.

3. (c)



As $PQRS$ is also cyclic quadrilateral then,

$\angle PQR + \angle RSP = 180°$

{Sum of opposite angles of cyclic quadrilateral}

4. (c) Both (1) and (2) are correct.
Diagonals of trapezium divides each other proportionally and any line parallel to the parallel sides of the trapezium divides the non-parallel sides proportionally.

5. (a) Sum of interior angles of polygon of n sides
$= (n-2) \times 180°$
Sum of exterior angles of any polygon = 360°
ATQ,

$(n-2)\times 180° = 2\times(360°)$

$\therefore (n-2) = 4 \Rightarrow n = 6.$

Hence, polygon is hexagon.

6. (a) $k^2 = x(x-1)(x-2)(x-3)$
$k^2 = x^4 - 6x^3 + 11x^2 - 6x + 1$
$k^2 = (x^2 - 3x + 1)^2$
Taking square root on both sides.
$k = x^2 - 3x + 1$

7. (d) $\frac{1}{bc(a-b)(a-c)} + \frac{1}{ca(b-c)(b-a)} + \frac{1}{ab(c-a)(c-b)}$

$$\frac{a(b-c)}{abc(a-b)(b-c)(a-c)} + \frac{b(c-a)}{abc(b-c)(b-a)(c-a)} + \frac{c(a-b)}{ab(a-b)(c-b)(c-a)}$$

$$= \frac{-[a(b-c)+b(c-a)+c(a-b)]}{abc(a-b)(b-c)(c-a)} = 0$$

8. (d) $\frac{2}{3} = 0.667, \frac{2+3}{3+3} = \frac{5}{6} = 0.833$ Difference = 0.167

$\frac{3}{4} = 0.74, \frac{3+3}{4+3} = \frac{6}{7} = 0.857$ Difference = 0.117

$\frac{4}{5} = 0.8, \frac{4+3}{5+3} = \frac{7}{8} = 0.875$ Difference = 0.075

$\frac{5}{6} = 0.833, \frac{5+3}{6+3} = \frac{8}{9} = 0.889$ Difference = 0.056

Hence, $\frac{5}{6}$ shows minimum change in its value

Circle

1. (d) Let $r = \frac{7}{2}$

Then, Surface Area $s = 4\times\frac{22}{7}\times\left(\frac{7}{2}\right)^2 = 154$ unit

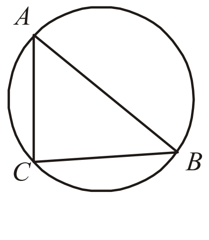
$\therefore$ (1) is not correct.

Again, when $r = \frac{21}{2}$

Then volume $= \frac{4}{3}\times\frac{22}{7}\times\left(\frac{21}{2}\right)^3 = 4851$ unit

$\therefore$ (2) is not correct.

2. (d) As AB is the diameter of the circle in which ΔABC is inscribed.



So,

ΔABC is a right angled triangle and $\angle C = 90°$

Now, $\cos(A+B) + \sin(A+B)$

$= \cos(180° - C°) + \sin(180° - C°)$

$= -\cos C° + \sin C°$

$= -\cos(90°) + \sin(90°) = 0 + 1 = 1.$

3. (a) $360° = 2\pi r$ (where r = radius)

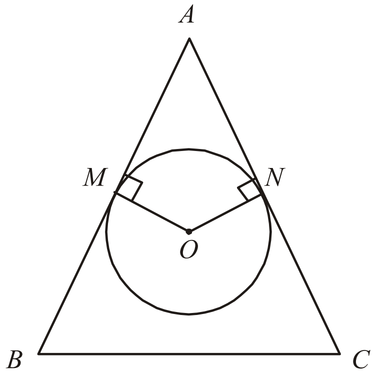
Then, $\frac{42°}{360°} = \frac{44}{2\pi r}$

$\therefore r = \frac{44 \times 360°}{2 \times \frac{22}{7} \times 42°} = 60$ m.

4. (d) Number of solid lead ball of diameter 2mm

$= \frac{(80)^3}{(1)^3} = 512000.$

5. (b) As, AB and AC touches the circle at point M and N respectively.



$\therefore \angle AMO = \angle ANO = 90°$

$\Rightarrow \angle A = 70°$

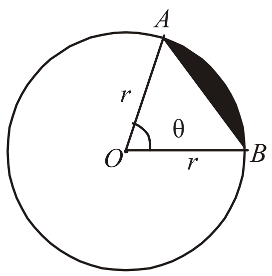
$\therefore$ From $AMON$ quadrilateral,

$\angle A + \angle M + \angle O + \angle N = 360°$

$\Rightarrow 70° + 90° + \angle MON + 90° = 360°$

$\therefore \angle MON = 110°.$

6. (b)



Required area is the shaded portion

= Area of sector $\widehat{AOB}$ – Area of triangle AOB

$= \frac{1}{2\pi} \cdot \pi r^2 \cdot (\theta) - \frac{1}{2} \cdot AO \cdot OB \cdot \sin(\theta)$

$= \frac{1}{2} r^2 (\theta) - \frac{1}{2} \cdot r^2 \cdot \sin\theta$

$= \frac{1}{2} r^2 (\theta - \sin\theta)$

$= \frac{1}{2} r^2 \left(\theta - 2 \cdot \sin\frac{\theta}{2} \cdot \cos\frac{\theta}{2}\right)$

7. (c) $\frac{x}{a} + \frac{y}{b} = a + b$ and $\frac{x}{a^2} + \frac{y}{b^2} = 2$

Since there are 4 variables and 2 equations.
So, we can suppose the value of two variables.
Let $a = b = 1$

$\therefore \quad \frac{x}{a} + \frac{y}{b} = a + b$

$\frac{x}{1} + \frac{y}{1} = 1 + 1$

$x + y = 2$

$\Rightarrow x = y = 1$

And $\frac{x}{a^2} + \frac{y}{b^2} = 2$

Be putting the values of a and b

$\frac{x}{1} + \frac{y}{1} = 2$

$x + y = 2$

$\Rightarrow x = y = 1$

Putting the values of a, b, x and y in $\frac{x}{a^2} - \frac{y}{b^2}$

$= \frac{1}{1} - \frac{1}{1} = 0$

8. (c) Statement 1 :

$x \propto z \quad \Rightarrow \quad x = kz \quad \ldots(1)$

$y \propto z \quad \Rightarrow \quad y = mz \quad \ldots(2)$

eq^n $(1)^2$ – eq^n $(2)^2$

$x^2 - y^2 = k^2z^2 - m^2z^2$

$x^2 - y^2 = z^2 (k^2 - m^2)$

So, $x^2 - y^2 \propto z^2$ (correct)

Statement 2 :

$x \propto \frac{1}{z} \quad \Rightarrow \quad x = \frac{k}{z} \quad \ldots(1)$

$y \propto \frac{1}{z} \quad \Rightarrow \quad y = \frac{m}{z} \quad \ldots(2)$

eq^n(1) multiply by eq^n(2)

$xy = \frac{km}{z^2}$

So, $xy \propto \frac{1}{z^2}$ (Correct)

Therefore, Both the statement 1 and 2 are correct.

9. (d)

$$\begin{array}{r|l} & x+5 \\ x^2-5x+6 & x^3-19x+30 \\ & \underline{\underset{-}{x^3}\ \underset{+}{-5x^2}\ \underset{-}{+6x}} \\ & 5x^2-25x+30 \\ & \underline{5x^2-25x+30} \end{array}$$

$(x+5)(x^2-5x+6) = x^3-19x+30$
$(x+5)(x-2)(x-3) = x^3-19x+30$
So, the HCF is $(x-2)(x-3)$

Statistics

1. (a) Here out of fifteen, five students failed and got zero marks.

Now, arranging data in ascending order :

0, 0, 0, 0, 0, 4, 5, 6, 6, 7, 7, 8, 8, 9, 9

Number of data $= n = 15$

Median marks $= \left(\frac{n+1}{2}\right)^{th} = \left(\frac{15+1}{2}\right)^{th} = 8^{th} = 6.$

2. (c) Arranging in ascending order :

111, 141, 154, 175, 176, 180, 191

Median yield $= \left(\frac{7+1}{2}\right)^{th} = 4^{th}$ data $= 175$ gm.

3. (d) Mode is used to measure average size of the shoe sold in the shop.

4. (d) Height of a rectangle in histogram represent frequency of the class.

5. (b) Mean age of the 7 member family

$= \frac{2+5+12+18+38+40+60}{7} = 25$ years.

New average when a new person of age x years added into the family after 5 years $= 25 + 1.5 = 26.5$

So, age of new member (x),

$26.5 \times 8 - 25 \times 7 = 2$ years.

6. (c) Let number of boys and girls in the class are m and $(100 - m)$ respectively.

Then, $100 \times 46 = 50m + 40(100 - m)$

$\Rightarrow 600 = 10m \Rightarrow m = 60.$
$\therefore$ Number of boys $= m = 60$
Number of girls $= n = 100 - 60 = 40.$
Difference, $m - n = 60 - 40 = 20.$

7. (b) Mean $(m) = \frac{\Sigma x_i}{n}$

Sum of Deviation $= \frac{\Sigma(x_i - m)}{n} = 0.$

8. (c) Sum of all 5 observations = 5 × mean
$\therefore 5m = x + x + 2 + x + 4 + x + 6 + x + 8$

$\therefore m = \frac{5}{5}(x+4) = x + 4.$

Now, mean of first three observation

$= \frac{x+x+2+x+4}{3} = x + 2 = m - 2.$

9. (d) Given data, 2, 4, 6, ... 100.
Here, number of Data = 50

Median $= \frac{\frac{50}{2} + \left(\frac{50}{2}+1\right)}{2} = \left(\frac{25^{th}+26^{th}}{2}\right)$ data

$= \frac{50+52}{2} = 51.$

10. (d) From (A.M.) (H.M.) = (G.M.)^2
(A.M.) (10) = $(12)^2$

$\therefore$ A.M. $= \frac{144}{10} = 14.4$

11. (d) For the divisibility of 13
If for a given number, form alternating sums of blocks of three numbers from the right and moving fowards the left, then if the number formed by the alternative sum and difference of blocks of 3-3 digits from right to the left is divisible by 13 then number is divisible by 13.
4, 132, 83P, 759, 387 $\rightarrow$ 387 – 759 + 83P – 132 + 4
$\Rightarrow$ 391 + 83P – 891
$\Rightarrow$ 83P – 500

With the help of options, if we put P = 8.

Then, 838 – 500 = 338

Which is divisible by 13.

So, value of P = 8.

12. (b) $2^{1000000} = 2^{(999999+1)} = (2^3)^{333333}.2^1$
$= (8)^{333333} \times 2$
$= \frac{8^{333333}}{7} \times 2$

When 8 is divided by 7
Then, Reminder = 1 $[\because 1^n = 1]$

$\therefore \frac{2}{7}$ = Remainder is 2.

13. (a) $\frac{x}{y} + \frac{y}{x} = 2,$
$x^2 + y^2 = 2xy$
$x^2 + y^2 - 2xy = 0$
$(x-y)^2 = 0$
$x - y = 0$
$x = y$
but from the question $x \neq y$
So, there is no such pair.

14. (d) Prime numbers between 50 and 100 = 53, 59, 61, 67, 71, 73, 79, 83, 89, 97
There are only 5 Pairs such that $(m - n = 6)$:
(59, 53), (67, 61), (73, 67), (79, 73) and (89, 83)

15. (a) $\because$ $a^n - b^n = (a - b)^n$, where n is an odd number
$\therefore$ $27^{27} - 15^{27} = (27 - 15)^{27} = 12^{27}$
$\frac{12^{27}}{6}$, Remainder is 0.

16. (d) 1. $a^3 + b^3 + c^3 - 3abc = (a + b + c)$
$(a^2 + b^2 + c^2 - ab - bc - ca)$
if $a + b + c = 0$
Then, $a^3 + b^3 + c^3 = 3abc$ [Correct]
2. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
if $a + b + c = 0$
Then, $a^2 + b^2 + c^2 = -2\ (ab + bc + ca)$ [Correct]
3. $(a + b + c)^3 = a^3 + b^3 + c^3 + 3\ (a + b)\ (b + c)\ (c + a)$
if $a + b + c = 0$
Then, $a^3 + b^3 + c^3 = -3ab\ (a + b)$ [Correct]
So, 1, 2 and 3 all are correct.

17. (b) $p = \frac{\sqrt{3q+2}+\sqrt{3q-2}}{\sqrt{3q+2}-\sqrt{3q-2}}$
By componendo – Devidendo
$$\frac{p+1}{p-1} = \frac{\sqrt{3q+2}}{\sqrt{3q-2}}$$
Squaring both sides, we get
$$\left(\frac{p+1}{p-1}\right)^2 = \frac{3q+2}{3q-2}$$
Again, by componendo – Dividendo
$$\frac{(p+1)^2+(p-1)^2}{(p+1)^2-(p-1)^2} = \frac{3q}{2}$$
$$\frac{p^2+1}{2p} = \frac{3q}{2}$$
$p^2 + 1 = 3pq$
$\Rightarrow$ $p^2 - 3pq + 1 = 0$
Hence, $p^2 - 3pq + 2 = 1$

18. (a) Unit digit of the expression 67^{32} is the same as 7^{32}
Unit digit in the expressons is:
$$\begin{bmatrix} 7^1 = 7 \\ 7^2 = 9 \\ 7^3 = 3 \\ 7^4 = 1 \end{bmatrix}$$
$$\begin{matrix} 7^5 = 7 \\ 7^6 = 9 \end{matrix}$$
We can clearly see that after the set of 4, the unit digit is repeating.
So, unit digit in
$7^{32} = 7^4 = 1$ ($\because$ 32 is divisible by 4 completely)
$\therefore$ Unit digit of $67^{32} = 1$

19. (c) $\frac{b+\sqrt{b^2-2bx}}{b-\sqrt{b^2-2bx}} = a$
By Componendo and Dividendo :
$$\frac{2b}{2\sqrt{b^2-2bx}} = \frac{a+1}{a-1}$$
$$b\frac{(a-1)}{(a+1)} = \sqrt{b^2-2bx}$$
Squaring on both sides :
$$b^2\frac{(a-1)^2}{(a+1)^2} = b^2 - 2bx$$
$$b^2 - b^2\frac{(a-1)^2}{(a+1)^2} = 2bx$$
$$b^2\left[1-\frac{(a-1)^2}{(a+1)^2}\right] = 2bx$$
$$b\left[\frac{4a}{(a+1)^2}\right] = 2x$$
$$x = \frac{2ab}{(a+1)^2}$$

CHAPTER

1 Number System

1. By adding x to 1254934, the resulting number becomes divisible by 11, while adding y to 1254934 makes the resulting number divisible by 3. Which one of the following is the set of values for x and y? **[2007-I]**
(a) $x = 1, y = 1$ (b) $x = 1, y = -1$
(c) $x = -1, y = 1$ (d) $x = -1, y = -1$
2. A ten-digit number is divisible by 4 as well as by 5. What could be the possible digit at the ten's place in the given number? **[2007-I]**
(a) 0, 1, 2, 4 or 6 (b) 1, 2, 4, 6 or 8
(c) 2, 3, 4, 6 or 8 (d) 0, 2, 4, 6 or 8
3. What least number should be subtracted from 26492518, so that the resulting number is divisible by 3 but not by 9 ? **[2007-I]**
(a) 1 (b) 3 (c) 4 (d) 7
4. Which one of the following numbers is a composite numbers? **[2007-I]**
(a) 589 (b) 571 (c) 569 (d) 563
5. Which one of the following is correct ?
$a^n + b^n$ is divisible by $a - b$ **[2009-II]**
(a) for all integral values of n
(b) when n is an even integer
(c) when n is an odd integer
(d) for no integral value of n
6. If x and y denote respectively, the area and the sum of the length of diagonals of a rectangle with length 1 unit and breadth $\frac{1}{2}$ unit, then which one on the following is correct? **[2009-II]**
(a) x and y are rational
(b) x is rational and y is irrational
(c) x is irrational and y is rational
(d) x and y are both irrational
7. In a divisible operation, the division is 5 times the quotient and twice the remainder. If the remainder is 15, then what is the dividend? **[2009-II]**
(a) 175 (b) 185 (c) 195 (d) 250
8. Which one of the following is correct?
The number 222222 is **[2009-II]**
(a) divisible by 3 but not divisible by 7
(b) divisible by 3 and 7 but not divisible by 11
(c) divisible by 2 and 7 but not divisible by 11
(d) divisible by 3, 7 and 11
9. Which one of the following is correct?
The sum of two irrational numbers **[2009-II]**
(a) is always a natural or irrational
(b) may be rational or irrational
(c) is always a rational number
(d) is always an irrational number
10. If the numbers q, $q + 2$ and $q + 6$ are all prime, then what can be the value of $3q + 9$? **[2009-II]**
(a) Only 18 (b) Only 42
(c) Only 60 (d) Both (b) and (c)
11. Let p denotes the product $2 \cdot 3 \cdot 5 \ldots 59 \cdot 61$ of all primes from 2 to 61. Consider the sequence $p + n$ $(2 \le n \le 59)$. What is the number of primes in this sequence (where n is a natural number)? **[2009-II]**
(a) 0 (b) 16 (c) 17 (d) 58
12. A three-digit number has digits h, t, u (from left to right) with $h > u$. If the digits are reversed and the number thus formed is subtracted from the original number, the unit's digit in the resulting number is 4. What are the other two digits of the resulting number from left to right? **[2009-II]**
(a) 5 and 9 (b) 9 and 5
(c) 5 and 4 (d) 4 and 5
13. **Assertion (A) :** Zero is a whole number.
Reason (R) : Every integer is a whole number. **[2009-II]**
(a) A and R are correct but R is correct explanation of A
(b) A and R are correct but R is not correct explanation of A
(c) A is correct but R is wrong
(d) A is wrong but R is correct
14. One dividing 4996 by a certain number, the quotient is 62 and the remainder is 36. What is the divisor? **[2009-II]**
(a) 80 (b) 85 (c) 90 (d) 95
15. Which of the following numbers is a prime? **[2009-II]**
(a) 667 (b) 861 (c) 481 (d) 331
16. A number, when divided by 987, gives a remainder 59. When the same number is divided by 21, what is the remainder? **[2009-II]**
(a) 21 (b) 19 (c) 17 (d) 15
17. Which one of the following statements is always correct? **[2009-II]**
(a) The square of a prime number is prime
(b) The sum of two square numbers is a square number
(c) The number of digits in a square number is even
(d) The product of two square numbers is square number
18. The set of integers is closed with respect to which one of the following ? **[2009-II]**
(a) Addition only
(b) Multiplication only
(c) Both addition and multiplication
(d) Division

19. What least value must be given to *, so that the number 8798546*5 is divisible by 11? **[2009-II]**
(a) 0 (b) 1 (c) 2 (d) 3

20. What is the sum of all prime numbers between 100 and 120? **[2009-I]**
(a) 652 (b) 650 (c) 644 (d) 533

21. What is the sum of positive integers less than 100 which leave a remainder 1 when divided by 3 and leave a remainder 2 when divided by 4? **[2009-II]**
(a) 416 (b) 620 (c) 1250 (d) 1314

22. What is the total number of three digit three digit numbers with unit digit 7 and divisible by 11? **[2009-II]**
(a) 6 (b) 7
(c) 8 (d) 9

23. If we divide a positive integer by another positive integer, what is the resulting number? **[2009-II]**
(a) It is always a natural number
(b) It is always an integer
(c) It is a rational number
(d) It is an irrational number

24. The product of two alternate odd integers exceeds three times the smaller by 12. What is the larger number? **[2009-II]**
(a) 3 (b) 5 (c) 7 (d) 9

25. Consider the following statements:
A number $a_1\ a_2\ a_3\ a_4\ a_5$ is divisible by 9, if
I. $a_1 + a_2 + a_3 + a_4 + a_5$ is divisible by 9.
II. $a_1 - a_2 + a_3 - a_4 + a_5$ is divisible by 9.
Which of the above statements is/are correct? **[2009-II]**
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

26. What is the last digit in the expansion of $(2457)^{754}$? **[2009-II]**
(a) 3 (b) 7 (c) 8 (d) 9

27. When a natural number n is divided by 4, the remainder is 3. What is the remainder when $2n$ is divided by 4? **[2009-II]**
(a) 1 (b) 2 (c) 3 (d) 6

28. There are four numbers forming a GP in which the third term is greater than the first by 9 and the second term is greater than the fourth by 18. What is the first term? **[2009-II]**
(a) 2 (b) 3 (c) – 2 (d) – 3

29. The angles of a triangle are in AP and the greatest angle is double the least. What is the ratio of angles in the radian measure? **[2009-II]**
(a) 2 : 3 : 4 (b) 1 : 2 : 3
(c) 3 : 3 : 6 (d) 4 : 5 : 7

30. If p is an integer, then every square integer is of the form **[2009-II]**
(a) $2p$ or $(4p - 1)$ (b) $4p$ or $(4p - 1)$
(c) $3p$ or $(3p + 1)$ (d) $4p$ or $(4p + 1)$

31. What least value must be given to ⊗, so that the number 84705⊗2 is divisible by 9? **[2009-II]**
(a) 0 (b) 1 (c) 2 (d) 3

32. When a polynomial is divided by a linear polynomial, then what is the remainder? **[2009-II]**
(a) Constant polynomial only
(b) Zero polynomial only
(c) Either constant or zero polynomial
(d) Linear polynomial

33. The remainder on dividing given integers a and b by 7 are, respectively 5 and 4. What is the remainder when ab is divided by 7? **[2010-I]**
(a) 3 (b) 4 (c) 5 (d) 6

34. If r and s are any real numbers such that $0 \le s \le 1$ and $r + s = 1$, then what is the maximum value of their product? **[2010-I]**
(a) 1 (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

35. Which one of the following numbers is not a square of any natural numbers ? **[2010-I]**
(a) 5041 (b) 9852 (c) 1936 (d) 6241

36. A three-digit number is divisible by 11 and has its digit in the unit's place equal to 1. The number is 297 more than the number obtained by reversing the digits. What is the number ? **[2010-I]**
(a) 121 (b) 231 (c) 561 (d) 451

37. Consider the following assumption and two statements:
Assumption: A number, '$ABCDE$' is divisible by 11.
Statement I: $E - D + C - B + A$ is divisible by 11.
Statement II: $E - D + C - B + A = 0$
Which one of the following is correct ? **[2010-I]**
(a) Only statement I can be drawn from the assumption
(b) Only statement II can be drawn from the assumption
(c) Both the statements can be drawn from the assumption
(d) Neither of the statements can be drawn from the assumption

38. What can be said about the expansion of $2^{12n} - 6^{4n}$, where n is positive integer ? **[2010-I]**
(a) Last digit is 4 (b) Last digit is 8
(c) Last digit is 2 (d) Last two digit are zero

39. What is the value of x for which x, $x + 1$, $x + 3$ are all prime numbers? **[2010-II]**
(a) 0 (b) 1 (c) 2 (d) 101

40. What is the last digit in the expansion of 3^{4798} ? **[2010-II]**
(a) 1 (b) 3 (c) 7 (d) 9

41. If k is any even positive integer, then $(k^2 + 2k)$ is **[2010-II]**
(a) divisible by 24
(b) divisible by 8 but may not be divisible by 24
(c) divisible by 4 but may not be divisible by 8
(d) divisible by 2 but may not be divisible by 4

42. If n is a positive integer, then what is the digit in the unit place of $3^{2n+1} + 2^{2n+1}$? **[2010-II]**
(a) 0 (b) 3 (c) 5 (d) 7

43. If the 14th term of an arithmetic series is 6 and 6th term is 14, then what is the 95th term? **[2010-II]**
(a) – 75 (b) 75 (c) 80 (d) – 80

44. The product of a rational number and an irrational number is **[2010-II]**
(a) a natural number (b) an irrational number
(c) a composite number (d) a rational number

45. For a positive integer n, define $d(n)$ = The number of positive divisors of n. What is the value of $d[d\{d(12)\}]$? **[2011-I]**
(a) 1 (b) 2
(c) 4 (d) None of these

46. The number 2784936 is divisible by which one of the follwing numbers ? **[2011-I]**
(a) 86 (b) 87 (c) 88 (d) 89

47. Consider the following numbers:
I. 247 II. 203
Which of the above numbers is/are prime ? **[2011-I]**
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

48. Which one of the following three-digit numbers divides 9238 and 7091 with the same remainder in each case ? **[2011-I]**
(a) 113 (b) 209 (c) 317 (d) 191

49. When a positive integer n is divided by 5, the remainder is 2. What is the remainder when the number $3n$ is divided by 5 ? **[2011-I]**
(a) 1 (b) 2 (c) 3 (d) 4

50. Consider the following statements:
If p is a prime such that $p + 2$ is also a prime, then
I. $p(p + 2) + 1$ is a perfect square.
II. 12 is a divisor of $p + (p + 2)$, if $p > 3$.
Which of the above statements is/are correct? **[2011-I]**
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

51. If three sides of a right angled triangle are integers in their lowest form, then one of its sides is always divisible by **[2011-I]**
(a) 6 (b) 5
(c) 7 (d) None of these

52. What is the number of prime factors of 30030? **[2011-I]**
(a) 4 (b) 5
(c) 6 (d) None of these

53. ABC is a triangle and AD is perpendicular to BC. It is given that the lenghts of AB, BC, CA are all rational numbers. Which one of the following is correct? **[2011-I]**
(a) AD and BD must be rational
(b) AD must be rational but BD need not be rational
(c) BD must be rational but AD need not be rational
(d) Neither AD nor BD need be rational

54. If n is a natural number, then $\sqrt{n}$ is **[2011-II]**
(a) always a natural number
(b) always a rational number
(c) always an irrational number
(d) either a natural number or an irrational number

55. The largest integer that divides product of any four consecutive integers is **[2011-II]**
(a) 4 (b) 6 (c) 12 (d) 24

56. Which among the following is the largest four digit number that is divisible by 88 ? **[2011-II]**
(a) 9988 (b) 9966 (c) 9944 (d) 8888

57. Which one of the following is a prime number? **[2011-II]**
(a) 161 (b) 171 (c) 173 (d) 221

58. Consider the following statements:
I. The product of any three consecutive integers is divisible by 6.
II. Any integer can be expressed in one of the three forms $3k$, $3k + 1$, $3k + 2$, where k is an integer.
Which of the above statements is/are correct ? **[2011-II]**
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

59. Consider the following statements:
I. Every composite number is a natural number.
II. Every whole number is a natural number.
Which of the statements given above is/are correct? **[2012-I]**
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

60. If a positive integer leaves remainder 28 when divided by 143, then what is the remainder obtained on dividing the same number by 13 ? **[2012-I]**
(a) 0 (b) 2 (c) 9 (d) 10

61. How many numbers between – 11 and 11 are multiples of 2 or 3 ? **[2012-I]**
(a) 11 (b) 14
(c) 15 (d) None of these

62. What number should be added to 231228 to make it exactly divisible by 33 ? **[2012-I]**
(a) 1 (b) 2 (c) 3 (d) 4

63. How many rational numbers are there between 1 and 1000? **[2012-I]**
(a) 998 (b) 999
(c) 1000 (d) Infinite

64. The pair of numbers which are relatively prime to each other is **[2012-I]**
(a) (68, 85) (b) (65, 91)
(c) (92, 85) (d) (102, 153)

65. The number 58129745812974 is divided by **[2012-I]**
(a) 11 (b) 8
(c) 4 (d) None of these

66. Consider the following statements:
I. If n is a prime number greater than 5, then $n^4 - 1$ is divisible by 2400.
II. Every square number is of the form $5n$, $(5n - 1)$ or $(5n + 1)$, where n is a whole number.
Which of the above statements is/are correct? **[2012-II]**
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

67. If the sum of an integer and its reciprocal is 10/3, then the integer is equal to **[2012-II]**
(a) 3 (b) 6 (c) 12 (d) 13

68. Which one of the following is neither prime number nor composite number ? **[2012-II]**
(a) 1 (b) 2
(c) 3 (d) None of these

69. Which one of the following has least number of divisors? **[2012-II]**
(a) 88 (b) 91 (c) 96 (d) 99

70. Consider the following statements:
I. Every integer is a rational number.
II. Every rational number is a real number.
Which of the above sttemetns is/are correct? **[2012-II]**
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

71. What is the harmonic mean of 10, 20, 25, 40 and 50? **[2012-II]**
(a) 25 (b) 30 (c) 26.1 (d) 21.3

72. The two digit number, which when divided by sum of the digit and product of the digits, respectively. The remainder is same and the differnce of quotients is one, is **[2013-I]**
(a) 14 (b) 23 (c) 32 (d) 41

73. Which one of the following numbers is divisible by 11? **[2013-I]**
(a) 45678940 (b) 54857266
(c) 87524398 (d) 93455120

74. If N, $(N + 2)$ and $(N + 4)$ are prime number, then the number of possible solution for N are **[2013-I]**
(a) 1 (b) 2
(c) 3 (d) None of these

75. The smallest positive prime (say p) such that $2^p - 1$ is not a prime is **[2013-I]**
(a) 5 (b) 11 (c) 17 (d) 29

76. Consider the following statements:
(I) There is a finite number of rational numbers between any two rational numbers.
(II) There is an infinite number of rational numbers between any two rational numbers.
(III) There is a finite number of irrational numbers between any two rational numbers.
Which of the above statements is/are correct? **[2013-I]**
(a) Only I (b) Only II
(c) Only III (d) Both I and II

77. If m and n are natural number, then $\sqrt[m]{n}$ is **[2013-II]**
(a) always irrational
(b) irrational unless n is the mth power of an integer
(c) irrational unless m is the nth power of an integer
(d) irrational unless m and n are coprime

78. If x is positive even integer and y is negative odd integer, then x^y is **[2013-II]**
(a) odd integer (b) even integer
(c) rational number (d) None of these

79. The pair of rational number that lies between $\frac{1}{4}$ and $\frac{3}{4}$ is **[2013-II]**
(a) $\frac{262}{1000}, \frac{752}{1000}$ (b) $\frac{24}{100}, \frac{78}{100}$
(c) $\frac{9}{40}, \frac{31}{40}$ (d) $\frac{252}{1000}, \frac{748}{1000}$

80. What is the last digit in $7^{402} + 3^{402}$? **[2013-II]**
(a) 0 (b) 4
(c) 8 (d) None of the above

81. What is $26^2 + 97^2$ equal to ? **[2013-II]**
(a) $27^2 + 93^2$ (b) $34^2 + 93^2$
(c) $82^2 + 41^2$ (d) $79^2 + 62^2$

82. Every prime number of the form $3k + 1$ can be represented in the form $6m + 1$ (where, k and m are integers), when **[2013-II]**
(a) k is odd
(b) k is even
(c) k can be both odd and even
(d) No such form is possible

83. If k is a positive integer, then every square integer is of the form **[2013-II]**
(a) only $4k$ (b) $4k$ or $4k + 3$
(c) $4k + 1$ or $4k + 3$ (d) $4k$ or $4k + 1$

84. If b is the largest square divisor of c and a^2 divides c, then which one of the following is correct (where a, b and c are integers) ? **[2013-II]**
(a) b divides a (b) a does not divide b
(c) a divided b (d) a and b are coprime

85. $19^5 + 21^5$ is divisible by **[2013-II]**
(a) Only 10 (b) Only 20
(c) Both 10 and 20 (d) Neither 10 nor 20

86. Consider the following statements:
I. 7710312401 is divisible by 11.
II. 173 is a prime number.
Which of the statements given above is/are correct ? **[2013-II]**
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

87. Consider the following statements:
I. To obtain prime numbers less than 121, we have to reject all the multiples of 2, 3, 5 and 7.
II. Every composite number less than 121 is divisible by a prime number less than 11.
Which of the statements given above is/are correct ? **[2013-II]**
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

88. Consider the following statements:
I. No integer of the form $4k + 3$, where k is an integer, can be expressed as the sum of two squares.
II. Square of an odd integer can expressed in the form $8k + 1$, where k is an integer.
Which of the above statements is/are correct? **[2014-I]**
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

89. If n is a whole number greater than 1, then $n^2 (n^2 - 1)$ is always divisible by **[2014-I]**
(a) 12 (b) 24 (c) 48 (d) 60

90. What is the remainder when 4^{1000} is divided by 7? **[2014-I]**

(a) 1 (b) 2
(c) 4 (d) None of these

91. $7^{10} - 5^{10}$ is divisible by **[2014-II]**

(a) 10 (b) 7 (c) 5 (d) 11

92. The multiplication of a three–digits number $XY5$, with digit Z yields X 215. What is $X + Y + Z$ equal to ? **[2014-II]**

(a) 13 (b) 15 (c) 17 (d) 18

93. If $N^2 - 33$, $N^2 - 31$ and $N^2 - 29$ are prime numbers, then what is the number of possible values of N, where N is an integer ? **[2014-II]**

(a) 1 (b) 2
(c) 6 (d) None of these

94. How many pairs of positive integers m and n satisfy the equation $\frac{1}{m}+\frac{4}{n}=\frac{1}{12}$, where n is an odd integer less than 60 ? **[2014-II]**

(a) 7 (b) 5 (c) 4 (d) 3

95. Consider all those two-digits positive integers less than 50, which when divided by 4 yield unity as remainder. What is their sum ? **[2014-II]**

(a) 310 (b) 314 (c) 218 (d) 323

96. How many pairs of X and Y are possible in the number 763X4Y2, if the number is divisible by 9? **[2014-II]**

(a) 8 (b) 9 (c) 10 (d) 11

97. What is the remainder when 4^{1012} is divided by 7? **[2014-II]**

(a) 1 (b) 2 (c) 3 (d) 4

98. What is the remainder when $(17^{23} + 23^{23} + 29^{23})$ is divided by 23 ? **[2014-II]**

(a) 0 (b) 1 (c) 2 (d) 3

99. p,q and r are prime numbers such that $p < q < r < 13$. In how many cases would $(p + q + r)$ also be a prime number ? **[2014-II]**

(a) 1 (b) 2
(c) 3 (d) None of these

100. What is the number of divisors of 360 ? **[2014-II]**

(a) 12 (b) 18
(c) 24 (d) None do these

101. If $\frac{37}{13}=2+\cfrac{1}{x+\cfrac{1}{y+\cfrac{1}{z}}}$ **[2015-I]**

where x, y, z are natural numbers, then what is z equal to?

(a) 1
(b) 2
(c) 3
(d) Cannot be determined due to insufficient data

102. What is $\frac{5+\sqrt{10}}{5\sqrt{5}-2\sqrt{20}-\sqrt{32}+\sqrt{50}}$ equal to? **[2015-I]**

(a) 5 (b) $5\sqrt{2}$ (c) $5\sqrt{5}$ (d) $\sqrt{5}$

103. The digit in the units place of the product 81 × 82 × 83 × 84 × ... × 99 is **[2015-I]**

(a) 0 (b) 4 (c) 6 (d) 8

104. A student was asked to multiply a number by 25. He instead multiplied the number by 52 and got the answer 324 more than the correct answer. The number to be multiplied was **[2015-I]**

(a) 12 (b) 15 (c) 25 (d) 32

105. What is the number of possible pairs of (P, Q) if the number 357P25Q is divisible by both 3 and 5? **[2015-I]**

(a) 7 (b) 6
(c) 5 (d) None of the above

106. The difference between the squares of two consecutive odd integers is always divisible by **[2015-I]**

(a) 3 (b) 7 (c) 8 (d) 16

107. What is the maximum value of m if the number $N = 35 \times 45 \times 55 \times 60 \times 124 \times 75$ is divisible by 5^m? **[2015-I]**

(a) 4 (b) 5 (c) 6 (d) 7

108. A person goes to a market between 4 p.m. and 5 p.m. When he comes back, he finds that the hour hand and minute hand have interchanged their positions. For how much time (approximately) was he out of his house? **[2015-I]**

(a) 55.25 minutes (b) 55.30 minutes
(c) 55.34 minutes (d) 55.38 minutes

109. When a ball bounces, it rises to $\frac{2}{3}$ of the height from which it fell. If the ball is dropped from a height of 36 m, how high will it rise at the third bounce? **[2015-I]**

(a) $10\frac{1}{3}$ m (b) $10\frac{2}{3}$ m (c) $12\frac{1}{3}$ m (d) $12\frac{2}{3}$ m

110. A light was seen regularly at an interval of 13 seconds. It was seen for the first time at 1 hour 54 minutes 50 seconds (a.m.) and the last time at 3 hours 17 minutes 49 seconds (a.m.). How many times was the light seen? **[2015-I]**

(a) 375 (b) 378 (c) 383 (d) 384

111. If n is a natural number and $n = p_1^{x_1} p_2^{x_2} p_3^{x_3}$, where p_1, p_2, p_3 are distinct prime factors, then the number of prime factors for n is **[2015-I]**

(a) $x_1 + x_2 + x_3$ (b) $x_1x_2x_3$
(c) $(x_1 + 1)(x_2 + 1)(x_3 + 1)$ (d) None of the above

112. Consider the following statements for the sequence of numbers given below : **[2015-I]**

11, 111, 1111, 11111, ...

1. Each number can be expressed in the form $(4m + 3)$, where m is a natural number.
2. Some numbers are squares.

Which of the above statements is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

113. Consider the following statements : **[2015-I]**
1. There exists only one prime number p such that $(17p + 1)$ is a square.
2. If x is the product of 10 consecutive prime numbers starting from 2, then $(x + 1)$ is also a prime number.

Which of the above statements is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

114. Out of 532 saving accounts held in a post office, 218 accounts have deposits over ₹ 10,000 each. Further, in 302 accounts, the first or sole depositors are men, of which the deposits exceed ₹ 10,000 in 102 accounts. In how many accounts the first or sole depositors are women and the deposits are up to ₹ 10,000 only? **[2015-I]**
(a) 116
(b) 114
(c) 100
(d) Cannot be determined from the given data

115. If $x + y + z = 0$
then $x^3 + y^3 + z^3 + 3xyz$ is equal to **[2015-I]**
(a) 0 (b) $6xyz$ (c) $12xyz$ (d) xyz

116. The last digit in the expansion of 17^{256} is **[2015-I]**
(a) 9 (b) 7 (c) 3 (d) 1

117. What is the remainder obtained **[2015-I]**
when $1421 \times 1423 \times 1425$ is divided by 12?
(a) 1 (b) 2 (c) 3 (d) 4

118. What is the remainder when 4^{96} is divided by 6? **[2015-I]**
(a) 4 (b) 3 (c) 2 (d) 1

119. The number of values of x satisfying $x + \frac{100}{x} > 50$, where x is a natural number less than or equal to 100 is **[2015-II]**
(a) 51 (b) 53 (c) 55 (d) 57

120. The largest natural number which divides every natural number of the form $(n^3 - n)(n - 2)$, where n is a natural number greater than 2 is **[2015-II]**
(a) 6 (b) 12 (c) 24 (d) 48

121. The digit in the units place of the resulting number of the expression $(234)^{100} + (234)^{101}$ is **[2015-II]**
(a) 6 (b) 4 (c) 2 (d) 0

122. The seven digit number 876p37q is divisible by 225. The values of p and q can be respectively **[2015-II]**
(a) 9, 0 (b) 0, 0 (c) 0, 5 (d) 9, 5

123. Let x and y be positive integers such that x > y. The expressions 3x + 2y and 2x + 3y when divided by 5 leave remainders 2 and 3 respectively. What is the remainder when (x – y) is divided by 5? **[2015-II]**
(a) 4 (b) 2 (c) 1 (d) 0

124. The sum of first 47 terms of the series
$\frac{1}{4}+\frac{1}{5}-\frac{1}{6}-\frac{1}{4}-\frac{1}{5}+\frac{1}{6}+\frac{1}{4}+\frac{1}{5}-\frac{1}{6}$..... is **[2015-II]**
(a) 0 (b) $-\frac{1}{6}$ (c) $\frac{1}{6}$ (d) $\frac{9}{20}$

125. A number consists of two digits, whose sum is 7. If the digits are reversed, the number is increased by 27. The product of digits of the number is **[2015-II]**
(a) 6 (b) 8 (c) 10 (d) 12

126. Consider all positive two digit numbers each of which when divided by 7 leaves a remainder 3. What is their sum? **[2015-II]**
(a) 661 (b) 666 (c) 676 (d) 777

127. Consider the following statements in respect of the expression **[2016-I]**
$$S_n = \frac{n(n+1)}{2}$$
where n is an integer.
1. There are exactly two values of n for which $S_n = 861$.
2. $S_n = S_{(n+1)}$ and hence for any integer m, we have two values of n for which $Sn = m$.

Which of the above statement is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

128. Consider the following statements in respect of two different non-zero integers p and q **[2016-I]**
1. For $(p + q)$ to be less than $(p - q)$, q must be negative.
2. For $(p + q)$ to be greater than $(p - q)$, both p and q must be positive.

Which of the above statements is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

129. $7^{10} - 5^{10}$ is divisible by **[2016-I]**
(a) 5 (b) 7 (c) 10 (d) 11

130. Let a two-digit number be k times the sum of its digits. If the number formed by interchanging the digits is m times the sum of the digits, then the value of m is **[2016-I]**
(a) $9 - k$ (b) $10 - k$ (c) $11 - k$ (d) $k - 1$

131. Let S be a set of first fourteen natural numbers. The possible number of pairs (a, b), where a, b, S and a, b such that ab leaves remainder 1 when divided by 15, is **[2016-I]**
(a) 3 (b) 5
(c) 6 (d) None of the above

132. A clock strickes once at 1 o'clock, twice at 2 o'clock and thrice at 3 o'clock and so on. If it takes 8 seconds to strike at 5 o'clock, the time taken by it to strike at 10 o'clock is **[2016-I]**
(a) 14 seconds (b) 16 seconds
(c) 18 seconds (d) None of the above

133. What is the maximum value of m, if the number $N = 90 \times 42 \times 324 \times 55$ is divisible by 3^m? **[2016-I]**
(a) 8 (b) 7 (c) 6 (d) 5

134. Consider the following statements: **[2016-I]**
1. Every natural number is a real number.
(b) Every real number is a rational number.
(c) Every integer is a real number.
(d) Every rational number is a real number

Which of the above statements are correct?
(a) 1, 2 and 3 (b) 1, 2 and 4
(c) 2 and 3 only (d) 3 and 4 only

135. Consider the following statements: **[2016-I]**
1. There exists a positive real number m such that cos $x = 2^{m+1}$.
2. $mn \geq m + n$ for all m, n belonging to set of natural numbers.

Which of the above statements is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

136. A person goes to a market between 4 p.m and 5 p.m. When he comes back, he finds that the hour hand and the minute hand of the clock have interchanged their positions. For how much time (approximately) was he out of his house? **[2016-I]**
(a) 55.38 minutes (b) 55.48 minutes
(c) 55.57 minutes (d) 55.67 minutes

137. If m and n are distinct natural numbers, then which of the following is/are integer/integers? **[2016-I]**
1. $\frac{m}{n}+\frac{n}{m}$
2. $mn\left(\frac{m}{n}+\frac{n}{m}\right)(m^2+n^2)$
3. $\frac{mn}{m^2+n^2}$

Select the correct answer using the code given below:
(a) 1 and 2 (b) 2 only (c) 2 and 3 (d) 3 only

138. Outside a meeting room, Madhukar was told by a person that each meeting takes place after 13/4 hours. The last meeting has been over just 45 minutes ago and the next meeting will take place at 2 p.m. At what time did Madhukar receive this information? **[2016-I]**
(a) 10 : 20 a.m. (b) 11 : 30 a.m.
(c) 11 : 40 a.m. (d) 11 : 50 a.m.

139. If $\sqrt{\frac{x}{y}}=\frac{10}{3}-\sqrt{\frac{y}{x}}$ and x – y = 8, then the value of xy is equal to **[2016-I]**
(a) 36 (b) 24 (c) 16 (d) 9

140. What would be the maximum value of Q in the equation 5P9 + 3R7 + 2Q8 = 1114 ? **[2016-II]**
(a) 9 (b) 8 (c) 5 (d) 4

141. In an examination, a student was asked to divide a certain number by 8. By mistake he multiplied it by 8 and got the answer 2016 more than the correct answer. What was the number? **[2016-II]**
(a) 252 (b) 256 (c) 258 (d) 260

142. A boy saves ₹4.65 daily. What is the least number of days in which he will be able to save an exact number of rupees ? **[2016-II]**
(a) 10 (b) 20 (c) 21 (d) 25

143. What is the unit digit of 7^{139} ? **[2016-II]**
(a) 9 (b) 7 (c) 6 (d) 3

144. What is the remainder when the number $(4444)^{4444}$ is divided by 9 ? **[2017-I]**
(a) 4 (b) 6 (c) 7 (d) 8

145. The number of prime numbers which are less than 100 is **[2017-I]**
(a) 24 (b) 25 (c) 26 (d) 27

146. Consider the following statements : **[2017-I]**
1. Of two consecutive integers, one is. even.
2. Square of an odd integer is of the form 8n + 1.

Which of the above statements is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

147. $(N^{P-1} - 1)$ is a multiple of p, if N is prime to p and p is a **[2017-I]**
(a) Prime number (b) Rational number
(c) Real number (d) Composite number

148. Three numbers which are co-prime to each other, are such that the product of the first two is 286 and that of the last two is 770. What is the sum of the three numbers? **[2018-1]**
(a) 85 (b) 80 (c) 75 (d) 70

149. All odd prime numbers upto 110 are multiplied together. What is the unit digit in this product. **[2018-1]**
(a) 0 (b) 3
(c) 5 (d) None of the above

150. The highest four-digit number which is divisible by each of the numbers 16, 36, 45, 48 is **[2018-II]**
(a) 9180 (b) 9360 (c) 9630 (d) 9840

151. How many five-digit numbers of the form XXYXX is/are divisible by 33? **[2018-II]**
(a) 1 (b) 3 (c) 5 (d) Infinite

152. A five-digit number XY235 is divisible by 3 where X and Y are digits satisfying X + Y ≤ 5. What is the number of possible pairs of values of (X, Y)? **[2018-II]**
(a) 5 (b) 6 (c) 7 (d) 9

153. The number of divisors of the number 38808, exclusive of the divisors 1 and itself is **[2018-II]**
(a) 74 (b) 72 (c) 70 (d) 68

154. Consider the following statements in respect of three 3-digit numbers XYZ, YZX and ZXY : **[2018-II]**
1. The sum of the numbers is not divisible by (X + Y + Z).
2. The sum of the numbers is divisible by 111.

Which of the above statements is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

155. The sum of all possible products taken two at a time out of the numbers ±1, ±2, ±3, ±4 is **[2018-II]**
(a) 0 (b) –30 (c) 30 (d) 55

156. What is the remainder when $(17^{29} + 19^{29})$ is divided by 18? **[2019-I]**
(a) 6 (b) 2 (c) 1 (d) 0

157. What is the largest value of n such that 10^n divides the product
$$2^5 \times 3^3 \times 4^8 \times 5^3 \times 6^7 \times 7^6 \times 8^{12} \times 9^9 \times 10^6 \times 15^{12} \times 20^{14} \times 22^{11} \times 25^{15} ?$$ **[2019-I]**
(a) 65 (b) 55 (c) 50 (d) 45

158. How many pairs (A, B) are possible in the number 479865AB if the number is divisible by 9 and it is given that the last digit of the number is odd ? **[2019-I]**
(a) 5 (b) 6 (c) 9 (d) 11

159. Consider the multiplication $999 \times abc = def132$ in decimal notation, where a, b, c, d, e and f are digits. What are the values of a, b, c, d, e and f respectively ? **[2019-I]**
(a) 6, 6, 8, 6, 8, 7 (b) 8, 6, 8, 6, 7, 8
(c) 6, 8, 8, 7, 8, 6 (d) 8, 6, 8, 8, 6, 7

160. Which of the following statements is ***not*** true? **[2019-I]**
(a) The difference of two prime numbers, both greater than 2, is divisible by 2.
(b) For two different integers m, n and a prime number p, if p divides the product $m \times n$, then p divides either m or n.
(c) If a number is of the form $6n - 1$ (n being a natural number), then it is a prime number.
(d) There is only one set of three prime numbers such that there is a gap of 2 between two adjacent prime numbers.

161. The sum of three prime numbers is 100. If one of them exceeds another by 36, then one of the numbers is **[2019-I]**
(a) 17 (b) 29
(c) 43 (d) None of the above

162. What is the unit place digit in the expansion of 7^{73} ? **[2019-I]**
(a) 1 (b) 3 (c) 7 (d) 9

163. Consider the following statements in respect of two integers p and q (both > 1) which are relatively prime : **[2019-I]**
1. Both p and q may be prime numbers.
2. Both p and q may be composite numbers.
3. One of p and q may be prime and the other composite.

Which of the above statements are correct ?
(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

164. The inequality $3^N > N^3$ holds when **[2019-I]**
(a) N is any natural number
(b) N is a natural number greater than 2
(c) N is a natural number greater than 3
(d) N is a natural number except 3

165. Which one of the following is an irrational number ? **[2019-I]**
(a) $\sqrt{59049}$ (b) $\dfrac{231}{593}$
(c) 0.45454545... (d) 0.1211221112222111122222...

166. The number 3^{521} is divided by 8. What is the remainder ? **[2019-I]**
(a) 1 (b) 3 (c) 7 (d) 9

167. A prime number contains the digit X at unit's place. How many such digits of X are possible ? **[2019-I]**
(a) 3 (b) 4 (c) 5 (d) 6

168. if 10^n divides $6^{23} \times 75^9 \times 105^2$, then what is the largest value of n ? **[2019-II]**
(a) 20 (b) 22 (c) 23 (d) 28

169. What is the digit in the unit's place of the number represented by $3^{98} - 3^{89}$? **[2019-II]**
(a) 3 (b) 6 (c) 7 (d) 9

170. The sum of the squares of four consecutive natural numbers is 294. What is the sum of the numbers ? **[2019-II]**
(a) 38 (b) 34 (c) 30 (d) 26

171. How many pairs of natural numbers are there such that the difference of their squares is 35? **[2019-II]**
(a) 1 (b) 2 (c) 3 (d) 4

172. Which one of the following is ***not*** correct? **[2019-II]**
(a) 1 is neither prime nor composite.
(b) 0 is neither positive nor negative.
(c) If $p \times q$ is even, then p and q are always even
(d) $\sqrt{2}$ is an irrational number

173. Consider the following statements: **[2019-II]**
1. If p is relatively prime to each of q and r, then p is relatively prime to the product qr.
2. If p divides the product qr and if p divides q, then p must divide r.

Which of the above statements is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

174. A person carries ₹500 and wants to buy apples and oranges out of it. If the cost of one apple is ₹5 and the cost of one orange is ₹7, then what is the number of ways in which a person can buy both apples and oranges using total amount? **[2019-II]**
(a) 10 (b) 14 (c) 15 (d) 17

175. Let x be the smallest positive integer such that when 14 divides x, the remainder is 7; and when 15 divides x, the remainder is 5. Which one of the following is correct ? **[2019-II]**
(a) $20 < x < 30$ (b) $30 < x < 40$
(c) $40 < x < 50$ (d) $x > 50$

176. Consider the following statements : **[2019-II]**
1. $\sqrt{75}$ is a rational number.
2. There exists at least a positive integer x such that $-\dfrac{4x}{5} < -\dfrac{7}{8}$.
3. $\dfrac{x-2}{x} < 1$ for all real values of x.
4. 4. 232323..... can be expressed in the form $\dfrac{p}{q}$ where p and q are integers.

Which of the above statements are correct ?
(a) 1 and 2 (b) 2 and 3 (c) 3 and 4 (d) 2 and 4

177. Consider the following statements : **[2019-II]**
1. Unit digit in 17^{174} is 7.
2. Difference of the squares of any two odd numbers is always divisible by 8.

3. Adding 1 to the product of two consecutive odd numbers makes it a perfect square.

Which of the above statements arc correct ?

(a) 1, 2 and 3 (b) 1 and 2 only
(c) 2 and 3 only (d) 1 and 3 only

178. When N is divided by 17, the quotient is equal to 182. The difference between the quotient and the remainder is 175. What is the value of N ? **[2019-II]**

(a) 2975 (b) 3094 (c) 3101 (d) 3269

179. The number $2 \times 3 \times 5 \times 7 \times 11 + 1$ **[2020-I]**

(a) a prime number
(b) not a prime, but power of a prime
(c) not a power of a prime, but a composite even number
(d) not a power of a prime, but a composite odd number

180. If the sum of the digits of a number $10^n - 1$, where n is a natural number, is equal to 3798, then what is the value of n? **[2020-I]**

(a) 421 (b) 422 (c) 423 (d) 424

181. If m is the number of prime numbers between 0 and 50; and n is the number of prime numbers between 50 and 100, then what is $(m - n)$ equal to ? **[2020-I]**

(a) 4 (b) 5 (c) 6 (d) 7

182. Which one of the following is the largest divisor of $3^x + 3^{x+1} + 3^{x+2}$, if x is any natural number? **[2020-I]**

(a) 3 (b) 13 (c) 39 (d) 117

183. A two-digit number is 9 more than four times of the number obtained by interchanging its digits. If the product of digits in the two-digit number is 8, then what is the number? **[2020-I]**

(a) 81 (b) 42 (c) 24 (d) 18

184. A number divides 12288, 28200 and 44333 so as to leave the same remainder in each case. What is that number ? **[2020-I]**

(a) 272 (b) 232 (c) 221 (d) 120

185. If the points P and Q represent real numbers $0.7\overline{3}$ and $0.5\overline{6}$ on the number line, then what is the distance between P and Q ? **[2020-I]**

(a) $\frac{1}{6}$ (b) $\frac{1}{5}$ (c) $\frac{16}{45}$ (d) $\frac{11}{90}$

186. The number of three digit numbers (all digits are different) which are divisible by 7 and also divisible by 7 on reversing the order of the digits, is **[2020-I]**

(a) Six (b) Five (c) Four (d) Three

187. Let XYZ be a 3-digit number. Let $S = XYZ + YZX + ZXY$. Which of the following statements is/are correct ? **[2020-I]**

1. S is always divisible by 3 and $(X + Y + Z)$
2. S is always divisible by 9
3. S is always divisible by 37

Select the correct answer using the code given below:

(a) 1 only (b) 2 only (c) 1 and 2 (d) 1 and 3

188. For two observations, the sum is S and product is P. What is the harmonic mean of these two observations ? **[2020-I]**

(a) $\frac{2S}{P}$ (b) $\frac{S}{(2P)}$ (c) $\frac{2P}{S}$ (d) $\frac{P}{(2S)}$

HINTS & SOLUTIONS

1. (b) Difference = Sum of digit at odd place – Sum of digit at even place
$= (1 + 5 + 9 + 4) - (2 + 4 + 3) = 19 - 9 = 10$
In 10, we must add at least 1 so that it is divisible by 11.
So $x = 1$
Also, the sum of digits of 1254934
$= 1 + 2 + 5 + 4 + 9 + 3 + 4 = 28$
1254934 will be divisible by 3, after adding y, if the value of y is – 1.
So, $x = 1$ and $y = -1$ is the set of values for x and y.

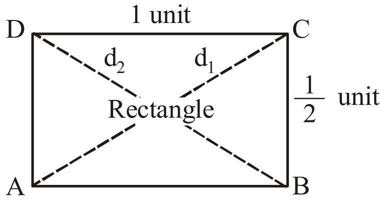
2. (d) **For divisible by 5:**
A number is divisible by 5 then its unit place must be 0 or 5.
For divisible by 4:
The last two digit of a number is divisible by 4, then the number is divisible by 4.
Possible digit at ten's place = 0, 2, 4, 6, 8.

3. (c) Sum of all the digits in the number (26492518)
$= 2 + 6 + 4 + 9 + 2 + 5 + 1 + 8 = 37$
When we subtract 4 from 37 then number must be divisible by 3.

4. (a) Here $(24)^2 < 589$
So, prime numbers less than 24 are 2, 3, 5, 7, 11, 13, 17, 19, 23.
Since, 589 is divisible by 19, then 589 is a composite number.

5. (d) A number of the form $a^n + b^n$ then it is divisible by $(a + b)$, if only n is odd number. So that $a^n + b^n$ is not divisible by integral value of n.

6. (b)



Area $= x = 1 \times \frac{1}{2} = \frac{1}{2}$ = Rational

$$d_1 = d_2 = \sqrt{1^2 + (1/2)^2} = \frac{\sqrt{5}}{2}$$

$$\therefore \quad y = d_1 + d_2 = \frac{\sqrt{5}}{2} + \frac{\sqrt{5}}{2}$$

$$= \frac{2\sqrt{5}}{2} = \sqrt{5} = \text{Irrational}$$

7. (c) Dividend = Quotient × Divisor + Remainder
According to the question
$D = 5Q$ and $D = 2R$
But $R = 15, D = 30$,
$5Q = 30, Q = 6$.
Dividend $= DQ + R = 30 \times 6 + 15 = 195$

8. (d) Given number is 222222.
Sum of digits = 2 + 2 + 2 + 2 + 2 + 2 = 12 which is divisible by 3.
So, number is also divisible by 3.
Sum of odd terms of digits – Sum of even terms of digits = 6 – 6 = 0, it is divisible by 11.
In a number a digit repeated six times, then this number is divisible by 7, 11 and 13.
Hence, the given number is divisible by 3, 7 and 11

9. (b) Let two rational numbers be $\sqrt{3}$ and $\sqrt{2}$.
Then, $(\sqrt{3} + \sqrt{2})$ is an irrational number.
Let two rational numbers be $3 - \sqrt{2}, \sqrt{2}$.
$\therefore (3 - \sqrt{2}) + \sqrt{2} = 3$ (rational)
Therefore, sum of two irrational numbers may be rational or irrational.

10. (d) By considering every option
(a) $3q + 9 = 18 \Rightarrow q = 3$
Then, numbers are 3, 5 and 9, which are not all prime.
(b) $3q + 9 = 42 \Rightarrow q = 11$
Then, numbers are 11, 13, 17, which are all prime.
(c) $3q + 9 = 60 \Rightarrow q = 17$
Then, numbers are 17, 19 and 23 which are all prime.
Hence, option (d) is correct.

11. (a) Given, $p = 2 \cdot 3 \cdot 5 \ldots 59 \cdot 61 = \ldots 0$
Also, $2 \le n \le 59$
Now, we check the sequence $p + n$
Since, unit digit of p is zero. Therefore, for every even value of n, $(p + n)$ is always divisible.
For odd value of $n = 2, 5, \ldots 59$
Take $n = 3$
$\therefore \quad p + n = p + 3 = (2 \cdot 3 \cdot 5 \ldots 59 \cdot 61 + 3)$
$= 3(2 \cdot 5 \ldots 59 \cdot 61 + 1)$ which is divisible.
Similarly, for even value of n, $p + n$ is divisible.
Hence, it is clear always $p + n$ is that divisible by any number. So, there is no prime number exist in this sequence.

12. (a) $\because$ Original number $= h \times 100 + t \times 10 + u$
Number obtained by reversing digits
$= u \times 100 + t \times 10 + h$
$\therefore$ Required number $= (h \times 100 + t \times 10 + u)$
$-(u \times 100 + t \times 10 + h) = 99\,(h - u)$
But the unit's place digit in above number is 4, therefore $(h - u)$ should be 6, then number is 594. Whose digits are 5, 9, 4 respectively.

13. (c) Zero is a whole number and every integer is not a whole number, because negative integers are not whole numbers.
Therefore, A is correct and R is wrong.

14. (a) Let the divisor be x.
So, Divided = Quotient × Divisor + Remainder
Dividend = 4996
Quotient = 62 and remainder = 36
$\therefore \quad 4996 = 62 \times x + 36$
$\Rightarrow \quad 62x = 4996 - 36$
$\Rightarrow \quad x = \frac{4990}{62} = 80$

15. (d) 667, 861, 481 is divided by 23, 21, 13 respectively.
But 331 is not divisible by any other numbers so 331 is a prime number.

16. (c) Number $= 987 \times k + 59$
Now, 987 is completely divided by 21.

$$21\overline{)59}\,(2$$
$$\underline{42}$$
$$17$$

Now, 17 is remainder.

17. (d) It is always correct that the product of two square numbers is a square number.
e.g., $4 \times 9 = 36$

18. (c) The set of integers is closed with respect of addition and multiplication.
e.g., Let Z = {..., – 3, – 2, – 1, 0, 1, 2, 3, ...}
$1 + 2 = 3$ and $2 + 1 = 3$ (for addition)
$1 \times 2 = 2$ and $2 \times 1 = 2$ (for multiplication)

19. (d) $N = 8\,7\,9\,8\,5\,4\,6 * 5$
Sum of odd digit places = 8 + 9 + 5 + 6 + 5 = 33
Sum of even digit places = 7 + 8 + 4 + * = 19 + *
Now, 33 – (19 + *) = 14 – *, it is divisible by 11, if value of * is 3.

20. (d) The prime numbers between 100 and 120 are 101, 103, 107, 109 and 113.
$\therefore$ Required sum = 101 + 103 + 107 + 109 + 113 = 533

21. (a) Required numbers are of the form of $12q - 2$
i.e., 10, 22, 34, 46, 58, 70, 82, 94
$\therefore$ Total sum = 10 + 22 + 34 + 46 + 58 + 70 + 82 + 94 = 416

22. (c) The total number of thee digit numbers with unit digit 7 and divisible by 11 are 187, 297, 407, 517, 627, 737, 847, 957.
$\therefore$ Total numbers = 8

23. (c) When we divide a positive integer by another positive integer, the resultant will be a rational number i.e., in the form of p/q, where p and q are positive integers and $q \neq 0$

24. (c) Let the first odd number be x and the alternate odd number is $x+4$
According to the question
$x(x+4) = 3x+12$
$\Rightarrow \; x^2+4x = 3x+12 \quad \Rightarrow \; x^2+x-12=0$
$\Rightarrow \; (x+4)(x-3)=0 \quad \Rightarrow \; x=3 \quad (\because x \neq -4)$
Hence, the larger number is $x+4 = 3+4 = 7$

25. (a) As we know that a number $a_1\,a_2\,a_3\,a_4\,a_5$ is divisible by 9, if sum of the digits, *i.e.*, $a_1+a_2+a_3+a_4+a_5$ is divisible by 9. Hence, only statement I is true.

26. (d) $(2457)^{754}$
We know that a unit digit of $(7)^4$ is 1.
Unit digit of $(2457)^{754} = (2457)^{188\times 4+2}$
$= (2457)^{188\times 4} . (2457)^2 \to [(2457)^4]^{188} . 9 \to (1)^{188} .9 \to 9$

27. (b) When a number n is divided by 4 then remainder is 3. Now, the number is double then remainder is also double.
So, remainder = 6
But remainder never greater than its divisor.
So, remainder = 6 – 4 = 2

28. (b) Let the GP series be $a, ar, ar^2, ar^3, ar^4, \ldots$
By given condition,
$T_3 = T_1 + 9$
$\Rightarrow \; ar^2 = a+9$
$\Rightarrow \; a(r^2-1) = 9 \quad \ldots(i)$
and $T_2 = T_4 + 18$
$\Rightarrow \; ar = ar^3 + 18$
$\Rightarrow \; ar(1-r^2) = 18 \quad \ldots(ii)$
On dividing Eq. (*i*) by Eq. (*ii*), we get
$$\therefore \; \frac{a(r^2-1)}{-ar(r^2-1)} = \frac{9}{18}$$
$\Rightarrow \; -r = 2$
From Eq. (*i*)
$a(4-1) = 9$
$\therefore \; a = 3$

29. (a) Let angles of a triangle in AP are $a, a+d, a+2d$.
Also, $a+2d = 2a$ (given condition)
$\Rightarrow a = 2d \quad \ldots(i)$
Also, $a+a+d+a+2d = 180^\circ$
($\because$ sum of angles of triangle = 180º)
$\Rightarrow 3a+3d = 180^\circ \; \Rightarrow 3a + 3\left(\frac{a}{2}\right)$
$= 180^\circ$ [from Eq. (*i*)]
$\Rightarrow 9a = 360^\circ$
$\Rightarrow a = 40^\circ, d = 20^\circ$
$\therefore$ Ratio of angles $= 40^\circ : 60^\circ : 80^\circ = 2:3:4$

30. (d) Let p be any positive number of the form $2m$, $2m+1$ for any whole number m.
Case I: $p = 2m$
$p^2 = 4m^2 = 4(m^2) = 4q$ where $q = m^2$
Case II: $p = 2m+1$
$p^2 = (2m+1)^2 = 4m^2+4m+1$
$= 4(m^2+m)+1$
$= 4q+1$ where $q = m^2+m$
From above we see that square of any positive integer is either of the form $4q$ or $(4q+1)$ for some integer q.

31. (b) **84705⊗2**
If sum of digit of a number is divisible by 9, then it is also devisible by 9.
$\Rightarrow \; 84705\otimes 2 = 8+4+7+0+5+\otimes+2 = 26+\otimes$
Now, we replace ⊗ by 1 then it become 27 and divided by 9.

32. (c) When a polynomial is divided by a linear polynomial, then the remainder is either constant or zero polynomial.

$$\text{e.g.,}\quad (ax+b)\overline{\big)\,ax^2+bx+c\,}^{\;x}$$
$$\underline{\;\overline{+}\,ax^2 \pm bx\;}$$
$$c = \text{constant}$$

OR

$$(ax+b)\overline{\big)\,ax^2+bx\,}^{\;x}$$
$$\underline{\;\overline{+}\,ax^2 \pm bx\;}$$
$$o = \text{zero}$$

33. (d) Let $a = 7p+5$
and $b = 7q+4$
where, p and q are natural numbers.
$\therefore \; ab = (7p+5)(7q+4)$
$ab = 49pq + (4p+5q)7 + 20$
$= 7(7pq+4p+5q) + 7\times 2 + 6$
when ab is divided by 7, we get the remainder 6.

34. (d) $P = rs = r(1-r) = r - r^2 \quad (\because r+s=1)$
$$\Rightarrow \; \frac{dP}{dr} = 1-2r = 0 \quad \text{(For max. and min.)}$$
$$\Rightarrow \; r = \frac{1}{2} \Rightarrow \frac{d^2P}{dr^2} = -2$$
= Negative ⇒ Maximum
$$\therefore \; \text{Maximum value of } rs = \frac{1}{4}$$

35. (b) Any number is not a square, if the unit's place digit of number may be 2, 3, 7, 8.
Hence, the number 9852 is not a square of any natural number.

36. (d) On taking option (d).
The reverse digit of 451 is 154.
Now, $154 + 297 = 451$ is equal to the original number.

37. (c) We know that, if the difference of the sum of odd digits and sum of even digits is either 0 or mlultiple of 11, then the number is divisible by 11.
Given number is $ABCDE$.
Here, $A + C + E - (B + D) = 0$ or divisible by 11
Hence, both statements are true.

38. (d) $2^{12n} - 6^{4n} = (2^{12})^n - (6^4)^n = (4096)^n - (1296)^n$
$= (4096 - 1296)\,[(4096)^{n-1} + (4096)^{n-2}(1296) + \ldots + (1296)^{n-1}] = 2800(k)$
Hence, last two digits are always be zero.

39. (c) If $x = 2$, then $x + 1$ and $x + 3$ are all prime numbers.

40. (d) Last digit of 3^{4798}
We know that 3 is cyclic of its unit digit at 4 times.
So, we devide 4798 by 4
$= 3^{4798} = 3^{1199 \times 4 + 2} = (3^{1199 \times 4}) \cdot 3^2 = 9$

41. (b) If k is any even positive integer, then $(k^2 + 2k)$ is divisible by 8 but may not be divisible by 24.
Let $k = 2m$, $m \in N$, then
$k^2 + k.2 = 4m^2 + 4m = 4m\,(m + 1)$
which is divisible by 4.

42. (c) $3^{2n+1} + 2^{2n+1} = 3 \times (3)^{2n} + 2 \times (2)^{2n}$
$= 3 \times (9)^n + 2 \times (4)^n$

n	Unit digit of $(9)^n$	Unit digit of $(4)^n$
1	9	4
2	1	6
3	9	4
4	1	6
5	9	4
⋮	⋮	⋮

Thus, when n is odd, then unit digit of $(9)^n = 9$ and $(4)^n = 4$
and when n is even, then unit digit of $(9)^n = 1$ and $(4)^n = 6$
Hence, when n is odd positive integer, then
$3 \times$ (unit digit of 9) $+ 2 \times$ (unit digit of 4)
$= 3 \times 9 + 2 \times 4 = 35$
Hence, unit digit of $(3)^{2n+1} + (2)^{2n+1} = 5$
Also, when n is even positive integer, then
$3 \times$ (unit digit of 9) $+ 2 \times$ (unit digit of 4)
$= 3 \times 1 + 2 \times 6 = 15$
Hence, unit digit of $(3)^{2n+1} + (2)^{2n+1} = 5$

43. (a) $\because\ T_{14} = 6$
$\Rightarrow\ a + 13d = 6$
and $T_6 = 14 \Rightarrow a + 5d = 14$
On solving equations (i) and (ii), we get
$a = 19, d = -1$
$\therefore\ T_{95} = a + 94d = 19 - 94 = -75$

44. (b) We know that the product of a rational number and an irrational number is an irrational number.

45. (d) $d\,[d\,\{d\,(12)\}] = d\,[d\,(6)]$
($\because$ positive integer divisor of 12 = 1, 2, 3, 4, 6, 12)
$= d\,(4)$ ($\because$ positive integer divisor of 6 = 1, 2, 3, 6)
$= 3$ ($\because$ positive integer divisor of 4 = 1, 2, 4)

46. (c) Factors of $88 = 11 \times 8$
2784936
= (Sum of odd place digit) – (Sum of even place digit)
$= 25 - 14 = 11$ (This is divisible by 11)
2784936:
Last three digit is 936 is divisible by 8.
So this number is also divisible by 8.
Now, 2784936 is divisible by 88.

47. (d) Here 247 is divisible by 13 and 203 is also divisible by 7. So, 247 and 203 are not prime number.

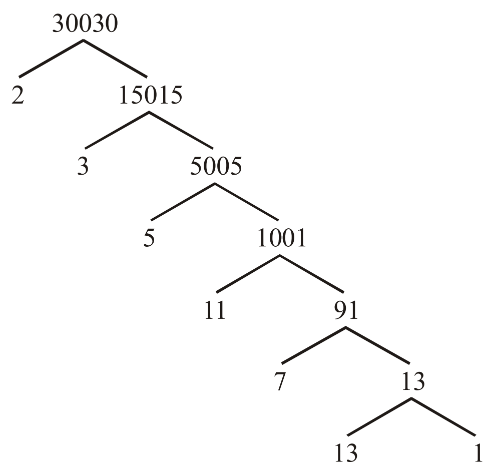
48. (a) Difference between the digit $= 9238 - 7091$
$= 2147$
2147 is completely divided by 113.

49. (a) Let $n = 5q + 2$
$3n = 3(5q + 2)$
$\Rightarrow\ 3n = 15q + 6 = 5\,(3q + 1) + 1$
When $3n$ is divided by 5, then remainder is 1.

50. (c) On taking $p = 11$,
$p + 2 = 13$ (prime number)
I. $11 \times 13 + 1 = 144$ (a square number)
II. $11 + 13 = 24$ (12 is a divisor of 24)
Hence, both statements I and II are correct.

51. (b) Let the lowest sides of a right triangle be 3, 4, 5.
By Pythagoras theorem, $(3)^2 + (4)^2 = (5)^2$
Hence, one of its sides is always divisible by 5.

52. (c)



So, prime factors of 30030 are
2, 3, 5, 11, 7, and 13
So, number of prime factors of 30030 is 6.

53. (c) Since, D is a point of BC. As BC is rational so BD must be rational but AD need not to be rational.

54. (d) If n is a natural number, then $\sqrt{n}$ is either a natural number or an irrational number.
e.g., when $n = 3 \Rightarrow \sqrt{3}$ = Irrational number
When $n = 9 \Rightarrow \sqrt{9} = 3$ = Rational number

55. (d) The largest integer that divides product of any four consecutive integers is 4! i.e., 24.
e.g., 1, 2, 3, 4 are four consecutive integers.
Multiplication = $1 \times 2 \times 3 \times 4 = 24$ which is divided by 24.

56. (c) If a number is divisible by 88, it should be divisible by 8 and 11.
In a given option, number 9944 and 8888 is divisible by 88.
Hence, largest number is 9944.

57. (c) 161, 171, 221, are divisible by 7, 3 and 13 respectively. But 173 is not divisible by any others numbers except 1 and 173.

58. (c) I. The product of any three consecutive integers is divisible by 3! i.e., 6.

II. Here, $3k = \{..., -6, -3, 0, 3, 6, ...\}$

$3k + 1 = \{..., -5, -2, 1, 4, 7, ...\}$

and $3k + 2 = \{..., -4, -1, 2, 5, 8, ...\}$

$\therefore \{3k, 3k+1, 3k+2\}$

$= \{..., -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, ...\}$

Hence, it is true.

59. (a) **I.** **Composite Number:** Natural numbers, which has more than 2 distinct factors are called composite number. It postulates that every composite number is a natural number.
II. Every whole number is not a natural number (zero is not a natural number).
So, only statement I is true.

60. (b) Quotient = $143K + 28$
Now, when same number is divided by 13. So, we check 143 is divided by 13 or not.
Here, 143 is completely divided by 13. Hence we divides 28 by 13 then remainder is 2.

61. (c) Following are the numbers between – 11 and 11 which are multiples of 2 or 3?
– 10, – 9, – 8, – 6, – 4, – 3, – 2, 0, 2, 3, 4, 6, 8, 9,10
So ,the numbers of multiples 2 or 3, between – 11 and 11 are 15.
Alternative Method:
Numbers between 0 and 11 which are multiples of 2 or 3

$$\frac{11}{2} + \frac{11}{3} - \frac{11}{6} = 5 + 3 - 1 = 7$$

Numbers between 0 and – 11

$$\frac{11}{2} + \frac{11}{3} - \frac{11}{6} = 7$$

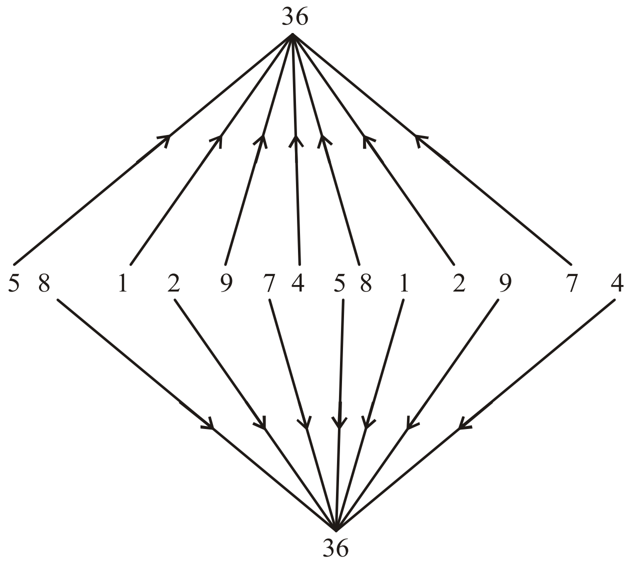
So, the numbers are 14 and 0.

62. (c) $\Rightarrow$ $231228 = 7006 \times 33 + 30$ ... (1)
Now, when the number divides by 33 its remainder is 30. Therefore, 3 must be added to 231228 to make it exactly divisible by 33.

63. (d) There are an infinite of rational numbers between any two given numbers.

64. (c) $92 = 2^2 \times 23$
$85 = 5 \times 17$
Since there is no common factor in 92 and 85, therefore they are relatively prime.

65. (a) **Divisibility Rule 11:** The difference between the sum of digit at odd place numbers and the sum of digit at even, place numbers is zero or multiple of 11.



Difference = 36 – 36 = 0, then it is divisible by 11.

66. (c) I. Given, n is a prime number greater than 5.
Now, $n^4 - 1 = (n^2 - 1)(n^2 + 1)$
$= (n - 1)(n + 1)(n^2 + 1)$
Put $n = 7$ (prime number greater than 5)
$n^4 - 1 = (7 - 1)(7 + 1)(49 + 1)$
$= 6 \cdot 8 \cdot 50 = 2400$
So, statements I is true.

II. Given, $n \in W$ (whole number)
i.e., $n = 0, 1, 2, 3, 4, 5, ...$
For $n = 0$,
$5n, (5n - 1), (5n + 1) = 0, -1, 1 = (0)^2, -1, (1)^2$
For $n = 1$,
$5n, (5n - 1), (5n + 1) = 5, 4, 6 = 5, (2)^2, 6$
For $n = 2$,
$5n, (5n - 1), (5n + 1) = 10, 9, 11$
$= 10, (3)^2, 11$
For $n = 3$,
$5n, (5n - 1), (5n + 1) = 15, 14, 16 = 15, 14, (4)^2$
For $n = 4$,
$5n, (5n - 1), (5n + 1) = 20, 19, 21$
For $n = 5$,
$5n, (5n - 1), (5n + 1) = 25, 24, 26 = (5)^2, 24, 26$ ...
so on.
So, statements II is true.

67. (a) Let the integer be x.

Integer + Its reciprocal $= \frac{10}{3}$

$\Rightarrow \quad x + \frac{1}{x} = \frac{10}{3}$

$\Rightarrow \quad \frac{x^2+1}{x} = \frac{10}{3}$

$\Rightarrow \quad 3x^2 + 3 = 10x$

$\Rightarrow \quad 3x^2 - 10x + 3 = 0$

$\Rightarrow \quad 3x^2 - 9x - x + 3 = 0$ (by splitting middle term)

$\Rightarrow \quad 3x(x-3) - 1(x-3) = 0$

$\Rightarrow \quad (x-3)(3x-1) = 0$

$\therefore \quad x = 3$ (since 1/3 is not an integer)

Alternative Method:

By Hook or by Crook.

From option (a),

$3 + \frac{1}{3} = \frac{9+1}{3} = \frac{10}{3}$

68. (a) **Prime number:** A number that is divisible by itself and one excluding one.

So, 1 is not prime number.

Composite Number: Natural numbers, which has more than 2 distinct factors are called composite numbers. 1 is not composite numbers.

69. (b) Here, $88 = 2 \times 2 \times 2 \times 11 = (2)^3 \times (11)^1$

$91 = (7)^1 \times (13)^1$

$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 = (2)^5 \times (3)^1$

and $99 = 3 \times 3 \times 11 = (3)^2 \times (11)^1$

So, 91 has least number of divisors.

70. (c) I. It is true that every integer is a rational number.

e.g., Set of integers,

$Z = \{..., \frac{-3}{1}, \frac{-2}{1}, \frac{-1}{1}, \frac{0}{1}, \frac{1}{1}, \frac{2}{1}, \frac{3}{1}, ...\}$

which of the form $\frac{p}{q}$, which also shown rational numbers.

II. It is also true that every rational number is a real number because a real number does not contain i (iota).

So, both satements are true.

71. (d) Let the number are $a_1 = 10, a_2 = 20, a_3 = 25, a_4 = 40$ and $a_5 = 50$.

$\therefore$ Harmonic mean $= \dfrac{\text{Number of observations}}{\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \frac{1}{a_4} + \frac{1}{a_5}}$

$= \dfrac{5}{\frac{1}{10} + \frac{1}{20} + \frac{1}{25} + \frac{1}{40} + \frac{1}{50}}$

$= \dfrac{5}{\frac{20+10+8+5+4}{200}}$

$= \frac{5 \times 200}{47} = \frac{1000}{47} = 21.27 \approx 21.3$

72. (c) From options,

(a) $\frac{14}{(1+4)} = \frac{14}{5} = 4$ (Rem) and

$\frac{14}{1 \times 4} = \frac{14}{4} = 2$ (Rem)

Since, remainder is not same.

(b) $\frac{23}{(2+3)} = \frac{23}{5} = 3$ (Rem) and

$\frac{23}{2 \times 3} = \frac{23}{6} = 5$ (Rem)

Since, remainder is not same.

(c) $\frac{32}{(3+2)} = \frac{32}{5} = 2$ (Rem) and

$\frac{32}{(3 \times 2)} = \frac{32}{6} = 2$ (Rem)

Since, remainder is same.

$\therefore$ Difference of quotients $= 6 - 5 = 1$

(d) $\frac{41}{(4+1)} = \frac{41}{5} = 1$ (Rem) and

$\frac{41}{(4 \times 1)} = \frac{41}{4} = 1$ (Rem)

Since, remainder is same.

But difference of quotients $= 10 - 8 = 2 \neq 1$

73. (d) We know that,

If the difference between the sum of digits at even places and sum of digits at odd places is (0) or multiple of 11, then the number is divisible by 11.

From options,

(a) 45678940

Sum of even places $= 5 + 7 + 9 + 0 = 21$

Sum of odd palaces $= 4 + 6 + 8 + 4 = 22$

Their difference $= 22 - 21 = 1 \neq 0$

(b) 54857266

Sum of even places $= 4 + 5 + 2 + 6 = 17$

Sum of odd palaces $= 5 + 8 + 7 + 6 = 26$

Their difference $= 26 - 17 = 9 \neq 0$

(c) 87524398

Sum of even places $= 7 + 2 + 3 + 8 = 20$

Sum of odd palaces $= 8 + 5 + 4 + 9 = 26$

Their difference $= 26 - 20 = 6 \neq 0$

(d) 93455120

Sum of even places $= 3 + 5 + 1 + 0 = 9$

Sum of odd palaces $= 9 + 4 + 5 + 2 = 20$

Difference $= 20 - 9 = 11$

It is divisible by 11.

74. (a) When N is a natural number, then there is only one possible case that N, $(N + 2)$, $(N + 4)$ are prime numbers when $N = 3$, then $N, (N + 2), (N + 4) = 3, 5, 7$ all are primes.

75. (b) On taking $p = 5$,
$2^p - 1 = 2^5 - 1 = 31$ which is prime
On taking $p = 11$,
$2^p - 1 = 2^{11} - 1 = 2047$
Since, 2047 is divisible by 23, so it is not prime.
Thus , the required least positive prime number is 11.

76. (b) We know that, between any two rational numbers, there are an infinite number of rational and irrational numbers.
Hence, statement II is correct.

77. (b) If m and n are natural numbers, then $\sqrt[m]{n}$ is irrational unless n is mth power of an integer.

78. (c) If x is a positive even integer and y is negative odd integer, then x^y is a rational number.

79. (d) $\frac{1}{4} = 0.25$ and $\frac{3}{4} = 0.75$
Only option (d) with $\frac{252}{1000} = 0.252$ and $\frac{748}{1000} = 0.748$ lies between 0.25 and 0.75.

80. (c) Here 7, and 3 both are repeat its unit digit after four times.
$= 7^{(4 \times 100 + 2)} + 3^{(4 \times 100 + 2)}$
$= 7^{4 \times 100} \times 7^2 + 3^{4 \times 100} \times 3^2$
$= 9 + 9$
$= 18$
So, its unit digit = 8

81. (d) Here we can check the unit digit
Unit digit of $26^2 + 97^2 = 6 + 9 \Rightarrow 5$
Now according to option
Unit digit of $79^2 + 62^2 = 1 + 4 \Rightarrow 5$
So, $26^2 + 97^2 = 79^2 + 62^2$

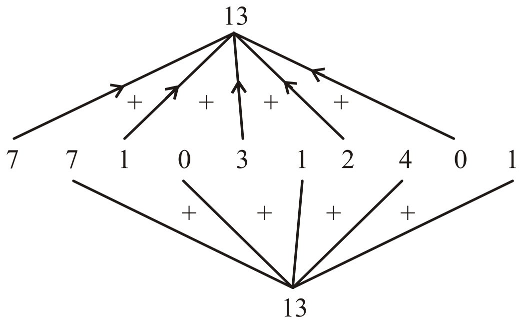
82. (b) Every prime number of the form $3k + 1$ can be represented in the form $6m + 1$ only, when k is even.

83. (d) For square integer 25, $4k + 1$ mean $4 \times 6 + 1$
and for 36
$4k$ mean 4×9
Now, if k is a positive integer, then every square integer is of the form $4k$ or $4k + 1$

84. (c) Here b is the largest square divisor of c and a^2 divides c then, it is sure that a divides b.

85. (c) We know that $a^n + b^n$ where n is odd numbers then it is divides by $a + b$.
So, $19^5 + 21^5 = 19 + 21 = 40$
Now, 40 is divided by both 10 and 20.
So that number is also divided by 10 and 20.

86. (c) **Statement I:**
Divisibility rule of 11: The difference between the sum of even places and the sum of odd places is 0 or that number is divisible by 11.



Difference = Sum of odd place – Sum of even place
= 13 – 13 = 0, it is divisible by 11.
Statement II:
173 is square of approximately 13.
So, below 13 prime number are -
2, 3, 5, 7, 11
Now, 173 is not divisible by 2, 3, 5, 7 and 11.
So, it is a prime number.
So, both the statements I and II are correct.

87. (c) Both the statements given are correct. As 121 is the square of 11. So, to obtain prime numbers less than 121, we reject all the multiples of prime numbers less than 11 *i.e.,* 2, 3, 5 and 7. Similarly, every composite number less than 121 is divisible by a prime number less than 11 *i.e.,* 2, 3, 5 or 7.

88. (a) I. $f(k) = 4k + 3$
For $k = 1, f(1) = 4 \times 1 + 3 = 7$
For $k = 2, f(2) = 4 \times 2 + 3 = 11$
For $k = 3, f(3) = 4 \times 3 + 3 = 15$
Values of $f(k)$ for $k = 1, 2, \ldots\ldots$ cannot be expressed as sum of squares, since
$1^2 + 2^2 = 5,\ 1^2 + 3^2 = 10,\ 2^2 + 3^2 = 13$
II. $f(k) = 8k + 1$
For $k = 1, f(1) = (8 \times 1) + 1 = 9 = (3)^2$
For $k = 2, f(2) = (8 \times 2) + 1 = 17$
For $k = 3, f(3) = (8 \times 3) + 1 = 25 = (5)^2$
For $k = 4, f(4) = (8 \times 4) + 1 = 33$
For $k = 5, f(5) = (8 \times 5) + 1 = 41$
$f(k) = 8k + 1$ is square of an odd integer only for some values of k. So, only Statement I is correct.

89. (a) Given that n is greater then 1, then $n^2(n^2 - 1)$ is always divisible by 12.
Example 1: Put $n = 2$, then
$n^2(n^2 - 1) = (2)^2 (2^2 - 1)$
$= 4 \times 3 = 12.$
Hence $n^2(n^2 - 1)$ is divisible by 12 but not by 24, 48 and 60 for n = 2.

90. (c) Remainder $4^{1000} = \frac{4^{(333 \times 3 + 1)}}{7}$

$$= \frac{(64)^{333} \times 4}{7}$$
$= 1 \times 4 = 4$

91. (d) $7^{10}-5^{10}$ is divisible by 11.

92. (a) Here, three–digits number = $XY5$

$$\begin{array}{r} XY5 \\ \times \quad Z \\ \hline X215 \end{array}$$

So, Z can take value 1, 3, 5, 7 and 9.
But only 9 satisfies it, then $X = 1$, $Y = 3$ and $Z = 9$

$$\because \begin{array}{r} 135 \\ \times 9 \\ \hline 1215 \end{array}$$

value of $X + Y + Z = 1 + 3 + 9 = 13$

93. (c) From option (c) By Hook and Crook $N = 6$
$N^2 - 33 = 6^2 - 33 = 36 - 33 = 3$, which is prime.
$N^2 - 31 = 6^2 - 31 = 36 - 31 = 5$, which is prime.
$N^2 - 29 = 6^2 - 29 = 36 - 29 = 7$, which is prime.
So, $N = 6$ only. possible value

94. (d) Given equation, $\frac{1}{m}+\frac{4}{n}=\frac{1}{12}$

$\Rightarrow \quad 12(n+4m) = mn$
$\Rightarrow \quad 12n + 48m = mn$
$\Rightarrow \quad m(48-n) = -12n$
$\Rightarrow \quad m(n-48) = 12n$

$$\therefore \quad m=\frac{12n}{n-48} \qquad ...(i)$$

Here, as m and n are positive integers, therefore > 48.
But n is an odd integer less than 60, therefore possible values of n = 49, 51, 53, 55, 57 and 59.
But on putting n = 53, 55, and
59 in Eq. (i), we get the non-integer values of m
On putting n = 49, 51 and 57 , we get the value of m = 588, 204 and 76, respectively.
Hence, there are three possible pairs of m and n that satisfy the equation.

95. (a) Let the two-digits numbers less than 50 which when divided by 4 yield unity as remainder be 13, 17, ..., 49. Here, first term, a = 13, common difference, $d = 4$ and $n = 10$.

$$\therefore \text{ Required sum } = \frac{n}{2}[2a+(n-1)d]$$

$$= \frac{10}{2}[2\times 13+(10-1)4]$$

$$= \frac{10}{2}[26+36] = \frac{10\times 62}{2} = 310$$

96. (d) If sum of all the digits is divisible by 9 then the number is divisible by 9
Given number is 76 3× 4Y2.
Given number is divisible by 9.
So, $7 + 6 + 3 + X + 4 + Y + 2 = 9k$
$\Rightarrow \quad 22 + X + Y = 9k$
It is clear that LHS is divisible by 9, if X + Y = 5, 14.
Now sum of X and Y is 5, then possible pairs are (1, 4), (4, 1), (2, 3) (3, 2), (0, 5) and (5, 0). When sum of X and Y is 11, then possible pairs are (5, 9), (9 , 5), (6, 8), (8, 6) and (7, 7).
Total possible pairs are 11.

97. (d) $= \frac{4^{1012}}{7} = \frac{4^{3\times 337+1}}{7} = \frac{4^{3\times 337}\times 4}{7}$

$$= \frac{(64)^{337}\times 4}{7} = \frac{(9\times 7+1)^{337}\times 4}{7}$$

We know that $= \frac{(ax+1)^n}{a}$ then its Remainder is 1
$= 1 \times 4 = 4$

98. (a) $\frac{17^{23}+23^{23}+29^{23}}{23}$

$$= \frac{17^{23}+(0)^{23}+(29)^{23}}{23}$$

$$= \frac{17^{23}+(29)^{23}}{23} = \frac{(17)^{23}+(6)^{23}}{23}$$

Here $\frac{a^n+b^n}{n}$
So remainder is always zero.

99. (b) The prime numbers less than 13 are 2, 3, 5, 7, 11.
Also, using the condition, $p < q < r < 13$ and $p + q + r$ is a prime number
Hence, only two possible pairs exist i.e. (3, 5, 11) and (5, 7, 11).

100. (c) $\because \quad 360 = 2^3 \times 3^2 \times 5$
$\therefore$ Number of divisors = (3 + 1) (2 + 1) (1 + 1)
$= 4 \times 3 \times 2 = 24$

101. (b) $\frac{37}{13} = 2+\cfrac{1}{x+\cfrac{1}{y+\cfrac{1}{z}}}$

$\Rightarrow \frac{37}{13}$ can be expressed as

$$= 2+\cfrac{1}{1+\cfrac{2}{11}} = 2+\cfrac{1}{\cfrac{1+1}{5+\cfrac{1}{2}}}$$

Now, this is compared by

$$2+\cfrac{1}{x+\cfrac{1}{y+\cfrac{1}{z}}} = 2+\cfrac{1}{1+\cfrac{1}{5+\cfrac{1}{2}}}$$

$\boxed{\therefore z = 2}$

102. (d) $\dfrac{5+\sqrt{10}}{5\sqrt{5}-2\sqrt{20}-\sqrt{32}+\sqrt{50}}$

$= \dfrac{5+\sqrt{10}}{5\sqrt{5}-4\sqrt{5}-4\sqrt{2}+5\sqrt{2}}$

$= \dfrac{5+\sqrt{10}}{\sqrt{5}+\sqrt{2}} = \dfrac{(5+\sqrt{10})\ (\sqrt{5}-\sqrt{2})}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})}$

$= \dfrac{5\sqrt{5}+\sqrt{50}-5\sqrt{2}-\sqrt{20}}{5-2}$

$= \dfrac{5\sqrt{5}+5\sqrt{2}-5\sqrt{2}-2\sqrt{5}}{3}$

$= \dfrac{3\sqrt{5}}{3} = \sqrt{5}$

103. (a) $81 \times 82 \times 83 \times 84 \times ... \times 99$
It can be written as
$= 81 \times 82 \times 83 \times 84 \times ... 90 \times 99$
When we multiply any number by multiple of 10, then resultant number always carry zero at unit place.

104. (a) Let *a* number be x
multiply by 25 we get another number y.
According to question
$x \times 25 = y$ ....(i)
Again $x \times 52 = 324 + y$ ....(ii)
Now subtract equation (i) from equation (ii)
$\Rightarrow x \times 52 - x \times 25 = 324 + y - y$
$\Rightarrow 27x = 324$

$x = \dfrac{324}{27} = 12$

105. (a) 357P25Q
If it is divided by 3, then sum of the digit must be divided by 3 and if it is divided by 5 then its unit digit must be 0 or 5.
$= 3 + 5 + 7 + P + 2 + 5 + Q$
1st Case Q = 0
$3 + 5 + 7 + P + 2 + 5 + 0 = 22 + P$
Possible values of P = 2, 5, 8
2nd Case when Q = 5
$3 + 5 + 7 + P + 2 + 5 + 5 = 27 + P$
Possible values of P = 0, 3, 6, 9
Possible pairs of (P,Q)
= (2, 0), (5, 0), (8, 0)
(0, 5), (3, 5) (6, 5) (9, 5)
Total no. of pairs = 7.

106. (c) Let two consecutive odd numbers = $(2x + 1)$ and $(2x + 3)$
According to question
$= (2x + 3)^2 - (2x + 1)^2$
$= 4x^2 + 12x + 9 - 4x^2 - 1 - 4x$
$= 8x + 8 = 8(x + 1)$
So, it is divisible by 8.

107. (c) $N = 35 \times 45 \times 55 \times 60 \times 124 \times 75$
$= \underbrace{7\times5}\times\underbrace{9\times5}\times\underbrace{11\times5}\times\underbrace{12\times5}\times124\times\underbrace{5\times5\times3}$
$= 5^6 \times 7 \times 9 \times 11 \times 12 \times 124 \times 3$
$m = 6$, The given number has maximum factor of 5 is 6.

108. (d) Let us assume that he was out of house for ‘t’ min.
So angle formed by min. hand = $6 \times t$
Angle formed by hour hand = $0.5 \times t$
Now, $0.5 \times t + 6 \times t = 360$
$\Rightarrow 6.5\,t = 360$

$t = \dfrac{360}{6.5} = 55.38$ min

109. (b) When a ball is dropped
from a height = 36 m

1st bounce back = $\dfrac{2}{3} \times 36 = 24$ m

2nd bounce back = $\dfrac{2}{3} \times 24 = 16$ m

3rd bounce back $\dfrac{2}{3} \times 16 = \dfrac{32}{3} = 10\dfrac{2}{3}$ m.

110. (d) 1st time seen
= 1 hour 54 min 50 sec
= 3600 sec + 54 × 60 sec + 50 sec
= 6890 sec
2nd time seen = 3 hour + 17 min + 49 sec
= 11869 sec.
Interval between light seen = 11869 – 6890
= 4979
Number of times light was seen

$= \dfrac{4979}{13} + 1 = 384$

111. (b) $n = P_1^{x_1}\ P_2^{x_2}\ P_3^{x_3}$
where P_1, P_2 and P_3 are distinct prime numbers
So no. of factors = $x_1 x_2 x_3$

112. (a) 11, 111, 1111, 11111, ...
1 Each number is divided by 4 then their remainder are always 3.
So, it can be written as = $4m + 3$
So statement is true.
2 No, this type of numbers are not squares. So it is not true.
Only 1 is true.

113. (c) **Statement 1**
Yes, there exists only one prime number p such that $(17p + 1)$ is a square.
Let $p = 19$
$\Rightarrow (17 \times 19 + 1) = 324$ which is square of 18, so it is true.

Statement 2
Product of first 10 consecutive number

$x(2, 3, 5, 7, 11, 13, 17, 19, 23, 29) = 6469693230$

$\therefore\ x + 1 = 6469693231$

Divisibility by 7: The difference of the numbers upto thousands place and remaning part of the number if it is divisible by 7 then the number is divisible by 7

$\underbrace{6\,4\,6\,9\,6\,9\,3}\qquad \underbrace{2\,3\,1}$

Thousand partRemaining part

Difference $= 6469693 - 231$

$= 6469462$

Again difference $= 6469 - 462 = 6007$

Again difference $7 - 6 = 1$ which is not divisible by 7.

Therefore, number 6469693231 is not divisible by 7

So it is a prime number

So both statements are true.

114. (b) Number of account up to ₹ 10,000

$= 532 - 218$

$= 314$ accounts.

Rest of accounts of men deposits

$= 302 - 102 = 200$ accounts

Number of accounts of women deposits $= 314 - 200$ $= 114$

115. (b) Given $x + y + z = 0$

$x^3 + y^3 + z^3 + 3xyz - 3xyz + 3xyz$

$= x^3 + y^3 + z^3 - 3xyz + 6xyz$

$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx) + 6xyz$

$(0)(x^2 + y^2 + z^2 - xy - yz - zx) + 6xyz$

$= 6xyz$

116. (d) 17^{256}

Last digit is 7 so

We know that 7 repeats its unit digit after 4 times.

$= (17)^{64\times 4}$

Unit of $7 \times 7 \times 7 \times 7 = 2401$

Now, 256 is completely divided by 4 so unit digit = 1

117. (c) $\dfrac{1421\times 1423\times 1425}{12}$

When we divide 1421, 1423 and 1425 then 5,7 at 9 are the remainders respectively.

$= \dfrac{5\times 7\times 9}{12} = \dfrac{315}{12} = 3$

118. (a) $\dfrac{4^{96}}{6}$

When 4^1 is divided by 6 then remainder = 4

4^2 is divided by 6 then remainder = 4

4^3 is divided by 6 then remainder = 4

4^4 is divided by 6 then remainder = 4

....

. ...

4^{96} divided by 6 then remainder = 4

119. (c) $x + \dfrac{100}{x} > 50$

$\Rightarrow x^2 - 50x + 100 > 0$

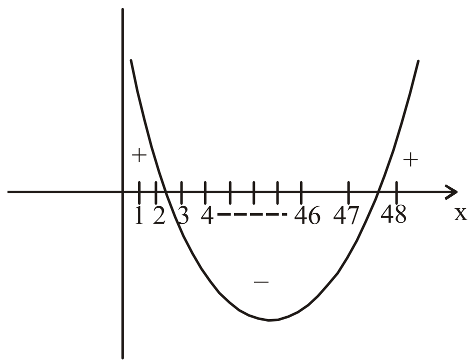
$$x = \frac{50 \pm \sqrt{(50)^2 - 4\times 100\times 1}}{2}$$

$$\left\{\text{using } x = \frac{\pm b + \sqrt{b^2 - 4ac}}{2a}\right\}$$

$x = 25 \pm 5\sqrt{21}$

so, $x^2 - 50x + 100 > 0$

$\left\{x - \left(25 + 5\sqrt{21}\right)\right\}\left\{x - \left(25 - 5\sqrt{21}\right)\right\} > 0$



As $x \le 100$ (Natural number)

So domain of x is = {1, 2, 48, 49, 50, 51------, 100}

Hence no. of values of x is 55.

Option (c) is correct.

120. (c) $(n^3 - n)(n - 2)$

$= n(n^2 - 1)(n - 2)$

$= n(n - 1)(n + 1)(n - 2)$

$(n - 2)(n - 1)n(n + 1)$

as $n > 2$ (natural number)

puting $n = 3$

$(n - 2)(n - 1)n(n + 1)$

$= (3 - 2)(3 - 1)3(3 + 1)$

$= 1 \times 2 \times 3 \times 4 = 24$

So, option (c) is correct

121. (d) $(234)^{100} + (234)^{101}$

See the pattern

$4^1 = 4$

$4^2 = 16$

$4^3 = 64$

$4^4 = 256$

so, at odd power of 4 we get unit digit as '4' and at even power of 4 we get unit digit as '6'.

$\Rightarrow (234)^{100}$ unit digit is 6

$\Rightarrow (234)^{101}$ unit digit is 4

$\Rightarrow (234)^{100} + (234)^{101}$ unit digit will be $(6 + 4 = 10)$ zero. So, option (d) is correct.

122. (*) 876p37q is divisible by 225 or 25×9. q has to be 5 and sum of all digits must be divided by 9.

$\therefore$ p = 0 or 9 and

Hence (c, d) are satisfying the condition.

123. (*) According to question.

$\Rightarrow 3x + 2y = 5k_1 + 2$ ---------(i)

$\Rightarrow 2x + 3y = 5k_2 + 3$ ---------(ii)

eq(i)-eq(ii)

$\Rightarrow x - y = 5(k_1 - k_2) - 1$

so when $(x - y)$ is divided by 5 remainder will be $5 + (-1) = 4$

At $x = 5, y = 1$, Remainder = 4

At $x = 6, y = 5$, Remainder = 1

So, option (a) and (c) both are is correct

124. (b) Given series is-

$$\underbrace{\frac{1}{4}+\frac{1}{5}-\frac{1}{6}-\frac{1}{4}-\frac{1}{5}+\frac{1}{6}}+\frac{1}{4}+\frac{1}{5}-\frac{1}{6}-\frac{1}{4}-\frac{1}{5}+\frac{1}{6}+----$$

These 6 terms are repeating which are resulting to zero.

so 1st 42 terms of this series will result is zero and after that series will be upto 47 terms –

$$\Rightarrow \frac{1}{4}+\frac{1}{5}-\frac{1}{6}-\frac{1}{4}-\frac{1}{5} \Rightarrow -\frac{1}{6}$$

So, option (b) is correct.

125. (c) Let the number be $(10x + y)$

According to question-

$\Rightarrow x + y = 7$ --- (i)

$(10y + x) - (10x + y) = 27$

$\Rightarrow 9(y - x) = 27$

$\Rightarrow y - x = 3$ --- (ii)

From eq (i) and (ii)

$x = 2; y = 5$

so $x \times y = 2 \times 5 = 10$

So, option (c) is correct.

126. (c) No.s divided by 7 leaing 3 as reminder are -

$$\left.\begin{array}{l} 7+3=10 \\ 14+3=17 \\ 21+3=24 \\ 28+3=31 \\ 35+3=38 \\ 42+3=45 \\ 49+3=52 \\ 56+3=59 \\ 63+3=66 \\ 70+3=73 \\ 77+3=80 \\ 84+3=87 \\ 91+3=94 \end{array}\right\} \text{Total 13 terms.}$$

Sum = $3 + 13 + 7[1 + 2 + 3 + \text{-----} + 13]$

$$= 39 + \left(7 \times \frac{13 \times 14}{2}\right) = 39 + 637 = 676$$

So, option (c) is correct.

127. (c) Given $S_n = \frac{n(n+1)}{2}$

Also $S_n = 861$

$$\Rightarrow 861 = \frac{n(n+1)}{2}$$

$n^2 + n - 1722 = 0$

$n = 41, -42$

(1) is true (i.e., there are exactly two values of n for which Sn = 861)

Given Sn = Sn + 1

$$\Rightarrow \frac{n(n+1)}{2} = \frac{(n+1)(n+2)}{2}$$

$n = n + 2$

$2 \neq 0$ (this is not possible)

$\Rightarrow$ 2 is true.

$\therefore$ Option (c) is correct.

128. (c) (If $p + q < p - q$ and if $p + q > p - q$

$\Rightarrow q + q < 0 \Rightarrow q > -q$

$\Rightarrow q < 0$

$\Rightarrow 2q > 0$

1 is true

$\Rightarrow q > 0$

$\Rightarrow$ q must be positive

$\Rightarrow$ p is also positive

$\Rightarrow$ (2) is true.

$\therefore$ option (c) is correct

129. (*) The dividend is in the form $a^n - b^n$

Here n is even

$\Rightarrow 7^{10} - 5^{10}$ is divisible by both $(7 + 5)$ and $(7 - 5)$

$\Rightarrow 7^{10} - 5^{10}$ is divisible by 12 and 2

Hence, it is divisible by 12 and 2. No options mathcing with the given answer.

130. (c) Let two digit number = $10y + x$

According to question,

Ist condition, $10y + x = k(x + y)$ ...(1)

2nd condition, $10x + y = m(x + y)$ ...(2)

Adding (1) and (2) we get

$11x + 11y = (k + m) \times (x + y)$

$11(x + y) = (k + m)(x + y)$

$k + m = 11$

$m = 11 - k$

$\therefore$ Option (c) is correct

131. (d) Here S be a set of 14 Natural Numbers i.e. {1, 2, 3, 4, 5. ---- 14}

Possible no. of pairs.

{a, b} is {(2, 8), (7, 13)}

$\Rightarrow$ Pairs = 2

$\Rightarrow$ Option (d) is correct.

132. (b) Time taken to reach 5'O Clock = 8 seconds

Time taken to reach 10'O Clock = $\frac{8}{5}$ seconds

Time taken to reach 10'O Clock = $\frac{8}{5}\times10$ =16 seconds

$\therefore$ Option (b) is correct.

133. (b) Here N = 90 × 42 × 3^24 × 55
Now 90 = 3 × 3 × 10 = $3^2 \times 10$
42 = 14 × 3 = 14×3^1
$3^24 = 3 \times 3 \times 3 \times 3 \times 4 = 3^4 \times 4$
$5^5 = 11 \times 5$
$N = 3^2 \times 3^1 \times 3^4 \times 10 \times 14 \times 4 \times 11 \times 5$
$N = 3^7 \times 10 \times 14 \times 4 \times 11 \times 5$
Maximum value of m = 7
$\therefore$ Option (b) is correct.

134. (b) Option (b) is correct.

135. (d) Option (d) is correct.

136. (a) Since two hands are interchange their positions so sum of the angles subtended at the centre by hour hand and minute hand = 360°
Let us suppose that she was out of house for t min.
So the sum of angles subtended at the centre by the hour hand and minute hand.
Angle made by hour hand.
i.e., 12h $\rightarrow$ 360°

$$\Rightarrow \quad 1\text{h} = \frac{360}{12} = 30°$$

$$\Rightarrow \quad 1\,\text{min} = \frac{30}{60} = 0.5\,\text{min}$$

Angle made by minute hand.
60 min = 360°
1 min = 60
$\Rightarrow$ (0.5)t + (6 × t)° = 360°
$\Rightarrow$ 6.5t = 360 $\Rightarrow$ t = 55.38 min.
$\therefore$ Option (a) is correct.

137. (b) Let m = 1 and n = 2 be two natural number

(1) $\frac{m}{n}+\frac{n}{m}=\frac{1}{2}+\frac{2}{1}=\frac{5}{2}$

which is not an integer

(2) $mn\left(\frac{m}{n}+\frac{n}{m}\right)\left(m^2+n^2\right)$

$= 1\times2\left(\frac{1}{2}+\frac{2}{1}\right)(1+4) = 2\times\frac{5}{2}\times5 = 25$

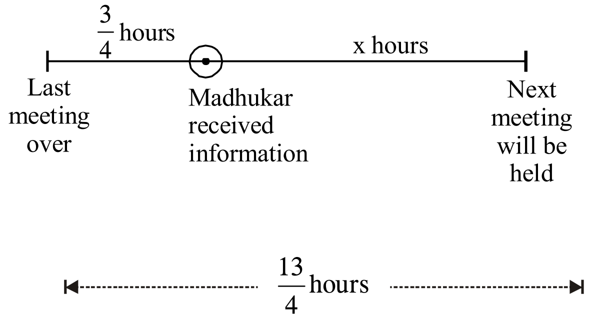
which is an integer

(3) $\frac{mn}{m^2+n^2}=\frac{1\times2}{1^2+2^2}=\frac{2}{5}$

which is not an integer

$\therefore$ Option (b) is correct.

138. (b) Let Madhukar received the informaton x hour before 2 p.m.



$$\frac{3}{4}+x=\frac{13}{4}\Rightarrow x=\frac{13}{4}-\frac{3}{4}=\frac{5}{2}=2\frac{1}{2}\text{ hours}$$

Hence Madhukar received information $2\frac{1}{2}$ hours before 2 p.m. i.e., 11 : 30 a.m.

139. (d) $\sqrt{\frac{x}{y}}=\frac{10}{3}-\sqrt{\frac{y}{x}}$

$$\sqrt{\frac{x}{y}}=\frac{10}{3}-\frac{1}{\sqrt{\frac{y}{x}}}$$

Let $\sqrt{\frac{x}{y}}=z$

$$\therefore \quad z=\frac{10}{3}-\frac{1}{z}\Rightarrow 3z^2-10z+3=0$$

$$\Rightarrow \quad z=3 \text{ or } \frac{1}{3}$$

$$\Rightarrow \quad \sqrt{\frac{x}{y}}=3 \quad \text{or} \quad \sqrt{\frac{x}{y}}=\frac{1}{3}$$

$\Rightarrow$ x = 9 y ...(i) or 9x = y ...(ii)
But x – y = 8 ...(iii)
$\therefore$ x > y
Hence eq. (ii) is rejected.
From eq. (i) and (iii),
9y – y = 8, $\Rightarrow$ y = 1 and x = 9
$\therefore$ xy = 9 × 1 = 9

140. (a)

$$\begin{array}{rccc} & \boxed{1} & 2 & \\ & 5 & P & 9 \\ & 3 & R & 7 \\ + & 2 & Q & 8 \\ \hline & 11 & \boxed{1} & 4 \end{array}$$

2 + P + R + Q = 11
P + R + Q = 9
For maximum value of Q
P and R be 0 then
Q = 9

141. (b) Let the original number be x
ATQ

$$8x - \frac{x}{8} = 2016$$

$$x\left[\frac{63}{8}\right] = 2016$$

8 by solving
x = 256

142. (b) Through option 4.65 when multiplied by = 20 gives whole number

143. (d) $(7)^{139} = 7^{4(34)+3}$
unit digit of 7^{4n} is 1 and $7^3 = 34$ is 3
than unit digit = 1 × 3 = 3

144. (c) $\frac{(4444)^{4444}}{9}$

$\frac{(7)^{4444}}{9}$ $\boxed{\text{when 4444 divided by 9}}$

$= \frac{(-2^4)^{1111}}{9}$ $\boxed{\text{remainder will be 7}}$

$= \frac{(16)^{1111}}{9}$ $\boxed{\text{or (–2) negative remainder}}$

$\frac{(-2)^{1110} \times (-2)}{9}$ $\boxed{\text{similarly (–2) negative}}$

$\frac{(-2^6)^{185} \times (-2)}{9} = \frac{(64)^{185} \times (-2)}{9}$ $\boxed{\text{remainder}}$

$\Rightarrow \frac{(1)^{185} \times (-2)}{9}$

$\frac{1 \times (-2)}{9} = \frac{7}{9}$ i.e Remainder = 7

145. (b) Number of prime numbers less than 100 are 25

146. (c) 1. Of two consecutive integer is always odd and other is even. So this is true statement.
2. Each odd no is in the form of (2n – 1)
$(2n + 1)^2 = 4x^2 + 1 + 4n$

147. (a) $(N^{P-1} - 1)$ where n is prime to P then
P is always a prime number.

148. (d) Let numbers be a, b and c respectively
According to the question
a × b = 286
b × c = 770
Value of b is H.C.F of 286 and 770 = 22
∴ a = 13 b = 22 c = 35
a + b + c = 13 + 22 + 35 = 70

149. (c) 1 × 3 × 5 × 7 .... × 109
If any odd number is multiplied by 5 it gives 5 on unit place
Hence final unit digit is 5

150. (b) A number divisible by 16, 36, 45, 48,
= L.C.M of 16, 36, 45 and 48 = 720
Highest four-digits number = 9999
Now, highest 4 digit number divisible by 720
Completely = 720 × 13 = 9360 < 9999

151. (b) Any number that are divisible by 33 must be divisible by 3 and 11.
A number that are divisible by 3 have sum of its digit divisible by 3.
∴ x + x + y + x + x = (4x + y) must be divisible by 3
And, For divisibility by 11, (x + y + x) – (x + x) = 0
⇒ y = 0
Now, 4x + y = 4x + 0 = 4x divisible by 3
Only when x = 3, 6 or 9.
Hence, three number are 33033, 66066, 99099.

152. (c) Any number that are divisible by 3 only when sum of its digits is divisible by 3
∴ (x + y + 2 + 3 + 5) = (10 + x + y)
As $x + y \le 5$,
then (10 + x + y) = 12 or 15
∴ (x, y) = (2, 0), (1, 1) (2, 3), (3, 2), (1, 4), (4, 1) and (5, 0)
Hence, number of possible pairs = 7

153. (c) $38808 = 2^3 \times 3^2 \times 7^2 \times 11$
∴ Number of divisors = (3 + 1)(2 + 1)(2 + 1)(1 + 1)
= 4 × 3 × 3 × 2 = 72
On excluding number '1' and '38808'.
Number of divisors = 72 – 2 = 70

154. (b) Statement 1
XYZ = 100X + 10Y + Z
YZX = 100Y + 10Z + Z
<u>ZXY = 100Z + 10X + Y</u>
= 111 (X + Y + Z), which is divisible by (X + Y + Z)
So, it is not correct
Statement 2
XYZ = 100X + 10Y + Z
YZX = 100Y + 10Z + X
<u>ZXY = 100Z + 10X + Y</u>
= 111 (X + Y + Z), which is divisible by 111
So, it is correct

155. (b) (+1) (–1)+ (+2)(–2)+ (+3)(–13+ (+4)(–4)
= –1–4–9–16 = – 30

156. (d) The number $17^{29} = (18 - 1)^{29}$ when divided by 18 leaves the remainder $(-1)^{29} = 18 - 1 = 17$
The number $19^{29} = (18 + 1)^{29}$ when divided by 18 leaves the remainder $(1)^{29} = 1$
Then after adding these two the remainder will be 17 + 1=18 which is divisible by 18 Hence the remainder will be 0

157. (a) For the number to be divisible by 10, it must contain the same powers for 2 and 5
Power of $2 = 2^{(5+16+7+36+6+28+11)} = 2^{109}$
Power of $5 = 5^{(3+6+12+14+30)} = 5^{(65)}$
Hence maximum possible power of 10 can be 65 only.

158. (a) If the number is divisible by 9 the sum of all its digit is divisible by 9
4 + 7 + 9 + 8 + 6 + 5 + A + B = 39 + A + B is divisible by 9
Possible values of B are 1, 3, 5, 7, 9 as it is given that

last digit is odd
For B = 1, A = 5
For B = 3, A = 3
For B = 5, A = 1
For B = 7, A = 8
For B = 9, A = 6

159. (d) $999 \times abc = def132$
We can write the above equation as
$(1000 - 1) \times abc = def132$
$abc000 - abc = def000 + 132 = (def + 1) \times 1000 - 868$
on comparing the LHS and RHS, we get
a = 8, b = 6, and c = 8 and d = a = 8
Now, $999 \times 868 = 867132$
$\therefore$ d = 8, e = 6, f = 7

160. (c) Option C is incorrect as 6n – 1 form can be a prime number but it is not necessarily true.

161. (d) Let the three prime numbers be $x, y, y + 36$
$x + y + y + 36 = 100 \Rightarrow x + 2y = 64$
$2y$ is an even number always
We know that
Even + even = even or odd + odd = even
So x has to be even to satisfy $x + 2y = 64$
The only even prime on is 2
Put $x = 2 \Rightarrow 2y = 62 \Rightarrow$ Or $y = 31$
So the numbers are 2, 31, 67

162. (c)

	Repitition values of unit digits according to their power								
Power	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1	1	2	3	4	5	6	7	8	9
2	1	4	9	6	5	6	9	4	1
3	1	8	7	4	5	6	3	2	9
4	1	6	1	6	5	6	1	6	1

From the above table we can see that the power 73 is of the form 4x + 1
Therefore the unit digit in $7^{73} = 7$

163. (d) All the given statements are true. The following are the examples for all the statements
Statement 1: Both p and q may be prime numbers. Ex – 7 and 11
Statement 2: Both p and q may be composite numbers. Ex – 8 and 12
Statement 3: One of p and q may be prime and the other composite.
Ex. 13 and 16

164. (d) $3^N > N^3$ holds for all the natural numbers except N = 3 at which $3^N = N^3 \Rightarrow 3^3 = 3^3$

165. (d) A number that cannot be represented in the form p/q where p and q are two integers, is known as lrrational number $\sqrt{59049} = 243$. Hence it is rational
231593 is already in the form of rational number
0.4545454545........ can be represented in the form of p/q as 5/9
0.12112211122211112222........cannot be represented in the form of p/q.
So that is an irrational number.

166. (b) $3^{521} = 3^{130 \times 4 + 1}$
As we know $3^1 = 3$ will leave remainder = 3 when divided by 8

167. (d) For prime no units place cannot be occupied by even number except for 2
Thus no of digits occupying unit digit of prime numbers = 6 (1, 2, 3, 5, 7, 9)

168. (a) $6^{23} \times 75^9 \times (105)^2$
$= 2^{23} \times 3^{23} \times 3^9 \times 5^9 \times 5^9 \times 3^2 \times 7^2 \times 5^2$
$= 2^{23} \times 5^{20} \times 3^{34} \times 7^2$
$= 2^{23} \times 5^{20}$
10 is divided by 2 and 5
Minimum of 20 and 23 is 20
$\therefore$ n = 20

169. (b) Unit digit of $(3^{18} - 3^{89})$
= Unit digit of $(3^{96} - 3^2 - 3^{88} - 3)$
= Unit digit of $(1 \times 9 - 1 \times 3)$
= 6

170. (b) $(n - 1)^2 + n^2 + (n + 1)^2 + (n + 2)^2 = 294$
$n^2 + 1 - 2n + n^2 + n^2 + 1 + 2n + n^2 + 4n + 4 = 294$
$4n^2 + 4n + 6 = 294$
$4n^2 + 4n - 288 = 0$
$n^2 + n - 77 = 0$
$n^2 + 9n - 8n - 72 = 0$
$\Rightarrow$ $(n - 8)(n + 9) = 0$
$\Rightarrow$ $\therefore$ n = 8, –9
$\therefore$ number are 7, 8, 9, 10
Sum of number = 7 + 8 + 9 + 10 = 34

171. (b) $a^2 - b^2 = 35$
$\Rightarrow$ $(a + b)(a - b) = 35 \times 1$ or 7×5
Hence, two such pairs are (1,35) and (5,7).

172. (c) If p × q is even
then at least one of p or q is even
So the statement both p and q are even is a false statement

173. (c)

174. (b) Let x and y be the number of apples and oranges bought by the person.
$$5x + 7y = 500 \Rightarrow y = \frac{500 - 5x}{7} = \frac{5(100 - x)}{7}$$
for x and y to be integers
For x = (2, 9, 16, 23, 30, 37, 46, 51, 58, 65, 72, 79, 86, 93)
y gives integer value
Hence, required number = 14

175. (b) ATQ, x = 14a + 7 = 15b + 5
For smallest value of x two number a and b should be minimum and when a = 2 = b,
then, x = 14 × 2 + 7 = 35

176. (d) Statement1: $\sqrt{75}$ is rational number is false
Statement 2: – there exerts at least a positive integer

x such that $-\frac{4x}{5} < -\frac{-7}{8}$

It is true for x > 1

Statement 3: $\frac{x-2}{x} < 1$ is not true for all real value of x.

Statement 4: $4.232323...... = \frac{423-4}{99} = \frac{419}{99}$

Hence, it is true

177. (c) 1. Number — Unit digit

7^1 — 7

7^2 — 9

7^3 — 3

7^4 — 1

7^5 — 7

7^{174} — $(7)^{43\times 4}$ — $1 \times 9 = 9$

2. Let n is a add number then (n+ 2) is also a odd number,

Now, $(n+2) - (n)^2 = (2n+2).2$
$= 4(n+1)$

As n is odd, so (n + 1) is even number and must be divisible by 2.

Hence difference of square of two odd number is away divisible by 8.

3. Let n and (n + 2) are two odd number.

Now, $n(n+2) + 1 = n^2 + 2n + 1 = (n+1)^2$

This is perfect square

Hence, statement 2 and 3 are true

178. (c) quotient = 182.

Remainder = 182 – 175 = 7

Number N = 17 × 182 + 7
= 3101

179. (a) 2 × 3 × 5 × 7 × 11 gives a number that are, divisible by 2, 3, 5, 7 and 11.

So, (2×3 × 5 × 7 × 11 + 1) gives a number that are not divisible by 2, 3, 5, 7 and 11.

Then, that number is not divisible by 4, 6 8, 9, and 10 as well.

Hence, number '(2 × 3 × 5 × 7 × 11 + 1)' is a prime number.

180. (b)

Value of n	Number	Sum of its digit
1	$10^1 - 1 = 9$	9
2	$10^2 - 1 = 99$	$9+9=2\times 9$
3	$10^3 - 1 = 999$	$9+9+9=3\times 9$
n	$10^n - 1 = 999....$ n times	$9+9$ n time $= n\times 9$

According to the question, Sum of its digits

$n \times 9 = 3798 \quad \Rightarrow \quad n = 422.$

181. (b) Number of prime number between 0 to 50.

m = 15; Number of prime number between 50 to 100

n = 10; $\therefore$ m – n = 5

182. (c) $3^x + 3^{x+1} + 3^{x+2} = 3^x(1+3+9) = 3^x \times 13$

Min value of x = 1

$\therefore$ Min divisor = 3 × 13 = 39

183. (a) Let two digits of the number are x and y, then

$xy = 8 \Rightarrow y = 8/x$ ...(i)

$(10x + y) = 4(10y + x) + 9$

$10x + y = 40y + 4x + 9$

$6x = 39y + 9$

$2x = 13y + 3$

$2x = \frac{104}{x} + 3 \qquad \left(\because \; y = \frac{8}{x}\right)$

$2x^2 - 3x - 104 = 0$

$\Rightarrow \quad x = 8, \frac{13}{2}$

$\therefore \quad x = 8, y = 1$

Required number = 81

184. (c) The greatest number that divide 28200, 44333 and 12288 is the H.C.F. of (28200 – 12288, 44333 – 28200, 44333 – 12288) = H.C.F of (15912, 16133, 32045) = 121.

185. (a) From equation we have

$P = 0.7\bar{3} = \frac{73-7}{90} = \frac{66}{90} = \frac{11}{15}$

$Q = 0.5\bar{6} = \frac{56-5}{90} = \frac{51}{90} = \frac{17}{30}$

$\therefore$ Distance between P and Q

$= \frac{11}{15} - \frac{17}{30} = \frac{22-17}{30} = \frac{5}{30} = \frac{1}{6}$

186. (c) 100x + 10y + z is divisible by 7.

100z + 10y + x is divisible by 7.

on solving above two equation

(99x – 99z) is divisible by 7.

x – z is divisible by 7.

four possible numbers. are

8 y 1 or 1 y 8 and 9 y 2 or 2 y 9

187. (d) S = XYZ + YZX + ZXY

S = 100X + 10Y + Z + 100Y + 10Z + X + 100Z + 10X + Y

= 111 (X + Y + Z) = 3 × 37 (X + Y + Z)

$\therefore$ 3 digit number s is always divisible by 3, (x + y + Z) and 37, but not always divided by 9

188. (c) $HM = \frac{2ab}{a+b} = \frac{2P}{S}$

CHAPTER

2 HCF and LCM

1. What is the HCF of the polynomials $x^3 + 3x^2y + 2xy^2$ and $x^4 + 6x^3y + 8x^2y^2$? **[2007-I]**
   (a) $x(x+2y)$ (b) $x(x+3y)$
   (c) $x+2y$ (d) None of these

2. If $(x + k)$ is the HCf of $(x^2 + ax + b)$ and $(x^2 + cx + d)$, then what is the value of k ? **[2007-II]**
   (a) $\frac{b+d}{a+c}$ (b) $\frac{b+d}{c+d}$
   (c) $\frac{a-b}{c-d}$ (d) $\frac{b-d}{a-c}$

3. If the HCF of $x^3 + mx^2 - x + 2m$ and $x^2 + mx - 2$ is a linear polynomial, then what is the value of m? **[2007-II]**
   (a) 1 (b) 2 (c) 3 (d) 4

4. A person has four iron bars whose lengths are 24 m, 36 m, 48 m and 72 m respectively. This person wants to cut pieces of same length from each of four bars. What is the least number of total pieces if he is to cut without any wastage? **[2007-II]**
   (a) 10 (b) 15 (c) 20 (d) 25

5. 5 bells start tolling together and toll at intervals of 2, 4, 6, 8 and 10 s, respectively. How many times do the five bells toll together in 20 min? **[2007-II]**
   (a) 10 (b) 11 (c) 12 (d) 15

6. LCM of two numbers is 16 times their HCF. The sum of LCM and HCF is 850. If one number is 50, then what is the other number ? **[2007-II]**
   (a) 800 (b) 1200 (c) 1600 (d) 2400

7. If HCF of m and n is 1, then what are the HCF of $m + n$, m and HCF of $m - n$, n, respectively ? $(m > n)$ **[2007-II]**
   (a) 1 and 2 (b) 2 and 1
   (c) 1 and 1 (d) Cannot be determined

8. Let p, q and r be natural numbers. If m is their LCM and n is their HCF, consider the following
   I. $mn = pqr$ if each p, q and r is prime.
   II. $mn = pqr$ if p, q and r are relatively prime in pairs.
   Which of the above statement is/are correct? **[2008-I]**
   (a) Only I (b) Only II
   (c) Both I and II (d) Neither I nor II

9. The LCM of three different numbers is 150. Which of the following cannot be their HCF? **[2008-I]**
   (a) 15 (b) 25 (c) 50 (d) 55

10. What is the HCF of $(x^2 + bx - x - b)$ and $[x^2 + x(a-1) - a]$? **[2008-I]**
    (a) $x+b$ (b) $x+a$
    (c) $x+1$ (d) $x-1$

11. If the HCF of $(x^2 + x - 12)$ and $(2x^2 - kx - 9)$ is $(x - k)$, then what is the value of k ? **[2008-I]**
    (a) -3 (b) 3 (c) -4 (d) 4

12. What is the LCM of $(6x^3 + 60x^2 + 150x)$ and $(3x^4 + 12x^3 - 15x^2)$? **[2008-I]**
    (a) $6x^2(x+5)^2(x-1)$ (b) $3x^2(x+5)^2(x-1)$
    (c) $6x^2(x+5)^2(x-1)^2$ (d) $3x^2(x+5)(x-1)^2$

13. What is the LCM of $(x + 2)^2 (x - 2)$ and $x^2 - 4x - 12$? **[2008-II]**
    (a) $(x+2)(x-2)$ (b) $(x+2)^3(x-2)(x-6)$
    (c) $(x+2)(x-2)^2$ (d) $(x+2)^2(x-2)(x-6)$

14. What is the value of k for which the HCF of $2x^2 + kx - 12$ and $x^2 + x - 2k - 2$ is $(x + 4)$? **[2008-II]**
    (a) 5 (b) 7 (c) 10 (d) -4

15. If $(x + k)$ is the HCF of $ax^2 + ax + b$ and $x^2 + cx + d$, then what is the value of k? **[2008-II]**
    (a) $\frac{b+d}{a+c}$ (b) $\frac{a+b}{c+d}$
    (c) $\frac{a-b}{c-d}$ (d) None of these

16. 21 mango trees, 42 apple trees and 56 orange trees have to be planted in rows such that each row contains the same number of trees of one variety only. What is the minimum number of rows in which the above trees may be planted? **[2008-II]**
    (a) 3 (b) 15 (c) 17 (d) 20

17. What is the greatest number which divides 392, 486 and 627 so as to leave the same remainder in each case? **[2009-I]**
    (a) 47 (b) 43 (c) 37 (d) 34

18. What is the LCM of $(x^2 - y^2 - z^2 - 2yz)$, $(x^2 - y^2 + z^2 + 2xz)$ and $(x^2 + y^2 - z^2 - 2xy)$? **[2009-I]**
    (a) $(x+y+z)(x+y-z)(x-y+z)$
    (b) $(x+y+z)(x-y-z)(x-y+z)$
    (c) $(x+y+z)(x+y-z)(x-y-z)$
    (d) $(x+y-z)(x-y-z)(x-y+z)$

19 If $(x + 2)$ is the HCF of $x^2 + ax + b$ and $x^2 + cx + d$ (where, $a \neq c$ and $b \neq d$), then which one of the following is correct? **[2009-I]**
   (a) $a+c=b+d$ (b) $2a+b=2c+d$
   (c) $b+2c=2a+d$ (d) $b-2c=2a-d$

20. What are the values of c when the HCF of $x^3 + cx^2 - x + 2c$ and $x^2 + cx - 2$ over the rationals is a linear polynomial ? **[2009-I]**

(a) ± 1 (b) ± 2 (c) ± 3 (d) ± 4

21. For two natural numbers m and n, let g_{mn} denote the greatest common factor of m and n. Consider the following in respect of three natural numbers k, m and n.

I. $g_{m(nk)} = g_{(mn)k}$ II. $g_{mn}\, g_{nk} = g_{mk}$

Which of the above statement is/are correct? **[2009-II]**

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

22. If the HCF of $x^3 - 27$ and $x^3 + 4x^2 + 12x + k$ is a quadratic polynomial, then what is the value of k ? **[2009-II]**

(a) 27 (b) 9 (c) 3 (d) -3

23. What is the LCM of $3(a^3 - b^3)$ and $11\,(a^4 - b^4)$? **[2009-II]**

(a) $33(a^3 - b^3)\,(a^2 + b^2)\,(a + b)$
(b) $(a^3 - b^3)\,(a^2 + b^2)\,(a - b)$
(c) $33(a - b)\,(a^2 + b^2 - ab)\,(a^3 - b^3)$
(d) $33(a^3 - b^3)\,(a^4 - b^4)$

24. What is the least number which when divided by 42, 72 and 84 leaves the remainders 25, 55 and 67, respectively? **[2009-II]**

(a) 521 (b) 512 (c) 504 (d) 487

25. For any integer n, what is the HCF of integers $m = 2n + 1$ and $k = 9n + 4$? **[2010-I]**

(a) 3 (b) 1 (c) 2 (d) 4

26. Consider the following statements:

I. The HCF of $x + y$ and $x^{10} - y^{10}$ is $x + y$.
II. The HCF of $x + y$ and $x^{10} + y^{10}$ is $x + y$.
III. The HCF of $x - y$ and $x^{10} + y^{10}$ is $x - y$.
IV. The HCF of $x - y$ and $x^{10} - y^{10}$ is $x - y$.

Which of the statement given above are correct ? **[2010-I]**

(a) I and II (b) II and III
(c) I and IV (d) II and IV

27. If the highest common factor of two positive integers is 24, then their least common multiple cannot be **[2010-I]**

(a) 72 (b) 216 (c) 372 (d) 600

28. Consider those number between 300 and 400 such that when each number is divided by 6, 9 and 12, it leaves 4 as remainder in each case. What is the sum of the numbers? **[2010-I]**

(a) 692 (b) 764 (c) 1080 (d) 1092

29. What is the smallest positive integer which when divided by 4, 5, 8 and 9 leaves remainder 3, 4, 7 and 8, respectively? **[2010-II]**

(a) 119 (b) 319 (c) 359 (d) 719

30. If $f\,(x)$ and $g\,(x)$ are two polynomials with integral co-efficients which vanish at $x = \frac{1}{2}$, then what is the factor of HCF of $f(x)$ and $g\,(x)$? **[2010-II]**

(a) $x - 1$ (b) $x - 2$
(c) $2x - 1$ (d) $2x + 1$

31. What is the sum of the digits of the least number which when divided by 52, leaves 33 as remainder, when divided by 78 leaves 59 and when divided by 117, leaves 98 as remainder ? **[2010-II]**

(a) 17 (b) 18 (c) 19 (d) 21

32. What is the HCF of $4x^3 + 3x^2y - 9xy^2 + 2y^3$ and $x^2 + xy - 2y^2$? **[2010-II]**

(a) $x - 2y$ (b) $x - y$
(c) $(x + 2y)\,(x - y)$ (d) $(x - 2y)\,(x - y)$

33. A number when dinded by 2, 3 or 5 given remainder 1. The number is **[2010-II]**

(a) 31 (b) 47 (c) 43 (d) 53

34. The HCF of two polynomials $p(x)$ and $q(x)$ is $2x\,(x + 2)$ and LCM is $24x\,(x + 2)^2\,(x - 2)$. If $p(x) = 8x^3 + 32x^2 + 32x$, then what is $q(x)$ equal to? **[2011-I]**

(a) $4x^3 - 16x$ (b) $6x^3 - 24x$
(c) $12x^3 + 24x$ (d) $12x^3 - 24x$

35. What is the least number of square tiles required to pave the floor of a room 9 m 99 cm long and 4 m 7 cm broad? **[2011-I]**

(a) 247 (b) 277 (c) 297 (d) 307

36. What is the HCF of 3.0, 1.2 and 0.06 ? **[2011-I]**

(a) 0.6 (b) 0.06 (c) 6.0 (d) 6.06

37. If the HCF of three numbers 144, x and 192 is 12, then the number x cannot be **[2011-I]**

(a) 180 (b) 84 (c) 60 (d) 48

38. What is the HCF of $(x^4 - x^2 - 6)$ and $(x^4 - 4x^2 + 3)$? **[2011-I]**

(a) $x^2 - 3$ (b) $x + 2$
(c) $x + 3$ (d) $x^2 + 3$

39. What is the LCM of $a^3b - ab^3$, $a^3b^2 + a^2b^3$ and $ab\,(a + b)$? **[2011-II]**

(a) $a^2b^2\,(a^2 - b^2)$ (b) $ab\,(a^2 - b^2)$
(c) $a^2b^2 + ab^3$ (d) $a^3b^3\,(a^2 - b^2)$

40. What is the HCF of the polynomials $x^4 - 3x + 2$, $x^3 - 3x^2 + 3x - 1$ and $x^4 - 1$? **[2011-II]**

(a) $x - 1$ (b) $x + 1$
(c) $x^2 - 1$ (d) None of these

41. The least number which when divided by 5, 6, 7 and 8 leaves a remainder 3 is **[2011-II]**

(a) 423 (b) 843 (c) 1683 (d) 2523

42. What is the LCM of $\frac{2}{3}, \frac{7}{9}$ and $\frac{14}{15}$? **[2011-II]**

(a) $\frac{7}{3}$ (b) $\frac{14}{3}$ (c) $\frac{2}{3}$ (d) $\frac{1}{3}$

43. The product of two numbers is 6912 and their GCD is 24. What is their LCM? **[2011-II]**
(a) 280 (b) 286 (c) 288 (d) 296

44. The sum of two numbers is 232 and their HCF is 29. What is the number of such pairs of numbers satisfying the above condition? **[2012-I]**
(a) One (b) Two
(c) Four (d) None of these

45. What is the HCF of $36(3x^4+5x^3-2x^2)$, $9(6x^3+4x^2-2x)$ and $54(27x^4-x)$? **[2012-I]**
(a) $9x(x+1)$ (b) $9x(3x-1)$
(c) $18x(3x-1)$ (d) $18x(x+1)$

46. The HCF and LCM of two natural numbers are 12 and 72, respectively. What is the difference between the two numbers, if one of the numbers is 24 ? **[2012-I]**
(a) 12 (b) 18 (c) 21 (d) 24

47. If $(x-6)$ is the HCF of $x^2-2x-24$ and x^2-kx-6, then what is the value of k ? **[2012-II]**
(a) 3 (b) 5 (c) 6 (d) 8

48. What is the greatest number that divides 13850 and 17030 and leaves a remainder 17? **[2012-II]**
(a) 477 (b) 159 (c) 107 (d) 87

49. Three planets revolve round the Sun once in 200, 250 and 300 days, respectively in their own orbits. When do they all come relatively to the same position as at a certain point of time in their orbits? **[2012-II]**
(a) After 3000 days (b) After 2000 days
(c) After 1500 days (d) After 1200 days

50. The product of HCF and LCM of 18 and 15 is **[2012-II]**
(a) 120 (b) 150 (c) 175 (d) 270

51. What is the HCF of the polynomials x^3+8, x^2+5x+6 and x^3+2x^2+4x+8? **[2013-I]**
(a) $x+2$ (b) $x+3$
(c) $(x+2)^2$ (d) None of these

52. The LCM of (x^3-x^2-2x) and (x^3+x^2) is **[2013-I]**
(a) x^3-x^2-2x (b) x^2+x
(c) $x^4-x^3-2x^2$ (d) $x-2$

53. The HCF of (x^4-y^4) and (x^6-y^6) is **[2013-I]**
(a) x^2-y^2 (b) $x-y$
(c) x^3-y^3 (d) x^4-y^4

54. The LCM of two numbers is 2376 while their HCF is 33. If one of the numbers is 297, then the other number is **[2013-I]**
(a) 216 (b) 264 (c) 642 (d) 792

55. What is the LCM of x^2+2x-8, x^3-4x^2+4x and x^2+4x ? **[2013-II]**
(a) $x(x+4)(x-2)^2$ (b) $x(x+4)(x-2)$
(c) $x(x+4)(x+2)^2$ (d) $x(x+4)^2(x-2)$

56. What is the HCF of $a^2b^4+2a^2b^2$ and $(ab)^7-4a^2b^9$? **[2013-II]**
(a) ab (b) a^2b^3 (c) a^2b^2 (d) a^3b^3

57. The HCF of two numbers is 98 and their LCM is 2352. The sum of the numbers may be **[2013-II]**
(a) 1372 (b) 1398 (c) 1426 (d) 1484

58. If for integers a, b and c, if HCF $(a, b) = 1$ and HCF $(a, c) = 1$, then which one of the following is correct? **[2013-II]**
(a) HCF $(a, bc) = 1$ (b) HCF $(a, bc) = a$
(c) HCF $(a, bc) = b$ (d) None of these

59. What is the HCF of $8(x^{5'}-x^3+x)$ and $28(x^6+1)$? **[2014-I]**
(a) $4(x^4-x^2+1)$ (b) $2(x^4-x^2+1)$
(c) (x^4-x^2+1) (d) None of these

60. In a fire range, 4 shooters are firing at their respective targets. The first, the second, the third and the fourth shooters hit the target once is every 5 s, 6 s, 7 s and 8 s, respectively. If all the them hit their target at 9:00 am, when will they hit their target together again? **[2014-I]**
(a) 9 : 04 am (b) 9 : 08 am
(c) 9 : 14 am (d) None of these

61. For any integer n, what is HCF $(22n+7, 33n+10)$ equal to? **[2014-I]**
(a) n (b) 1
(c) 11 (d) None of these

62. For any integers 'a' and 'b' with HCF $(a, b) = 1$, what is HCF $(a+b, a-b)$ equal to ? **[2014-I]**
(a) It is always 1 (b) It is always 2
(c) Either 1 or 2 (d) None of these

63. If a and b be positive integers, then HCF

$$\left(\frac{a}{\text{HCF}(a,b)}, \frac{b}{\text{HCF}(a,b)}\right)$$ equal to ? **[2014-I]**

(a) a (b) b
(c) 1 (d) $\dfrac{a}{\text{HCF}(a,b)}$

64. What is the number of integral solutions of the equations HCF $(a, b) = 5$ and $a+b=65$? **[2014-I]**
(a) None (b) Infinitely many
(c) Less than 65 (d) Exactly one

65. The LCM of two intergers is 1237. What is their HCF ? **[2014-II]**
(a) 37 (b) 19
(c) 1 (d) Cannot be determined

66. There are 48 cricket balls, 72 hockey balls and 84 tennis balls and they have to be arranged in several rows in such a way that every row contains the same number balls of balls of one type. What is the minimum number of rows required for this to happen ? **[2014-II]**
(a) 12 (b) 16 (c) 17 (d) 19

67. The HCF of two natural numbers m and n is 24 and their product is 552. How many sets of values of m and n are possible ? **[2014-II]**

(a) 1
(b) 2
(c) 4
(d) No set of m and n is possible satisfying the given conditions

68. The LCM of two numbers is 90 times their HCF. The sum of LCM and HCF is 1456. If one of the numbers is 160, then what is the other number ? **[2014-II]**

(a) 120 (b) 136
(c) 144 (d) 184

69. The HCF and LCM of two polynomials are $(x + y)$ and $(3x^5 + 5x^4y + 2x^3y^2 - 3x^2y^3 - 5xy^4 - 2y^5)$ respectively. If one of the polynomials is $(x^2 - y^2)$, then the other polynomial is **2015-I]**

(a) $3x^4 - 8x^3y + 10x^2y^2 + 7xy^3 - 2y^4$
(b) $3x^4 - 8x^3y - 10x^2y^2 + 7xy^3 + 2y^4$
(c) $3x^4 + 8x^3y + 10x^2y^2 + 7xy^3 + 2y^4$
(d) $3x^4 + 8x^3y - 10x^2y^2 + 7xy^3 + 2y^4$

70. What is the sum of digits of the least multiple of 13, which when divided by 6, 8 and 12 leaves 5, 7 and 11 respectively as the remainders ? **[2015-II]**

(a) 5 (b) 6 (c) 7 (d) 8

71. A number when divided by 7 leaves a remainder 3 and the resulting quotient when divided by 11 leaves a remainder 6. If the same number when divided by 11 leaves a remainder m and the resulting quotient when divided by 7 leaves a remainder n. What are the values of m and n respectively? **[2015-II]**

(a) 1 and 4 (b) 4 and 1 (c) 3 and 6 (d) 6 and 3

72. If $(x+1)$ is the HCF of $Ax^2 + Bx + C$ and $Bx^2 + Ax + C$ where $A \neq B$, then the value of C is **[2015-II]**

(a) A (b) B (c) A–B (d) 0

73. The LCM of two numbers is 12 times their HCF. The sum of HCF and LCM is 403. If one of the numbers is 93, then the other number is **[2015-II]**

(a) 124 (b) 128 (c) 134 (d) 138

74. The sum and difference of two expressions are $5x^2 - x - 4$ and $x^2 + 9x - 10$ respectively. The HCP of the two expression will be **[2016-I]**

(a) $(x+1)$ (b) $(x-1)$
(c) $(3x+7)$ (d) $(2x-3)$

75. Consider the following in respect of natural numbers a, b and c: **[2016-I]**

1. LCM $(ab, ac) = a$ LCM (b, c)
2. HCF $(ab, ac) = a$ HCF (b, c)
3. HCF $(a, b) <$ LCM (a, b)
4. HCF (a, b) divides LCM (a, b)

Which of the above are correct?

(a) 1 only (b) 3 and 4 only
(c) 1, 2 and 4 only (d) 1, 2, 3 and 4

76. There are two numbers p and q such that their HCF is 1. Which of the following statements are correct? **[2016-II]**

1. Both p and q may be prime.
2. One number may be prime and the other composite.
3. Both the numbers may be composite.

Select the correct answer using the code given below : ,

(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

77. Consider the following statements : **[2017-I]**

1. If a = bc with HCF (b, c) = 1, then HCF (c, bd) = HCF (c, d).
2. If a = bc with HCF (b, c) = 1, then LCM (a, d) = LCM (3, bd).

Which of the above statements is/are correct ? **[2017- I]**

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

78. What is the LCM of $x^3 + 8$, $x^2 + 5x + 6$ and $x^3 + 4x^2 + 4x$?

(a) $x(x+2)^2(x+3)(x^2-2x+4)$
(b) $x(x-2)^2(x-3)(x^2+2x+4)$
(c) $(x+2)^2(x+3)(x^2-2x+4)$
(d) $(x-2)^2(x-3)(x^2-2x+4)$

79. The HCF of two expressions p and q is 1. What is the reciprocal of their LCM ? **[2017- I]**

(a) p + q (b) p – q
(c) pq (d) $(pq)^{-1}$

80. How many numbers from 1 to 1000 are divisible by 2, 3, 4 and 5? **[2017- II]**

(a) 16 (b) 17
(c) 32 (d) None of the above

81. The product of two non-zero expressions is $(x + y + z)p^3$. If their HCF is p^2, then their LCM is **[2017- II]**

(a) $(x+y+z)$ (b) $(x+y+z)p^2$
(c) $(x+y+z)p^5$ (d) $(x+y+z)p$

82. If the HCF of polynomials **[2017- II]**
$f(x) = (x-1)(x^2+3x+a)$ and
$g(x) = (x+2)(x^2+2x+b)$ is (x^2+x-2),
then what are the values of a and b respectively?

(a) 2, 2 (b) 2, –3
(c) –1, –3 (d) –2, –1

83. How many numbers between 500 and 1000 are divisible by 13? **[2017- II]**

(a) 36 (b) 37 (c) 38 (d) 39

84. There are two numbers which are greater than 21 and their LCM and HCF are 3003 and 21 respectively. What is the sum of these numbers? **[2018-I]**

(a) 504 (b) 508 (c) 514 (d) 528

85. Walls (excluding their roofs and floors) of 5 identical rooms having length, breadth and height 6 m, 4 m and 2.5 m respectively are to be painted. Paints are available only in cans of 1 L and one litre of paint can be used for painting 20 square metres. What is the number of cans required for painting? **2018-I]**
(a) 10 (b) 12 (c) 13 (d) 14

86. HCF and LCM of two polynomials are $(x + 3)$ and $(x^3 - 9x^2 - x + 105)$ respectively. If one of the two polynomials is $x^2 - 4x - 21$, then the other is **[2018-II]**
(a) $x^2 + 2x - 21$ (b) $x^2 + 2x + 15$
(c) $x^2 - 2x - 15$ (d) $x^2 - x - 15$

87. The product of two integers p and q, where p > 60 and q > 60, is 7168 and their HCF is 16. The sum of these two integers is **[2018-II]**
(a) 256 (b) 184 (c) 176 (d) 164

88. What is the LCM of the polynomials **[2019-II]**
$x^3 + 3x^2 + 3x + 1$,
$x^3 + 5x^2 + 5x + 4$ and
$x^2 + 5x + 4$?
(a) $(x+1)^3(x+4)(x^2+x+1)$ (b) $(x + 4)(x^2 + x + 1)$
(c) $(x+1)(x^2 + x + 1)$ (d) $(x+1)^2(x+4)(x^2+x+1)$

89. HCF of two numbers is 12. Which one of the following can never be their LCM ? **[2019-II]**
(a) 80 (b) 60 (c) 36 (d) 24

90. X, Y and Z start at same point and same time in the same direction to run around a circular stadium. X completes a round in 252 seconds, Y in 308 seconds and Z in 198 seconds. After what time will they meet again at the starting point ? **[2019-II]**
(a) 26 minutes 18 seconds (b) 42 minutes 36 seconds
(c) 45 minutes (d) 46 minutes 12 seconds

91. What is the LCM of $\frac{1}{3}, \frac{5}{6}, \frac{2}{9}, \frac{4}{27}$? **[2019-II]**
(a) $\frac{5}{18}$ (b) $\frac{1}{27}$ (c) $\frac{10}{27}$ (d) $\frac{20}{3}$

92. Two unequal pairs of numbers satisfy the following conditions: **[2020-I]**
(i) The product of the two numbers in each pair is 2160.
(ii) The HCF of the two numbers in each pair is 12.
If x is the mean of the numbers in the first pair and y is the mean of the numbers in the second pair, then what is the mean of x and y ?
(a) 60 (b) 72 (c) 75 (d) 78

93. If $(x + k)$ is the HCF of $x^2 + 5x + 6$ and $.x^2 + 8x + 15$, then what is the value of k ? **[2020-I]**
(a) 5 (b) 3 (c) 2 (d) 1

94. What is the least perfect square which is divisible by 3, 4, 5, 6 and 7 ? **[2020-I]**
(a) 1764 (b) 17640 (c) 44100 (d) 176400

HINTS & SOLUTIONS

1. (a) Let $f_1(x) = x^3 + 3x^2y + 2xy^2$
$= x(x^2 + 3yx + 2y^2)$
$= x(x^2 + 2xy + xy + 2y^2)$
$= x[x(x + 2y) + y(x + 2y)]$
$= x(x + y)(x + 2y)$
and $f_2(x) = x^4 + 6x^3y + 8x^2y^2$
$= x^2(x^2 + 6xy + 8y^2)$
$= x^2(x^2 + 2xy + 4xy + 8y^2)$
$= x^2[x(x + 2y) + 4y(x + 2y)]$
$= x^2(x + 2y)(x + 4y)$
$\therefore$ HCF of $f_1(x)$ and $f_2(x)$ $= x(x + 2y)$

2. (d) $(x + k)$ is the HCF of $(x^2 + ax + b)$ and $(x^2 + cx + d)$.
$\therefore$ $(-k)^2 + a(-k) + b = 0$
$= (-k)^2 + c(-k) + d$
$\Rightarrow$ $(a - c)k = (b - d)$
$\Rightarrow$ $k = \frac{b-d}{a-c}$

3. (a) Let $f_1(x) = x^3 + mx^2 - x + 2m$
and $f_2(x) = x^2 + mx - 2$
Let $m = 1$
$\therefore$ $f_1(x) = x^3 + x^2 - x + 2$
and $f_2(x) = x^2 + x - 2 = (x + 2)(x - 1)$
When $x = 1$,
$f(1) = 1 + 1 - 1 + 2 \neq 0$
When $x = -2$,
$f(-2) = (-2)^3 + (-2)^2 - (-2) + 2 = 0$
Required value of m is 1.

4. (b) $24 = 12 \times 2$,
$36 = 12 \times 3$,
$48 = 12 \times 4$,
and $72 = 12 \times 6$
$\therefore$ HCF (24, 36, 48, 72) = 12
Total pieces = 2 + 3 + 4 + 6 = 15

5. (a) LCM of 2, 4, 6, 8 and 10 is 120s.
i.e., 2 min after tolling together.
Total in 20 min $= \frac{\text{Total time}}{\text{LCM } t \text{ intervals}}$
In 20 min tolling $= \frac{20 \text{ min}}{2 \text{ min}} = 10$ times.

6. (a) Let first number = x, second number = y
$\therefore$ LCM × HCF = Product of numbers = $x \times y$
Also, LCM = 16 HCF, LCM + HCF = 850 and
$x = 50$

$\therefore$ 17 HCF = 850
$\Rightarrow$ HCF = 50
Now, LCM = 16 × 50 = 800
$\therefore$ 800 × 50 = 50 × y
$\therefore$ $y = 800$

7. (c) HCF of m and n is 1.
$\therefore$ HCF $(m+n, m) = 1$
and HCF $(m-n, n) = 1$

8. (c) Let $P = 10, q = 11, r = 13$ (Co-prime numbers)
LCM of (10, 11, 13) = 1430
HCF (10, 11, 13) = 1
$mn = 1430 \times 1 = 1430$
Also, $pqr = 10 \times 11 \times 13 = 1430$
So, $mn = pqr$

9. (d) We know that LCM is the multiple of HCF. So that 55 cannot be HCF because it is not divisor of 150.

10. (d) Let $f_1(x) = x^2 + bx - x - b$
$= x(x+b) - 1(x+b)$
$= (x-1)(x+b)$
and $f_2(x) = x^2 + xa - x - a$
$= x(x+a) - 1(x+a)$
$= (x+a)(x-1)$
$\therefore$ HCF of $f_1(x)$ and $f_2(x) = (x-1)$

11. (b) HCF of $x^2 + x - 12$ and $2x^2 - kx - 9$ is $(x-k)$,
then $x = k$ will be the factor of $2x^2 - kx - 9$
$\therefore$ $2k^2 - k^2 - 9 = 0$
$\Rightarrow$ $k^2 - 9 = 0$
$k = \pm 3$
and factor of $x^2 + x - 12$ are $(x+4)(x-3)$.
Hence, the value of k is 3.

12. (a) Let $f_1(x) = 6x^3 + 60x^2 + 150x$
$= 6x(x^2 + 10x + 25)$
$= 3 \times 2 \times x \times (x+5)^2$
and $f_2(x) = 3x^4 + 12x^3 - 15x^2$
$= 3x^2(x^2 + 4x - 5)$
$= 3x^2(x^2 + 5x - x - 5)$
$= 3x^2(x+5)(x-1)$
$\therefore$ LCM of $f_1(x)$ and $f_2(x)$
$= 3 \times 2 \times x^2 \times (x+5)^2(x-1)$
$= 6x^2(x+5)^2(x-1)$

13. (d) Let $f_1(x) = (x+2)^2(x-2)$
and $f_2(x) = x^2 - 4x - 12 = (x-6)(x+2)$
$\therefore$ LCM of $f_1(x), f_2(x) = (x+2)^2(x-2)(x-6)$

14. (a) $(x+4)$ is HCF, so it will be common in both expression.
$x = -4$ will make each one zero.
$\therefore$ $2(-4)^2 + k(-4) - 12 = 0$
and $(-4)^2 + (-4) - 2k - 2 = 0$
$\Rightarrow$ $32 - 12 = 4k$
and $16 - 6 = 2k$
$\therefore$ $k = 5$

15. (d) $x + k$ is the HCF of given expression, then
$x = -k$, root of the function
Now, $a(-k)^2 - ak + b = 0 = k^2 - ck + d$
$\Rightarrow$ $k^2(a-1) - (a-c)k + b - d = 0$
$$\Rightarrow \quad k = \frac{(a-c) \pm \sqrt{(a-c)^2 - 4(a-1)(b-d)}}{2(a-1)}$$

16. (c) Minimum number of rows $= \frac{21}{7} + \frac{42}{7} + \frac{56}{7} = 17.$

17. (a) Given numbers are 392, 486 and 627.
For same remainder
486 – 392 = 94
627 – 486 = 141
627 – 392 = 235
HCF of (94, 141, 235) = 47

18. (b) LCM of $[\{x^2 - (y+z)^2\}, \{(x+z)^2 - y^2\}, \{(x-y)^2 - z^2\}]$
$=$ LCM of $[(x+y+z)(x-y-z), (x+z+y)$
$(x+z-y), (x-y+z)(x-y-z)]$
$= (x+y+z)(x-y-z)(x-y+z)$

19. (c) Let $f(x) = x^2 + ax + b$ and $g(x) = x^2 + cx + d$
$\therefore$ $f(-2) = 4 - 2a + b = 0 \Rightarrow b - 2a = -4$
and $g(-2) = 4 - 2c + d = 0 \Rightarrow d - 2c = -4$
$\therefore$ $b - 2a = d - 2c \Rightarrow b + 2c = d + 2a$

20. (a) Let $f_1(x) = x^3 + cx^2 - x + 2c$
and $f_2(x) = x^2 + cx - 2$
Let $c = 1$
$\therefore$ $f_1(x) = x^3 + x^2 - x + 2 = (x+2)(x^2 - x + 1)$
$f_2(x) = x^2 + x - 2 = (x+2)(x-1)$
Here, $(x+2)$ is HCF of $f_1(x)$ and $f_2(x)$ which is linear.

21. (d) Let 2 natural numbers m and n are 12 are 18.
$\therefore$ HCF of 12 and 18 $= g_{mn} = 6$
Let third natural number k is 20.
So, $g_{nk} =$ HCF (18, 20) = 2 and $g_{mk} = 4$
So, $g_{m(nk)} =$ HCF (12, 360) = 12
So, $g_{(mn)k} =$ HCF (216, 20) = 4
From I statement, $g_{m(nk)} \neq g_{(mn)k}$
From II statement, $g_{mn}\, g_{nk} \neq g_{mk}$
So, both statements are false.

22. (b) Given that $x^3 - 27 = (x-3)(x^2 + 9 + 3x)$

$$\begin{array}{r|l} & \quad x+1 \\ x^2+9+3x & x^3+4x^2+12x+k \\ & \underline{-x^3 \pm 3x^2 \pm 9x} \\ & x^2+3x+k \\ & \underline{-x^2 \pm 3x \pm 9} \\ & k-9 \end{array}$$

Hence the value of k should be 9.

23. (a) Given expression can be written as
$3(a^3 - b^3) = 3(a-b)(a^2+b^2+ab)$
and $11(a^4 - b^4) = 11(a-b)(a+b)(a^2+b^2)$
$\therefore$ Required LCM $= 33(a^3 - b^3)(a+b)(a^2+b^2)$

24. (d) LCM of 42, 72 and 84 = 504
Difference between divisor and remainder
$= 25 - 42 = 55 - 72 = 67 - 84$
$= -17$
Required number $= 504 - 17 = 487$.

25. (b) Here n = Integer.
$m = 2n + 1$ and $k = 9n + 4$
HCF of (m, k) = HCF $(2n+1, 9n+4)$

$$\begin{array}{l} 2n+1\overline{)\,9n+4\,}(4 \\ \quad\;\;\underline{-8n\pm 4} \\ \qquad\; n\overline{)\,2n+1\,}(2 \\ \qquad\quad\;\underline{-2n} \\ \qquad\qquad 1\overline{)\,n\,}(n \\ \qquad\qquad\quad\underline{n} \\ \qquad\qquad\quad \times \end{array}$$

HCF of $(m, k) = 1$

26. (c) If we put $x = y$ in the expression $x^{10} - y^{10}$, then $x^{10} - y^{10} = 0$. So $(x - y)$ is factor of $x^{10} - y^{10}$
Similarly, if we put $x = -y$ in the expression $x^{10} - y^{10} = 0$. So $(x + y)$ is a factor of $x^{10} - y^{10}$
So, statement I and IV are true.

27. (c) In the given options, only 372 is not divisible by 24. Therefore, LCM of numbers cannot be 372.

28. (a) LCM of $(6, 9, 12) = 36$
Then, number is the form of $36p + 4$.
Since, the required number between 300 and 400.
$\therefore\; p = 9$ and 10
$\therefore$ Required sum $= 328 + 364 = 692$

29. (c) LCM of $(4, 5, 8, 9) = 360$
Difference between divisor and remainder.
$= 3 - 4 = 4 - 5 = 7 - 8$.
$= 8 - 9 = -1$
Hence, Required value $= 360 - 1 = 359$.

30. (c) $\because f(x)$ and $g(x)$ vanish at $x = \dfrac{1}{2}$
So, $(2x - 1)$ is a factor of $f(x)$ and $g(x)$ both.
Hence, a factor of $f(x)$ and $g(x) = 2x - 1$.

31. (a) Here, $52 - 33 = 78 - 59 = 117 - 98 = 19$
Now, $52 = 13 \times 2 \times 2$
$78 = 13 \times 2 \times 3$
$117 = 13 \times 3 \times 3$
$\therefore$ LCM $= 13 \times 2 \times 2 \times 3 \times 3 = 468$
$\therefore$ Required number $= 468 - 19 = 449$
Hence, the sum of digits is 17.

32. (c) Given function
$f_1(x) = 4x^3 + 3x^2y - 9xy^2 + 2y^3$
$= (x - y)(4x^2 + 7xy - 2y^2)$
$= (x - y)(x + 2y)(4x - y)$
and another function
$f_2(x) = x^2 + xy - 2y^2 = (x - y)(x + 2y)$
HCF of $f_1(x)$ and $f_2(x)$ $= (x - y)(x + 2y)$

33. (a) LCM of 2, 3, 5 = 30
30 is least number which is divided by 2, 3 and 5.
The number of which gives reminder $= 30 + 1 = 31$.

34. (b) $\because\; p(x) \times q(x) = \text{LCM} \times \text{HCF}$
$\therefore\; (8x^3 + 32x^2 + 32x) \times q(x)$
$= 2x(x+2) \times 24x(x+2)^2(x-2)$

$$\Rightarrow q(x) = \frac{48x^2(x+2)^3(x-2)}{8x(x^2+4x+4)}$$

$$= \frac{6x(x+2)^3(x-2)}{(x+2)^2}$$

$= 6x(x^2 - 4) = 6x^3 - 24x$

35. (c) For the least number of tiles to pave the floor, the size of tiles should be maximum.
$\therefore$ Side of tiles = HCF of 999 and 407 = 37 cm

$$\therefore \text{ Required number of tiles} = \frac{999 \times 407}{37 \times 37} = 297$$

36. (b) Multiply by 100
$3.0 = 300$
$1.2 = 120$
$0.06 = 6$
Now, HCF of $(300, 120, 6) = 6$
So, HCF $(3.0, 1.2, 0.06) = 0.06$

37. (d) Here given the HCF of 144, x, 192 is 12.

$$\begin{array}{l} \quad\;\; 1 \\ 144\overline{)192} \\ \quad\;\; \underline{144} \\ \quad\;\; \times 48\overline{)x} \end{array}$$

x can not be 48 because HCF is 12.

38. (a) $x^4 - x^2 - 6 = x^4 - 3x^2 + 2x^2 - 6$
$= x^2(x^2 - 3) + 2(x^2 - 3)$
$= (x^2 - 3)(x^2 + 2)$
and $x^4 - 4x^2 + 3 = x^4 - 3x^2 - x^2 + 3$
$= x^2(x^2 - 3) - 1(x^2 - 3)$
$= (x^2 - 3)(x^2 - 1)$
$\therefore$ HCF $= (x^2 - 3)$

39. (a) Here, $a^3b - ab^3 = ab(a^2 - b^2) = ab(a-b)(a+b)$
$a^3b^2 + a^2b^3 = a^2b^2(a+b)$ and
$ab(a+b) = ab(a+b)$
$\therefore$ LCM $[(a^3b - ab^3), (a^3b^2 + a^2b^3), ab(a+b)]$
$= a^2b^2(a+b)(a-b) = a^2b^2(a^2 - b^2)$

40. (a) Let $f_1(x) = x^4 - 3x + 2 = (x-1)(x^3 + x^2 + x - 2)$
$f_2(x) = x^3 - 3x^2 + 3x - 1 = (x-1)^3$
and $f_3(x) = x^4 - 1 = (x-1)(x+1)(x^2+1)$
$\therefore$ HCF of $f_1(x), f_2(x), f_3(x) = x - 1$

41. (b) We find out the LCM of given numbers.
$\therefore$ LCM (5, 6, 7, 8) $= \text{LCM}(5, 2\times 3, 7, 2\times 4)$
$= 2 \times 5 \times 3 \times 7 \times 4 = 840$
$\therefore$ Required number $= 840 + 3 = 843$

42. (b) LCM $\left(\frac{2}{3}, \frac{7}{9}, \frac{14}{15}\right) = \frac{\text{LCM}(2,7,14)}{\text{HCF}(3,9,15)} = \frac{14}{3}$

43. (c) Let the numbers be $24x$ and $24y$.
$\therefore$ $24x \times 24y = 6912 \Rightarrow xy = \frac{6912}{(24)^2} = 12$
$\therefore$ LCM is $24 \times 12 = 288$

44. (b) Let two numbers by $29x$ and $29y$.
$\therefore$ $29x + 29y = 232 \Rightarrow x + y = 8$
$\Rightarrow$ $(x, y) = (1, 7), (3, 5)$
Since, one such pair is 87 and 145.
Hence, the other pairs is 203 and 29.

45. (c) Let $f_1(x) = 36(3x^4 + 5x^3 - 2x^2)$
$= 36x^2(3x^2 + 5x - 2)$
$= 36 \cdot x^2\{3x^2 + 6x - x - 2\}$
$= 36 \cdot x^2\{3x(x+2) - 1(x+2)\}$
$= 2 \times 2 \times 3 \times 3 \times x \times x \times (x+2)(3x-1)$
$f_2(x) = 9(6x^3 + 4x^2 - 2x)$
$= 9x(6x^2 + 4x - 2)$
$= 18x(3x^2 + 2x - 1)$
$= 18x(3x^2 + 3x - x - 1)$
$= 3 \times 3 \times 2 \times x(3x-1)(x+1)$
$f_3(x) = 54(27x^4 - x)$
$= 54x(27x^3 - 1)$
$= 2 \times 3 \times 3 \times 3 \times x \times (3x-1)(9x^2 + 3x + 1)$
$\therefore$ HCF of $f_1(x), f_2(x), f_3(x)$
$= 2 \times 3 \times 3 \times x \times (3x-1)$
$= 18x(3x-1)$

46. (a) Second number $= \frac{\text{LCM} \times \text{HCF}}{\text{First number}} = \frac{72 \times 12}{24} = 36$
$\therefore$ Difference between two numbers $= 36 - 24 = 12$

47. (b) Here, $(x-6)$ is the HCF of $x^2 - 2x - 24$ and $x^2 - kx - 6$
So, that $(x-6)$ is a factor of both expression.
According to question,
$\Rightarrow$ $f(x_1) = f(x_2)$ at $(x_1 = x_2 = 6)$
$\Rightarrow$ $(6)^2 - 2(6) - 24 = (6)^2 - k(6) - 6$ (By condition)
$\Rightarrow$ $36 - 12 - 24 = 36 - 6k - 6$
$\Rightarrow$ $0 = 30 - 6k \Rightarrow 6k = 30$
$\therefore$ $k = 5$

48. (b) Here 13850 and 17030 are two numbers which leaves remainder 17.
Now, $13850 - 17 = 13833$
$17013 - 17 = 17013$
$13833 = 159 \times 3 \times 29$
$17013 = 107 \times 159$
$\therefore$ HCF = 159.
It is the greatest number.

49. (a) Given that, three planets revolves the Sun once in 200, 250, 300 days,.
$\therefore$ Required time = LCM of (200, 250, 300) = 3000 days
Now, after 3000 days they all come relatively to the same position as at a certain point of time in their orbits.

50. (d) HCF of 18 and 15 = 3
LCM of 18 and 15 $= 2 \times 3 \times 3 \times 5 = 90$
$\therefore$ Product of HCF and LCM of both numbers
$= 3 \times 90 = 270$

51. (a) Let $f_1(x) = x^3 + 8$
$= x^3 + 2^3 = (x+2)(x^2 - 2x + 4)$
$= (x+2)(x-2)^2 = (x+2)(x-2)(x-2)$
$f_2(x) = x^2 + 5x + 6 = x^2 + 3x + 2x + 6$
$= x(x+3) + 2(x+3)$
$= (x+3)(x+2)$
and $f_3(x) = x^3 + 2x^2 + 4x + 8$
$= x^2(x+2) + 4(x+2)$
$= (x+2)(x^2+4)$
$\therefore$ HCF of $[f_1(x), f_2(x), f_3(x)] = x + 2$

52. (c) Let $f_1(x) = x^3 - x^2 - 2x = x(x^2 - x - 2)$
$= x\{x^2 - 2x + x - 2\}$
$= x\{x(x-2) + 1(x-2)\} = x(x+1)(x-2)$
and $f_2(x) = x^3 + x^2 = x^2(x+1) = x \cdot x(x+1)$
$\therefore$ LCM of $[f_1(x), f_2(x)] = x(x+1) \cdot x(x-2)$
$= x^2(x+1)(x-2) = x^2(x^2 - x - 2)$
$= x^4 - x^3 - 2x^2$

53. (a) Let $f_1(x) = (x^4 - y^4) = [(x^2)^2 - (y^2)^2]$
$= (x^2 - y^2)(x^2 + y^2)$
$= (x-y)(x+y)(x^2+y^2)$
and $f_2(x) = (x^6 - y^6) = (x^3)^2 - (y^3)^2$
$= (x^3 + y^3)(x^3 - y^3)$
$= (x+y)(x^2 - xy + y^2)(x-y)(x^2 + xy + y^2)$
$= (x-y)(x+y)(x^2 - xy + y^2)(x^2 + xy + y^2)$
$\therefore$ HCF of $[f_1(x), f_2(x)] = (x-y)(x+y) = x^2 - y^2$

54. (b) From formula,
$\therefore$ (HCF to two numbers) × (LCM of two numbers)
= (First number) × (Second number)
$\therefore$ Second number $= \frac{33 \times 2376}{297} = 264$

55. (a) $x^2 + 2x - 8 = x^2 + 4x - 2x - 8$
$= x(x+4) - 2(x+4) = (x-2)(x+4)$
$x^3 - 4x^2 + 4x = x^3 - 2x^2 - 2x^2 + 4x$
$= x^2(x-2) - 2x(x-2)$
$= (x^2 - 2x)(x-2) = x(x-2)(x-2)$
$x^2 + 4x = x(x+4)$
So, LCM of $(x^2 + 2x - 8)$, $(x^3 - 4x^2 + 4x)$ and $(x^2 + 4x) = x(x-2)(x+4)(x-2)$
$= x(x+4)(x-2)^2$

56. (c) $a^2b^4 + 2a^2b^2 = a^2b^2(b^2 + 2)$
and $(ab)^7 - 4a^2b^9 = a^7b^7 - 4a^2b^9$
$= a^2b^2(a^5b^5 - 4b^7)$
HCF of $[(a^2b^4 + 2a^2b^2), ((ab)^7 - 4a^2b^9) = a^2b^2$

57. (a) HCF of two numbers is 98. It means that 98 is common in both the numbers. Therefore, the sum of these two numbers also be multiple of 98. So, 1372 is divided by 98.

58. (a) For integers a, b and c, if HCF $(a, b) = 1$ and HCF $(a, c) = 1$, then HCF $(a, bc) = 1$

59. (a) Let $f_1(x) = 8(x^5 - x^3 + x)$
$= 4 \times 2 \times x(x^4 - x^2 + 1)$
and $f_2(x) = 28(x^6 + 1) = 7 \times 4[(x^2)^3 + (1)^3]$
$= 4 \times 7 \times (x^2 + 1)(x^4 - x^2 + 1)$
$\therefore$ HCF of $f_1(x)$ and $f_2(x) = 4(x^4 - x^2 + 1)$

60. (c) Time after which they will hit the target again together = LCM (5, 6, 7 and 8)

$$\begin{array}{c|l} 2 & 5,6,7,8 \\ \hline & 5,3,7,4 \end{array}$$

$= 5 \times 3 \times 7 \times 2 \times 4 = 840$ s

They hit after target together $= \dfrac{840}{60} = 14$ min.

So, next time target = 9 : 00 am + 14 min
= 9 : 14 am

61. (b) HCF of $(22n + 7, 33n + 10)$ is always 1
Examples
For $n = 1$, HCF $(29, 43) \Rightarrow$ HCF = 1
For $n = 2$, HCF $(51, 76) \Rightarrow$ HCF = 1
For $n = 3$, HCF $(73, 109) \Rightarrow$ HCF = 1
since $22n$ and $33n$ are multiples of 11, therefore $22n + 7$ and $33n + 10$ are not the multiple of 11.
Hence, HCF of $22n + 7$ and $33 + 10$ will not be equal to 11 or n.

62. (c) Given that HCF $(a, b) = 1$ means that a and b are co-prime numbers.
So, HCF $(a + b, a - b)$
Let $a = 4, b = 3$
HCF $(4, 3) = 1$
Now, HCF $(3 + 4, 4 - 3) =$ HCF $(7, 1)$
HCF is equal = 1
Let $a = 23$ and $b = 17$
HCF $(23, 17) = 1$
HCF $(23 + 17, 23 - 17) =$ HCF $(40, 6) = 2$
So, HCF $(a + b, a - b) =$ Either 1 or 2

63. (c) HCF $\left(\dfrac{a}{\text{HCF}(a,b)}, \dfrac{b}{\text{HCF}(a,b)}\right)$

$$= \frac{\text{HCF (a,b)}}{\text{LCM(HCF (a,b), HCF (a,b))}} = \frac{\text{HCF (a,b)}}{\text{HCF (a,b)}} = 1$$

Example 1: $a = 12$ and $b = 24$

HCF of $\left(\dfrac{12}{\text{HCF}(12,24)}, \dfrac{24}{\text{HCF}(12,24)}\right)$

$= \text{HCF}\left(\dfrac{12}{12}, \dfrac{24}{12}\right) = \text{HCF}(1, 2) = 1.$

Example 2: $a = 19, b = 23$

HCF of $\left(\dfrac{19}{\text{HCF}(19,23)}, \dfrac{23}{\text{HCF}(19,23)}\right)$

$= \text{HCF of}\left(\dfrac{19}{1}, \dfrac{23}{1}\right)$

$= \text{HCF of}(19, 23) = 1$

64. (c) $\because$ HCF $(a, b) = 5$
Let $a = 5x$ and $b = 5y$
$\therefore$ $5x + 5y = 65$
$\Rightarrow$ $x + y = 13$
$\therefore$ Number of pairs of (x, y)
$= (1, 12), (2, 11) ((3, 10), (4, 9), (5, 8), (6, 7)$
Hence, total number of solution is less than 65.

65. (c) Given, LCM of two intergers is 1237, which is a prime number.
So, their HCF is 1

66. (c) HCF of 48, 72 and 84 $= 2^2 \times 3 = 12$

minimum number of rows $= \dfrac{48}{12} + \dfrac{72}{12} + \dfrac{84}{12}$

$= 4 + 6 + 7 = 17$

67. (d) HCF of two natural numbers m and $n = 24$
$m \times n = 552$
LCM of two natural numbers

$= \dfrac{\text{Product of } m \text{ and } n}{\text{HCF of } m \text{ and } n}$

$= \dfrac{552}{24} = 23$

Therefore, no set of m and n is possible satisfying the given condition.

68. (c) Let the HCF of two number = x
The LCM of two numbers = $90x$
According to question
$LCM + HCF = 1456$
$90x + x = 1456$
$x = 16$
HCF of two numbers = 16,
LCM of two number = 1440
LCM × HCF = I^{st} numbers × 2^{nd} number

$\Rightarrow$ 2^{nd} Number = $\dfrac{1440 \times 16}{160} = 144$

69. (c) HCF × LCM = 1^{st} polynomial × 2^{nd} polynomial

$\Rightarrow$ 2^{nd} polynomial $= \dfrac{HCF \times LCM}{1^{st} \text{ Polynomial}}$

$= \dfrac{(x+y)\times(3x^5+5x^4y+2x^3y^2-3x^2y^3-5xy^4-2y^5)}{(x^2-y^2)}$

$= 3x^4 + 8x^3y + 10x^2y^2 + 7xy^3 + 2y^4$

70. (d) LCM of 6, 8, 12 is 24
Number is when divided by 6, 8 and 12 leaves 5, 6 and 11 as remainders, as 6 – 5 = 1, 8 – 7 = 1, 12 – 11 = 1
So (24 k – 1) will be divisible by 13.

$\Rightarrow \dfrac{24k-1}{13}$ is divisible for $k_{min} = 6$

at k = 6
Number is = 24 k – 1
= 24 × 6 – 1 = 143
Sum of its digit = 1 + 4 + 3 = 8
So, option (d) is correct

71. (a) This is an example of successive division. Let the number be N. The number and successive quotients, the successive divisors and the corresponding remainders are tabulated below :

Quotients	N	q_1	q_2
Dinsors	7	11	
Remainder	3	6	

One value of N is 6(7) + 3 = 45
In general, N = 77K + 45
$\therefore$ N = 11 (7k + 4) + 1 i.e. m = 1
and q_1 = 7k + 4, q_2 = k and n = 4
$\therefore$ (m, n) = (1, 4)
So, option (a) is correct.

72. (d) If (x + 1) is HCF then x = –1 will satisfy both the equation-
$A(-1)^2 + B(-1) + C = 0$
$\Rightarrow$ A – B + C = 0 ----------(a)
$B(-1)^2 + A(-1) + C = 0$
$\Rightarrow$ B – A + C = 0 ----------(b)
Adding equ. (a) and (b)
A – B + C + B – A + C = 0
2C = 0

$\boxed{C = 0}$

So, option (d) is correct.

73. (a) LCM × HCF = 1^{st} number × 2^{nd} number
Let the 1^{st} number be 'A' and 2^{nd} number be 'B'.
L × H = A × B
L × 12 L = 93 × B
$12 L^2 = 93 B$ -------------(i)
L + H = 403 ------------(ii)
L + 12L = 403

$L = \dfrac{403}{13} = 31$

12 × 31 × 31 = 93 B

$B = \dfrac{12 \times 31 \times 31}{93}$

B = 4 × 31
B = 124
So, option (a) is correct.

74. (b) Let p(x) and q(x) be two expressions,
then $p(x) + q(x) = 5x^2 - x - 4$ ...(i)
and $p(x) - q(x) = x^2 + 9x - 10$ ...(ii)
Solving (i) and (ii), we get
$2p(x) = 6x^2 + 8x - 14$ and $2q(x) = 4x^2 - 10x + 6$
$p(x) = 3x^2 + 4x - 7 \Rightarrow q(x) = 2x^2 - 5x + 3$
$\Rightarrow$ p(x) = (3x + 7) (x – 1) and q(x) = (x – 1) (2x – 3)
H.C.F. of two expressions p(x) and q(x) is (x – 1)
$\therefore$ Option (b) is correct.

75. (a)

76. (d) All the statement are true example for each statement.
1. take number 2 and 3
2. take number 7 and 25
3. take number 25 and 16
and many more.

77. (a) Putting any random number
eg a = 6, b = 2, c = 3 and d = 6 in 1 statement
it can be varified that this statement is correct and statement 2 is incorrect.

78. (a) $x^3 + 2^3 = (x + 2)(x^2 - 2x + 4)$
$x^2 + 5x + 6 = x^2 + 2x + 3x + 6$
$= (x + 2)(x + 3)$
$x^3 + 4x^2 + 4x = x(x^2 + 4x + 4)$
$x(x + 2)(x + 2)$
$LCM = x(x + 2)^2(x + 3)(x^2 - 2x + 4)$

79. (d) LCM of p and q is 1
LCM = pq

Reciprocal = $\dfrac{1}{pq} = pq^{-1}$

80. (a) LCM of 2, 3, 4 and 5 is 60.
Number of numbers divisible by 60 from 1 to 600 = 10
Number of numbers divisible by 60 from 601 to 900 i.e. from 300 numbers = 5
Number of numbers divisible by 60 from 901 to 1000 i.e. from 100 numbers = 1
Total numbers = 10 + 5 + 1 = 16

81. (d) We are given that HCF = p^2 and the product of two non-zero expressions = $(x + y + z)p^3$
We know that HCF × LCM = Product of two numbers

Therefore,

$p^2 \times \text{LCM} = (x+y+z)p^3 \Rightarrow \text{LCM}$

$= \dfrac{(x+y+z)p^3}{p^2} = (x+y+z)p$

82. (b) Since HCF of two polynomials is x^2+x-2, therefore splitting this polynomial by middle term, we get

$x^2+x-2 = x^2+2x-x-2 = x(x+2)-(x+2)$
$= (x-1)(x+2)$

Being the HCF of the given polynomials, we conclude that $(x-1)(x+2)$ is a factor of $f(x)$ and $g(x)$. Therefore, dividing $f(x)$ by the HCF and dividing $g(x)$ by the HCF will give the values of a and b.

Now, $\dfrac{(x-1)(x^2+3x+a)}{(x-1)(x+2)} = \dfrac{(x^2+3x+a)}{(x+2)}$

and $\dfrac{(x+2)(x^2+2x+b)}{(x-1)(x+2)} = \dfrac{(x^2+2x+b)}{(x-1)}$

Since $(x+2)$ is a factor of (x^2+3x+a), therefore, $x=-2$will satisfy this polynomial. Thus,

$x^2+3x+a=0 \Rightarrow (-2)^2+3.-2+a=0$

$\Rightarrow 4-6+a=0 \Rightarrow -2+a=0 \Rightarrow a=2$

Also, since $(x-1)$ is a factor of x^2+2x+b, therefore, $x=1$ will satisfy this polynomial. Thus,

r $x^2+2x+b=0 \Rightarrow 1+2+b=0 \Rightarrow b+3=0 \Rightarrow b=-3$

Hence, $a=2, b=-3$

83. (c) Consider $a=507, b=988, d=13$

$n = \dfrac{b-a}{d}+1$

$n = \dfrac{988-507}{13}+1$

$n = \dfrac{481}{13}+1$

$n = 37+1$
$n = 38$

84. (a) Let numbers be 21x and 21y
According to the question
$21x \times 21y = 21 \times 3003$

$x+y = \dfrac{21 \times 3003}{21 \times 21} = 143$

Possible pairs of values of x and y are (1, 143) and (11, 13)
Both numbers are greater than 21. Hence, (11, 13) is right pair of co-prime factors
Sum of numbers $= 11 \times 21 + 13 \times 21 = 231 + 273 = 504$

85. (c) length of each room = 6 m
breadth of each room = 4 m
height of each room = 2.5 m
area of walls of each room $= 2(6+4) \times 2.5 = 50\ m^2$
Total area of walls of 5 rooms $= 5 \times 50 = 250\ m^2$
According to the question
$1\ \ell = 20\ m^2$

Required paint $= \dfrac{250}{20} = 12.5\ \ell$

Hence, 13 cans are required.

86. (c) We know that,
Product of two polynomial = HCF × LCM
$\therefore$ One polynomial $\times (x^2-4x-21)$
$= (x+3) \times (x^3-9x^2-x+105)$

$\therefore$ Polynomial $= \dfrac{(x+3)(x-7)(x^2-2x-15)}{(x+3)(x-7)}$

$= (x^2-2x-15)$
$\therefore$ Required polynomial $= (x^2-2x-15)$

87. (c) Let p = 16a and q = 16b
Where a and b are co-prime.
Now, 16a × 16b = 7168
ab = 28
Possible pair of co-prime (a, b) such that p, q > 0,
= (4, 7), or (7, 4)
Now, p = 16 × 4 = 64,
q = 16 × 7 = 112
$\therefore$ (p + q) = 64 + 112 = 176

88. (a) $x^3+3x^2+3x+1 = (x+1)^3$
$x^3+5x^2+5x+4 = (x+4)(x^2+x+1)$
$x^2+5x+4 = (x+1)(x+4)$
L.C.M $= (x+1)^3(x+4)(x^2+x+1)$

89. (a) Let two numbers are 12a and 12b.
Such that H.C.F = 12.
then L.C.M = 12.a.b
Here, L.C.M of these two number must be divisible by 12.
'80' is not divisible by 12, so can not be L.C.M

90. (d) X completes a round in inch 252 sec.
Y completes a round in inch 308 sec.
Z completes a round in inch 198 sec.
L.C.M of 252, 308 and 198 = 2 × 2× 3 × 3 × 7 × 11
= 2772 sec.
= 46 min. 12 sec.

91. (d) L.C.M of $\dfrac{1}{3}, \dfrac{5}{6}, \dfrac{2}{9}, \dfrac{4}{27}$

$= \dfrac{\text{L.C.M of } 1,5,2,4}{\text{H.C.F of } 3,6,9,27} = \dfrac{20}{3}$

92. (b) As, HCF of two numbers are 12.
Let two numbers are12a and 12b
then,12a × 12b = 2160 ⇒ ab = 15
$\therefore$ sets of numbers are (1, 15) and (3, 5)
means of set of numbers are 8 and 4
means of means = 6
$\therefore$ Req. number = 6 × 12 = 72

93. (b) As (x + k) is HCF
$\therefore$ $F(-k) = k^2-5k+6 = k^2-8k+15$
$\Rightarrow$ 3k = 9 $\Rightarrow$ k = 3

94. (c) Reqd. Square = (3 × 2 × 2 × 5 × 7) × (3 × 5 × 7)
= 9 × 49 × 100 = 44100

CHAPTER

3 Decimal Fractions

1. Consider the following decimal numbers **[2007-I]**

I. 1.16666666... II. 1.181181118...

III. 2.010010001... IV. 1.454545...

Which of the above numbers represent(s) rational number (s)?

(a) Only IV (b) II and III

(c) I and IV (d) None of these

2. If $2.5252525... = \frac{p}{q}$ (in the lowest form), then what is the value of $\frac{q}{p}$? **[2007-II]**

(a) 0.4 (b) 0.42525 (c) 0.0396 (d) 0.396

3. Which one of the following is correct ?

3.292929... is **[2007-II]**

(a) an integer (b) a rational number

(c) an irrational number (d) not a real number

4. Consider the following statements: **[2008-I]**

I. $\frac{1}{22}$ cannot be written as a terminating decimal.

II. $\frac{2}{15}$ can be written as a terminating decimal.

III. $\frac{1}{16}$ can be written as a terminating decimal.

Which of the statements given above is/are correct?

(a) Only I (b) Only II (c) Only III (d) II and III

5. If 1 is subtracted from the numerator of a fraction it becomes (1/3) and if 5 is added to the denominator the fraction becomes (1/4). Which fraction shall result, if 1 is subtracted from the numerator and 5 is added to the denominator ? **[2008-I]**

(a) $\frac{5}{12}$ (b) $\frac{7}{23}$ (c) $\frac{1}{8}$ (d) $\frac{2}{3}$

6. Which one of the following is correct? **[2008-I]**

(a) $-\frac{7}{10} < -\frac{2}{3} < -\frac{5}{8}$ (b) $-\frac{5}{8} < -\frac{2}{3} < -\frac{7}{10}$

(c) $-\frac{5}{8} < -\frac{7}{10} < -\frac{2}{3}$ (d) $-\frac{7}{10} < -\frac{5}{8} < -\frac{2}{3}$

7. Which one is the largest among the following ? **[2009-II]**

(a) 0.725 (b) $0.72\overline{5}$

(c) $0.7\overline{25}$ (d) $0.\overline{725}$

8. What is the value of $1.\overline{34} + 4.1\overline{2}$? **[2010-I]**

(a) $\frac{133}{90}$ (b) $\frac{371}{90}$ (c) $5\frac{219}{990}$ (d) $5\frac{461}{990}$

9. Which one of the following is a non-terminating and repeating decimal? **[2010-II]**

(a) $\frac{13}{8}$ (b) $\frac{3}{16}$ (c) $\frac{3}{11}$ (d) $\frac{137}{25}$

10. What is the value of $2.\overline{6} - 1.\overline{9}$? **[2011-I]**

(a) $0.\overline{6}$ (b) $0.\overline{9}$ (c) $0.\overline{7}$ (d) 0.7

11. What is $27 \times 1.\overline{2} \times 5.526\overline{2} \times 0.\overline{6}$ equal to? **[2011-I]**

(a) $121.\overline{57}$ (b) $121.\overline{75}$

(c) $121.7\overline{5}$ (d) None of these

12. What is $3.\overline{76} - 1.4\overline{576}$ equal to ? **[2011-II]**

(a) $2.3\overline{100191}$ (b) $2.3\overline{101091}$

(c) $2.3\overline{110091}$ (d) $2.3\overline{110901}$

13. What is the value of $0.00\overline{7} + 17.\overline{83} + 310.020\overline{2}$? **[2012-I]**

(a) $327.86\overline{638}$ (b) $327.866\overline{38}$

(c) $327.866\overline{83}$ (d) $327.866\overline{8}$

14. What is the value of 0.242424... ? **[2012-II]**

(a) 23/99 (b) 8/33 (c) 7/33 (d) 47/198

15. Representation of $0.2\overline{341}$ in the form $\frac{p}{q}$, where p and q are integers, $q \neq 0$, is **[2013-I]**

(a) $\frac{781}{3330}$ (b) $\frac{1171}{4995}$ (c) $\frac{2341}{9990}$ (d) $\frac{2339}{9990}$

16. Let p be a prime number other than 2 or 5. One would like to express the vulgar fraction l/p in the form of a recurring decimal. Then the decimal will be **[2015-I]**

(a) a pure recurring decimal and its period will be necessarily $(p - 1)$

(b) a mixed recurring decimal and its period will be necessarily $(p - 1)$

(c) a pure recurring decimal and its period will be some factor of $(p - 1)$

(d) a mixed recurring decimal and its period will be some factor of $(p - 1)$

17. The value of $(0.\overline{63} + 0.\overline{37})$ is **[2015-II]**

(a) 1 (b) $\frac{100}{91}$ (c) $\frac{100}{99}$ (d) $\frac{1000}{999}$

18. If **[2016-I]**

$$\frac{61}{19}=3+\cfrac{1}{x+\cfrac{1}{y+\cfrac{1}{z}}}$$

where x, y and z are natural numbers, then what is z equal to ?

(a) 1 (b) 2 (c) 3 (d) 4

19. Which one of the following rational numbers has non–terminating and repeating decimal expansion ? **[2016-II]**

(a) $\frac{15}{1600}$ (b) $\frac{23}{8}$ (c) $\frac{35}{50}$ (d) $\frac{17}{6}$

20. What is $\sqrt{\frac{0.064\times 6.25}{0.081\times 4.84}}$ equal to ? **[2017-I]**

(a) $\frac{10}{99}$ (b) $\frac{100}{99}$ (c) 9 (d) 99

21. What number must be subtracted from both the numerator and the denominator of the fraction $\frac{27}{35}$ so that it becomes $\frac{2}{3}$? **[2017-I]**

(a) 6 (b) 8 (c) 9 (d) 11

22. Which one among the following is the largest ? **[2017-I]**

(a) $\frac{7}{9}$ (b) $\frac{11}{14}$ (c) $\frac{3}{4}$ (d) $\frac{10}{13}$

23. What is the value of $\sqrt[3]{4\frac{12}{125}}$? **[2017-I]**

(a) $1\frac{3}{5}$ (b) $1\frac{2}{5}$ (c) $1\frac{4}{5}$ (d) $2\frac{2}{5}$

24. If the points P and Q represent the real numbers $0.\overline{83}$ and $0.\overline{62}$ on the number line, then the distance between P and Q is **[2017-II]**

(a) $\frac{21}{90}$ (b) $\frac{19}{90}$ (c) $\frac{21}{100}$ (d) $\frac{56}{90}$

25. Which one of the following is correct? **[2018-I]**

(a) Decimal expansion of a rational number is terminating.

(b) Decimal expansion of a rational number is non-terminating.

(c) Decimal expansion of an irrational number is terminating.

(d) Decimal expansion of an irrational number is non-terminating and non-repeating.

26. What is the difference between $0.\overline{9}$ and 0.9? **[2018-I]**

(a) 0 (b) 0.099 (c) 0.1 (d) 0.09

27. Which one of the following decimal numbers is a rational number with denominator 37? **[2018-I]**

(a) 0.459459459 … (b) 0.459459459 (c) 0.0459459459… (d) 0.00459459…

28. What is $0.\overline{53}+0.5\overline{3}$ equal to ? **[2019-I]**

(a) $1.0\overline{68}$ (b) $1.06\overline{8}$ (c) $1.\overline{068}$ (d) 1.068

HINTS & SOLUTIONS

1. (c) Since, 1.16666... and 1.454545... are recurring numbers and we know that recurring numbers represent rational numbers.
So, that I and IV are rational numbers.

2. (d) $\frac{p}{q}=2.\overline{52}\Rightarrow\frac{p}{q}=\frac{252-2}{99}\Rightarrow\frac{p}{q}=\frac{250}{99}$

$\therefore\quad \frac{q}{p}=\frac{99}{250}=0.396$

3. (b) 3.292929... = $3.\overline{29}$ is a non-terminating repeating decimal. Then, it is a rational number.

4. (c) $\frac{1}{22}=0.04545...$

$\frac{2}{15}=0.1333...$

$\frac{1}{16}=0.0625$

$\frac{1}{16}$ can be written as terminating decimal.

5. (c) Let the numerator and denominator of a fraction are x and y, respectively,
According to question,

$\frac{x-1}{y}=\frac{1}{3}\Rightarrow 3x-3=y\Rightarrow 3x-y=3 \quad ...(i)$

and $\frac{x}{y+5}=\frac{1}{4}\Rightarrow 4x-y=5 \quad ...(ii)$

On solving eqs. (i) and (ii), we get

$x=2$ and $y=3$

$\therefore$ Required fraction $=\frac{x-1}{y+5}=\frac{2-1}{3+5}=\frac{1}{8}$

6. (a) By option (a),

$\frac{-7}{10} < \frac{-2}{3} < \frac{-5}{8}$

Here LCM of (3, 8, 10) = 120

$\frac{-7}{10} \times 120 < \frac{-2}{3} \times 120 < \frac{-5}{8} \times 120$

$-84 < -80 < -75$

So this is correct.

7. (d) (a) 0.725

(b) $0.72\overline{5} = 0.7255...$

(c) $0.7\overline{25} = 0.7252525...$

(d) $0.\overline{725} = 0.725725725...$

Largest number is $0.\overline{725}$.

8. (d) $\because 1.\overline{34} = \frac{134-1}{99} = \frac{133}{99}$

and $4.1\overline{2} = \frac{412-41}{90} = \frac{371}{90}$

$\therefore \quad 1.\overline{34} + 4.1\overline{2} = \frac{133}{99} + \frac{371}{90} = \frac{1330+4081}{990}$

$= \frac{5411}{990} = 5\frac{461}{990}$

9. (c) $\because \frac{13}{8} = \frac{13}{8} \times \frac{125}{125} = \frac{1625}{1000} = 1.625$

$\frac{3}{16} = \frac{3 \times 625}{16 \times 625} = \frac{1875}{10000} = 0.1875$

$\frac{137}{25} = \frac{137 \times 4}{25 \times 4} = \frac{548}{100} = 5.48$

It is clear that all of these are terminating and decimals. Hence, $\frac{3}{11}$ is a non-terminating and repeating decimal.

10. (a) $2.\overline{6} - 1.\overline{9} = 2\frac{6}{9} - 2 = (2-2) + \frac{6}{9} = \frac{6}{9} = 0.\overline{6}$

11. (d) $27 \times 1.\overline{2} \times 5.526\overline{2} \times 0.\overline{6} = 27 \times 1\frac{2}{9} \times 5\frac{4736}{9000} \times \frac{6}{9}$

$= 27 \times \frac{11}{9} \times \frac{49736}{9000} \times \frac{6}{9}$

$= \frac{11 \times 49736 \times 2}{9000} = \frac{1094192}{9000} = 121.577$

12. (a) $3.\overline{76} - 1.4\overline{576} = 3 + 0.\overline{76} - 1 - 0.4\overline{576}$

$= 3 + \left(\frac{76-0}{99}\right) - 1 - \left(\frac{4576-4}{9990}\right)$

$= 3 + \frac{76}{99} - 1 - \frac{4572}{9990} = 2 + \left(\frac{76}{99} - \frac{4572}{9990}\right)$

$= 2 + \frac{1}{9}\left(\frac{76}{11} - \frac{4572}{1110}\right) = 2 + \frac{1}{9} \times \frac{(84360 - 50292)}{12210}$

$= 2 + \frac{1}{9} \times \frac{34068}{12210} = 2 + \frac{11356}{36630}$

$= 2 + 0.3\overline{100191} = 2.3\overline{100191}$

13. (b) $0.00\overline{7} + 17.\overline{83} + 310.020\overline{2}$

$= \frac{7}{900} + \frac{1783-17}{99} + \frac{3100202 - 310020}{9000}$

$= \frac{7}{900} + \frac{1766}{99} + \frac{2790182}{9000}$

$= \frac{770 + 1766000 + 30692002}{99000}$

$= \frac{32458772}{99000} = 327.866\overline{38}$

14. (b) Given that,

$0.242424...? = 0.\overline{24} = \frac{24}{99} = \frac{8}{33}$

Write down as many 9's in the denominator as the number of digits in the period of decimal number.

15. (d) Let $x = 0.2\overline{341}$

Here multiply by 10 both sides,

$10x = 2.\overline{341}$ .... (i)

Now, multiply by 1000 both sides,

$10000x = 2341.\overline{341}$ ...... (ii)

Now, substract equation (i) from equation (ii),

$9990x = 2341 - 2 = 2339$

$\therefore \quad x = \frac{2339}{9990}$.

- **Shortcut:**

$0.2\overline{341} = \frac{2341-2}{9990} = \frac{2339}{9990}$

16. (a) **Pure recurring decimal:-**

A decimal fraction in which all the figures occur repeatedly is called a pure recurring decimal as 7.4444 .., 2.666 ...., etc.

Let P be prime number

So P = 7, 11, 13,...

$\frac{1}{7} = .142857142857....$

$\frac{1}{11} = .09090909....$

$\frac{1}{13} = 0.0769230769230...$

All above example are pure recurring decimal and its period will (p – 1)

17. (c) $0.\overline{63} = 0.636363$ ------
Let $x = 0.636363$ ------
$\Rightarrow 100x = 63.6363$ ------
$\Rightarrow 99x = 63$
$\Rightarrow x = \frac{63}{99}$
$0.\overline{37} = 0.373737$ ------
Let $y = 0.37\ 37\ 37$ ------
$\Rightarrow 100y = 37.\ 37\ 37$ ------
$\Rightarrow 99y = 37$
$\Rightarrow y = \frac{37}{99}$
$x + y = \frac{63}{99} + \frac{37}{99} = \frac{100}{99}$
So, option (c) is correct.

18. (*) $\frac{61}{19} = 3 + \cfrac{1}{x + \cfrac{1}{y + \cfrac{1}{z}}}$

In real $\frac{61}{19} = 3 + \frac{4}{19}$

$\Rightarrow \frac{61}{19} = 3 + \cfrac{1}{x + \cfrac{1}{3 + \cfrac{1}{1}}}$

$= 3 + \cfrac{1}{x + \cfrac{1}{4}}$

$= 3 + \cfrac{1}{\frac{9}{2} + \frac{1}{4}}$

$= 3 + \cfrac{1}{\frac{19}{4}}$

$= 3 + \frac{4}{19}$

$\Rightarrow x = \frac{9}{2}, y = 3, z = 1$
Also if $z = 3$, then

$\frac{61}{19} = 3 + \cfrac{1}{x + \cfrac{1}{1 + \cfrac{1}{3}}} = 3 + \cfrac{1}{x + \cfrac{3}{4}}$

$= 3 + \cfrac{1}{4 + \cfrac{3}{4}} = 3 + \cfrac{1}{\frac{19}{4}}$

$= 3 + \frac{4}{19}$

$\therefore$ Option (a) & (c) is correct.

19. (d) $\frac{17}{6}$ only fraction whose dinominator is not in form of 2^m5^n when simplified

20. (b) $\sqrt{\frac{0.064}{0.081} \times \frac{6.25}{4.84}} = \frac{8}{9} \times \frac{25}{22}$

$= \frac{100}{99}$

21. (d) ATQ
$\frac{27 - x}{35 - x} = \frac{2}{3}$
$81 - 3x = 70 - 2x$
$x = 11$

22. (b) $\frac{11}{14}$

23. (a) $\sqrt[3]{4\frac{12}{125}} = \sqrt[3]{\frac{512}{125}} = \frac{8}{5} = 1\frac{3}{5}$

24. (b) We have $P = 0.8\overline{3}$ and $Q = 0.6\overline{2}$
The distance between P and Q is
$0.8\overline{3} - 0.6\overline{2} = 0.2\overline{1}$
Expressing this distance in the form of rational number, we assume $0.2\overline{1} = x$
No. of digits with bar = 1
No. of digits without bar = 1
Therefore, the denominator would be 90.
No. of digits after the decimal = 2
Therefore, the numerator would be 21 – 2 = 19
Thus, $x = \frac{19}{90}$

25. (a)

26. (c) Value of $0.\overline{9} = \frac{9}{9} = 1$

Value of $0.9 = \frac{9}{10}$

difference $= 1 - \frac{9}{10} = \frac{1}{10} = 0.1$

27. (a)

28. (a) $.\overline{53} + 0.5\overline{3}$
$= 0.5353535353.... + 0.5333333333$
$= 1.068686868 = 1.0\overline{68}$

CHAPTER

4 Powers and Roots

1. Which one of the following numbers is an integer? **[2007-I]**

(a) $\left[\left(\sqrt{2}+\sqrt{3}\right)/\left(\sqrt{3}-\sqrt{2}\right)\right]+\sqrt{6}$

(b) $\left[\left(\sqrt{2}+\sqrt{3}\right)/\left(\sqrt{3}-\sqrt{2}\right)\right]+2\sqrt{6}$

(c) $\left[\left(\sqrt{2}+\sqrt{3}\right)/\left(\sqrt{2}-\sqrt{3}\right)\right]+2\sqrt{6}$

(d) $\left[\left(\sqrt{2}+\sqrt{3}\right)/\left(\sqrt{2}-\sqrt{3}\right)\right]+\sqrt{6}$

2. What is the square root of $\left(\dfrac{x^5-1}{x-1}\right)+\left(x^3+2x^2+x\right)$? **[2007-I]**

(a) x^2+x+1 (b) x^2-x+1

(c) x^2-x-1 (d) x^2+x-1

3. What is the square root of $9+2\sqrt{14}$? **[2007-II]**

(a) $1+2\sqrt{2}$ (b) $\sqrt{3}+\sqrt{6}$

(c) $\sqrt{2}+\sqrt{7}$ (d) $\sqrt{2}+\sqrt{5}$

4. Which is the largest number among $\sqrt{2}, \sqrt[3]{3}, \sqrt[6]{6}$ and $\sqrt[12]{12}$? **[2007-II]**

(a) $\sqrt{2}$ (b) $\sqrt[3]{3}$ (c) $\sqrt[6]{6}$ (d) $\sqrt[12]{12}$

5. The sum of the square of a number and the square of the reciprocal of the number, is thrice the difference of the square of the number and the square of the reciprocal of the number. What is the number? **[2007-II]**

(a) 1 (b) $(2)^{1/4}$ (c) $(3)^{1/3}$ (d) $(4)^{1/4}$

6. What is/are the real value(s) of $(256)^{0.16} \times (16)^{0.18}$? **[2007-II]**

(a) Only – 4 (b) Only 4

(c) 4, – 4 (d) 2, – 2

7. If $(x)^{\frac{1}{m}}=(y)^{\frac{1}{n}}=(z)^{\frac{1}{p}}$ and $xyz = 1$, then what is the value of $m + n + p$? **[2007-II]**

(a) 0 (b) 1

(c) 2 (d) –1

8. **Assertion (A):** $\sqrt{\dfrac{5041}{6889}}$ is rational **[2007-II]**

Reason (R): The square root of a rational number is always rational.

(a) A and R are correct and R is correct explanation of A

(b) A and R are correct but R is not correct explanation of A

(c) A is correct but R is wrong

(d) A is wrong but R is correct

9. What is the square of $\left(2+\sqrt{2}\right)$? **[2008-I]**

(a) A rational number (b) An irrational number

(c) A natural number (d) A whole number

10. If $\left(ab^{-1}\right)^{2x-1}=\left(ba^{-1}\right)^{x-2}$, then what is the value of x? **[2008-I]**

(a) 1 (b) 2 (c) 3 (d) 4

11. What is the value of **[2008-I]**

$\sqrt{7.84}+\sqrt{0.0784}+\sqrt{0.000784}+\sqrt{0.00000784}$?

(a) 3.08 (b) 3.108

(c) 3.1008 (d) 3.1108

12. If $\sqrt{1+\dfrac{93}{196}}=1+\dfrac{x}{14}$, then what does x equal to? **[2008-II]**

(a) 1 (b) 2 (c) 3 (d) 4

13. If $y=\left(a^x\right)^{\left(a^x\right)^{\dots\infty}}$, then which one of the following is correct? **[2008-II]**

(a) $\log y = x\, y \log a$ (b) $\log y = x + y \log a$

(c) $\log y = y + x \log a$ (d) $\log y = (y + x) \log a$

14. What is the value of

$\dfrac{1}{1+\sqrt{2}}+\dfrac{1}{\sqrt{2}+\sqrt{3}}+\dots+\dfrac{1}{\sqrt{15}+\sqrt{16}}$? **[2008-II]**

(a) 0 (b) 1 (c) 2 (d) 3

15. A ball is dropped from a height 64 m above the ground and every time it hits the ground it rises to a height equal to half of the previous. What is the height attained after it hits the ground for the 16th time? **[2009-I]**

(a) 2^{-12} m (b) 2^{-11} m (c) 2^{-10} m (d) 2^{-9} m

16. If $a^x = c^q = b$ and $c^y = a^z = d$, then which one of the following is correct ? **[2009-I]**

(a) $x/y = q/z$ (b) $x + y = q + z$

(c) $xy = qz$ (d) $x^y = q^z$

17. If $27 \times (81)^{2n+3} - 3^m = 0$, then what is m equal to? **[2009-II]**

(a) $2n + 5$ (b) $5n + 6$

(c) $8n + 3$ (d) $8n + 15$

18. If $x = \left(a + \sqrt{a^2 + b^3}\right)^{1/3} + \left(a - \sqrt{a^2 + b^3}\right)^{1/3}$, then what is the value of $x^3 + 3bx - 2a$? **[2009-II]**

(a) $2a^3$ (b) $-2a^3$ (c) 1 (d) 0

19. If $(3.7)^x = (0.037)^y = 10000$, then what is the value of $\frac{1}{x} - \frac{1}{y}$? **[2009-II]**

(a) 1 (b) 2 (c) 1/2 (d) 1/4

20. Out of a group of swans 7/2 times the square root of the number are swimming in the pool while the two remaining are playing outside the pool. What is the total number of swans? **[2009-II]**

(a) 4 (b) 8 (c) 12 (d) 16

21. The number $\sqrt{0.0001}$ is **[2009-II]**

(a) a rational number less than 0.01

(b) a rational number

(c) an irrational number

(d) neither a rational number nor an irrational number

22. If $a^x = b^y = c^z$ and $abc = 1$, then $xy + yz + zx$ equal to **[2009-II]**

(a) xyz (b) x + y + z

(c) 0 (d) 1

23. What is the value of **[2010-I]**

$\sqrt{29.16} + \sqrt{0.2916} + \sqrt{0.002916} + \sqrt{0.00002916}$?

(a) 5.9949 (b) 5.9894

(c) 5.9984 (d) 5.9994

24. If $p^x = r^y = m$ and $r^w = p^z = n$, then which one of the following is correct ? **[2010-II]**

(a) $xw = yz$ (b) $xz = yw$

(c) $x + y = w + z$ (d) $x - y = w - z$

25. $\frac{1}{\sqrt{9}-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-\sqrt{4}}$ equal to **[2010-II]**

(a) 0 (b) 1 (c) 5 (d) $\frac{1}{3}$

26. If $3^x \times 27^x = 9^{x+4}$, then x equal to **[2011-I]**

(a) 4 (b) 5 (c) 6 (d) 7

27. What is one of the square roots of $9 - 2\sqrt{14}$? **[2011-I]**

(a) $\sqrt{7} - \sqrt{3}$ (b) $\sqrt{6} - \sqrt{3}$

(c) $\sqrt{7} - \sqrt{5}$ (d) $\sqrt{7} - \sqrt{2}$

28. What is the square root of $\frac{0.324 \times 0.64 \times 129.6}{0.729 \times 1.024 \times 36}$? **[2011-I]**

(a) 4 (b) 3 (c) 2 (d) 1

29. What is the smallest number that must be added to 1780 to make it a perfect square? **[2011-II]**

(a) 39 (b) 49 (c) 59 (d) 69

30. If $2^m + 2^{1+m} = 24$, then what is the value of m ? **[2011-II]**

(a) 0 (b) $\frac{1}{3}$ (c) 3 (d) 6

31. What is the value of $\frac{\sqrt{0.0032}}{\sqrt{0.32}}$? **[2011-II]**

(a) 0.0001 (b) 0.001 (c) 0.01 (d) 0.1

32. What is $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$ equal to ? **[2011-II]**

(a) 16 (b) 8 (c) 4 (d) $\sqrt{15}$

33. If $3^{x+y} = 81$ and $81^{x-y} = 3$, then what is the value of x ? **[2012-I]**

(a) $\frac{17}{16}$ (b) $\frac{17}{8}$ (c) $\frac{17}{4}$ (d) $\frac{15}{4}$

34. What is one of the square roots of **[2012-I]**

$16x^6 - 24x^5 + 25x^4 - 20x^3 + 10x^2 - 4x + 1$?

(a) $4x^3 - 3x^2 + 2x + 1$ (b) $4x^3 - 3x^2 - 2x - 1$

(c) $4x^3 - 3x^2 + 2x - 1$ (d) $4x^3 - 3x^2 - 2x + 1$

35. If $a^x = b, b^y = c$ and xyz = 1, then what is the value of c^z ? **[2012-I]**

(a) a (b) b (c) ab (d) $\frac{a}{b}$

36. If $196\,x^4 = x^6$, then x^3 is equal to which one of the following? **[2012-I]**

(a) $x^6/14$ (b) $14x^4$ (c) $x^2/14$ (d) $14x^2$

37. If $a = 2 + \sqrt{3}$, then what is the value of $\left(a^2 + a^{-2}\right)$? **[2012-II]**

(a) 12 (b) 14 (c) 16 (d) 18

38. If $\sqrt{10+\sqrt[3]{x}} = 4$, then what is the value of x ? **[2012-II]**
(a) 150 (b) 216 (c) 316 (d) 450

39. The least number of four digits which is a perfect square is **[2012-II]**
(a) 1204 (b) 1024 (c) 1402 (d) 1420

40. What is the value of $\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+...}}}}$? **[2013-I]**
(a) 2 (b) 3 (c) 3.5 (d) 4

41. If $16\times 8^{n+2} = 2^m$, then m is equal to **[2013-I]**
(a) $n + 8$ (b) $2n + 10$ (c) $3n + 2$ (d) $3n + 10$

42. The expression $\left[\left(\sqrt{2}\right)^{\sqrt{2}}\right]^{\sqrt{2}}$ gives **[2013-I]**
(a) a natural number
(b) an integer and not a natural number
(c) a rational number but not an integer
(d) a real number but not a rational number

43. Which is the smallest number among the following? **[2013-II]**
(a) $\left[\left(5^{-2}\right)^{-2}\right]^{-2}$ (b) $\left[\left(5^{-2}\right)^{2}\right]^{-2}$
(c) $\left[\left(2^{-5}\right)^{-2}\right]^{-2}$ (d) $\left[\left(2^{-5}\right)^{2}\right]^{-2}$

44. Consider the following in respect of the numbers $\sqrt{2}, \sqrt[3]{3}$ and $\sqrt[6]{6}$ **[2014-I]**
I. $\sqrt[6]{6}$ is the greatest number.
II. $\sqrt{2}$ is the smallest number.
Which of the above statements is/are correct?
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

45. The product of four consecutive natural numbers plus one is **[2014-I]**
(a) a non-square
(b) always sum of two square numbers
(c) a square
(d) None of these

46. The difference of cubes of two consecutive integers **[2014-I]**
(a) is odd or even (b) is never divisible by 2
(c) is always even (d) None of these

47. What is
$$\frac{1}{a-b}-\frac{1}{a+b}-\frac{2b}{a^2+b^2}-\frac{4b^3}{a^4+b^4}-\frac{8b^7}{a^8-b^8} \text{ equal to ?}$$
[2014-II]
(a) $a + b$ (b) $a - b$
(c) 1 (d) 0

48. The square root of $\frac{(0.75)^3}{1-0.75} + [0.75 + (0.75)^2 + 1]$ is **[2015-I]**
(a) 1 (b) 2 (c) 3 (d) 4

49. What are the possible solutions for x of the equation $x^{\sqrt{x}} = \sqrt[n]{x^x}$, where x and n are positive integers? **[2015-I]**
(a) $0, n$ (b) $1, n$
(c) n, n^2 (d) $1, n^2$

50. If $a - b = 4$ and $a^2 + b^2 = 40$, where a and b are positive integers, then $a^3 + b^6$ is equal to **2015-I]**
(a) 264 (b) 280
(c) 300 (d) 324

51. What is $\sqrt{4+\sqrt{4-\sqrt{4+\sqrt{4-...............}}}}$ equal to? **[2015-II]**
(a) 3 (b) $\frac{\sqrt{13}-1}{2}$ (c) $\frac{\sqrt{13}+1}{2}$ (d) 0

52. If $x = \sqrt{3}+\sqrt{2}$, then the value of $x^3 + x + \frac{1}{x} + \frac{1}{x^3}$ is **[2015-II]**
(a) $10\sqrt{3}$ (b) $20\sqrt{3}$ (c) $10\sqrt{2}$ (d) $20\sqrt{2}$

53. Which one of the following is correct? **[2015-II]**
(a) $\sqrt{2} < \sqrt[4]{6} < \sqrt[3]{4}$ (b) $\sqrt{2} > \sqrt[4]{6} > \sqrt[3]{4}$
(c) $\sqrt[4]{6} < \sqrt{2} < \sqrt[3]{4}$ (d) $\sqrt[4]{6} > \sqrt{2} > \sqrt[3]{4}$

54. If $x = 2^{\frac{1}{3}} + 2^{-\frac{1}{3}}$, then the value of $2x^3 - 6x - 5$ is euqal to **[2016-I]**
(a) 0 (b) 1 (c) 2 (d) 3

55. If $4^x 2^y = 128$ and $3^{3x}3^{2y} - 9^{xy} = 0$, then the value of $x + y$ can be equal to **[2016-I]**
(a) 7 (b) 5 (c) 3 (d) 1

56. The value of the expression is equal to **[2016-I]**
$$\frac{(243+647)^2+(243-647)^2}{243\times 243+647\times 647}$$
(a) 0 (b) 1 (c) 2 (d) 3

57. What is $\frac{6^2+7^2+8^2+9^2+10^2}{\sqrt{7+4\sqrt{3}}-\sqrt{4+2\sqrt{3}}}$ equal to? **[2016-II]**
(a) 330 (b) 340 (c) 355 (d) 366

58. The sum of the squares of two positive integers is 208. If the square of the larger number is 18 times the smaller number, then what is the difference of the larger and smaller numbers ? **[2016-II]**
(a) 2 (b) 3 (c) 4 (d) 6

59. What is the remainder when 2^{100} is divided by 101 ? **[2016-II]**
(a) 1 (b) 11 (c) 99 (d) 100

60. Which one of the following is correct in respect of the number 1729 ? **[2016-II]**

(a) It cannot be written as the sum of the cubes of two positive integers
(b) It can be written as the sum of the cubes of two positive integers in one way only
(c) It can be written as the sum of the cubes of two positive integers in two ways only
(d) It can be written as the sum of the cubes of two positive integers in three ways only

61. What is $\sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}}+\sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}}+\ldots+$ **[2016-II]**

$\sqrt{1+\frac{1}{2007^2}+\frac{1}{2008^2}}$ equal to ?

(a) $2008-\frac{1}{2008}$ (b) $2007-\frac{1}{2007}$
(c) $2007-\frac{1}{2008}$ (d) $2008-\frac{1}{2009}$

62. Consider the following statements in respect of positive odd integers x and y : **[2016-II]**

1. $x^2 + y^2$ is even integer.
2. $x^2 + y^2$ is divisible by 4

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

63. What is the remainder when $13^5 + 14^5 + 15^5 + 16^5$ is divided by 29 ? **[2016-II]**

(a) 8 (b) 5 (c) 3 (d) 0

64. What is the difference between the sum of the cubes and that of squares of first ten natural numbers? **[2016-II]**

(a) 2280 (b) 2640 (c) 3820 (d) 4130

65. What is the square root of **[2017-I]**

$$\frac{(0.35)^2+0.70+1}{2.25}+0.19\ ?$$

(a) 1 (b) 2 (c) 3 (d) 4

66. What is the value of **[2017-I]**

$$\frac{(443+547)^2+(443-547)^2}{443\times 443+547\times 547}?$$

(a) 0 (b) 1 (c) 2 (d) 3

67. If $a^3 = 335 + b^3$ and $a = 5 + b$, then what is the value of $a + b$ (given that $a > 0$ and $b > 0$) ? **[2017-I]**

(a) 7 (b) 9 (c) 16 (d) 49

68. If $9^x\, 3^y = 2187$ and $2^{3x}\, 2^{2y} - 4^{xy} = 0$, then what can be the value of $(x + y)$? **[2017-I]**

(a) 1 (b) 3 (c) 5 (d) 7

69. The values of x which satisfy the equation $5^{1+x} + 5^{1-x} = 26$ are **[2017-I]**

(a) 1, 1 (b) 0, 1
(c) 1, 2 (d) – 1, 0

70. If each of the dimensions of a rectangle is increased by 200%, the area is increased by

(a) 300% (b) 400%
(c) 600% (d) 800%

71. Let f(x) and g(x) be two polynomials (with real coefficients) having degrees 3 and 4 respectively. What is the degree of f(x) g(x)? **[2017-II]**

(a) 12 (b) 7
(c) 4 (d) 3

72. Consider the following numbers : **[2017-II]**

1. 2222
2. 11664
3. 343343
4. 220347

Which of the above are not perfect squares? **[2017-II]**

(a) 1, 2 and 3 (b) 1, 2 and 4
(c) 2, 3 and 4 (d) 1, 3 and 4

73. The value of $\sqrt{1+\sqrt{1+\sqrt{1+\ldots}}}$ **[2017-II]**

(a) Equals to 1
(b) Lies between 0 and 1
(c) Lies between 1 and 2
(d) Is greater than 2

74. If $x = y^{1/a}$, $y = z^{1/b}$ and $z = x^{1/c}$ where $x \neq 1, y \neq 1, z \neq 1$, then what is the value of abc? **[2018-I]**

(a) –1 (b) 1 (c) 0 (d) 3

75. If $2b = a + c$ and $y^2 = xz$, then what is $x^{b-c}\, y^{c-a}\, z^{a-b}$ equal to? **[2018-I]**

(a) 3 (b) 2 (c) 1 (d) –1

76. The smallest integer with 4 digits which is a perfect square is **[2018-I]**

(a) 1000 (b) 1024
(c) 1089 (d) None of the above

77. What is the value of $\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}=\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$? **[2018-I]**

(a) $-2\sqrt{15}$ (b) $2\sqrt{15}$ (c) $\sqrt{15}$ (d) $-\sqrt{15}$

78. What is the value of **[2018-I]**

$$\frac{1}{1+x^{b-a}+x^{c-a}}+\frac{1}{1+x^{a-b}+x^{c-b}}+\frac{1}{1+x^{a-c}+x^{b-c}}$$

where $x \neq 0$?

(a) –1 (b) 0 (c) 1 (d) 3

79. The sum of a number and its square is 20. Then the number is **[2018-I]**
(a) –5 or 4 (b) 2 or 3
(c) –5 only (d) 5 or –4

80. What is the largest power of 10 that divides the product $1 \times 2 \times 3 \times 4 \ldots\ldots\ldots\ldots \times 23 \times 24 \times 25$? **[2018-I]**
(a) 2 (b) 4
(c) 5 (d) None of the above

81. If $x = y^a$, $y = z^b$ and $z = x^c$, then the value of abc is **[2018-II]**
(a) 1 (b) 2 (c) –1 (d) 0

82. If $x = 2 + 2^{2/3} + 2^{1/3}$, then the vlaue of the expression $x^3 - 6x^2 + 6x$ will be **[2018-II]**
(a) 2 (b) 1 (c) 0 (d) –2

83. If $x^6 + \frac{1}{x^6} = k\left(x^2 + \frac{1}{x^2}\right)$, then k is equal to **[2018-II]**
(a) $\left(x^2 - 1 + \frac{1}{x^2}\right)$ (b) $\left(x^4 - 1 + \frac{1}{x^4}\right)$
(c) $\left(x^4 + 1 + \frac{1}{x^4}\right)$ (d) $\left(x^4 - 1 - \frac{1}{x^4}\right)$

84. If $a^x = b^y = c^z$ and abc = 1, then the value of $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ will be equal to **[2018-II]**
(a) –1 (b) 0 (c) 1 (d) 3

85. If $a = xy^{p-1}$, $b = yz^{q-1}$, $c = zx^{r-1}$, then $a^{q-r}\, b^{r-p}\, c^{p-q}$ is equal to **[2018-II]**
(a) abc (b) xyz
(c) 0 (d) None of the above

86. Suppose n is a positive integer such that $(n^2 + 48)$ is a perfect square. What is the number of such n ? **[2019-I]**
(a) One (b) Two (c) Three (d) Four

87. For $x = \frac{4\sqrt{6}}{\sqrt{2} + \sqrt{3}}$, what is the value of

$\frac{x + 2\sqrt{2}}{x - 2\sqrt{2}} + \frac{x + 2\sqrt{3}}{x - 2\sqrt{3}}$? **[2019-I]**
(a) 1 (d) $\sqrt{2}$ (c) $\sqrt{3}$ (d) 2

88. If $3^x = 4^y = 12^z$, then z is equal to **[2019-I]**
(a) xy (b) $x + y$ (c) $\frac{xy}{x + y}$ (d) $4x + 3y$

89. What is the value of $2 + \sqrt{2 + \sqrt{2 + \sqrt{\ldots\ldots\ldots}}}$? **[2019-I]**
(a) 1 (b) 2 (c) 3 (d) 4

90. The expression $5^{2n} - 2^{3n}$ has a factor **[2019-I]**
(a) 3 (b) 7
(c) 17 (d) None of the above

91. If $a = \sqrt{7 + 4\sqrt{3}}$, then what is the value of $a + \frac{1}{a}$? **[2019-II]**
(a) 2 (b) 3 (c) 4 (d) 7

92. What is the sum of all integer values of n for which $n^2 + 19n + 92$ is a perfect square ? **[2019-II]**
(a) 21 (b) 19 (c) 0 (d) –19

93. What is the square root of $16 + 6\sqrt{7}$? . **[2019-II]**
(a) $4 + \sqrt{7}$ (b) $4 - \sqrt{7}$ (c) $3 + \sqrt{7}$ (d) $3 - \sqrt{7}$

94. Let a and b be two positive real numbers such that $a\sqrt{a} + b\sqrt{b} = 32$ and $a\sqrt{b} + b\sqrt{a} = 31$. What is the value of $\frac{5(a + b)}{7}$? **[2019-II]**
(a) 5 (b) 7
(c) 9 (d) Cannot be determined

95. Which one of the following is a set of solutions of the equation $x^{\sqrt{x}} = \sqrt[n]{x^x}$, if n is a positive integer? **[2020-I]**
(a) $\{1, n^2\}$ (b) $\{1, \sqrt{n}\}$
(c) $\{1, n\}$ (d) $\{n, n^2\}$

96. Which one of the following is the largest number among $2222^2, 222^{22}, 22^{222}\ 2^{2222}$? **[2020-I]**
(a) 2^{2222} (b) 22^{222} (c) 222^{22} (d) 2222^2

97. If $5^{x+1} - 5^{x-1} = 600$, then what is the value of 10^{2x} ? **[2020-I]**
(a) 1 (b) 1000 (c) 100000 (d) 1000000

98. If m and n are positive integers such that $m^n = 1331$, then what is the value of $(m - 1)^{n-1}$? **[2020-I]**
(a) 1 (b) 100 (c) 121 (d) 125

99. If $x = \sqrt{2}$, $y = \sqrt[3]{3}$ and $z = \sqrt[6]{6}$, then which one of the following is correct ? **[2020-I]**
(a) $y < x < z$ (b) $z < x < y$
(c) $z < y < x$ (d) $x < y < z$

HINTS & SOLUTIONS

1. (c) (a) $\dfrac{\sqrt{2}+\sqrt{3}}{\sqrt{3}-\sqrt{2}}+\sqrt{6}=\dfrac{\left(\sqrt{2}+\sqrt{3}\right)^2}{3-2}+\sqrt{6}$

$=2+3+2\sqrt{6}+\sqrt{6}$

$=5+3\sqrt{6}$

It is not an integer.

(b) $\dfrac{\sqrt{2}+\sqrt{3}}{\sqrt{3}-\sqrt{2}}+2\sqrt{6}=\dfrac{\left(\sqrt{2}+\sqrt{3}\right)^2}{3-2}+2\sqrt{6}$

$=2+3+2\sqrt{6}+2\sqrt{6}$

$=5+4\sqrt{6}$

It is also not an integer.

(c) $\dfrac{\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}}+2\sqrt{6}=\dfrac{\left(\sqrt{2}+\sqrt{3}\right)^2}{2-3}+2\sqrt{6}$

$=-\left(2+3+2\sqrt{6}\right)+2\sqrt{6}$

$=-5$

It is an integer.

(d) $\dfrac{\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}}+\sqrt{6}=\dfrac{\left(\sqrt{2}+\sqrt{3}\right)^2}{\left(\sqrt{2}\right)^2-\left(\sqrt{3}\right)^2}+\sqrt{6}$

$=\dfrac{2+3+2\sqrt{2}\sqrt{3}}{2-3}+\sqrt{6}$

$=-\left(5+2\sqrt{6}\right)+\sqrt{6}$

$=-5-\sqrt{6}$

It is not an integer

2. (a) $\left(\dfrac{x^5-1}{x-1}\right)+\left(x^3+2x^2+x\right).$

$\dfrac{x^5-1}{x-1}+x^3+2x^2+x$

$=\dfrac{x^5-1+x^4+2x^3+x^2-x^3-2x^2-x}{x-1}$

$=\dfrac{x^5+x^4+x^3-x^2-x-1}{x-1}$

$=\dfrac{x^3\left(x^2+x+1\right)-1\left(x^2+x+1\right)}{(x-1)}$

$=\dfrac{\left(x^2+x+1\right)\left(x^3-1\right)}{x-1}$

$=\dfrac{\left(x^2+x+1\right)(x-1)\left(x^2+x+1\right)}{(x-1)}$

$=\left(x^2+x+1\right)^2$

$\therefore$ Square root of $\left\{\dfrac{x^5-1}{x-1}+x^3+2x^2+x\right\}$

$=\sqrt{\left(x^2+x+1\right)^2}=x^2+x+1$

3. (c) $9+2\sqrt{14}=\left(\sqrt{7}\right)^2+\left(\sqrt{2}\right)^2+2\sqrt{7}\times\sqrt{2}$

$=\left(\sqrt{7}+\sqrt{2}\right)^2$

$\therefore \quad \sqrt{9+2\sqrt{14}}=\left(\sqrt{7}+\sqrt{2}\right)$

4. (b) $\sqrt{2}, \sqrt[3]{3}, \sqrt[6]{6}$ and $\sqrt[12]{12}$

LCM of 2, 3, 6 and 12 is 12

It can be written as

$\sqrt[12]{2^6}$, $\sqrt[12]{3^4}$, $\sqrt[12]{6^2}$ and $\sqrt[12]{12}$.

So $\sqrt[3]{3}$ is largest number.

5. (b) Let number be x, then its reciprocal be $\dfrac{1}{x}$.

According to question,

$x^2+\dfrac{1}{x^2}=3\left(x^2-\dfrac{1}{x^2}\right)$

$\therefore \quad x^2+\dfrac{1}{x^2}=3x^2-\dfrac{3}{x^2} \Rightarrow 2x^2=\dfrac{4}{x^2}$

$\Rightarrow \quad x^4=2 \Rightarrow x=(2)^{1/4}$

6. (b) $(256)^{0.16}\times(16)^{0.18}=\left[(16)^2\right]^{0.16}\times(16)^{0.18}$

$=(16)^{0.32}\times(16)^{0.18}$

$=(16)^{0.5}=\left[4^2\right]^{0.5}=4^1=4$

7. (a) Let, $(x)^{\frac{1}{m}} = (y)^{\frac{1}{n}} = (z)^{\frac{1}{p}} = k$ (say)

$\Rightarrow \quad x = k^m, y = k^n$ and $z = k^p$

Now multiply x, y and z

$\therefore \quad xyz = k^{m+n+p}$ ($\because xyz = 1$, given)

$\Rightarrow \quad 1 = k^{m+n+p} = k^0$

Here, base is same,

$m + n + p = 0$

8. (c) (A) $\sqrt{\frac{5041}{6889}} = \frac{71}{83}$ = rational number

(R) Now, 2 is a rational number but $\sqrt{2}$ is not a rational number. i.e., irrational number.

9. (b) Square of $2+\sqrt{2} = (2+\sqrt{2})^2$

$= (2)^2 + (\sqrt{2})^2 + 2.2.\sqrt{2} = 4 + 2 + 4\sqrt{2}$

$= 6 + 4\sqrt{2}$

$6 + 4\sqrt{2}$ is an irrational number.

10. (a) Given that, $(ab^{-1})^{2x-1} = (ba^{-1})^{x-2}$

$\Rightarrow \quad \left(\frac{a}{b}\right)^{2x-1} = \left(\frac{b}{a}\right)^{x-2}$

$\Rightarrow \quad \left(\frac{a}{b}\right)^{2x-1} \left(\frac{a}{b}\right)^{x-2} = 1$

$\Rightarrow \quad \left(\frac{a}{b}\right)^{2x-1+x-2} = \left(\frac{a}{b}\right)^0$

Base is same

So, $3x - 3 = 0 \Rightarrow x = 1$

11. (d) $\sqrt{7.84} + \sqrt{0.0784} + \sqrt{0.000784} + \sqrt{0.00000784}$

$= \sqrt{\frac{784}{100}} + \sqrt{\frac{784}{10000}} + \sqrt{\frac{784}{1000000}} + \sqrt{\frac{784}{100000000}}$

$= \frac{28}{10} + \frac{28}{100} + \frac{28}{1000} + \frac{28}{10000}$

$= 2.8 + 0.28 + 0.028 + 0.0028 = 3.1108$

12. (c) Given that, $\sqrt{1 + \frac{93}{196}} = 1 + \frac{x}{14}$

$\Rightarrow \quad \sqrt{\frac{289}{196}} = 1 + \frac{x}{14} \Rightarrow \frac{17}{14} = 1 + \frac{x}{14}$

$\Rightarrow \quad \frac{x}{14} = \frac{17}{14} - 1 \Rightarrow x = 3$

13. (a) Given, $y = (a^x)^{(a^x)^{\ldots\infty}}$

$\therefore \quad y = (a^x)^y$

Taking log on both sides

$\log y = y \log a^x \Rightarrow \log y = x\,y \log a$

14. (d) $\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \ldots + \frac{1}{\sqrt{15}+\sqrt{16}}$

(on rationalisation)

$= \frac{1-\sqrt{2}}{1-2} + \frac{\sqrt{2}-\sqrt{3}}{2-3} + \ldots + \frac{\sqrt{15}-\sqrt{16}}{15-16}$

$= -1\left(1-\sqrt{2}+\sqrt{2}-\sqrt{3}+\ldots+\sqrt{15}-\sqrt{16}\right)$

$= -1(1-4) = 3$

15. (c) After 1st hit ball height will be = $\frac{1}{2}(64)$

After 2nd hit ball height will be = $\left(\frac{1}{2}\right)^2 (64)$

....

....

....

After 16th hit ball height will be = $\left(\frac{1}{2}\right)^{16} (64)$

$= \frac{1}{2^{16}}(2^6) = 2^{-10}$ m

16. (c) Given, $a^x = c^q = b$ and $c^y = a^z = d$

Now, $a^x = c^q$

$\Rightarrow \quad a^{x \cdot z} = c^{q \cdot z} \Rightarrow (a^z)^x = (c^z)^q$

Here, $a^z = c^y \quad \Rightarrow \quad (c^y)^x = c^{zq}$

$\Rightarrow yx = zq$

17. (d) Given, $27 \times (81)^{2n+3} - 3^m = 0$

$\Rightarrow \quad 3^3 \times (3)^{8n+12} = 3^m$

$\Rightarrow \quad 3^{8n+15} = 3^m \quad \Rightarrow \quad m = 8n + 15$

(on comparing)

18. (d) Given, $x = \left(a + \sqrt{a^2 + b^3}\right)^{1/3} + \left(a - \sqrt{a^2 + b^3}\right)^{1/3}$

On cubing both sides, we get

$x^3 = \left(a + \sqrt{a^2+b^3}\right) + \left(a - \sqrt{a^2+b^3}\right)$

$+3\left(a + \sqrt{a^2+b^3}\right)^{1/3}\left(a - \sqrt{a^2+b^3}\right)^{1/3}$

$\left\{\left(a + \sqrt{a^2+b^3}\right)^{1/3} + \left(a - \sqrt{a^2+b^3}\right)^{1/3}\right\}$

$\Rightarrow \quad x^3 = 2a - 3b(x)$

$\Rightarrow \quad x^3 + 3bx - 2a = 0$

19. (c) Given, $(3.7)^x = (0.037)^y = 10000$

$\Rightarrow \quad (3.7)^x = 10^4$ and $(0.037)^y = 10^4$

$\Rightarrow \quad 37 = 10^{\frac{4}{x}+1}$ and $37 = 10^{\frac{4}{y}+3}$

$\Rightarrow \quad 10^{\frac{4}{x}+1} = 10^{\frac{4}{y}+3} \quad \Rightarrow \quad \frac{4}{x}+1 = \frac{4}{y}+3$

$\therefore \quad \frac{4}{x} - \frac{4}{y} = 3 - 1 \quad \Rightarrow \quad \frac{1}{x} - \frac{1}{y} = \frac{1}{2}$

20. (d) Let the total number of swans = x

Number of swans swimming in the root $= \frac{7}{2}\sqrt{x}$

Remaining swans = 2

According to question,

$\frac{7}{2}\sqrt{x} + 2 = x \Rightarrow \frac{7}{2}\sqrt{x} = x - 2$

On squaring both sides, we get

$\frac{49}{4}x = x^2 + 4 - 4x$

$\Rightarrow \quad 4x^2 - 65x + 16 = 0$

$= 4x^2 - 64x - x + 16 = 0$

$\Rightarrow \quad 4x(x-16) - 1(x-16) = 0$

$\Rightarrow \quad (x-16)(4x-1) = 0$

$\therefore \quad x = 16 \left(\because x \neq \frac{1}{4}\right)$

21. (b) $\sqrt{0.0001} = \sqrt{\frac{00001}{10000}} = \frac{1}{100} = 0.01$

So, it is a rational number.

22. (c) Given, $a^x = b^y = c^z = k$

$\Rightarrow \quad a = k^{1/x}, b = k^{1/y}$ and $c = k^{1/z}$

$\therefore \quad abc = k^{\frac{1}{x}+\frac{1}{y}+\frac{1}{z}}$

$\Rightarrow \quad 1 = k^{\frac{1}{x}+\frac{1}{y}+\frac{1}{z}} = \text{k} \quad (\because abc = 1, \text{given})$

On comparing, we get

$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$

$\Rightarrow \quad xy + yz + zx = 0$

23. (d) $\sqrt{29.16} + \sqrt{0.2916} + \sqrt{0.002916} + \sqrt{0.00002916}$

$= 5.4 + 0.54 + 0.054 + 0.0054$

$= 5.9994$

24. (a) Given that, $p^x = r^y = m$ and $r^w = p^z = n$

Now, $p^x = r^y$

$\Rightarrow \quad \left(p^x\right)^w = \left(r^y\right)^w \Rightarrow p^{xw} = r^{yw}$

$\Rightarrow \quad p^{xw} = \left(r^w\right)^y$ ...(i)

Here, $r^w = p^z$ put in eq. (i)

then, $p^{xw} = \left(p^z\right)^y \Rightarrow p^{xw} = p^{zy}$ (Base is same)

$\therefore xw = zy$

25. (c) $\frac{1}{\sqrt{9}-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-\sqrt{4}}$

(on rationalisation)

$= \left(\sqrt{9}+\sqrt{8}\right) - \left(\sqrt{8}+\sqrt{7}\right) + \left(\sqrt{7}+\sqrt{6}\right) - \left(\sqrt{6}+\sqrt{5}\right) + \left(\sqrt{5}+\sqrt{4}\right)$

$= \sqrt{9} + \sqrt{4} = 3 + 2 = 5$

26. (a) $3^x \times 27^x = 9^{x+4}$

$\therefore 3^x \times 3^{3x} = 3^{2(x+4)}$

$\Rightarrow 3^{x+3x} = 3^{2(x+4)}$

Here base is same

So, $x + 3x = 2(x+4) \Rightarrow 4x - 2x = 8$

$\therefore \quad x = \frac{8}{2} = 4$

27. (d) $\sqrt{9 - 2\sqrt{14}} = \sqrt{7 + 2 - 2 \times \sqrt{7} \times \sqrt{2}}$

$= \sqrt{\left(\sqrt{7}-\sqrt{2}\right)^2} = \sqrt{7} - \sqrt{2}$

28. (d) $\sqrt{\frac{0.324 \times 0.64 \times 129.6}{0.729 \times 1.024 \times 36}}$

$= \sqrt{\frac{324 \times 64 \times 1296}{729 \times 1024 \times 36}}$

$= \frac{18 \times 8 \times 36}{27 \times 32 \times 6} = 1$

29. (d) We know that, $(42)^2 = 1764$ and $(43)^2 = 1849$

But 1780 lies between 1764 and 1849.

Now, smallest number = 1849 – 1780 = 69

30. (c) Given, $2^m + 2^{1+m} = 24$

$\therefore \quad 2^m(1+2) = 24 \quad \Rightarrow \quad 2^m \times 3 = 24$

$\Rightarrow \quad 2^m = 8 = 2^3$

$\therefore \quad m = 3$

31. (d) $\dfrac{\sqrt{0.0032}}{\sqrt{0.32}} = \dfrac{\sqrt{0.32}}{\sqrt{100}} \times \dfrac{1}{\sqrt{0.32}} = \dfrac{1}{10} = 0.1$

32. (b) $\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \dfrac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$= \dfrac{\left(\sqrt{5}+\sqrt{3}\right)^2 + \left(\sqrt{5}-\sqrt{3}\right)^2}{\left(\sqrt{5}\right)^2 - \left(\sqrt{3}\right)^2}$

$= \dfrac{2\left\{\left(\sqrt{5}\right)^2 + \left(\sqrt{3}\right)^2\right\}}{5-3} = \dfrac{2(5+3)}{2} = 8$

33. (b) Given that $3^{x+y} = 81$ or $3^{x+y} = 3^4$

$\Rightarrow x + y = 4$ .....(i)

and $81^{x-y} = 3$ or $(3^4)^{x-y} = 3^1$

$\Rightarrow x - y = \dfrac{1}{4}$ .....(ii)

On solving eqs. (i) and (ii), we get

$x + y = 4 \quad \Rightarrow \quad x - y = \dfrac{1}{4}$

$2x = \dfrac{17}{4} \quad \Rightarrow \quad x = \dfrac{17}{8}$

34. (c) From option (c),

$\left(4x^3 - 3x^2 + 2x - 1\right)^2 = \left[\left(4x^3 - 3x^2\right) + (2x-1)\right]^2$

$= \left(4x^3 - 3x^2\right)^2 + (2x-1)^2 + 2\left(4x^3 - 3x^2\right)(2x-1)$

$= 16x^6 + 9x^4 - 24x^5 + 4x^2 + 1 - 4x$

$+16x^4 - 12x^3 - 8x^3 + 6x^2$

$= 16x^6 - 24x^5 + 25x^4 - 20x^3 + 10x^2 - 4x + 1$

35. (a) equ $a^x = b$

Multiplying both sides by y in power

$\left(a^x\right)^y = b^y$

$\Rightarrow \quad a^{xy} = c \quad \left(\because b^y = c\right)$

Again multiplying both sides by z in power

$\left(a^{xy}\right)^z = c^z$

$\Rightarrow \quad a^{xyz} = c^z$

But $xyz = 1$ Given

So $a = c^z$

36. (d) Given, $196x^4 = x^6$

$\Rightarrow \left(14x^2\right)^2 = \left(x^3\right)^2 \Rightarrow 14x^2 = x^3$

37. (b) Given that, $a = 2 + \sqrt{3}$, $\dfrac{1}{a} = 2 - \sqrt{3}$

Now, $a^2 + a^{-2} = \left(a + \dfrac{1}{a}\right)^2 - 2$

$= \left(2 + \sqrt{3} + 2 - \sqrt{3}\right)^2 - 2$

$= (4)^2 - 2 = 16 - 2 = 14$

38. (b) Given, $\sqrt{10 + \sqrt[3]{x}} = 4$

On squaring both sides

$10 + \sqrt[3]{x} = 16$

$\Rightarrow \sqrt[3]{x} = 6$

On cubic both sides

$x = (6)^3 = 216$

39. (b) Factor of 1024 = 32 × 32 = 32

So, 1024 is a perfect square number.

40. (b) Factorisation of 6 = 2 × 3

Given that $\sqrt{6 + \sqrt{6 + \sqrt{6 + \sqrt{6 + \ldots}}}}$

It has positive sign

Hence, its value will be greater number of the factors = 3

41. (d) Given that, $16 \times 8^{n+2} = 2^m$

$\Rightarrow \quad (2)^4 \times 2^{3(n+2)} = 2^m$

$\Rightarrow \quad (2)^{(4 + 3n + 6)} = 2^m$

$\Rightarrow 2^{(3n+10)} = 2^m$

Here base is same, so

$3n + 10 = m$

$\Rightarrow \quad m = 3n + 10$

42. (d) $\left[\left(\sqrt{2}\right)^{\sqrt{2}}\right]^{\sqrt{2}} = \left(\sqrt{2}\right)^2 = 2$

43. (c) From option (a) $\left[\left(5^{-2}\right)^{-2}\right]^{-2} = 5^{-8} = \dfrac{1}{5^8}$

From option (b) $\left[\left(5^{-2}\right)^2\right]^{-2} = 5^8 = 5^8$

From option (c) $\left[\left(2^{-5}\right)^{-2}\right]^{-2} = 5^{-20} = \frac{1}{5^{20}}$

From option (d) $\left[\left(2^{-5}\right)^{2}\right]^{-2} = 5^{20} = 5^{20}$

Now, smallest number $\left[\left(2^{-5}\right)^{-2}\right]^{-2}$

44. (d) LCM of 2, 3 and 6 = 12

Now, $\sqrt{2} = 2^{\frac{1}{2}\times\frac{12}{12}} = 12\sqrt{2^6} = \sqrt[12]{64}$

$\sqrt[3]{3} = 3^{\frac{1}{3}\times\frac{12}{12}} = 12\sqrt{3^4} = \sqrt[12]{81}$

$\sqrt[6]{6} = 6^{\frac{1}{6}\times\frac{12}{12}} = \sqrt[12]{6^2} = \sqrt[12]{36}$

So, $\sqrt{2}$ is not smallest and $\sqrt[6]{6}$ is not greatest. So neither I nor II correct.

45. (c) **Example 1:** Let consider four consecutive natural numbers = 4, 5, 6 and 7 product = 4 × 5 × 6 × 7 = 840
Now, 840 + 1 = 841
It is square of 29.
Example 2: Let other four consecutive natural number = 13, 14, 15 and 16
Product = 13 × 14 × 15 × 16 = 43680
Now, 43680 + 1 = 43681
It is square of 43681
So this is always a square.

46. (b) One of the two consecutive integers is even and other is odd.
(even integer)^3 – (odd integer)^3 = (even integer) – (odd integer) = odd integer
(odd integer)^3 – (even integer)^3 = (odd integer) – (even integer) = odd integer

47. (d) $\frac{1}{a-b}-\frac{1}{a+b}-\frac{2b}{a^2+b^2}-\frac{4b^3}{a^4+b^4}-\frac{8b^7}{a^8-b^8}$

$= \frac{(a+b)-(a-b)}{(a-b)(a+b)}-\frac{2b}{a^2+b^2}-\frac{4b^3}{a^4+b^4}-\frac{8b^7}{a^8-b^8}$

$= \frac{2b}{a^2-b^2}-\frac{2b}{a^2+b^2}-\frac{4b^3}{a^4+b^4}-\frac{8b^7}{a^8-b^8}$

$= \frac{2b(a^2+b^2)-2b(a^2-b^2)}{(a^2-b^2)(a^2+b^2)}-\frac{4b^3}{a^4+b^4}-\frac{8b^7}{a^8-b^8}$

$= \frac{2a^2b+2b^3-2a^2b+2b^3}{(a^4-b^4)}-\frac{4b^3}{a^4+b^4}-\frac{8b^7}{a^8-b^8}$

$= \frac{4b^3(a^4+b^4)-4b^3(a^4-b^4)}{(a^4-b^4)(a^4+b^4)}-\frac{8b^7}{a^8-b^8}$

$= \frac{4a^4b^3+4b^7-4a^4b^3+4b^7}{a^8-b^8}-\frac{8b^7}{a^8-b^8}$

$= \frac{8b^7}{a^8-b^8}-\frac{8b^7}{a^8-b^8} = 0$

48. (d) $\frac{(.75)^3}{1-0.75} + [.75 + (.75)^2 + 1]$

$= \frac{(.75)^3+(1-.75)(.75+(.75)^2+1)}{(1-.75)}$

$= \frac{(.75)^3+(1)^3-(.75)^3}{.25}$

$= \frac{1}{.25}\times 1 = 4$

49. (d) $x^{\sqrt{x}} = \sqrt[n]{x^x}$

$\Rightarrow \left(x^x\right)^{\frac{1}{2}} = x^{\frac{x}{n}}$

Take log on both the sides,

$\Rightarrow \log(x^{\sqrt{x}}) = \log(x)^{x/n}$

$\Rightarrow \sqrt{x}\log x = \frac{x}{n}.\log x$

$\Rightarrow \sqrt{x}\log x - \frac{x}{n}\log x = 0$

$\Rightarrow \log x(\sqrt{x}-\frac{x}{n}) = 0$

$\Rightarrow (\log x)\ (\sqrt{x})(1-\frac{\sqrt{x}}{n}) = 0$

$\Rightarrow \log x = 0 \Rightarrow x = 1,$

$\sqrt{x} = 0 \Rightarrow x = 0$ or $\sqrt{x} = n \Rightarrow x = n^2$

Thus, $x = 0, 1, n^2$
$x = 0$ is not admissible
Since, log$x$ is not defined
$\therefore$ $x = 1, n^2$

50. (b) Given
$a - b = 4$ ... (i)
$a^2 + b^2 = 40$
Now, we know that
$(a - b)^2 = a^2 + b^2 - 2ab$
$\Rightarrow 16 = 40 - 2ab$
$2ab = 24$
$ab = 12$
$(a + b)^2 = 40 + 2ab$
$= 40 + 24$
$a + b = \sqrt{64} = 8$ ...(ii)
Now, on solving equations (i) and (ii)
$2a = 12$

$\therefore\ a = 6$
$b = 2$
$a^3 + b^6 = 216 + 64 = 280$

51. (c) Let the given enpsession be x

$\therefore\ x = \sqrt{4+\sqrt{4-x}}$

$x^2 - 4 = \sqrt{4-x}$

$(x^2-4)^2 = 4 - x$

$x^4 + 16 - 8x^2 = 4 - x$

$x + (x^2-4)^2 = 4$

All the choices are positive. By trial, $x = \frac{\sqrt{13}+1}{2}$

$\therefore\ x^2 = \frac{14+2\sqrt{3}}{4} = \frac{7+\sqrt{13}}{2}$

$x^2 - 4 = \frac{\sqrt{13}-1}{2}$ and $(x^2-4)^2 = \frac{14-2\sqrt{3}}{4}$

$= \frac{7-\sqrt{13}}{2}$

So, option (c) is correct.

52. (b) $x = \sqrt{3}+\sqrt{2}$

$\frac{1}{x} = \frac{1}{\sqrt{3}+\sqrt{2}} = \frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})}$

$= \sqrt{3}-\sqrt{2}$

$\Rightarrow x^3 + x + \frac{1}{x} + \frac{1}{x^3}$

$= x + \frac{1}{x} + x^3 + \frac{1}{x^3}$

$= \left(x+\frac{1}{x}\right) + \left(x+\frac{1}{x}\right)^3 - 3\left(x+\frac{1}{x}\right)$

$= \left(\sqrt{3}+\sqrt{2}+\sqrt{3}-\sqrt{2}\right) + \left(\sqrt{3}+\sqrt{2}+\sqrt{3}-\sqrt{2}\right)^3$
$\quad -3\left(\sqrt{3}+\sqrt{2}+\sqrt{3}-\sqrt{2}\right)$

$= 2\sqrt{3} + (2\sqrt{3})^3 - 3(2\sqrt{3})$

$= 2\sqrt{3} + 24\sqrt{3} - 6\sqrt{3}$

$= 20\sqrt{3}$

So, option (b) is correct

53. (a) $\sqrt{2} = 2^{\frac{1}{2}} = \left(2^6\right)^{\frac{1}{12}} = (64)^{\frac{1}{12}}$

$\sqrt[4]{6} = 6^{\frac{1}{4}} = \left(6^3\right)^{\frac{1}{12}} = (216)^{\frac{1}{12}}$

$\sqrt[3]{4} = 4^{\frac{1}{3}} = \left(4^4\right)^{\frac{1}{12}} = (256)^{\frac{1}{12}}$

so, correct order is-

$\Rightarrow\ (264)^{\frac{1}{12}} < (216)^{\frac{1}{12}} < (256)^{\frac{1}{12}}$

$\Rightarrow\ \sqrt{2} < \sqrt[4]{6} < \sqrt[3]{4}$

So, option (a) is correct.

54. (a) Given $x = 2^{\frac{1}{3}} + 2^{-\frac{1}{3}}$

$x = 2^{\frac{1}{3}} + \frac{1}{2^{\frac{1}{3}}}$

Cubing on both sides, we get

$x^3 = \left(2^{\frac{1}{3}}\right)^3 + \left(\frac{1}{2^{\frac{1}{3}}}\right)^3 + 3.2^{\frac{1}{3}}.\frac{1}{2^{\frac{1}{3}}}\left(2^{\frac{1}{3}} + \frac{1}{2^{\frac{1}{3}}}\right)$

$\Rightarrow\ x^3 = 2 + \frac{1}{2} + 3(x)$

$\Rightarrow\ x^3 = \frac{4+1+3x\times 2}{2}$

$\Rightarrow 2x^3 = 5 + 6x$

$\Rightarrow 2x^3 - 6x - 5 = 0$

$\therefore$ option (a) is correct.

55. (b) $4^x.2^y = 128$ and $3^{3x}.3^{2y} - 9^{xy} = 0$

$2^{2x}.\ 2^y = 128 = (2)^7 \quad 3^{3x+2y} = 3^{2xy} \left[\because 9 = 3^2\right]$

$\Rightarrow\ 2x + y = 7 \ \Rightarrow\ 3x + 2y = 2xy \quad ...(2)$

$\Rightarrow y = 7 - 2x \quad ...(1)$

Substitute this value of y in (2) we get

$3x + 2(7-2x) = 2x(7-2x)$

$3x + 14 - 4x = 14x - 4x^2$

$4x^2 - 15x + 14 = 0$

$(4x-7)(x-2) = 0$

either $4x - 7 = 0$ or $x - 2 = 0$

$\Rightarrow\ x = \frac{7}{4}$ or $x = 2$

$x \neq \frac{7}{4} \Rightarrow x = 2$

$y = 7 - 2(2) = 3$

$\Rightarrow x + y = 2 + 3 = 5$

$\therefore$ Option (b) is correct.

56. (c) $\frac{(243+647)^2 + (243-647)^2}{243\times 243 + 647\times 647}$

$= \frac{(243)^2 + (647)^2 + 2\cdot(243)\cdot(647) + (243)^2 + (647)^2 - 2\cdot(243)(647)}{(243)^2 + (647)^2}$

$$= \frac{2(243)^2 + 2(647)^2}{(243)^2 + (647)^2}$$

$$= \frac{2\left[(243)^2 + (647)^2\right]}{(243)^2 + (647)^2}$$

$= 2$

$\therefore$ Option (c) is correct.

57. (a) $\dfrac{6^2 + 7^2 + 8^2 + 9^2 + 10^2}{\sqrt{7+4\sqrt{3}} - \sqrt{4+2\sqrt{3}}}$

$$\Rightarrow \frac{330}{\sqrt{(2+\sqrt{3})^2 - \sqrt{(1+\sqrt{3})^2}}}$$

$$\Rightarrow \frac{300}{2+\sqrt{3}-1-\sqrt{3}} = 330$$

58. (c) let two no. 8 be x and y

$x^2 + y^2 = 208$

$x^2 + 18x = 208 \qquad \therefore \; y^2 = 18x$

$x^2 + 18x - 208 = 0$

$x^2 + 26x - 8x - 208 = 0$

$(x-8)(x+26) = 0$

$x = 8$

$y^2 = 18 \times 8$

$y^2 = 144$

$y = 12$

$y - x = 12 - 8 = 4$

59. (a) 2^1, devided by $(1+1)$ $R = 0$

2^2, devided by $(2+1)$ $R = 1$

2^3, devided by $(3+1)$ $R = 0$

2^4, devided by $(4+1)$ $R = 1$

Same for even paper 100

2^{100}, is devided by $(100+1)$, $R = 1$

60. (c) 1729 can be written as

$12^3 + 1^3, 10^3 + 9^3$

61. (a) $\sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}} + \sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}} + ... \sqrt{1+\frac{1}{2007^2}+\frac{1}{2008^2}}$

$$\Rightarrow \sqrt{\left(\frac{3}{2}\right)^2} + \sqrt{\left(\frac{7}{6}\right)^2} \; .... \; \sqrt{\frac{(2007\times 2008+1)^2}{(2007)^2(2008)^2}}$$

$$\frac{3}{2} + \frac{7}{6} + \frac{13}{12} .... \frac{(2007\times 2008+1)}{(2007)(2008)}$$

$$\Rightarrow 1 + \frac{1}{1\times 2} + 1 + \frac{1}{2\times 3} + 1 + \frac{1}{3\times 4} .... 1 + \frac{1}{2007\times 2008}$$

$$\Rightarrow 2007 + 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} \; .... \; \frac{1}{2007} - \frac{1}{2008}$$

$$\Rightarrow 2008 - \frac{1}{2008}$$

62. (a) Only statement 1 is correct it can be easily varified by taking any 2 odd integer.

ex $\to$ 3 and 5

or

the two odd number be $2n + 1$ and $2n + 3$

squaring and adding by we get

$\Rightarrow 4n^2 + 4n + 1 + 4n^2 + 12n + 9$

$\Rightarrow 8n^2 + 16n + 10$

$8n^2 + 16n + 8 + 2$

$[42n^2 + 4n + 2] + 2$

i.e. $x^2 + y^2$ is an odd number but not necessarily a multiple of 4.

63. (d) $\dfrac{13^5 + 14^5 + 15^5 + 16^5}{29} = \dfrac{13^5 + 16^5}{29} + \dfrac{14^5 + 15^5}{29}$

$$\Rightarrow \frac{13^5 + 16^5}{13+16} + \frac{15^5 + 14^5}{15+14}$$

no remainder

as $x^n + a^n$ when n is odd is completely devisibel by $x + a$

64. (b) $\Sigma n^3 - \Sigma n^2 = \left[\dfrac{n(n+1)^2}{2}\right] - \left(\dfrac{n(n+1)(2n+1)}{6}\right)$

$n = 10$

$$\left[\frac{10\,[11]}{2}\right]^2 - \left[\frac{10\,(11)\,(21)}{6}\right]$$

$3025 - 385 = 2640$

65. (a) $\dfrac{(0.35)^2 + (0.70) + 1}{2.25} + 0.19$

$$\Rightarrow \frac{(0.35)^2 + 2(0.35)1 + 1^2 + 2.25\times 0.19}{2.25}$$

$$\Rightarrow \frac{(0.35+1)^2 + 0.4275}{2.25}$$

$$\Rightarrow \frac{1.8225 + 0.4275}{2.25} \Rightarrow \frac{2.25}{2.25} = 1$$

66. (c) $\dfrac{(443+547)^2 + (443-547)^2}{443\times 443 + 547\times 547}$

$$\Rightarrow \frac{2[(443)^2 + (547)^2]}{(443)^2 + (547)^2}$$

$\because (a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$

$= 2$

67. (b) $a - b = 5$, $a^3 - b^3 = 335$

$(a-b)^3 = 125$

$a^3 - b^3 - 3ab\,[a-b] = 125$

$3ab\,[a-b] = 335 - 125$

$$ab = \frac{210}{5\times 3} = 14$$

$(a-b)^2 + 4ab = 25 + 4\,(14)$

$(a+b)^2 = 81$

$a + b = 9$

68. (c) $9^x\, 3^y = 2187,\ 2^{3x}\, 2^{2y} = 4^{xy}$

$9^x 3^y = 9^1 3^5$ or $9^2\, 3^3$ or $9^3 3^1$ in all the condition $x + y = 5$

69. (a) $5^{1+x} + 5^{1-x} = 26$

$5^{1+x} + 5^{1-x} = 5^2 + 5^0$

Case I

$1 + x = 2,\qquad 1 - x = 0$

$x = 1\qquad x = 1$

Case II

$1 + x = 0,\qquad 1 - x = 2$

$x = -1,\qquad x = -1$

70. (d) length and breadth when increased by 200%

$$\text{net effect in area} = 200 + 200 + \frac{200 \times 200}{100} = 800\%$$

$$\left\{x + y + \frac{xy}{100}\right\}$$

71. (b) Let $f(x) = ax^3 + bx^2 + cx + d$

and $g(x) = ax^4 + bx^3 + cx^2 + dx + e$

Then, $f(x).g(x)$

$= (ax^3 + bx^2 + cx + d)(ax^4 + bx^3 + cx^2 + dx + e)$

$= a^2x^7 + abx^6 + acx^5 \ldots d^2x + de$

Thus, it is clear that degree of $f(x)\, g(x)$ is 7.

72. (d) According to the rule, the numbers having 2, 3, 7 and 8 at their unit's place are not perfect squares. Therefore, here, we can see that options 1, 3 and 4 have 2, 3 and 7, respectively at their unit's place and so, consequently, these are not perfect squares.

73. (c) If $\sqrt{x + \sqrt{x + \sqrt{x + \ldots\ldots}}}$ then its value is $\dfrac{\sqrt{4x+1}+1}{2}$

Therefore, according to the question,

$$\frac{\sqrt{4x+1}+1}{2} = \frac{\sqrt{4.1+1}+1}{2} = \frac{\sqrt{5}+1}{2} = \frac{2.236+1}{2}$$

$= \dfrac{3.236}{2} = 1.618$ which lies between 1 and 2.

74. (b) $x = y^{\frac{1}{a}}$ (given)

$y = z^{\frac{1}{b}}$ (given)

$z = x^{\frac{1}{c}}$ (given)

Now

$y = z^{\frac{1}{b}}$

$x = y^{\frac{1}{a}} = z^{\frac{1}{b} \times \frac{1}{a}}\qquad (\because y = z^{\frac{1}{b}})$

$z = x^{\frac{1}{c}}$

$z = z^{\frac{1}{ab} \times \frac{1}{c}}\qquad (\because x = z^{\frac{1}{ab}})$

$z = z^{\frac{1}{abc}}$

$\therefore \dfrac{1}{abc} = 1\qquad \therefore abc = 1$

75. (c) $2b = a + c$ (given)

$\Rightarrow b + b = a + c$

$\Rightarrow a - b = b - c$

$y^2 = xz$ (given)

$x^{(b-c)} \times y^{c-a} \times z^{(a-b)} = x^{a-b} \times y^{c-a} \times z^{a-b}$

$(\because a - b = b - c)$

$(xz)^{a-b} \times y^{c-a}$ {because $a^n \times b^n = (ab)^n$}

$(y^2)^{a-b} \times y^{c-a}$ $\{\because y^2 = xz\}$

$y^{2a-2b+c-a} = y^{a+c-2b} = y^{2b-2b}$ $(\because a + c = 2b)$

$y^0 = 1$

76. (b) Smallest 4 digits integer = 1000

$31^2 < 1000$

$\therefore$ smallest 4 digits integer which is a perfect square is $32^2 = 1024$

77. (a) $\dfrac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} - \dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

$$= \frac{\left(\sqrt{5}-\sqrt{3}\right)^2 - \left(\sqrt{5}+\sqrt{3}\right)^2}{\left(\sqrt{5}+\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)}$$

$$= \frac{\left(\sqrt{5}\right)^2 + \left(\sqrt{3}\right)^2 - 2\times\sqrt{5}\times\sqrt{3} - \left\{\left(\sqrt{5}\right)^2 + \left(\sqrt{3}\right)^2 + 2\times\sqrt{5}\times\sqrt{3}\right\}}{\left(\sqrt{5}\right)^2 - \left(\sqrt{3}\right)^2}$$

$$= \frac{5 + 3 - 2\sqrt{15} - 5 - 3 - 2\sqrt{15}}{5 - 3} = \frac{-4\sqrt{15}}{2} = -2\sqrt{15}$$

78. (c) $\dfrac{1}{1 + x^{b-a} + x^{c-a}} + \dfrac{1}{1 + x^{a-b} + x^{c-b}} + \dfrac{1}{1 + x^{a-c} + x^{b-c}}$

$$= \frac{1}{1 + \frac{x^b}{x^a} + \frac{x^c}{x^a}} + \frac{1}{1 + \frac{x^a}{x^b} + \frac{x^c}{x^b}} + \frac{1}{1 + \frac{x^a}{x^c} + \frac{x^b}{x^c}}$$

$$= \frac{1}{\frac{x^a + x^b + x^c}{x^a}} + \frac{1}{\frac{x^b + x^a + x^c}{x^b}} + \frac{1}{\frac{x^c + x^a + x^b}{x^c}}$$

$$= \frac{x^a}{x^a + x^b + x^c} + \frac{x^b}{x^a + x^b + x^c} + \frac{x^c}{x^a + x^b + x^c}$$

$$= \frac{x^a + x^b + x^c}{x^a + x^b + x^c} = 1$$

79. (a) Let number be x
According to the question
$x^2 + x = 20$
$x^2 + x - 20 = 0$
$x^2 + 5x - 4x - 20 = 0$
$x(x + 5) - 4(x + 5) = 0$
$(x - 4)(x + 5) = 0$
$x - 4 = 0 \qquad \therefore x = 4$
$x + 5 = 0 \qquad \therefore x = -5$

80. (d) In these type of questions number of the pair of 5 and even number makes a multiple of 10
in the product of $1 \times 2 \times 3 \times 4 \times .... \times 24 \times 25$
Number of multiples of 5 = 5 (5, 10, 15, 20, 25)
Here 25 is a pair of 5 $(25 = 5 \times 5)$
Number of 5 in this product = 6
Number of even numbers in this product = 12
Hence required power of 10 is 6

81. (a) $x = y^a$, $y = z^b$ and $z = x^c$ {given}
From $x = y^a$
$\Rightarrow x = (z^b)^a$
$x = z^{ab}$
$\Rightarrow x = (x^c)^{ab}$
$\therefore x = x^{abc}$
Hence, abc = 1.

82. (a) $x = 2 + 2^{2/3} + 2^{1/3} + \Rightarrow (x - 2) = (2^{2/3} + 2^{1/3})$ ...(i)
on cutting both sides, we get.

$$(x - 2)^3 = \left(2^{\frac{1}{3}} + 2^{\frac{2}{3}}\right)^3$$

$$x^3 - 8 - 6x^2 + 12x = 2 + 2^2 + 3.2^{\frac{1}{2}}.2^{\frac{2}{3}}\left(2^{\frac{1}{3}} + 2^{\frac{2}{3}}\right)$$

$x^3 - 8 - 6x^2 + 12x = 6 + 6(x - 2)$
$x^3 - 2 - 6x^2 + 6x = 0$
$\therefore x^3 - 6x^2 + 6x = 2$

83. (b) $\left(x^6 + \frac{1}{x^6}\right) = \left\{(x^2)^3 + \left(\frac{1}{x^2}\right)^3\right\}$

$$\left(x^2 + \frac{1}{x^2}\right)\left(x^4 + \frac{1}{x^4} - 1\right)$$

$$\therefore \quad K = \left(x^4 + \frac{1}{x^4} - 1\right)$$

84. (b) $a^x = b^y = c^z = k$ (let)
$a = (k)^{1/x}$, $b = (k)^{1/y}$, $c = (k)^{1/2}$

Now, abc = 1 $\Rightarrow (k)^{\frac{1}{x}+\frac{1}{y}+\frac{1}{z}} = (k)^{\circ}$

$$\therefore \quad \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$$

85. (d) $a = xy^{P-1} \Rightarrow a^{q-r} = (xy^{P-1}) = x^{(q-r)}.y^{(P-1)(q-r)}$
$b = yz^{q-1} \Rightarrow b^{r-p} = (yz^{(q-1)})^{(r-p)} = y^{(r-P)}.z^{(q-1)(r-p)}$.
$c = zx^{r-1} \Rightarrow c^{P-q} = (zx^{(r-1)})^{P-q} = z^{(p-q)}.x^{(r-1)(p-q)}$.
Now, a^{q-r}, $b^{r-p}.c^{p-q} = \{x^{(q-r)}.y^{(pq-pr-q+r)}\}.\{y^{(r-p)}.z^{(qr-pq-r+p)}\}.\{.z^{(p-q)}.x^{(rp-qr-p+q)}\}$
$= x^{(q-r+rp-p+q-qr)}.y^{\{pq-pr-q+r+r-p)}.z^{(qr-pq-r+P+P-q)}$
Hence, correct option is (d)

86. (c) $(n^2 + 48) = x^2$
$48 = x^2 - n^2$
$48 = (x - n)(x + n)$
So the possible number of pairs of $(x - n)$ and $(x + n)$ are (1,48), (2,24), (3,16), (4,12), (6,8)
On solving the above pairs for $(x - n)$ and $(x + n)$, we get the integer values of n and x as
$n = 1, x = 7$
$n = 4, x = 8$
$n = 11, x = 13$
So, the total possible values of n are 3.

87. (d) $x = \frac{4\sqrt{6}}{\sqrt{2} + \sqrt{3}}$
on rationalizing.

$$x = \frac{4\sqrt{6}}{\sqrt{3} + \sqrt{2}} \times \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} - \sqrt{2}}$$

$$x = 12\sqrt{2} - 8\sqrt{3}$$

putting the value of x in the equation

$$\frac{14\sqrt{2} - 8\sqrt{3}}{10\sqrt{2} - 8\sqrt{3}} + \frac{12\sqrt{2} - 6\sqrt{3}}{12\sqrt{2} - 10\sqrt{3}}$$

$$= \frac{7\sqrt{2} - 4\sqrt{3}}{5\sqrt{2} - 4\sqrt{3}} + \frac{6\sqrt{2} - 3\sqrt{3}}{6\sqrt{2} - 5\sqrt{3}}$$

$$\frac{2\sqrt{2}}{5\sqrt{2} - 4\sqrt{3}} + 1 + 1 + \frac{2\sqrt{3}}{6\sqrt{2} - 5\sqrt{3}}$$

$$2 + \frac{2\sqrt{2}(6\sqrt{2} - 5\sqrt{3}) + 2\sqrt{3}(5\sqrt{2} - 4\sqrt{3})}{(5\sqrt{2} - 4\sqrt{3})(6\sqrt{2} - 5\sqrt{3})}$$

$$2 + \frac{24 - 10\sqrt{6} + 10\sqrt{6} - 24}{(5\sqrt{2} - 4\sqrt{3})(6\sqrt{2} - 5\sqrt{3})} = 2 + 0 = 2$$

88. (c) $3^x = 4^y = 12^z$
Taking log of all 3 we get
$x \log 3 = y \log 4 = z \log 12 = k$

$$z = \frac{k}{log 12} = \frac{k}{log(3 \times 4)}$$

$$= \frac{k}{log 3 + log\ 4} = \frac{k}{\frac{k}{x} + \frac{k}{y}} \quad = \frac{xy}{(x + y)}$$

89. (d) $2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + ...}}} = x$

$$\Rightarrow \sqrt{2 + \sqrt{2 + \sqrt{2 + ...}}} = x - 2$$

Squaring both side

$$2+\sqrt{2+\sqrt{2+\sqrt{2+}}}\ldots=(x-2)^2$$
$\Rightarrow x = x^2 - 4x + 4$
$\Rightarrow x^2 - 5x + 4 = 0$
$\Rightarrow x^2 - 4x - x + 4 = 0$
$\Rightarrow x(x-4) - 1(x+4) = 0$
$\Rightarrow (x-1)(x-4) = 0$
$\therefore x = 1, 4$
As x always take greater than 2
$\therefore x = 4$

90. (c) Given expression
$5^{2n} - 2^{3n} = (5^2)^n - (2^3)^n = (25)^n - (8)^n$
We know that $a^n - b^n$ always have a common factor $(a-b)$
Therefore one of the factor is $25 - 8 = 17$

91. (c) $a = \sqrt{7+4\sqrt{3}} = \sqrt{(2)^2+(\sqrt{3})^2+4\sqrt{3}}$
$= \sqrt{(2+\sqrt{3})^2} = 2+\sqrt{3}$
$= \frac{1}{a} = \frac{1}{2+\sqrt{3}} = 2-\sqrt{3} \Rightarrow a+\frac{1}{a} = 4$

92. (d) $n^2 + 19n + 92 = (n^2 + 18n + 81) + n + 11$
To be perfect square
$n + 11 = 0 \quad \therefore \quad n = -11$
$n^2 + 19n + 92 = (n^2 + 20n + 100) - n - 8$
To be perfect square, $-n - 8 = 0$
$n = -8$
ATQ, Sum $= -11 - 8 = -19$

93. (c) $(a+b)^2 = 16 + 6\sqrt{7}$
$\Rightarrow \; 2ab = 6\sqrt{7} \; \Rightarrow \; ab = 3\sqrt{7}$
$\Rightarrow \; a = 3, b = \sqrt{7}$
$\therefore \; (16 + 6\sqrt{7}) = (9 + 7 + 6\sqrt{7})$
$= (3^2 + (\sqrt{7})^2 + 2 \times 3\sqrt{7})$
$= (\sqrt{7} + 3)^2$
$\therefore$ Square root of $(16+6\sqrt{7})$ is $(3+\sqrt{7})$

94. (a) $a\sqrt{a} + b\sqrt{b} = 32$ ...(i)
$a\sqrt{b} + b\sqrt{a} = 31$ ...(ii)
Squaring and subtracting equation (i) and (ii), we have
$$a^2a + b^2b + 2ab\sqrt{ab} = 32^2$$
$$a^2b + b^2a + 2ab\sqrt{ab} = 31^2$$
$$a^2(a-b) - b^2(a-b) = 32^2 - 31^2$$
$(a-b)^2(a+b) = 63$
$(a-b)^2(a+b) = 3^2 \cdot 7$
$\therefore \; (a-b) = 3$ and $a + b = 7$
$\Rightarrow \; a = 5, b = 2$
Now, $\frac{5(a+b)}{7} = \frac{5(5+2)}{7} = 5$

95. (a) From question we have $x^{\sqrt{x}} = x^{x/n}$
$\Rightarrow$ On equating the powers on both end
$= \sqrt{x} = \frac{x}{n} \Rightarrow n = \sqrt{x} \; \Rightarrow \; x = n^2$
So possible solution are $x = \{1, n^2\}$

96. (a) Number in ascending orders are
$2222^2 < 222^{22} < 22^{222} < 2^{2222}$

97. (d) $5^{x+1} - 5^{x-1} = 600$
$5^{x-1}(25-1) = 600 \quad \Rightarrow 5^{x-1} = 25 \quad \Rightarrow x = 3$
$10^{2x} = 10^6 = 1000000$

98. (b) $1331 = 11^3 = m^n$
$\Rightarrow \; m = 11, n = 3$
$(m-1)^{n-1} = 10^2 = 100$

99. (b) $x = 2^{1/2} = 8^{1/6}$
$y = 3^{1/3} = 9^{1/6}$
and $z = \sqrt[6]{6} = 6^{1/6}$
Hence $= y > x > z$

CHAPTER 5

Percentage

1. If the length of a rectangle is increased by 10% and the area is unchanged, then by how much per cent does the breadth decrease? **[2007-I]**
(a) 100/11% (b) 100/9% (c) 9% (d) 10%

2. The population of a village increases by 20% in one year and decrease by 20% by the next year. If at the begining of the third year, the population is 5184, what was the population in the first year? **[2007-II]**
(a) 5400 (b) 5500 (c) 5600 (d) 5800

3. There are some coins and rings of either gold or silver in a box. 60% of the objects are coins, 40% of the rings are of gold and 30% of the coins are of silver. What is the percentage of gold articles? **[2009-I]**
(a) 16% (b) 27% (c) 58% (d) 70%

4. if the radius of the base and the height of a right circular cone are increased by 20%, then what is the approximate percentage increase in volume? **[2009-II]**
(a) 60% (b) 68% (c) 73% (d) 75%

5. 10% of the inhabitants of a certain city left that city. Later on 10% of the remaining inhabitants of that city again left the city. What is the remaining percentage of population of that city? **[2009-II]**
(a) 80% (b) 80.4% (c) 80.6% (d) 81%

6. The number of workers in the employment guarantee scheme increased by 15 which resulted into an increase of 20%. What was the initial number of workers? **[2009-II]**
(a) 60 (b) 75 (c) 80 (d) 90

7. If 50% of $(x - y)$ = 40% of $(x + y)$, then what per cent of x is y? **[2011-I]**
(a) $10\frac{1}{9}\%$ (b) $11\frac{1}{9}\%$ (c) $13\frac{1}{9}\%$ (d) $21\frac{1}{9}\%$

8. A person spends 30% of monthly salary on rent, 25% on food, 20% on children's education and 12% on electricity and the balance of ₹ 1040 on the remaining items. What is the monthly salary of the person? **[2011-I]**
(a) ₹ 8000 (b) ₹ 9000 (c) ₹ 9600 (d) ₹ 10600

9. If salary of X is 20% more than salary of Y, then by how much percentage is salary of Y less than X? **[2011-I]**
(a) 25 (b) 20 (c) $\frac{50}{3}$ (d) $\frac{65}{4}$

10. If the height of a cone is increased by 50%, then what is the percentage increase in the volume of the cone? **[2011-I]**
(a) $\frac{100}{3}\%$ (b) 40 % (c) 50 % (d) $\frac{200}{3}\%$

11. 38L of milk was poured into a tub and the tub was found to be 5% empty. To completely fill the tub, what amount of additional milk must be poured? **[2011-I]**
(a) 1 ℓ (b) 2 ℓ (c) 3 ℓ (d) 4 ℓ

12. Water contains $14\frac{2}{7}\%$ of hydrogen and the rest is oxygen. In 350 g of water, oxygen will be **[2011-I]**
(a) 300g (b) 250g
(c) 200g (d) None of these

13. To an examination, a candidate needs 40% marks. All questions carry equal marks. A candidate just passed by getting 10 answers correct by attempting 15 of the total questions. How many questions are there in the examination? **[2011-I]**
(a) 25 (b) 30 (c) 40 (d) 45

14. The income of 'A' is 20% higher than that of 'B'. The income of 'B' is 25% less than of 'C'. What percent less is A's income from C's income? **[2011-I]**
(a) 7% (b) 8% (c) 10% (d) 12.5%

15. What is the number whose 20% is 30% of 40? **[2011-I]**
(a) 90 (b) 80 (c) 60 (d) 50

16. An employee is required to contribute 10% of his payment to General Provident Fund. If he gets ₹13500 as net pay in a month, then what is the monthly General Provident Fund contribution (assuming no other deductions)? **[2011-II]**
(a) ₹ 1215 (b) ₹ 1350 (c) ₹ 1500 (d) ₹ 1650

17. A man losses 20% of his money. After spending 25% of the remaining, he has ₹480 left. What is the amount of money he originally had? **[2012-I]**
(a) ₹ 600 (b) ₹ 720 (c) ₹ 800 (d) ₹ 840

18. The radius of the base of a right circular cone is increased by 15% keeping the height fixed. The volume of the cone will be increased by **[2012-II]**
(a) 30% (b) 31% (c) 32.25% (d) 34.75%

19. The price of an article is ₹ 25. After two successive cuts by the same percentage, the price becomes ₹20.25. If each time the cut was x%, then **[2012-II]**
(a) $x = 9$ (b) $x = 10$ (c) $x = 11$ (d) $x = 11.5$

20. What is 5% of 50% of 500? **[2012-II]**
(a) 12.5 (b) 25 (c) 1.25 (d) 6.25

21. X, Y and Z had taken a dinner together. The cost of the meal of Z was 20% more than that of Y and the cost of the meal of X was 5/6 as much as the cost of the meal

of Z. If Y paid ₹100, then what was the total amount that all the three of them had paid? **[2013-II]**

(a) ₹285 (b) ₹300
(c) ₹355 (d) None of these

22. A person's salary has increased from ₹7200 to ₹ 8100. What is the percentage increase in his salary? **[2013-II]**

(a) 25% (b) 18%
(c) $16\frac{2}{3}\%$ (d) $12\frac{1}{2}\%$

23. A person sold an article from ₹ 3600 and got a profit of 20%. Had he sold the article for ₹ 3150, how much profit would he have got? **[2013-II]**

(a) 4% (b) 5% (c) 6% (d) 10%

24. A water pipe is cut into two pieces. The longer piece is 70% of the length of the pipe. By how much percentage is the longer piece longer than the shorter peice? **[2014-I]**

(a) 140% (b) $\frac{400}{3}\%$
(c) 40% (d) None of these

25. On a 20% discount sale, an article costs 596. What was the original price of the article? **[2014-I]**

(a) ₹ 720 (b) ₹ 735 (c) ₹ 745 (d) ₹ 775

26. A person could save 10% of his income. But 2 years later, when his income increased by 20%, he could save the same amount only as before. By how much percentage has his expenditure increased? **[2015-I]**

(a) $22\frac{2}{9}\%$ (b) $23\frac{1}{3}\%$ (c) $24\frac{2}{9}\%$ (d) $25\frac{2}{9}\%$

27. A milkman claims to sell milk at its cost price only. Still he is making a profit of 20% since he has mixed some amount of water in the milk. What is the percentage of milk in the mixture? **[2015-II]**

(a) $\frac{200}{3}\%$ (b) 75% (c) 80% (d) $\frac{250}{3}\%$

28. 20% of a number when added to 20 becomes the number itself, then the number is **[2015-II]**

(a) 20 (b) 25 (c) 50 (d) 80

29. A's salary was increased by 40% and then decreased by 20%. On the whole A's salary is increased by **[2015-II]**

(a) 60% (b) 40% (c) 20% (d) 12%

30. In an election 10% of the voters on the voter list did not cast their vote and 60 voters cast their ballot papers blank. There were only two candidates. The winner was supported by 47% of total voters in the voter list and he got 308 voters more than his rival. The number of voters on the voter list is **[2015-II]**

(a) 3600 (b) 6200 (c) 6028 (d) 6400

31. The salary of a person is increased by 10% of his original salary. But he received the same amount even after increment. What is the percentage of his salary he did not receive? **[2016-I]**

(a) 11% (b) 10%
(c) (100/11)% (d) (90/11)%

32. The expenditure of a household for a certain month is ₹ 20,000, out of which ₹ 8,000 is spent on education, ₹ 5,900 on food, ₹ 2,800 on shopping and the rest on personal care. What percentage of expenditure is spent on personal care? **[2016-I]**

(a) 12% (b) 16.5% (c) 18% (d) 21.8%

33. A candidate scoring x% marks in an examination fails by a marks, while another candidate who scores y% marks gets b marks more than the minimum required pass marks. What is the maximum marks for the examination ? **[2016-II]**

(a) $\frac{100(a+b)}{x-y}$ (b) $\frac{100(a-b)}{x+y}$
(c) $\frac{100(a+b)}{y-x}$ (d) $\frac{100(a-b)}{x-y}$

34. If a% of a + b% of b = 2% of ab, then what percent of a is b ? **[2017-I]**

(a) 50% (b) 75%
(c) 100% (d) Cannot be determined

35. $\frac{5}{9}^{th}$ part of the population in a village are males. If 30% of the males are married, the percentage of unmarried females in the total population is **[2017-I]**

(a) $20\frac{2}{9}\%$ (b) $27\frac{2}{9}\%$ (c) $27\frac{7}{9}\%$ (d) $29\frac{2}{9}\%$

36. A fruit seller has a certain number of mangoes of which 5% are rotten. He sells 75% of the remainder and he is left with 95 mangoes. How many mangoes did he have originally? **[2017-II]**

(a) 500 (b) 450 (c) 400 (d) 350

37. A student has to secure 40% of marks to pass an examination. He gets only 45 marks and fails by 5 marks. The maximum marks are **[2017-II]**

(a) 120 (b) 125 (c) 130 (d) 150

38. When prices rise by 12%, if the expenditure is to be the same, what is the percentage of consumption to be reduced? **[2017-II]**

(a) $16\frac{2}{3}\%$ (b) $10\frac{2}{7}\%$ (c) $16\frac{3}{5}\%$ (d) $10\frac{5}{7}\%$

39. If the price of wheat rises by 25%, then by how much percent must a man reduce his consumption in order to keep his budget the same as before? **[2018-I]**

(a) 15% (b) 20% (c) 25% (d) 30%

40. Out of 85 children playing badminton or table tennis or both, the total number of girls in the group is 70% of the total number of boys in the group. The number of boys playing only badminton is 50% of the number of boys and the total number of boys playing badminton is 60% of the total number of boys. The number of children playing only table tennis is 40% of the total number of children and a total of 12 children play badminton and table tennis both. The number of girls playing only badminton is **[2018-II]**

(a) 14 (b) 16 (c) 17 (d) 35

41. A person bought two articles X and Y from a departmental store. The sum of prices before sales tax was ‘ 130. There was no sales tax on the article X and 9% sales tax on the article Y. The total amount the person paid, including the sales tax was ₹136.75. What was the price of the article Y before sales tax? **[2018-II]**

(a) ₹ 75 (b) ₹ 85 (c) ₹ 122 (d) ₹ 125

42. In a hostel the rent per room is increased by 20%. If number of rooms in the hostel is also increased by 20% and the hostel is always full, then what is the percentage change in the total collection at the cash counter ? **[2019-I]**

(a) 30% (b) 40% (c) 44% (d) 48%

43. *x*, *y* and *z* are three numbers such that *x* is 30% of z and *y* is 40% of *z*. If *x* is *p*% of *y*, then what is the value of *p* ? **[2019-I]**

(a) 45 (b) 55 (c) 65 (d) 75

44. The train fare and bus fare between two stations is in the ratio 3 : 4. If the train fare increases by 20% and bus fare increases by 30%, then what is the ratio between revised train fare and revised bus fare ? **[2019-II]**

(a) $\frac{9}{13}$ (b) $\frac{17}{12}$ (c) $\frac{32}{43}$ (d) $\frac{19}{21}$

45. The price of an article *X* increases by 20% every year and price of article *Y* increases by 10% every year. In the year 2010, the price of article *X* was ₹5000 and price of article *Y* was ₹2000. In which year the difference in their prices exceeded ₹5000 for the first time ? **[2020-I]**

(a) 2012 (b) 2013 (c) 2014 (d) 2015

46. If the annual income of *X* is 20% more than that of *Y*, then the income of *Y* is less than that of *X* by *p*%. What is the value of *p* ? **[2020-I]**

(a) 10 (b) $16\frac{2}{3}$ (c) $17\frac{1}{3}$ (d) 20

HINTS & SOLUTIONS

1. (a) Area of Rectangle = length × breadth
Here, length is increased by 10%. But area is constant so that breadth is decreased.

$$L \times B = K \text{ (Constant Area)}$$

(+10%) (Decrease)

$$\frac{110}{100}L \times \frac{b\times 100}{110} = K \text{ (Constant Area)}$$

Percentage decrease by $= \dfrac{b-\dfrac{100b}{110}}{b}\times 100 = \dfrac{100}{11}\%$

2. (a) Change in population at begining of third year

$$= x + y + \frac{xy}{100}$$

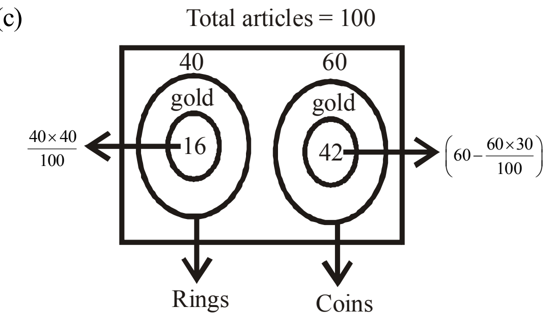
$$= 20 - 20 + \frac{(20)\times(-20)}{100} = -4\%$$

Let Initial population = x

Now, $x \times \dfrac{(100-4)}{100} = 5184$

$$x = \frac{5184\times 100}{96} = 5400.$$

3. (c)



Total of gold articles = 42 + 16 = 58 of gold articles

$$= \frac{58}{100}\times 100 = 58\%$$

4. (c) Change in volume

$$= x + y + z + \frac{xy + yz + zx}{100} + \frac{xyz}{10000}$$

Here, $x = y = z = 20\%$

Change in volume

$$= 20 + 20 + 20 + \frac{20\times 20 + 20\times 20 + 20\times 20}{100} + \frac{20\times 20\times 20}{10000}$$

$$= 60 + \frac{1200}{100} + \frac{8}{10} = 72.8 \approx 73\%$$

5. (d) Here two times decrease in the population of certain city. so net rate of decrement in population

$$= x + y + \frac{xy}{100}$$

$$= -10 - 10 + \frac{10\times 10}{100}$$

$= -19\%$

Rest of Remaining population = (100 - 19)%

= 81%.

6. (b) Let initial number of workers = x

$$\therefore 20\% \text{ of } x = 15$$

$$\Rightarrow \frac{20}{100}\times x = 15$$

$$\therefore x = 75$$

7. (b) Given that, 50% of $(x - y) = 40\%$ of $(x + y)$

$$\Rightarrow \frac{50}{100}\times(x - y) = \frac{40}{100}\times(x + y)$$

$$\Rightarrow 5x - 5y = 4x + 4y$$

$$\Rightarrow x = 9y \qquad \text{...(i)}$$

Let r % of x = y

$$\Rightarrow \frac{r}{100}\times 9y = y \qquad \text{[from Eq. (i)]}$$

$$\therefore r = \frac{100}{9} = 11\frac{1}{9}\%$$

8. (a) Let the monthly salary of person = ₹x.

Total spends = (30% + 25% + 20% + 12%) = 87%.

Now,

$$x\times\frac{13}{100} = 1040$$

$$\therefore x = \frac{1040\times 100}{13} = 8000.$$

9. (c) $\underset{120}{X} \xleftarrow{(+20\%)} \underset{100}{Y}$

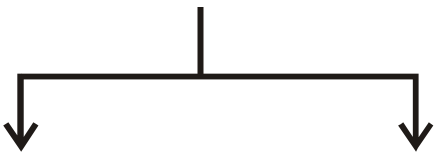
Less percentage $= \frac{20}{120}\text{x}100 = \frac{50}{3}$

10. (c) Volume of cone $= \frac{1}{3}\pi r^2 h$

Here radius is constant, then volume is directly proportional to height. Now height increase 50%. So percentage increase in volume is 50%.

11. (b) Let tub capacity xL.

Tube (x)



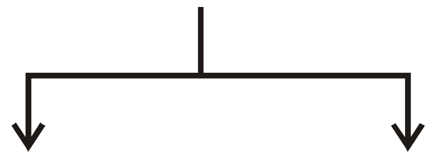
(x – 38) (ℓ empty) 38 (Milk)

Now, $x\times\frac{95}{100} = 38$

x = 40 L,

Additional milk = 40 L – 38 L = 2L.

12. (a) Water (100%)



Hydrogen $\frac{98+2}{7} = \frac{100}{7}\%$ Oxygen $\frac{600}{7}\%$

Ratio of Hydrogen and oxygen = 1 : 6

Now, oxygen in 350g of water $= \frac{6}{7}\times 350 = 300\text{g}$

13. (a) Let the number of questions in examination = x

By given condition, 40% of x = 10

$$\Rightarrow \frac{x \text{ x } 40}{100} = 10$$

$$\therefore x = \frac{1000}{40} = 25$$

14. (c) A $\xleftarrow{(+20\%)}$ B C

120 : 100

75 $\xrightarrow{(-25\%)}$ 100

Making B equal

A	:	B	:	C
360	:	300		
		300		400

Now,

A	:	B	:	C
18	:	15	:	20

A income less $= \frac{2}{20}\times 100 = 10\%$

15. (c) Let the number be x

According to question 20% of x = 30% of 40

$$\Rightarrow \frac{x\times 20}{100} = \frac{40\times 30}{100}$$

$$\Rightarrow x = \frac{40\times 30}{20} = 60$$

16. (c) Let the net pay of employee = x

After contributing 10% = $x \times \frac{90}{100} = \frac{9x}{10}$

According to question $\frac{9x}{10} = 13500$

$\because \quad x = \frac{13500 \times 10}{9} = 15000$

$\because$ General provident fund = 10% of basic pay

$= \frac{10 \times 15000}{100} = 1500$

17. (c) Let man has originally ₹ x

After 20% loss = $\frac{x \times 80}{100} = \frac{8x}{10}$

After spending 25% = $\frac{8x}{10} \times \frac{75}{100} = \frac{8x}{10} \times \frac{3}{4}$

According to the question,

$\frac{8x}{10} \times \frac{3}{4} = 480$

$\Rightarrow \quad 8x \times 3 = 480 \times 4 \times 10$

$\therefore \quad x = \frac{480 \times 4 \times 10}{8 \times 3} = 800$

18. (c) $x + y + \frac{xy}{100}$

$= 15 + 15 + \frac{15 \times 15}{100}$

$= 30 + \frac{225}{100}$ = 32. 25%

19. (b) According to the question,

$\Rightarrow 25 \times \left(\frac{100 - x}{100}\right)\left(\frac{100 - x}{100}\right) = 20.25$

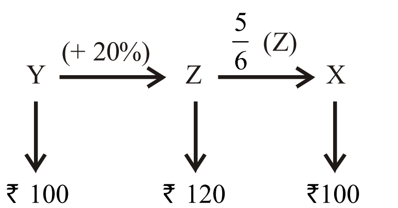
$\Rightarrow (100 - x)^2 = \frac{202500}{25} \Rightarrow (100 - x)^2 = 8100$

$\Rightarrow 100 - x = 90$

$\therefore$ x = 10

20 (a) 5% of 50% of 500 = $\frac{5}{100} \times \frac{50}{100} \times 500 = 12.5$

21. (d) According to question.



So total amount = 100 + 100 + 120

= ₹ 320.

22. (d) Percentage increase in salary

$= \frac{8100 - 7200}{7200} \times 100$

$= \frac{900}{7200} \times 100 = 12.5\% = 12\frac{1}{2}\%$

23. (b) Let the cost price of the article be ₹ x

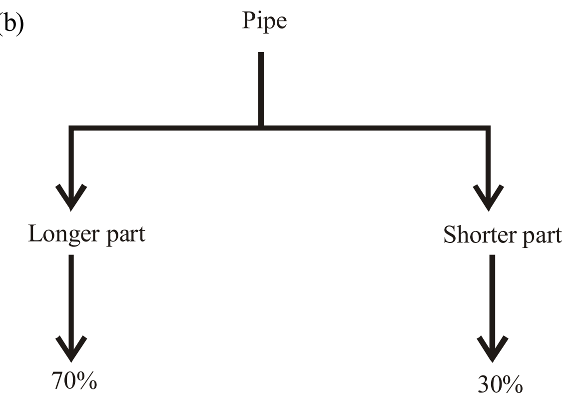
After 20% profit $\Rightarrow \quad \frac{120x}{100} = 3600$

$x = 3000$

Now, profit percentage, when the article is sold for ₹3150

$= \frac{3150 - 3000}{3000} \times 100 = \frac{150}{3000} \times 100 = 5\%$

24. (b)



Percentage of longer pipe as compare to shorter pipe

$= \frac{70 - 30}{30} \times 100 = \frac{40}{30} \times 100 = \frac{400}{3}\%$

25. (c) Let the original price of article be ₹ x

Now, After 20% discount article costs ₹ 596

So, $596 = \frac{80}{100} \times x \quad \Rightarrow \quad x = \frac{596 \times 100}{80} = 745$

$\therefore$ Original price = ₹ 745

26. (a) Let income be ₹ 100

Expenditure amount = $100 \times \frac{90}{100}$ = ₹ 90

Now, income increased by 20% = $100 \times \frac{120}{100}$

= ₹ 120

Expenditure amount = (120 – 10) = ₹ 110

Increase in expenditure = 110 – 90 = ₹ 20

Increase in % of expenditure = $\frac{20}{90} \times 100$

$= \frac{200}{9} = 22\frac{2}{9}\%$

27. (d) Milkman is getting 20% profit by selling the milk mixed with water.

So, the quantity of milk he is selling less is

$= \left(\frac{20}{100 + 20}\right) \times 100$

$= \frac{100}{6}\%$

This quantity is the percentage of water in milk that is

$= \frac{100}{6}\%$ Percentage of milk in the mixture

$= 100 - \frac{100}{6}$

$= \frac{500}{6}\% = \frac{250}{3}\%$

So, option (d) is correct.

28. (b) Let the number be x.
According to question-

$\Rightarrow \frac{20}{100}x + 20 = x$

$\Rightarrow \frac{x}{5} + 20 = x$

$\Rightarrow x - \frac{x}{5} = 20 \Rightarrow \frac{4x}{5} = 20$

$x = 25$

So, option (b) is correct

29. (d) Let the salary of A be 100.
$\Rightarrow$ A's salary after 40% increase will be

$100 + \frac{100+40}{100} = 140\%$

$\Rightarrow$ A's salary after 20% decrease will be

$140 - \frac{140 \times 20}{100} = 112$

On a whole A's salary is increased by

$= \left(\frac{112-100}{100}\right) \times 100 = 12\%$

So, option (d) is correct.

Short cut method

$+40 - 20 + \frac{(40 \times -20)}{100} = 20 - 8 = 12\% \uparrow$ se.

30. (b) Let the number of voters on voter list is 'x'.
Valid votes = (0.9 x – 60)
Votes in support of 2nd candidate
= (0.9 x – 60) – 0.47 x
According to question-
$\Rightarrow 0.47x - [(0.9x - 60) - 0.47x] = 308$
$\Rightarrow 0.94x - 0.9x + 60 = 308$
$\Rightarrow 0.04x = 248$
$\Rightarrow x = 6200$
So, option (b) is correct.

31. (c) Let original salary of a person = x ₹
Increase in salary = 10% of x
Then New Salary = x + 10% of x

$x + \frac{10}{100} \times x = \frac{11}{10}x$

But he received same amount even after increment.
Let y be the percentage salary he did not receive.

then $\frac{11}{10}x - \frac{11}{10}x \times \frac{y}{100} = x$

$\Rightarrow \frac{11}{10}x\left[1 - \frac{y}{100}\right] = x$

$\Rightarrow 1 - \frac{y}{100} = \frac{10}{11}$

$\Rightarrow \frac{y}{100} = 1 - \frac{10}{11} = \frac{1}{11}$

$\Rightarrow y = \frac{100}{11}\%$

$\therefore$ Option (c) is correct.

32. (b) Total Expenditure = 2000 ₹
Education Expenditure = 8000
Food Expenditure = 5900
Shopping Expenditure = 2800
Personal care Expenditure
= 20000 – (8000 + 5900 + 2800)
= 20000 – 16700
= 3300 ₹
Percentage of expenditure on personal care

$= \frac{\text{Personal care expenditure}}{\text{total expenditure}} \times 100$

$= \frac{3300}{2000} \times 100 = 16.5\%$

$\therefore$ Option (b) is correct.

33. (c) Let maximum mark be = M
ATQ

$\frac{xM}{100} + a = \frac{yM}{100} - b$ [Passing marks]

$a + b = \frac{M}{100}(y - x)$

$\frac{100(a+b)}{(y-x)} = M$

34. (c) $\frac{a \times a}{100} + \frac{b \times b}{100} = \frac{2}{100}ab$

$a^2 + b^2 = 2ab$

$a^2 + b^2 - 2ab = 0$

$(a - b)^2 = 0$

$a = b$

$\therefore$ a is 100% of b

35. (c) Male Population = $\frac{5}{9}$

Females population = $\frac{4}{9}$

Married males = $\frac{30}{100} \times \frac{5}{9} = \frac{1}{6}$

Married Female = $\frac{1}{6}$ Part

unmarried female = $\frac{4}{9} - \frac{1}{6} = \frac{5}{18}$

$\therefore = \frac{5}{18}\times 100$

$\therefore = 27\frac{7}{9}\%$.

36. (c) Let the number of mangoes the fruit seller had originally be x.
We are given that, 5% of the total mangoes are rotten i.e. 5% of x i.e. $\frac{x}{20}$ are rotten.
We are also given that 75% of the remainder are sold i.e. 75% of (x – 5% of x)
25% of 95% of x = 95
x = 400

37. (b) Here, the maximum marks are 100% and according to the question, 45 + 5 = 40% i.e. 50 = 40%.
Therefore, by unitary method,

$$1\% = \frac{50}{40}$$

$$100\% = \frac{50}{40}\times 100 = 125.$$

Thus, maximum marks are 125 marks.

38. (d) If the price rises by r% and expenditure remains same then the percentage of consumption to be reduced is given by $\frac{100r}{100+r}\%$.

$$\frac{100r}{100+r}\% = \frac{100\times 12}{100+12}\% = \frac{1200}{112}\% = 10\frac{5}{7}\%$$

39. (b) Let price of 1 kg wheat be 100 and total consumption be 100 kg
Total cost at starting = 100 × 100 = 10000
Total new cost = 10000

New price of wheat/kg = $100\times\frac{125}{100} = 125$

New consumption = $\frac{10000}{125} = 80$ kg

decrease in consumption

$$= \frac{100-80}{100}\times 100 = 20\%$$

40. (a) Total Children = 85
Let, boys are x, then girls are (85 – x)
ATQ,

$$(85-x) = \frac{70}{100}\times x$$

x = 50
Boys = 50 and Girls = 35
Now,
No. of boys playing badminton only 25
Total no. of boys playing badminton = 30
And,
No. of children playing only table tennis = 34
Children play both badminton and table tennis are 12
Number of girls play both games = 12 – 5 = 7
Number of boys play only table tennis = 50 – 30 = 20
$\therefore$ Number of girls play only Table tennis = 34 – 20 = 14
and so, Number of girls play only badminton
= 35 – 14 – 7 = 14.

41. (a) ATQ
Let cost of X = x
Then, cost of Y = 130 – x
Now,

$$x + (130-x)\frac{109}{100} = 136.75$$

100x + 14170 – 109x = 13675
9x = 495 ⇒ x = 55
Thus, rate of article y = (130 – x) = 130 – 55 = ₹ 75

42. (c) Let initial rent be ₹ 100 and initial rooms be 100
So initial collection = 100 × 100 = ₹10000
Now new rent = 100 × 20% = 120
New no of rooms = 100 × 20% = 120
So new collection = 120 × 120 = 14400
% change in collection

$$= \frac{(14400-10000)}{10000}\times 100 = \frac{4400}{10000}\times 100 = 44\%$$

43. (d) x = 30% of z = 30z/100 = 3z/10
y = 40% of z = 40z/100 = 4z/10
According to the question,

$$x = p\%\times y \Rightarrow p = \frac{x}{y}\times 100 = \frac{\frac{3z}{10}}{\frac{4z}{100}}\times 100 = \frac{3z}{4z}\times 100$$

$$= \frac{3}{4}\times 100 = 75\%$$

44. (a)

Train fire	:	Bus fire
3	:	4
↓12%	:	↓13%
3.6		5.2

Ratio = 3.6 : 5.2 ⇒ $\boxed{9 : 13}$

45. (b)

	X	Y
2010	5000	2000
2011	6000	2200
2012	7200	2420
2013	8640	2662

Difference in price in 2013 = 8640 – 2662 = 5978 > 5000

46. (b) Let income y = 100 % and difference = 20 %
$\therefore$ x = 120
Different = (120 – 100) = 20

Diff. percent = $100\times\frac{20}{120} = \frac{100}{6} = 16\frac{2}{3}\%$

CHAPTER

6 Ratio and Proportion

1. x varies inversely as the square of y in such a way that, if $x = 1$, then $y = 6$. If $y = 3$, then what is the value of x? **[2007-I]**
(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) 2 (d) 4

2. 6 years hence a father's age will be three times his son's age and three years ago father was nine times as old as his son. What is the present age of father? **[2007-I]**
(a) 48 years (b) 42 years
(c) 36 years (d) 30 years

3. In a mixture of 80 l, the ratio of milk and water is 3 : 1. If the ratio of milk and water is to be 2 : 3 the how much amount of water is to be further added ? **[2007-II]**
(a) 70 l (b) 80 l (c) 100 l (d) 140 l

4. The age of a mother, before two years, was eight times the age of there daughter. After 1 year, mother's age will be five times the daughter's age. After how many years from now the mother's age will become three times the daughter's age? **[2007-II]**
(a) 6 years (b) 8 years
(c) 10 years (d) 12 years

5. The monthly incomes of A and B are in the ratio 4 : 3. Each of them saves ₹ 600. If the ratio of their expenditure is 3 : 2, then what is the monthly income of A? **[2007-II]**
(a) ₹ 2400 (b) ₹ 1800 (c) ₹ 2000 (d) ₹ 3600

6. A bag contains 50 paise, ₹ 1 and ₹ 2 coins in the ratio 2 : 3 : 4. If the total amount is ₹ 240, what is the total number of coins ? **[2008-I]**
(a) 90 (b) 150 (c) 180 (d) 200

7. If 78 is divided into three parts which are proportional to $1, \frac{1}{2}, \frac{1}{6}$, then what is the middle part ? **[2008-I]**
(a) $\frac{28}{3}$ (b) 13 (c) $\frac{52}{3}$ (d) $\frac{117}{5}$

8. x varies directly as y and inversely as square of z. When $y = 4$ and $z = 14$, $x = 10$. If $y = 16$ and $z = 7$, then what is value of x ? **[2008-I]**
(a) 180 (b) 160 (c) 154 (d) 140

9. What is the number which has to be added to each term of the ratio 49 : 68, so that it becomes 3 : 4? **[2008-II]**
(a) 3 (b) 5 (c) 8 (d) 9

10. What number must be added to each of 4, 10, 12 and 24, so that the resulting numbers are in proportion? **[2008-II]**
(a) 6 (b) 8 (c) 10 (d) 4

11. 10 years ago, Ram was 5 times as old as Shyam but 20 years later from now he will be only twice as old as Shyam. How many years old is Shyam? **[2008-II]**
(a) 20 years (b) 30 years
(c) 40 years (d) 50 years

12. The sum of the age of a father and the age of a son is 75 years. If the product of their ages before 5 years was 750, then what is the present age of the father? **[2008-II]**
(a) 60 years (b) 55 years
(c) 52 years (d) 50 years

13. Let y is equal to the sum of two quantities of which one varies directly as x and the other inversely as x. If $y = 6$ when $x = 4$ and $y = 10/3$, when $x = 3$, then what is the relation between x and y? **[2009-I]**
(a) $y = x + (4/x)$
(b) $y = -2x + (4/x)$
(c) $y = 2x + (8/x)$
(d) $y = 2x - (8/x)$

14. A bag contains ₹ 114 in the form of ₹ 1, 50 paise and 10 paise coins in the ratio 3 : 4 : 10. What is the number of 50 paise coins? **[2009-I]**
(a) 76 (b) 72 (c) 56 (d) 48

15. If $a : b = 1\frac{1}{2} : 2\frac{1}{4}$ and $b : c = 2 : 3\frac{1}{2}$, then what is $a : b : c$ equal to ? **[2009-I]**
(a) 12 : 8 : 21 (b) 8 : 21 : 12
(c) 8 : 12 : 21 (d) 21 : 8 : 12

16. If the ratio of x to y is 25 times the ratio of y to x, then what is the ratio of x to y? **[2009-II]**
(a) 1 : 5 (b) 5 : 1
(c) 25 : 1 (d) 1 : 25

17. If $x : y = 1 : 3$, $y : z = 5 : k$, $z : t = 2 : 5$ and $t : x = 3 : 4$, then what is the value of k? **[2009-II]**
(a) 1/2 (b) 1/3
(c) 2 (d) 3

18. A person P started a business with a capital of ₹ 2525 and another person Q joined P after some months with a capital of ₹ 1200. Out of the total annual profit of ₹ 1644, P's share was ₹ 1212. When did Q join as partners ? **[2009-II]**
(a) After 2 months
(b) After 3 months
(c) After 4 months
(d) After 5 months

19. If a quantity y varies as the sum of three quantities of which the first varies as x, the second varies as $-x + x^2$, the third varies as $x^3 - x^2$, then what is y equal to? **[2010-I]**
(a) kx^3, where k is a constant
(b) $kx + lx^2 + mx^3$, where k, l, m are constants
(c) kx^2, where k is a constant
(d) kx, where k is a constant

20. The wages of labourers in a factory has increased in the ratio 22 : 25 and their number decreased in the ratio 3 : 2. What was the original wages bill of the factory, if the present bill is ₹ 5000? **[2010-I]**
(a) ₹ 4000 (b) ₹ 6000
(c) ₹ 8000 (d) None of these

21. A mixture contains milk and water in the ratio 5 : 1. On adding 5*l* of water, the ratio of milk and water becomes 5 : 2. What is the quantity of milk in the original mixture? **[2010-II]**
(a) 5 *l* (b) 25 *l*
(c) 27.5 *l* (d) 32.5 *l*

22. The ratio between the ages of *A* and *B* is 2 : 5. After 8 years their ages will be in the ratio 1 : 2. What is the difference between their present ages? **[2010-II]**
(a) 20 years (b) 22 years
(c) 24 years (d) 25 years

23. If *x* varies as the *m*th power of *y*, *y* varies as the *n*th power of *z* and *x* varies as the *p*th power of *z*, then which one of the following is correct? **[2010-II]**
(a) $p = m + n$ (b) $p = m - n$
(c) $p = mn$ (d) None of theses

24. If $(a - b) : (a + b) = 1 : 5$, then what is $(a^2 - b^2) : (a^2 + b^2)$ equal to? **[2010-II]**
(a) 2 : 3 (b) 3 : 2
(c) 5 : 13 (d) 13 : 15

25. Three numbers are in the ratio 3 : 2 : 5 and the sum of their squares is 1862. What are the three numbers? **[2010-II]**
(a) 18, 12, 30 (b) 24, 16, 40
(c) 15, 10, 25 (d) 21, 14, 35

26. In a class, the number of boys is more than the number of girls by 12% of the total students. What is the ratio of number of boys to that of girls? **[2010-II]**
(a) 11 : 14 (b) 14 : 11 (c) 28 : 25 (d) 25 : 28

27. The ratio of *A* to *B* is *x* : 8 and the ratio of *B* to *C* is 12 : *z*. If the ratio of *A* to *C* is 2 : 1, then what is the ratio of *x* : *z*? **[2010-II]**
(a) 2 : 3 (b) 3 : 2 (c) 4 : 3 (d) 3 : 4

28. If ₹ 2600 is divided among three persons *A*, *B* and *C* in the ratio $\frac{1}{2}:\frac{1}{3}:\frac{1}{4}$, then how much does *A* get? **[2011-I]**
(a) ₹ 600 (b) ₹ 800 (c) ₹ 1000 (d) ₹ 1200

29. A certain amount of money has be divided between two persons *P* and *Q* in the ratio 3 : 5. But it was divided in the ratio 2 : 3 and thereby *Q* loses ₹ 10. What was the amount? **[2011-I]**
(a) ₹ 250 (b) ₹ 300 (c) ₹ 350 (d) ₹ 400

30. '*X*' is twice as old as '*Y*' 3 years ago, when '*X*' was as old as '*Y*' today. If the difference between their ages as present is 3 years, how old is '*X*' at present? **[2011-I]**
(a) 18 years (b) 12 years (c) 9 years (d) 8 years

31. Two vessels are full with milk and water mixed in the ratio 1 : 3 and 3 : 5 respectively. If both are mixed in the ratio 3 : 2, what is the ratio of milk and water in the new mixture? **[2011-I]**
(a) 4 : 15 (b) 3 : 7
(c) 6 : 7 (d) None of these

32. A bag contains ₹ 112 in the form of ₹ 1,50 paise and 10 paise coins in the ratio 3 : 8 : 10. What is the number of 50 paise coins? **[2011-I]**
(a) 112 (b) 108 (c) 96 (d) 84

33. If *p*% of ₹ *x* is equal to *t* times *q*% of ₹ *y*, then what is the ratio of *x* to *y*? **[2011-I]**
(a) *pt* : *q* (b) *p* : *qt* (c) *qt* : *p* (d) *q* : *pt*

34. If $P : Q = \frac{3}{5}:\frac{5}{7}$ and $Q : R = \frac{3}{4}:\frac{2}{5}$, then what is $P : Q : R$ equal to? **[2011-II]**
(a) $\frac{3}{5}:\frac{5}{7}:\frac{2}{5}$ (b) $\frac{9}{20}:\frac{15}{28}:\frac{2}{7}$
(c) $\frac{3}{5}:\frac{3}{4}:\frac{2}{5}$ (d) $\frac{3}{5}:\frac{5}{7}:\frac{3}{4}$

35. If ₹ 8400 is divided among *A*, *B* and *C* in the ratio $\frac{1}{5}:\frac{1}{6}:\frac{1}{10}$, what is the share of *A*? **[2011-II]**
(a) 3200 (b) 3400 (c) 3600 (d) 4200

36. Two numbers are in the ratio 3 : 5. If 9 is subtracted from each number, then they are in the ratio of 12 : 23. What is the second number ? **[2011-II]**
(a) 44 (b) 55 (c) 66 (d) 77

37. The fourth proportional to 7, 11, 14 is **[2011-II]**
(a) 16 (b) 18 (c) 20 (d) 22

38. If *x* : *y* = 7 : 5, then what is the value of (5*x* – 2*y*) : (3*x* + 2*y*)? **[2012-I]**
(a) 5/4 (b) 6/5 (c) 25/31 (d) 31/42

39. What is the mean proportional between $(15 + \sqrt{200})$ and $(27 - \sqrt{648})$? **[2012-I]**
(a) 4 (b) $14\sqrt{7}$ (c) $3\sqrt{5}$ (d) $5\sqrt{3}$

40. Two numbers are in the ratio 2 : 3. If 9 is added to each number, they will be in the ratio 3 : 4. What is the product of the two numbers? **[2012-I]**
(a) 360 (b) 480 (c) 486 (d) 512

41. Sex ratio is defined as the number of females per 1000 males. In a place, the total inhabitants are 1935000 out of which 935000 are females. What is the sex ratio for the place ? **[2012-II]**
(a) 935 (b) 1000 (c) 1935 (d) 9350

42. A milkman bought 15 kg of milk and mixed 3 kg of water in it. If the price per kg of the mixture becomes ₹ 22, what is cost price of the milk per kg? **[2012-II]**
(a) ₹ 28.00 (b) ₹ 26.40 (c) ₹ 24.00 (d) ₹ 22.00

43. Age of *X* is six times that of *Y*. After 4 years, *X* is four times elder to *Y*. What is the present age of *Y* ? **[2012-II]**
(a) 4 years (b) 5 years (c) 6 years (d) 7 years

44. In a certain school, the ratio of boys to girls is 7 : 5. If there are 2400 students in the school, then how many girls are there ? **[2013-I]**
(a) 500 (b) 700 (c) 800 (d) 1000

45. If *a*, *b*, *c*, *d* and *e* are in continued proportion, then *a*/*e* is equal to **[2013-I]**
(a) a^3 / b^3 (b) a^4 / b^4 (c) b^3 / a^3 (d) b^4 / a^4

46. If $A : B = 2 : 3$, $B : C = 5 : 7$ and $C : D = 3 : 10$, then what is $A : D$ equal to ? **[2014-I]**
(a) 1 : 7 (b) 2 : 7 (c) 1 : 5 (d) 5 : 1

47. The height of a tree varies as the square root of its age (between 5 to 17 yr). When the age of the tree is 9 yr, its height is 4 ft. What will be the height of the tree at the age of 16 yr ? **[2014-II]**
(a) 5 *ft* 4 inch (b) 5 *ft* 5 inch
(c) 4 *ft* 4 inch (d) 4 *ft* 5 inch

48. The ratio of ages of *A* and *B* is 2 : 5 and the ratio of ages of *B* and *C* is 3 : 4, What is the ratio of ages of *A*, *B* and *C* ? **[2014-II]**
(a) 6 : 15 : 20 (b) 8 : 5 : 3
(c) 6 : 5 : 4 (d) 2 : 15 : 4

49. The sides of a triangle are in the ratio $\frac{1}{2}:\frac{1}{3}:\frac{1}{4}$. If its perimeter is 52 cm, then what is the length of the smallest side? **[2014-II]**
(a) 9 cm (b) 10 cm (c) 11 cm (d) 12 cm

50. $(x + y) : (x - y) = 3 : 5$ and xy = positive imply that **[2014-II]**
(a) *x* and *y* are both positive
(b) *x* and *y* are both negative
(c) one of them is positive and one of them is negative
(d) no real solutions for *x* and *y* exist

51. A milkman claims to sell milk at its cost price only, but he is making a profit of 20% since he has mixed some amount of water in the milk. What is the percentage of milk in the mixture? **[2015-I]**
(a) 80% (b) $\frac{250}{3}$% (c) 75% (d) $\frac{200}{3}$%

52. 16 litres of a mixture contains milk and water in the ratio 5:3. If 4 litres of milk is added to this mixture, the ratio of milk to water in the new mixture would be **[2015-I]**
(a) 2 : 1 (b) 7 : 3 (c) 4 : 3 (d) 8 : 3

53. A tin of oil was $\frac{4}{5}$ full. When 6 bottles of oil were taken out from this tin and 4 bottles of oil were poured into it, it was $\frac{3}{4}$ full. Oil of how many bottles can the tin contain ? (All bottles are of equal volume) **[2015-II]**
(a) 35 (b) 40 (c) 45 (d) 50

54. If a : b = 3 : 5 and b : c = 7 : 8, then 2a : 3b : 7c is equal to **[2015-II]**
(a) 42 : 105 : 320 (b) 15 : 21 : 35
(c) 6 : 15 : 40 (d) 30 : 21 : 350

55. In a mixture of milk and water of volume 30 litre, the ratio of milk and water is 7 : 3. The quantity of water to be added to the mixture to make the ratio of milk and water 1 : 2 is **[2015-II]**
(a) 30 (b) 32 (c) 33 (d) 35

56. The annual incomes of two persons are in the ratio 9 : 7 and their expenses are in the ratio 4 : 3. If each of them saves ₹ 2,000 per year, what is the difference in their annual income? **[2016-I]**
(a) ₹ 4,000 (b) ₹ 4,500 (c) ₹ 5,000 (d) ₹ 5,500

57. In an office, one third of the workers are women, half of the women are married and one third of the married women have children. If three fourth of the men are married and one third of the married men have children, then what is the ratio of married women to married men ? **[2016-II]**
(a) 1 : 2 (b) 2 : 1 (c) 3 : 1 (d) 1 : 3

58. There are twelve friends A, B, C, D, E, F, G, H, I, J, K and L who invested money in some business in the ratio of 1 : 2 : 3 : 4 : 5 : 6 : 7 : 8 : 9 : 10 : 11 : 12 and the duration for which they invested the money is in the ratio of 12 : 11 : 10 : 9 : 8 : 7 : 6 : 5 : 4 : 3 : 2 : 1 respectively. Who will get the maximum profit at the end of the year ? **[2016-II]**
(a) F only (b) G only
(c) Both F and G (d) Neither F nor G

59. Incomes of Mahesh and Kamal are in the ratio 1 : 2 and their expenses are in the ratio 1 : 3. Which one of the following statements is correct? **[2016-II]**
(a) Mahesh saves more than what Kamal saves
(b) Savings of both of them are equal
(c) Kamal saves more than what Mahesh saves
(d) It is not possible to determine who saves more

60. X and Y entered into partnership with ₹700 and ₹600 respectively. After 3 months X withdrew 2/7 of his stock but after 3 months, he puts back 3/5 of What he had withdrawn. The profit at the end of the year is ₹726. How much of this should X receive ? **[2016-II]**
(a) ₹336 (b) ₹366 (c) ₹633 (d) ₹663

61. The cost of a diamond varies directly as the square of its weight. A diamond broke into four pieces with their weights in the ratio of 1 : 2 : 3 : 4. If the loss in total value of the diamond was ₹ 70,000, what was the price of the original diamond ? **[2017-I]**
(a) ₹ 1,00,000 (b) ₹ 1,40,000
(c) ₹ 1,50,000 (d) ₹ 1,75,000

62. Leela got married 6 years ago. Today her age is $1\frac{1}{4}$ times her age at the time of her marriage. Her son's age is $\frac{1}{10}$ times her age. What is the present age of her son ? **[2017-I]**
(a) 1 year (b) 2 years (c) 3 years (d) 4 years

63. In a class of 49 students, the ratio of girls to boys is 4 : 3. If 4 girls leave the class, the ratio of girls to boys would be **[2017-I]**
(a) 11 : 7 (b) 8 : 7 (c) 6 : 5 (d) 9 : 8

64. The ratio of two numbers is 1 : 5 and their product is 320. What is the difference between the squares of these two numbers ? **[2017-I]**
(a) 1024 (b) 1256
(c) 1536 (d) 1640

65. 25 kg of alloy X is mixed with 125 kg of alloy Y. If the amount of lead and tin in the alloy X is in the ratio 1 : 2 and the amount of lead and tin in the alloy Y is in the ratio 2 : 3, then what is the ratio of lead to tin in the mixture? **[2017-I]**

(a) 1 : 2 (b) 2 : 3 (c) 3 : 5 (d) 7 : 11

66. To maintain 8 cows for 60 days, a milkman has to spend ₹ 6,400. To maintain 5 cows for n days, he has to spend ₹ 4,800. What is the value of n? **[2017-II]**

(a) 46 days (b) 50 days
(c) 58 days (d) 72 days

67. Five years ago, Ram was three times as old as Shyam. Four years from now, Ram will be only twice as old as Shyam. What is the present age of Ram? **[2017-II]**

(a) 30 years (b) 32 years
(c) 36 years (d) 40 years

68. If 78 is divided into 3 parts which are proportional to $1, \frac{1}{3}, \frac{1}{6}$, then the middle part is **[2017-II]**

(a) $\frac{28}{3}$ (b) 13 (c) $\frac{52}{3}$ (d) $\frac{55}{3}$

69. There are 350 boys in the first three standards. The ratio of the number of boys in first and second standards is 2 : 3, while that of boys in second and third standards is 4 : 5. What is the total number of boys in first and third standards? **[2017-II]**

(a) 302 (b) 280 (c) 242 (d) 230

70. $\frac{1}{25}$ of the students who registered did not appear for the examination, $\frac{11}{20}$ of those who appeared passed. If the number of registered students is 2000, the number who passed is **[2018-I]**

(a) 1920 (b) 1056 (c) 1020 (d) 864

71. If A : B = 1 : 2, B : C = 3 : 4, C : D = 2 : 3 and D : E = 3 : 4, then what is B : E equal to? **[2018-I]**

(a) 3 : 2 (b) 1 : 8 (c) 3 : 8 (d) 4 : 1

72. ₹ 120 is distributed among A, B and C so that A's share is ₹ 20 more than B's and ₹ 20 less than C's. What is B's share? **[2018-I]**

(a) ₹10 (b) ₹15 (c) ₹20 (d) ₹25

73. A gentleman left a sum of ₹39,000 to be distributed after his death among his widow, five sons and four daughters. If each son receives 3 times as much as a daughter receives, and each daughter receives twice as much as their mother receives, then what is the widow's share? **[2018-I]**

(a) ₹ 1,000 (b) ₹ 1,200
(c) ₹ 1,500 (d) None of the above

74. The age of a woman is a two-digit integer. On reversing this integer, the new integer is the age of her husband who is elder to her. The difference between their ages is one-eleventh of their sum. What is the differences between their ages? **[2018-I]**

(a) 8 years (b) 9 years
(c) 10 years (d) 11 years

75. An alloy A contains two elements, copper and tin in the ratio of 2 : 3, whereas an alloy B contains the same elements in the ratio of 3 : 4. If 20 kg of alloy A, 28 kg of alloy B and some more pure copper are mixed to form a third alloy C which now contains copper and tin in the ratio of 6 : 7, then what is the quantity of pure copper mixed in the alloy C? **[2018-I]**

(a) 3 kg (b) 4 kg (c) 5 kg (d) 7 kg

76. A, B, C, D and E start a partnership firm. Capital contributed by A is three times that contributed by D. E contributes half of A's contribution, B contributes one-third of E's contribution and C contributes two-third of A's contribution. If the difference between the combined shares of A, D and E and the combined shares of B and C in the total profit of the firm is ₹13,500, what is the combined share of B, C and E? (The shares are supposed to be proportional to the contributions) **[2018-I]**

(a) ₹ 13,500 (b) ₹ 18,000
(c) ₹ 19,750 (d) ₹ 20,250

77. According to Mr. Sharma's will, half of his property goes to his wife and the rest is equally divided between his two sons, Ravi and Raj. Some years later, Ravi dies and leaves half of his property to his widow and rest to his brother Raj. When Raj dies he leaves half of his property to his widow and remaining to his mother, who is still alive. The mother now owns ₹ 88,000 worth of the property. The total worth of the property of Mr. Sharma was **[2018-II]**

(a) ₹1,00,000 (b) ₹1,24,000
(c) ₹1,28,000 (d) ₹1,32,000

78. X bought 4 bottles of lemon juice and Y bought one bottle of orange juice. Orange juice per bottle costs twice the cost of lemon juice per bottle. Z bought nothing but contributed ' 50 for his share of the drink which they mixed together and shared the cost equally. If Z's '50 is covered from his share, then what is the cost of one bottle of orange juice? **[2018-II]**

(a) ₹75 (b) ₹50 (c) ₹46 (d) ₹30

79. The ratio of the sum and difference of the ages of the father and the son is 11 : 3. Consider the following statements: **[2018-II]**

1. The rato of their ages is 8 : 5.
2. The ratio of their ages after the son attains twice the present age will be 11 : 8.

Which of the statements given above is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

80. If $a : b = c : d = 1 : 6$, then what is the value of $\frac{a^2 + c^2}{b^2 + d^2}$? **[2019-I]**

(a) $\frac{1}{600}$ (b) $\frac{1}{60}$ (c) $\frac{1}{36}$ (d) $\frac{1}{6}$

81. If $(4a+7b)(4c-7d)=(4a-7b)(4c+7d)$, then which one of the following is correct ? **[2019-I]**

(a) $\frac{a}{b}=\frac{c}{d}$ (b) $\frac{a}{d}=\frac{c}{b}$ (c) $\frac{a}{b}=\frac{d}{c}$ (d) $\frac{4a}{7b}=\frac{c}{d}$

82. Given y is inversely proportional to $\sqrt{x}$, and $x=36$ when $y=36$. What is the value of x when $y=54$? **[2019-II]**

(a) 54 (b) 27 (c) 16 (d) 8

83. The monthly incomes of A and B are in the ratio 4 : 3. Each saves ₹ 600. If their expenditures are in the ratio 3 : 2, then what is the monthly income of A ? **[2019-II]**

(a) ₹1800 (b) ₹2000 (c) ₹2400 (d) ₹3600

84. If $\frac{a}{b}=\frac{1}{3},\frac{b}{c}=2,\ \frac{c}{d}=\frac{1}{2},\frac{d}{e}=3$ and $\frac{e}{f}=\frac{1}{4}$, then what is the value of $\frac{abc}{def}$? **[2020-I]**

(a) $\frac{1}{4}$ (b) $\frac{3}{4}$ (c) $\frac{3}{8}$ (d) $\frac{27}{4}$

85. If $x^2+9y^2=6xy$, then what is $y:x$ equal to? **[2020-I]**

(a) 1 : 3 (b) 1 : 2 (c) 2 : 1 (d) 3 : 1

86. If x varies as yz, then y varies inversely as **[2020-I]**

(a) xz (b) $\frac{x}{z}$ (c) $\frac{z}{x}$ (d) $\frac{1}{(xz)}$

87. The population of two villages is 1525 and 2600 respectively. If the ratio of male to female population in the first village is 27 : 34 and the ratio of male to female population in the second village is 6 : 7, then what is the ratio of male to female population of these two villages taken together? **[2020-I]**

(a) $\frac{33}{41}$ (b) $\frac{85}{82}$ (c) $\frac{71}{90}$ (d) $\frac{5}{6}$

HINTS & SOLUTIONS

1. (d) $\because \quad x \propto \frac{1}{y^2} \Rightarrow x=\frac{k}{y^2}$ ...(*i*)

$x=1$ and $y=6$

From Eq. (*i*),

$\therefore \quad 1=\frac{k}{6^2} \Rightarrow k=36$

On putting the value of k in Eq. (*i*), we get

$x=\frac{36}{y^2}$ ...(*ii*)

On putting $y=3$ in Eq. (*ii*), we get

$x=\frac{36}{9}=4$

2. (d) Let the age of son and father is x and $3x$ years respectively 6 years hence.

$\therefore$ Present age of father = $(3x-6)$ years

Present age of son = $(x-6)$ years

3 years ago, age of father = $3x-6-3$

$=(3x-9)$ years

Age of son = $x-6-3=(x-9)$ years

According to given condition,

$3x-9=9(x-9) \Rightarrow 3x-9=9x-81$

$\Rightarrow 81-9=9x-3x \Rightarrow 6x=72$

$\therefore \quad x=\frac{72}{6}=12$

$\therefore$ Present age of father = $3\times 12-6=30$ years

3. (a) Volume of mixture of milk and water = 80 l

Ratio of milk and water = 3 : 1

$\therefore$ Volume of milk $=\frac{3}{3+1}\times 80$

$=\frac{3}{4}\times 80=60\ l$

Volume of water = $80-60=20\ l$

Let amount of water added = $x\ l$

According to given condition,

$\frac{60}{20+x}=\frac{2}{3}$

$\Rightarrow 40+2x=180$

$\Rightarrow 2x=180-40$

$\Rightarrow 2x=140$

$\therefore \quad x=70\ l$

4. (b) Let present age of mother and daughter be x and y.

2 years ago, $x-2=8(y-2)$

$\Rightarrow \quad x-8y=-14$ ...(*i*)

1 year after, $x+1=5(y+1)$

$\Rightarrow \quad x-5y=4$ ...(*ii*)

On solving equations. (*i*) and (*ii*), we get

$y=6$ years and $x=34$ years

Let after z years mother will be three times of her daughter.

$34+z=3(6+z) \Rightarrow z=8$ years

5. (a) Let monthly incomes of A's and B's are $4x$ and $3x$. And monthly expenditures of A's and B's are $3y$ and $2y$.

Each saving = ₹ 600

$\therefore$ Income – Expenditure = Saving

$\therefore \quad 4x-3y=600$ ...(*i*)

$3x-2y=600$ ...(*ii*)

On solving equations. (*i*) and (*ii*), we get

$x=600$

$\therefore$ A's income = $4x=4\times 600=$ ₹ 2400

6. (c) Let the number of coins of 50 paise, ₹ 1 and ₹ 2 coins are $2x$, $3x$ and $4x$ respectively.
Value of 50 paise coins = ₹ x
Value of ₹ 1 coins = ₹ $3x$
Value of ₹ 2 coins = ₹ $8x$
According to the question,
$$x + 3x + 8x = 240$$
$$\Rightarrow \quad x = \frac{240}{12} = 20$$
Hence, total number of coins = $2x + 3x + 4x$ = $9x = 9 \times 20 = 180$

7. (d) Ratio of three parts = $1:\frac{1}{2}:\frac{1}{6}$ = 6 : 3 : 1
$$\therefore \quad \text{Middle part} = \frac{3}{6+3+1} \times 78$$
$$= \frac{3\times 78}{10} = \frac{3\times 39}{5} = \frac{117}{5}$$

8. (b) Given, $x \propto y$ and $x \propto \frac{1}{z^2}$
Now, $x \propto \frac{y}{z^2} \Rightarrow x = \frac{ky}{z^2}$
$\because$ $x = 10$ at $y = 4$ and $z = 14$
$$\therefore \quad 10 = \frac{k \cdot 4}{196} \Rightarrow k = \frac{1960}{4} = 490$$
Now, at $z = 7$ and $y = 16$
$$\therefore \quad x = \frac{490 \times 16}{7 \times 7} = 160$$

9. (c) Let x number be added to 49 : 68, then it becomes 3:4.
$$\therefore \quad \frac{49+x}{68+x} = \frac{3}{4}$$
$\Rightarrow$ $196 + 4x = 204 + 3x$
$\therefore$ $x = 8$

10. (d) Suppose x number is added to each term, they become in proportion i.e., $(4 + x)$, $(10 + x)$, $(12 + x)$ and $(24 + x)$ are in proportion.
$\therefore$ $(4 + x) : (10 + x) :: (12 + x) : (24 + x)$
$$\Rightarrow \quad \frac{4+x}{10+x} = \frac{12+x}{24+x}$$
$\Rightarrow$ $(4 + x)(24 + x) = (10 + x)(12 + x)$
$\Rightarrow$ $96 + 28x + x^2 = 120 + 22x + x^2$
$\Rightarrow$ $28x - 22x = 120 - 96 \Rightarrow 6x = 24$
$\therefore$ $x = 4$

11. (a) Let the age of Shyam and Ram was x and $2x$ years respectively 20 years later from now.
$\therefore$ Present age of Ram = $(2x - 20)$ years
and present age of Shyam = $(x - 20)$ years
10 years ago, the age of Ram = $2x - 20 - 10$ = $2x - 30$
Age of Shyam = $x - 20 - 10 = (x - 30)$ years
According to the question
$2x - 30 = 5(x - 30)$
$\Rightarrow$ $2x - 30 = 5x - 150$
$\Rightarrow$ $150 - 30 = 5x - 2x$
$\Rightarrow$ $3x = 120$
$$\therefore \quad x = \frac{120}{3} = 40$$
$\therefore$ The present age of Shyam = $40 - 20 = 20$ years.

12. (b) Let the present age of father and the son is x years and $(75 - x)$ years respectively.
5 years ago, age of father = $(x - 5)$ years
and age of son = $75 - x - 5 = (70 - x)$ years
According to the question,
$(x - 5)(70 - x) = 750$
$\Rightarrow$ $70x - x^2 - 350 + 5x = 750$
$\Rightarrow$ $-x^2 + 75x - 350 = 750$
$\Rightarrow$ $x^2 - 55x - 20x + 1100 = 0$
$\Rightarrow$ $(x - 55)(x - 20) = 0$
$\therefore$ $x = 20, 55$
$\therefore$ Present age of father = 55 years

13. (d) According to the question,
$$y = lx + \frac{m}{x} \quad \text{...}(i)$$
where, l and m are proportionality constant.
When $y = 6$, $x = 4$, then $6 = 4l + \frac{m}{4}$
$\Rightarrow$ $16l + m = 24$ ...(*ii*)
When $y = \frac{10}{3}$, $x = 3$, then
$$\frac{10}{3} = 3l + \frac{m}{3}$$
$\Rightarrow$ $9l + m = 10$ ...(*iii*)
From equations. (*ii*) and (*iii*), we get.
$$\begin{array}{l} 16l + m = 24 \\ 9l + m = 10 \\ - \quad - \quad - \\ \hline 7l + 0 = 14 \end{array}$$
$$\Rightarrow \quad l = \frac{14}{7} = 2$$
From Eq. (*iii*),
$9 \times 2 + m = 10 \Rightarrow m = 10 - 18 = -8$
From Eq. (*i*),
$$y = 2x - \frac{8}{x}$$

14. (a) Let the number of ₹ 1, 50 paise and 10 paise coins are $3x$, $4x$ and $10x$ respectively.
Given, total money = ₹ 114
$\Rightarrow$ $(3x) + 0.5\,(4x) + 0.1\,(10x) = 114$
$\Rightarrow$ $3x + 2x + x = 114$
$\Rightarrow$ $6x = 114$
$$\therefore \quad x = \frac{114}{6} = 19$$
$\therefore$ Number of 50 paise coins = $4 \times 19 = 76$

15. (c) $a : b = \frac{3}{2} : \frac{9}{4}$ and $b : c = 2 : \frac{7}{2}$

$\Rightarrow$ $a : b = 6 : 9$ and $b : c = 4 : 7$

LCM of 9 and 4 = 36

$\Rightarrow$ $a : b = 24 : 36$ and $b : c = 36 : 63$

$\therefore$ $a : b : c = 24 : 36 : 63$

$\Rightarrow$ $a : b : c = 8 : 12 : 21$

16. (b) Given, $\frac{x}{y} = 25\left(\frac{y}{x}\right)$

$\Rightarrow$ $\frac{x^2}{y^2} = \frac{25}{1} \Rightarrow \frac{x}{y} = \frac{5}{1}$

17. (a) Given, $x : y = 1 : 3$, $y : z = 5 : k$, $z : t = 2 : 5$

$t : x = 3 : 4$

$\frac{x}{y} \times \frac{y}{z} \times \frac{z}{t} \times \frac{t}{x} = 1$

$\Rightarrow$ $\frac{1}{3} \times \frac{5}{k} \times \frac{2}{5} \times \frac{3}{4} = 1 \Rightarrow \frac{1}{2} = k$

$\therefore$ $k = \frac{1}{2}$

18. (b) Let Q join for x month.

$\therefore$ Ratio of capital $= 2525 \times 12 : 1200 \times x$

$= 2525 : 100x = 101 : 4x$

$\therefore$ P's profit $= \frac{101}{101+4x} \times 1644$

$\Rightarrow$ $1212 = \frac{101 \times 1644}{101+4x}$

$\Rightarrow$ $\frac{1212}{101 \times 1644} = \frac{1}{101+4x}$

$\Rightarrow$ $\frac{1}{137} = \frac{1}{101+4x}$

$\Rightarrow$ $101 + 4x = 137 \Rightarrow 4x = 36$

$\therefore$ $x = 9$

Q joined for 9 month i.e., he joined after 3 months.

19. (b) Since, first term $\propto x \Rightarrow$ First term $= c_1 x$

Second term $\propto (-x + x^2)$

$\Rightarrow$ Second term $= c_2(-x + x^2)$

Third term $\propto (x^3 - x^2)$

$\Rightarrow$ Third term $= c_3(x^3 - x^2)$

Also, $y \propto [c_1 x + c_2(-x + x^2) + c_3(x^3 - x^2)]$

$\Rightarrow$ $y = c_4[c_1 - c_2)x + (c_2 - c_3)x^2 + c_3 x^3]$

$= c_4(c_1 - c_2)x + (c_2 - c_3)c_4 x^2 + c_3 c_4 x^3$

$= kx + lx^2 + mx^3$

where $k = c_4(c_1 - c_2)$,

$l = (c_2 - c_3)c_4$ and $m = c_3 c_4$

20. (d) Let initial salary = ₹ $22x$

Final salary = ₹ $25x$

Let initial number of employees = $3y$

Final number of employees = $2y$

$\therefore$ Present bill = Final salary × Final number of employees

$\Rightarrow$ $5000 = 25x \times 2y$

$\Rightarrow$ $\frac{5000}{50} = xy$

$\Rightarrow$ $xy = 100$

$\therefore$ Original bill = Initial salary × Initial number of employees = $22x \times 3y$

$= 66xy = 66 \times 100 =$ ₹ 6600

21. (b) Let quantities of milk and water are $5x$ and x l.

According to the question,

$\frac{5x}{x+5} = \frac{5}{2}$

$\Rightarrow$ $10x = 5x + 25$

$\Rightarrow$ $5x = 25$

$\therefore$ $x = 5$

Hence, the quantity of milk in the original mixture $= 5 \times 5 = 25$ l

22. (c) Let the ages of A and B are $2x$ and $5x$ years.

According to the question,

$\frac{2x+8}{5x+8} = \frac{1}{2}$

$\Rightarrow$ $4x + 16 = 5x + 8$

$\Rightarrow$ $x = 8$

$\therefore$ Difference between their present ages

$= 5x - 2x = 3x = 3 \times 8 = 24$ years

23. (c) $\because$ $x \propto y^m$ ...(*i*)

$y \propto z^n$ ...(*ii*)

$x \propto z^p$ ...(*iii*)

On putting the values if x and y from equations (*ii*) and (*iii*) in Eq. (*i*) we get

$z^p \propto (z^n)^m$

$\Rightarrow$ $z^p \propto z^{mn}$

$\therefore$ $p = mn$

24. (c) Given, $\frac{a-b}{a+b} = \frac{1}{5}$

$\Rightarrow$ $\frac{a+b}{a-b} = \frac{5}{1}$

$\Rightarrow$ $\frac{a+b+a-b}{a+b-a+b} = \frac{5+1}{5-1}$

(using componendo and dividendo rule)

$\Rightarrow$ $\frac{2a}{2b} = \frac{6}{4} = \frac{3}{2} \Rightarrow \frac{a}{b} = \frac{3}{2}$

$\Rightarrow$ $\frac{a^2}{b^2} = \frac{9}{4}$

$\Rightarrow$ $\frac{a^2+b^2}{a^2-b^2} = \frac{13}{5}$

(using componendo and dividendo rule)

$\Rightarrow$ $\frac{a^2-b^2}{a^2+b^2} = \frac{5}{13}$

$\therefore$ $(a^2 - b^2) : (a^2 + b^2) = 5 : 13$

25. (d) Let the numbers be $3x$, $2x$ and $5x$.

$\therefore \quad (3x)^2 + (2x)^2 + (5x)^2 = 1862$

$\Rightarrow \quad 9x^2 + 4x^2 + 25x^2 = 1862$

$\Rightarrow \quad x^2 = \dfrac{1862}{38} = 49$

$x = 7$

Hence, the required numbers are 3×7, 2×7 and 5×7. i.e., 21, 14 and 35

26. (b) Let the number of boys = x

The number of girls = y

According to the question,

$x - y = \dfrac{(x+y)\times 12}{100}$

$\Rightarrow \quad 25x - 25y = 3x + 3y$

$\Rightarrow \quad 22x = 28y \Rightarrow x : y = 14 : 11$

27. (c) Given, $A : B = x : 8$, $B : C = 12 : z$

$\Rightarrow \quad \dfrac{A}{C} = \dfrac{A}{B} \times \dfrac{B}{C}$

$= \dfrac{x}{8} \times \dfrac{12}{z} = \dfrac{3x}{2z}$

But $A : C = 2 : 1$

$\Rightarrow \quad \dfrac{3x}{2z} = \dfrac{2}{1} \Rightarrow x : z = 4 : 3$

28. (d) A's part = $\dfrac{\frac{1}{2}}{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}} \times 2600$

$= \dfrac{1}{2} \times \dfrac{12}{13} \times 2600 =$ ₹ 1200

29. (d) Let the amount be ₹ x.

In first condition,

Q's part = $\dfrac{5x}{5+3} = \dfrac{5}{8}x$

In second condition,

Q's part = $\dfrac{3x}{2+3} = \dfrac{3}{5}x$

According to the question,

$\dfrac{5}{8}x - \dfrac{3}{5}x = 10$

$\therefore \quad \dfrac{x}{40} = 10 \Rightarrow x =$ ₹ 400

30. (c) Let present age of X = x years

Present age of $Y = (x - 3)$ years

3 years ago, age of $X = (x - 3)$ years

Age of $Y = (x - 6)$ years

According to the question,

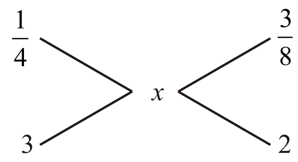
$x - 3 = 2(x - 6)$

$\Rightarrow \quad x - 3 = 2x - 12$

$\Rightarrow \quad 12 - 3 = 2x - x$

$x = 9$ years

31. (d) By alligation method,



$\therefore \quad \dfrac{\frac{3}{8} - x}{x - \frac{1}{4}} = \dfrac{3}{2}$

$\Rightarrow \quad \dfrac{3}{4} - 2x = 3x - \dfrac{3}{4}$

$\Rightarrow \quad 5x = \dfrac{6}{4} = \dfrac{3}{2}$

$\therefore \quad x = \dfrac{3}{10}$

32. (a) Let the number of ₹ 1,50 paise and 10 paise coins are $3x$, $4x$ and x respectively.

According to the question,

$3x + 4x + x = 112$

$\Rightarrow \quad x = \dfrac{112}{8} = 14$

$\therefore$ Number of 50 paise coins = $14 \times 8 = 112$

33. (c) $p\%$ of $x = t$ ($q\%$ of y)

$\Rightarrow \quad \dfrac{xp}{100} = \dfrac{yq}{100} \times t$

$\Rightarrow \quad \dfrac{x}{y} = \dfrac{qt}{p}$

$\therefore \quad x : y = qt : p$

34. (b) Given, $P : Q = \dfrac{3}{5} : \dfrac{5}{7}$ ...(i)

$Q : R = \dfrac{3}{4} : \dfrac{2}{5}$ ...(ii)

From Eq. (i),

$P : Q = \dfrac{3}{5} \times \dfrac{3}{4} : \dfrac{5}{7} \times \dfrac{3}{4}$

$= \dfrac{9}{20} : \dfrac{15}{28}$ ...(iii)

From Eq. (ii),

$Q : R = \dfrac{3}{4} \times \dfrac{5}{7} : \dfrac{2}{5} \times \dfrac{5}{7}$

$= \dfrac{15}{28} : \dfrac{2}{7}$ ...(iv)

From equations (iii) and (iv),

$P : Q : R = \dfrac{9}{20} : \dfrac{15}{28} : \dfrac{2}{7}$

35. (c) Given, $A : B : C = \frac{1}{5} : \frac{1}{6} : \frac{1}{10} = 6 : 5 : 3$

$\therefore$ Share of $A = \frac{6}{6+5+3} \times 8400 = \frac{6}{14} \times 8400$

$=$ ₹ 3600

36. (b) Let two numbers are $3x$ and $5x$.

According to the question,

$\frac{3x-9}{5x-9} = \frac{12}{23}$

$\Rightarrow$ $69x - 207 = 60x - 108$

$69x - 60x = 207 - 108$

$\Rightarrow$ $9x = 99$

$\therefore$ $x = 11$

$\therefore$ Second number $= 5 \times 11 = 55$

37. (d) Let fourth proportional be x.

$7 : 11 :: 14 : x$

$\Rightarrow \frac{7}{11} = \frac{14}{x} \quad \Rightarrow x = 2 \times 11$

$\therefore$ $x = 22$

38. (c) $\frac{x}{y} = \frac{7}{5}$ (Given that)

$\therefore \frac{5x-2y}{3x+2y} = \frac{(5\times7-2\times5)}{(3\times7+2\times5)} = \frac{35-10}{21+10} = \frac{25}{31}$

39. (c) Mean proportional between $(15 + \sqrt{200})$ and $(27 - \sqrt{648})$

$= \sqrt{(15+\sqrt{200})(27-\sqrt{648})}$

$= \sqrt{(15\times27)-(15\times\sqrt{648})+27\sqrt{200}-(\sqrt{200}\times\sqrt{648})}$

$=$

$\sqrt{405-(15\times18\sqrt{2})+(27\times10\sqrt{2})-10\sqrt{2}\times18\sqrt{2})}$

$= \sqrt{405-\left(270\sqrt{2}\right)+\left(270\sqrt{2}\right)-360}$

$= \sqrt{405-360} = \sqrt{45} = 3\sqrt{5}$

40. (c) Let the two numbers are $2x$ and $3x$,

According to the question,

$\frac{2x+9}{3x+9} = \frac{3}{4}$

$\Rightarrow$ $9x + 27 = 8x + 36$

$\Rightarrow$ $9x - 8x = 36 - 27$

$\therefore$ $x = 9$

So the numbers are 18 and 27.

Their product = 18 × 27 = 486.

41. (a) Total number of inhabitants = 1935000

Total number of females = 935000

$\therefore$ Total number of males = 1935000 – 935000

= 1000000

$\therefore$ Sex ratio $= \frac{935000}{1000000} \times 1000 = 935$

42. (b) Let cost price of milk ₹ x per kg.

Price of 15kg of milk = ₹ $15x$.

Now, mix 3kg of water, therefore quantity of mixture = (15 + 3) kg = 18 kg

So, price of mixture is ₹22 per kg

According to question.

$15x = 22 \times 18$

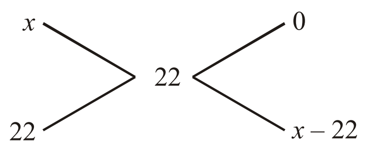
$x = \frac{22\times18}{15} = \frac{132}{5} = 26.40$

Alternate Method :

Let CP of milk be ₹ x per kg.

By Alligation method

price of milk price of water



$\therefore$ $22 : (x - 22) = 15 : 3$

$\Rightarrow \frac{22}{x-22} = \frac{15}{3}$

$\Rightarrow \frac{22}{x-22} = 5$

$\Rightarrow$ $22 = 5x - 110$

$\Rightarrow$ $5x = 132$

$\therefore$ $x =$ ₹ 26.40

43. (c) Let the age of X and Y are x years and y years respectively.

Then, $(x + 4) = 4(y + 4)$

$\Rightarrow$ $6y + 4 = 4y + 16$

$\Rightarrow$ $2y = 12$

$\therefore$ $y = 6$

$\therefore$ The present age of y = 6 years

44. (d) Let the number of boys and girls are $7x$ and $5x$, respectively.

According to question,

$\Rightarrow$ $7x + 5x = 2400 \Rightarrow 12x = 2400$

$\therefore$ $x = 200$

$\therefore$ Number of girls $= 5x$

$= 5 \times 200 = 1000$

45. (b) If a, b, c, d and e are continued proportion.

$\therefore \frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \frac{d}{e} = \text{k}(\text{let})$

$\Rightarrow$ d = ek, c = dk = ek^2, b = ek^3, a = ek^4

$\therefore \frac{a}{e} = \frac{ek^4}{e} = k^4 = \frac{a^4}{b^4} \qquad [\because \text{k} = \frac{a}{b}]$

46. (a) Given, $A : B = 2 : 3$, $B : C = 5 : 7$ and $C : D = 3 : 10$

$\therefore \frac{A}{D} = \frac{A}{B} \times \frac{B}{C} \times \frac{C}{D} = \frac{2}{3} \times \frac{5}{7} \times \frac{3}{10} = \frac{1}{7}$

$\therefore$ $A : D = 1 : 7$

47. (a) Let height of tree be h ft and age be a yr.
Now, according to the question,

$h \propto \sqrt{a} \quad \Rightarrow \quad h = k\sqrt{a}$

Here k is a constant.
Now, age = 9 yr, So, height = 4 ft

$\Rightarrow$ From Eq. (i), we get $4 = k\sqrt{9}$

$\Rightarrow \quad 4 = k \times 3 \qquad \therefore \quad k = \frac{4}{3}$

Again of age = 16 yr, then $h = k\sqrt{16} = \frac{4}{3} \times 4 \quad \left[\because k = \frac{4}{3}\right]$

$\therefore \quad h = \frac{16}{3} ft = 5\frac{1}{3} ft = 5 ft$ 4 inch

48. (a) $A : B = 2 : 5 \rightarrow \times 3$
$B : C = 3 : 4 \rightarrow \times 5$
Now, $A : B = 6 : 15$
$B : C = 15 : 20$
So $A : B : C = 6 : 15 : 20$

49. (d) Sides of a triangle are in the ratio $= \frac{1}{2} : \frac{1}{3} : \frac{1}{4}$
$= 6 : 4 : 3$
x involves in ratio so that sides of a triangle $6x$, $4x$ and $3x$ respectively.
Perimeter of a triangle = Sum of all sides of a triangle
$\Rightarrow \quad 52 = 6x + 4x + 3x \Rightarrow 52 = 13x$

$\therefore \quad x = \frac{52}{13} = 4$

$\therefore$ Smallest side of a triangle $= 3x = 3 \times 4 = 12$ cm

50. (d) $\therefore \quad \frac{x+y}{x-y} = \frac{3}{5}$

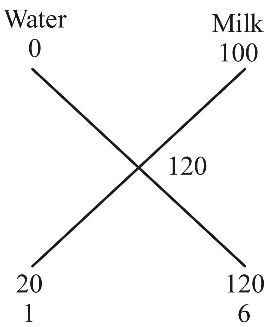
Apply componendo and dividendo rule, we get

$$\frac{(x+y)+(x-y)}{(x+y)-(x-y)} = \frac{3+5}{3-5}$$

$\Rightarrow \quad \frac{2x}{2y} = \frac{8}{-2} \qquad \Rightarrow \quad \frac{x}{y} = -4 \qquad \therefore \quad x = -4y$

But given that, xy = Positive
$\therefore \quad -4y \times y$ = Positive
$\Rightarrow \quad -4y^2$ = Positive, it is not possible.
Therefore, no real solution for x and y exist.

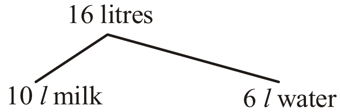
51. (b) Let milkman buy milk of ₹100.
20% profit then = ₹120



percentage of milk $= \frac{6-1}{6} \times 100$

$= \frac{5}{6} \times 100 = \frac{250}{3}\%$

52. (b)



If $4l$ milk is added in mixture then

New ratio $= \frac{(10+4)}{6}$

$= \frac{14}{6} = \frac{7}{3} = 7 : 3$

53. (b) Let the number of bottles in the tin be 20 n.
[LCM of (5, 4) = 20]. Initially it had 16 n bottles. 6 bottles were removed and 4 were poured into the tin.

Then it was $\frac{3}{4}$ full.

$16n - 6 + 4 = 15n \Rightarrow n = 2$
$\therefore \quad 20n = 20 \times 2 = 40$
So, option (b) is correct.

54. (c) $\frac{a}{b} = \frac{3}{5} \Rightarrow a = \frac{3}{5}b$

$\frac{b}{c} = \frac{7}{8} \Rightarrow b = \frac{7}{8}c = \frac{8}{7}b$

$a : b : c = \frac{3}{5}b : b : \frac{8}{7}b = 21 : 35 : 40$

$2a : 3b : 7c = 42 : 105 : 280 = 6 : 15 : 40$
So, option (c) is correct

55. (c) Milk in the mixture $= 30 \times \frac{7}{10} = 21$ litres

Water in the mixture $= 30 \times \frac{3}{10} = 9$ litres

Let the quantity of water added be x.
According to question-

$\frac{21}{9+x} = \frac{1}{2} \Rightarrow 9 + x = 42 \Rightarrow x = 33$ litres

So, option (c) is correct.

56. (a) Let annual income of one person = 9x
Let annual income of second person = 7x
Let Expenses of Ist person = 4y
Let Expenses of 2nd person = 3y
Saving of Ist person = 2000
Saving of 2nd person = 2000
According to condition,
$9x - 4y = 2000$
$7x - 3y = 2000$
On solving, we get
$x = 2000$

Annual income of 1st person = 9 × 2000 = 18000
Annual income of 2nd person = 7 × 2000 = 14000
Difference between their incomes = ₹ 4000
∴ Option (a) is correct.

57. (d)

Men $\frac{2x}{3}$ → Married $\frac{3}{4}\left(\frac{2x}{3}\right)$, Unmarried $\frac{1}{4}\left(\frac{2x}{3}\right)$

Women $\frac{x}{3}$ → Married $\frac{x}{6}$, Unmarried $\frac{x}{6}$

$$\frac{\text{married women}}{\text{married men}} = \frac{x/6}{x/2} = 1:3$$

58. (c) Multiplying ratio of investment and time we get ratio
12 : 22 : 30 : 36 : 40 : 42 : 42 : 40 : 36 : 30 : 22 : 12
where F and G have equal maximum profit

59. (d) Not possible to determine.

60. (b) x invested = ₹700, y invested ₹600

x after 3 month = $700 - \frac{2}{7}(700)$ = ₹500

again after 3 months deposited = $500 \oplus \frac{3}{5}(200)$
= ₹620
effective money invested by x = 700 × 3 + 500 × 3 + 620 × 6 = 2100 + 1500 + 3720 = ₹ 7320
effective money invested by y = 600 × 12 = ₹7200

Profit of x = $\frac{7320}{[7320+7200]} \times 726$

$\frac{7320}{14520} \times 726$ = ₹366

61. (a) Let price of diamond be kx^2 where K is constant
total price for 4 pieces
$Kx^2\,[1+4+9+16] = 30\,kx^2$
Price of original diamond = $100\,kx^2$
difference = $70kx^2 = 70000$ or $kx^2 = 1000$
original price
100 × 1000 = ₹ 100000

62. (c) Let present age of Leela = x years
Leela's age at the time of Marriage = x – 6
ATQ

$x = 1\frac{1}{4}(x-6)$

$x = \frac{5}{4}(x-6)$

$4x = 5x - 30$
$x = 30$

Child's age = $\frac{1}{10} \times 30 = 3$ years.

63. (b) No. of boys = $\frac{3}{7} \times 49 = 21$

No. of girls = $\frac{4}{7} \times 49 = 28$

ATQ

$$\frac{28-4}{21} = \frac{24}{21} = \frac{8}{7}$$

64. (c) Let the 2 numbers be x and 5x
ATQ
$5x(x) = 320$
$5x^2 = 320$
$x^2 = 64$
$x = 8$
number are 8 and 40
$40^2 - 8^2 = (40+8)(40-8)$
$48 \times 32 = 1536$

65. (d)

	Alloy X	Alloy Y
Total	25	125
Ratio of lead and tin	1 : 2	2 : 3
Lead	$\frac{1}{3} \times 25$	$\frac{2}{5} \times 125$
Tin	$\frac{2}{3} \times 25$	$\frac{3}{5} \times 125$

When mixed

Lead = $\frac{25}{3} + 50$; Tin = $\frac{50}{3} + 75$

$= \frac{175}{3}$ $= \frac{275}{3}$

Ratio = $\dfrac{\frac{175}{3}}{\frac{275}{3}} = \frac{175}{275} = 7:11$

66. (d) If to maintain M1 cows for D1 days a milkman spends □W1 and to maintain M2 cows for D2 days, a milkman spends □ W2, then $\frac{M_1D_1}{W_1} = \frac{M_2D_2}{W_2}$

$$\frac{M_1D_1}{W_1} = \frac{M_2D_2}{W_2} \Rightarrow \frac{8 \times 60}{6400} = \frac{5 \times n}{4800}$$

$$\Rightarrow \frac{3}{40} = \frac{n}{960}$$

$$\Rightarrow n = \frac{3 \times 960}{40} = 3 \times 24 = 72$$

Hence, the milkman needs 72 days for maintenance.

67. (b) Using the options and the first given condition, we get $x - 5 = 3(y-5)$
Using the option (a),
$x - 5 = 3(y-5) \Rightarrow 30 - 5 = 3(y-5)$

$\Rightarrow 25 = 3y - 15 \Rightarrow 40$
which gives value of y as a fraction.
Using the option (b),
$x - 5 = 3(y - 5) \Rightarrow 32 - 5 = 3(y - 5)$
$\Rightarrow 27 = 3(y - 5) \Rightarrow 9 = y - 5 \Rightarrow y = 14$
Using the option (c),
$x - 5 = 3(y - 5) \Rightarrow 36 - 5 = 3(y - 5)$
$\Rightarrow 31 = 3y - 15 \Rightarrow 46 = 3y$
which gives value of y as a fraction.
Using the option (d),
$x - 5 = 3(y - 5) \Rightarrow 40 - 5 = 3(y - 5)$
$\Rightarrow 35 = 3y - 15 \Rightarrow 50 = 3y$
which also gives the value of y as a fraction.
Since only option (b) gives the final value of y as an integer, therefore option (b) is the correct answer

68. (c) We are given that the 3 parts are proportional to 1, $\frac{1}{3}, \frac{1}{6}$.
LCM of the denominators is 6.
Therefore, the ratio will be $\frac{1\times 6}{1\times 6} = \frac{6}{6} : \frac{1\times 2}{3\times 2} = \frac{2}{6} : \frac{1}{6}$
i.e. 6 : 2 : 1.
Sum of the ratio parts is 6 + 2 + 1 i.e. 9.
Now, the middle part of 78 is $\frac{2}{9} \times 78 = \frac{2}{3} \times 26 = \frac{52}{3}$

69. (d) We are given that the ratio of the number of boys in the first and the second standards is 2 : 3 and the ratio of the number of boys in the second and third standards is 4 : 5.
Now, we calculate a common ratio for all the three standards.
2 : 3 and 4 : 5 will be 2 × 4 : 3 × 4 = 8 : 12 and 4 × 3 : 5 × 3 = 12 : 15.
Therefore, the common ratio for all the three standards is 8 : 12 : 15.
Sum of the ratio parts = 8 + 12 + 15 = 35
Number of boys in first standard = $\frac{8}{35} \times 350 = 80$
Number of boys in third standard = $\frac{15}{35} \times 350 = 150$
Total number of boys in both standards = 80 + 150 = 230.

70. (b) Total number of registered students = 2000
Total number of appeared students
$= 2000 - 2000 \times \frac{1}{25} = 1920$
Total number of passed students
$= 1920 \times \frac{11}{20} = 1056$

71. (c) A : B = 1 : 2
B : C = 3 : 4
C : D (2 : 3)2 = 4 : 6
D : E = (3 : 4)2 = 6 : 8
∴ B : E = 3 : 8

72. (c) Let share of A be x
share of B = x – 20
share of C = x + 20
According to the question
x + x – 20 + x + 20 = 120
$\Rightarrow 3x = 120 \qquad \therefore x = \frac{120}{3} = 40$
Hence, share of B = 40 – 20 = 20

73. (a) Let share of mother be x
share of each daughter = 2x
share of each son = 2x × 3 = 6x
Total amount = 5 × 6x + 4 × 2x + x = 39x
According to the question
$39x = 39000 \qquad \therefore \quad x = \frac{39000}{39} = 1000$
Hence, widow's share is 1000.

74. (b) Let unit digit of age of women be x and ten's digit be y
Age of woman = y × 10 + x × 1 = 10y + x
Age of her husband = x × 10 + y × 1 = 10x + y
According to the question
$(10x + y + 10y + x)\frac{1}{11} = 10x + y - (10y + x)$
$(11x + 11y)\frac{1}{11} = 9x - 9y$
x + y = 9x – 9y
8x = 10y ∴ x : y = 5 : 4
Hence possible values of x and y are 5 and 4 respectively
Required difference = 9 × 5 – 9 × 4 = 9 years

75. (b) Copper in alloy $A = \frac{2}{5} \times 20 = 8$ kg
Tin in alloy $A = \frac{3}{5} \times 20 = 12$ kg
Copper in alloy $B = \frac{3}{7} \times 28 = 12$ kg
Tin in alloy $B = \frac{4}{7} \times 28 = 16$ kg
Let here copper x kg be mixed
According to the question,
$\frac{8 + 12 + x}{12 + 16} = \frac{6}{7} \quad ; \quad \frac{20 + x}{28} = \frac{6}{7}$
20 + x = 24 ∴ x = 24 – 20 = 4 kg

76. (b) Let contribution of D be x
Contribution of A = x × 3 = 3x
Contribution of E $= 3x \times \frac{1}{2} = \frac{3x}{2}$
Contribution of $B = \frac{3x}{2} \times \frac{1}{3} = \frac{x}{2}$

Contribution of $C = 3x \times \frac{2}{3} = 2x$

According to the question

$$\left(3x + x + \frac{3x}{2}\right) - \left(\frac{x}{2} + 2x\right) = 13500$$

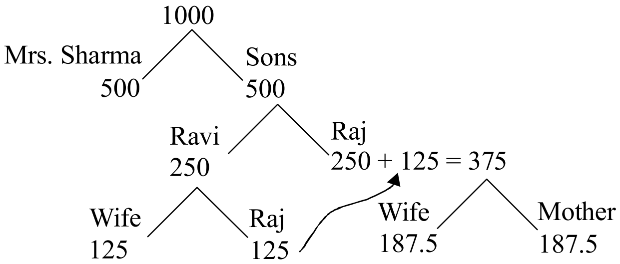
$$\frac{11x}{2} - \frac{5x}{2} = 13500$$

$$\frac{6x}{2} = 13500 \qquad \therefore \; x = 4500$$

Total share of B, C and E

$$= \frac{4500}{2} + 2 \times 4500 + \frac{3 \times 4500}{2} = 18000$$

77. (c) Let Mr. Sharma's property = 1000 units
From his will chart,



Now, property of Mrs. Sharma = 500 + 187.5 = 687.5 units

687.5 units $\rightarrow$ Rs. 88,000

Then,

1000 units $\rightarrow \frac{88000}{687.5} \times 1000$

= ₹1,28000

78. (b) Let, cost of lemon Juice bottle = x
cost of orange Juice bottle = 2x
ATQ, 4 bottle of lemon juice = 2 bottle of orange juice
Total bottle of orange juice bought = x + 2x = 3x
Now, Z's share = 50.
$\therefore$ cost of one bottle of orange juice = 50

79. (b) Let age of father and son be 'F' and 'S' resp.
ATQ,

$$\frac{F+S}{F-S} = \frac{11}{3}$$

$$3F + 3S = 11F - 11S$$

$$14S = 8F$$

$$\frac{F}{S} = \frac{7}{4}$$

Statement 1: Ratio of their ages = 7 : 4
So, it is incorrect
Statement 2: Let son, s age = 4 years and father's age = 7 years
ATQ,
When son's age = 8 yrs
Then father's age = 11 yrs
So, it is correct

80. (c) $\frac{a}{b} = \frac{c}{d} = \frac{1}{6}$

$a = c = 1$
$b = d = 6$

$$\frac{a^2 + c^2}{b^2 + d^2} = \frac{1+1}{36+36} = \frac{2}{72} = \frac{1}{36}$$

81. (a) $(4a + 7b)(4c - 7d) = (4a - 7b)(4c + 7d)$
$(4a + 7b)/(4a - 7b) = (4c + 7d)/(4c - 7d)$
Using componendo and dividendo
$\{(4a + 7b) + (4a - 7b)\} / \{(4a + 7b) - (4a - 7b)\}$
$= \{(4c + 7d) + (4c - 7d)\} / \{(4c + 7d) - (4c - 7d)\}$
$8a / 14b = 8c / 14d$
$a/b = c/d$

82. (c) $y = \frac{k}{\sqrt{x}}$

$$36 = \frac{k}{\sqrt{36}} = \frac{k}{6} \Rightarrow k = 36 \times 6 = 216$$

Now $\sqrt{x} = \frac{216}{54} = 4 \Rightarrow x = 16$

83. (c) Income - A : B
4 : 3
Expenditure - 3 : 2
Each saving - 600
As Income – Expenditures = 4x – 3x = 600
Income – Expenditures x = 600
Income - 4x $\rightarrow 600 \times 4 = 2400$

84. (c) We have
a : b = 1 : 3
b : c = 2 : 1
c : d = 1 : 2
d : e = 3 : 1
e : f = 1 : 4
a : b : c : d : e : f = 2 : 6 : 3 : 6 : 2 : 8

$$\frac{abc}{def} = \frac{2 \times 6 \times 3}{6 \times 2 \times 8} = \frac{3}{8}$$

85. (a) We have, $x^2 + 9y^2 = 6xy$

$$9\left(\frac{y}{x}\right)^2 - 6\left(\frac{y}{x}\right) + 1 = 0$$

$$\Rightarrow \left(3\frac{y}{x} - 1\right)^2 = 0 \qquad \Rightarrow \frac{y}{x} = \frac{1}{3}$$

86. (c) As, $x \propto yz \Rightarrow \frac{1}{y} \propto z/x$

87. (d) $$\frac{\text{Male}}{\text{Total}} = \frac{\frac{1525 \times 27}{61} + \frac{2600 \times 6}{13}}{1525 + 2600} = \frac{1875}{4125} = \frac{5}{11}$$

$$\therefore \frac{\text{Male}}{\text{Female}} = \frac{5}{11-5} = \frac{5}{6}$$

CHAPTER 7

Average

1. A batsman scores 80 runs in his sixth innings and thus increases his average by 5. What is his average after six innings?` **[2011-I]**
(a) 50 (b) 55 (c) 60 (d) 65

2. An aeroplane flies along the four sides of a square at a speed of 100, 200, 300 and 400 km/h, respectively. What is the average speed of the plane in its flight around the square? **[2011-I]**
(a) 196 km/h (b) 200 km/h
(c) 250 km/h (d) None of these

3. How much tea at ₹ 9 per kg must be mixed with 100 kg of superior tea at ₹ 13.50 per kg to give an average price of ₹ 11 per kg? **[2011-I]**
(a) 85 kg (b) 120 kg
(c) 125 kg (d) 130 kg

4. The population of a state increased from 100 million to 169 million in two decades. What is the average increase in population per decade? **[2011-I]**
(a) 20% (b) 34.5% (c) 69% (d) 30%

5. A person invested part of ₹ 45000 at 4% and the rest at 6%. If his annual income from both are equal, then what is the average rate of interest? **[2011-I]**
(a) 4.6% (b) 4.8% (c) 5.0% (d) 5.2%

6. Nine numbers are written in ascending order. The middle number is the average of the nine numbers. The average of the first five larger numbers is 68 and that of five smaller numbers is 44. What is the sum of all nine numbers? **[2011-I]**
(a) 450 (b) 501 (c) 504 (d) 540

7. In an examination, 40% of the candidates wrote their answers in Hindi and the others in English. The average marks of the candidates written in Hindi is 74 and the average marks of the candidates written in English is 77. What is the average marks of all the candidates ? **[2011-I]**
(a) 75.5 (b) 75.8 (c) 76.0 (d) 76.8

8. The mean weight of 150 students in a certain class is 60 kg. The mean weight of the boys from the class is 70 kg, while that of girls is 55 kg. What is the number of girls in the class ? **[2011-I]**
(a) 105 (b) 100 (c) 95 (d) 60

9. Out of 250 observations, the first 100 observations have mean 5 and the average of the remaining 150 observations is $\frac{25}{3}$. What is the average of the whole group of observations ? **[2011-I]**
(a) 6 (b) 7 (c) 8 (d) 9

10. The mean weight of 150 students in a class is 60 kg. The mean weight of boys is 70 kg and that of girls is 55 kg, what is the number of boys in the class ? **[2012-II]**
(a) 50 (b) 60 (c) 75 (c) 100

11. The average weight of a class of 15 boys and 10 girls is 38.4 kg. If the average weight of the boys is 40 kg, then what is the average weight of the girls? **[2013-I]**
(a) 36.5 kg (b) 35 kg (c) 36 kg (d) 34.6 kg

12. If the average of A and B is 30, the average of C and D is 20, then which of the following is/are correct?
I. The average of B and C must be greater than 25.
II. The average of A and D must be less than 25.
Select the correct answer using the codes given below. **[2014-I]**
(a) Only I (b) Only II
(c) Either I or II (d) Neither I or II

13. The price of a commodity increased by 5% from 2010 to 2011, 8% from 2011 to 2012 and 77% from 2012 to 2013. What is the average price increase (approximate) from 2010 to 2013? **[2014-II]**
(a) 26% (b) 32% (c) 24% (d) 30%

14. The average of m numbers is n^4 and the average of n numbers is m^4. The average of (m + n) numbers is **[2015-II]**
(a) mn (b) $m^2 + n^2$
(c) $mn\,(m^2 + n^2)$ (d) $mn(m^2 + n^2 - mn)$

15. The average weight of students in a class is 43 kg. Four new students are admitted to the class whose weights are 42 kg, 36.5 kg, 39 kg and 42.5 kg respectively. Now the average weight–of the students of the class is 42.5 kg, The number of students in the beginning was **[2015-II]**
(a) 10 (b) 15 (c) 20 (b) 25

16. Four years ago, the average age of A and B was 18 years. Now the average age of A, B and C is 24 years. After 8 years, the age of C will be. **[2015-II]**
(a) 32 years (b) 28 years
(c) 36 years (d) 40 years

17. If the mean age of combined group of boys and girls is 18 years and the mean of age of boys is 20 and that of girls is 16, then what is the percentage of boys in the group ? **[2016-II]**
(a) 60 (b) 50 (c) 45 (d) 40

18. A cricketer has a certain average of 10 innings. In the eleventh inning he scored 108 runs, thereby increasing his average by 6 runs. What is his new average? **[2016-II]**
(a) 42 (b) 47 (c) 48 (d) 60

19. The mean of 20 observations is 17. On checking it was found that the two observations were wrongly copied as 3 and 6. If wrong observations are replaced by correct values 8 and 9, then what is the correct mean ? **[2016-II]**
(a) 17.4 (b) 16.6 (c) 15.8 (d) 14.2

20. In a class of 100 students, there are 70 boys whose average marks in a subject are 75. If the average marks of the complete class is 72, then what is the average marks of the girls ? **[2016-II]**

(a) 64 (b) 65 (c) 68 (d) 74

21. Sunil wants to spend ₹ 200 on two types of sweets, costing ₹ 7 and ₹ 10 respectively. What is the maximum number of sweets he can get so that no money is left over ? **[2017-I]**

(a) 25 (b) 26 (c) 27 (d) 28

22. The mean of 5 numbers is 15. If one more number is the mean of the 6 numbers becomes 17. What is the included number? **[2017-I]**

(a) 24 (b) 25 (c) 26 (d) 27

23. The mean marks obtained by 300 students in a subject are 60. The mean of top 100 students was found to be 80 and the mean of last 100 students was found to be 50. The mean marks of the remaining 100 students are **[2017-I]**

(a) 70 (b) 65 (c) 60 (d) 50

24. Let a, b, c, d, e, f, g be consecutive even numbers and j, k, ℓ, m, n be consecutive odd numbers. What is the average of all the numbers ? **[2017-I]**

(a) $\frac{3(a+n)}{2}$ (b) $\frac{(5l+7d)}{4}$

(c) $\frac{(a+b+m+n)}{4}$ (d) None of the above

25. At present the average of the ages of a father and a son is 25 years. After seven years the son will be 17 years old. What will be the age of the father after 10 years? **[2018-I]**

(a) 44 years (b) 45 years
(c) 50 years (d) 52 years

26. If the average of 9 consecutive positive integers is 55, then what is the largest integer? **[2018-I]**

(a) 57 (b) 58 (c) 59 (d) 60

27. The average of the ages of 15 students in a class is 19 years. When 5 new students are admitted to the class, the average age of the class becomes 18.5 years. What is the average age of the 5 newly admitted students? **[2018-I]**

(a) 17 years (b) 17.5 years
(c) 18 years (d) 18.5 years

28. The sum of ages of a father, a mother, a son Sonu and daughters Savita and Sonia is 96 years. Sonu is the youngest member of the family. The year Sonu was born, the sum of the ages of all the members of the family was 66 years. If the father's age now is 6 times that of Sonu's present age, then 12 years hence, the father's age will be **[2018-II]**

(a) 44 years (b) 45 years
(c) 46 years (d) 48 years

29. Ten (10) years before, the ages of a mother and her daugther were in the ratio 3 : 1. In another 10 years from now, the ratio of their ages will be 13 : 7. What are their present ages? **[2018-II]**

(a) 39 years, 21 years (b) 55 years, 25 years
(c) 75 years, 25 years (d) 49 years, 31 years

30. The average marks of section A are 65 and that of section B are 70. If the average marks of both the sections combined are 67, then the ratio of number of students of section A to that of section B is **[2018-II]**

(a) 3 : 2 (b) 1 : 3 (c) 3 : 1 (d) 2 : 3

31. Priya's age was cube of an integral number (different from 1) four years ago and square of an integral number after four years. How long should she wait so that her age becomes square of a number in the previous year and cube of a number in the next year ? **[2019-I]**

(a) 7 years (b) 12 years (c) 14 years (d) 21 years

32. In a class of 100 students, the average weight is 30 kg. If the average weight of the girls is 24 kg and that of the boys is 32 kg, then what is the number of girls in the class ? **[2019-I]**

(a) 25 (b) 26 (c) 27 (d) 28

33. A race has three parts. The speed and time required to complete the individual parts for a runner is displayed on the following chart: **[2019-I]**

	Part I	Part II	Part III
Speed (kmph)	9	8	7. 5
Time (minutes)	50	80	100

What is the average speed of this runner ?

(a) 8. 17 kmph (b) 8. 00 kmph
(c) 7. 80 kmph (d) 7. 77 kmph

34. The average of 50 consecutive natural numbers is x. What will be the new average when the next four natural numbers are also included ? **[2019-I]**

(a) $x+1$ (b) $x+2$ (c) $x+4$ (d) $x+(x/54)$

35. Consider two-digit numbers which remain the same when the digits interchange their positions. What is the average of such two-digit numbers? **[2019-I]**

(a) 33 (b) 44 (c) 55 (d) 66

36. Radha and Rani are sisters. Five years back, the age of Radha was three times that of Rani, but one year back the age of Radha was two times that of Rani. What is the age difference between them? **[2019-II]**

(a) 8 (b) 9 (c) 10 (d) 11

37. A library has an average number of 510 visitors on Sunday and 240 on other days. What is the average number of visitors per day in a month of 30 days beginning with Saturday ? **[2019-II]**

(a) 276 (b) 282 (c) 285 (d) 375

38. In a class room the ratio of number of girls to that of boys is 3 : 4. The average height of students in the class is 4.6 feet. If the average height of the boys in the class is 4.8 feet, then what is the average height of the girls in the class ? **[2020-I]**

(a) Less than 4.2 feet
(b) More than 4.2 feet but less than 4.3 feet
(c) More than 4.3 feet but less than 4.4 feet
(d) More than 4.4 feet but less than 4.5 feet

HINTS & SOLUTIONS

1. (b) Let the average of 5 innings = x
Scores in sixth inning = 80
$\therefore$ Total of 5 innings = $5x$
According to the question,
$$\frac{5x+80}{6} = x + 5$$
$\Rightarrow$ $5x + 80 = 6x + 30$
$\Rightarrow$ $x = 80 - 30 = 50$
$\therefore$ His average after six innings = 50 + 5 = 55

2. (d) Average speed = $\dfrac{\text{Total distance}}{\text{Total time}}$
$$= \frac{x+x+x+x}{\frac{x}{100}+\frac{x}{200}+\frac{x}{300}+\frac{x}{400}}$$
$$= \frac{4x}{\frac{12x+6x+4x+3x}{1200}}$$
$$= \frac{4x \times 1200}{25x} = 192 \text{ km/h}$$

3. (c) Let x kg of tea of ₹ 9 per kg.
$$\frac{9 \times x + 13.5 \times 100}{x+100} = 11$$
$\Rightarrow$ $9x + 1350 = 11x + 1100$ $\Rightarrow$ $2x = 250$
$\therefore$ $x = 125$ kg.

4. (b) The difference of population in two decades
= 169 – 100 = 69 million
$\therefore$ Increase in population in first decade
$$= \frac{69\%}{2} = 34.5\%$$

5. (b) Let a person invest 4% of x.
According to question
$$\therefore \frac{x \times 4}{100} = \frac{(45000-x)}{100} \times 6$$
$\Rightarrow$ $2x = 45000 \times 3 - 3x$
$$\Rightarrow x = \frac{45000 \times 3}{5} = ₹\ 27000$$
Another part is ₹ 18000.
Let r = Average rate of interest
Interest for 1st part in one year = $\dfrac{27000 \times 4}{100}$ = ₹ 1080
Similarly, interest for rest part in one year = 1080
$\because$ Total interest = ₹ 2160
$$\therefore \frac{45000 \times r}{100} = 2160$$
$$\Rightarrow r = \frac{216}{45} = 4.8\%$$

6. (c) Since, 5th term = average of 9 numbers = x
Sum of first five larger numbers = 68 × 5 = 340
Sum of first five smaller numbers = 44 × 5 = 220
Average of nine numbers = $\dfrac{340+220-x}{9}$
(since, x is subtracted because 5th term repeated twice)
$$\therefore x = \frac{560-x}{9}$$
$\Rightarrow$ $9x + x = 560$ $\Rightarrow$ $x = 56$
$\therefore$ Sum of 9 numbers = 56 × 9 = 504

7. (b) Let total number of candidates in Hindi = 100
$\therefore$ Total marks of 40 candidates in English = 40 × 74
Total marks of 60 candidates = 60 × 77
$\therefore$ Required average marks = $\dfrac{40 \times 74 + 60 \times 77}{100}$
$$= \frac{2960+4620}{100} = \frac{7580}{100} = 75.80$$

8. (b) Let the no. of boys = x
No. of girls = $150 - x$
Average weight = $\dfrac{70x + (150-x) \times 55}{150}$
$$\Rightarrow 60 = \frac{70x + (150-x) \times 55}{150}$$
No. of boys (x) = 50
No. of girls = 150 – x = 150 – 50 = 100

9. (b) Given, $n_1 = 100$, $\bar{x}_1 = 5$ and $n_2 = 150$, $\bar{x}_2 = \dfrac{25}{3}$
Average of whole group of observations
$$\bar{x}_1 = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1+n_2}$$
$$= \frac{100 \times 5 + 150 \times \frac{25}{3}}{100+150} = \frac{1750}{250} = 7$$

10. (a) Total weight of 150 students
= 150 × 60 = 9000 kg
Let total no. of boys = x
No. of girls = $150 - x$
Average weight = $\dfrac{x \times 70 + (150-x)55}{150}$
$$\Rightarrow 60 = \frac{70x + 150 \times 55 - 55x}{150}$$
$\Rightarrow$ $60 \times 150 = 15x + 150 \times 55$
$\Rightarrow$ $15x = 60 \times 150 - 150 \times 55$
$$\therefore x = \frac{750}{15} = 50$$
$\therefore$ No. of boys in the class = 50

11. (c) Let average weight of girls = x
Total weight of the boys = 40 kg × 15 = 600 kg.
Average weight

$$= \frac{\text{Total weight of girls + Total weight of boys}}{\text{No. of boys + No. of girls}}$$

$$\Rightarrow \quad 38.4 = \frac{600 + 10 \times x}{15 + 10}$$

$$\Rightarrow \quad 38.4 = \frac{600 + 10x}{25}$$

$\Rightarrow \quad 38.4 \times 25 = 600 + 10x$

$\therefore \quad x = 36$ kg

12. (d) Average of A and B = 30

$$\Rightarrow \quad \frac{A+B}{2} = 30 \Rightarrow A + B = 60$$

Average of C and D = 20

$$\Rightarrow \quad \frac{C+D}{2} = 20 \Rightarrow C + D = 40$$

Here, we can't find the avg of B and C, A and D so that Neither I or II are follows.

13. (d) Average price increase = $\left(\frac{5+8+77}{3}\right)\%$

$$= \frac{90}{3}\% = 30\%$$

14. (d) Sum of 'm' numbers = $m.n^4$
Sum of 'n' numbers = $n.m^4$
Sum of 'm + n' numbers = $mn^4 + nm^4$

Average of 'm + n' numbers = $\dfrac{mn^4 + nm^4}{(m+n)}$

$$= \frac{mn\left(n^3 + m^3\right)}{(m+n)}$$

$$= \frac{mn(m+n)\left(m^2 + n^2 - mn\right)}{(m+n)}$$

$= mn(m^2 + n^2 - mn)$
So, option (d) is correct.

15. (c) Let the no. of students in the beginning be x.
According to question-

$$\Rightarrow \frac{43x + 42 + 36.5 + 38 + 42.5}{(x+4)} = 42.5$$

$$\Rightarrow \frac{43x + 160}{(x+4)} = 42.5$$

$\Rightarrow 43x + 160 = 42.5\,x + 170$
$\Rightarrow 0.5x = 10$
$\Rightarrow x = 20$
So, option (c) is correct.

16. (c) Let their age be A, B and C respectively.

$$\Rightarrow \frac{A+B+C}{3} = 24$$

$\Rightarrow A + B + C = 72$ --- (i)

$$\Rightarrow \frac{A-4+B-4}{2} = 18$$

$$\Rightarrow \frac{A+B-8}{2} = 18$$

$A + B = 44$ --- (ii)
eq(i) – eq (ii) -
$(A+B+C) - (A+B) = 72 - 44$
$C = 28$ years
After 8 years age of C
$= 28 + 8 = 36$ years
So, option (c) is correct.

17. (b) Let number of boys and girls be x and y
ATQ

$$\frac{20x + 16y}{x + y} = 18$$

$20x + 16y = 18x + 18y$
$2x = 2y$
$x = y$
i.e. boys are 50% of group

18. (c) Let the cricketer's average be = x run per match
ATQ

$$\frac{10x + 108}{11} = x + 6$$

$11x + 66 = 10x + 108$
$x = 42$
New average = $42 + 6 = 48$

19. (a) Mean of 20 observation = 17
total sum = $17 \times 20 = 340$
ATQ

$$\frac{340 - 3 - 6 + 8 + 9}{20} = \frac{348}{20} = 17.4$$

20. (b) Total student = 100, Boy = 70, Girls = 30
Boys average = 75
Total marks = $75 \times 70 = 5250$
Class average = 72
Total class marks = $72 \times 100 = 7200$

Total marks of girls = $\dfrac{7200 - 5250}{30} = \dfrac{1950}{30} = 65$

21. (b) Let the two types of sweets be x and y
ATQ
$7x + 10y = 200$
$\therefore$ it is possible when
7x is multiple of 10
i.e. 70 and 140 .....
$7 \times 10 + 10 \times 13 = 200;\quad 10 + 13 = 23$
$7 \times 20 + 10 \times 6 = 200;\quad 20 + 6 = 26$
Maximum sweet went x = 20 and y = 6 i.e. Total 26.

22. (d) Mean of 5 number = 15
Total = $15 \times 5 = 75$
Let number included = $75 + x$
ATQ

$$\frac{75 + x}{6} = 17$$

$x = 102 - 75 = 27$

23. (d) Mean marks of 300 student = 60
Total mark = 300×60
Mean marks of 100 students = 80
Mean marks of another 100 students = 50
let Mean mark of remaining 100 student = x
ATQ
$100x + 100 \times 50 + 100 \times 80 = 300 \times 60$
$100x = 5000$
$x = 50$

24. (d) when a, b, c, d, e, f, g are consecutive evenno then no or
$d-6, d-4, d-2, d+2, d+4, d+6$
Total = 7d
Similarly,
When J, K, L, m and n be consecutive odd no. then numbers are $\ell-4, \ell-2, \ell, \ell+2, \ell+4$
Total = 5ℓ
Average = $\dfrac{5\ell + 7d}{12}$

25. (c) Total age of father and son at present
$= 2 \times 25 = 50$ years
7 years later total age of father and son
$= 50 + 2 \times 7 = 64$ years
7 years later age of son = 17 years
7 years later age of father = 64 – 17 = 47 years
Present age of father = 47 – 7 = 40 years
age of father 10 years later = 40 + 10 = 50 years

26. (c) Let integers be x, x + 1, x + 2, x + 3, x + 4, x + 5, x + 6, x + 7 and x + 8 respectively
According to the question
$$\frac{x + x + 1 + x + 2 + x + 3 + x + 4 + x + 5 + x + 6 + x + 7 + x + 8}{9} = 55$$
$9x + 36 = 495$
$9x = 495 - 36 = 459$
$\therefore \quad x = \dfrac{459}{9} = 51$
Hence, largest integer = 51 + 8 = 59

27. (a) Total age of 15 students of the class = 15 × 19 =285
Total age of 20 students of the class = 18.5 × 20=370
Total age of 5 new students = 370 – 285 = 85 years
average age of new students $= \dfrac{85}{5} = 17$ years

28. (d) Let present age of son is x years
Then, Sum of present age of all 5 members of the family = 96
Now, from question, at the time the son sonu was born i.e. a time before x years, sum of ages of 4 member of the family = 66.
So, $66 + 5x = 96$
$5x = 30 \quad \Rightarrow \quad x = 6.$
Sonu's present age = 6 years
Sonu's father present age = 6 × 6 = 36 years
12 years hence father's age = 36 + 12 = 48 years

29. (b) Let present age of mother and daughter are M and D respectively.
According to the question
$$\frac{M-10}{D-10} = \frac{3}{1} \Rightarrow M - 3D = -20 \quad \text{...(i)}$$
and,
$$\frac{M+10}{D+10} = \frac{13}{7} \Rightarrow 7M - 13D = 60 \quad \text{...(ii)}$$
From (i) and (ii), we have
$21D - 13D = 140 + 60$
$80D = 200 \quad \Rightarrow \quad D = 25$ and $M = 55$
Hence, Mother's present age = 55 yrs.
Daughter's present age = 25 years

30. (a) Section A
$$\frac{x_1 + x_2 .... + x_n}{n} = 65$$
$x_1 + x_2 + ... + x_n = 65 \times n \quad$ ...(1)
Section B
$$\frac{y_1 + y_2 .... + y_m}{m} = 7\tilde{0}$$
$y_1 + y_2 + ... + y_m = 70 \times m \quad$ ...(2)
Combined Section
$$\frac{y_1 + y_2 .... + y_m}{m} = 70$$
$65n + 70m = 67n + 67m$; $3m = 2n$; $\dfrac{3}{2} = \dfrac{n}{m}$

31. (c) Let present age of Priya be x
$x - 4 = n^3$
$x + 4 = (b)^2$
since n is a no > 1 on putting n = 2 we get $x = 12$
So $x + 4 = 16$ which is square of an integral number thus, consistent with given information after how many years her age becomes such that age before one year is a square and age after one year is a cube
Using option if we add 14 years to current age, we get age = 26 years
Here 25 is a square and 27 is a cube thus making 14 the correct answer.

32. (a) By alligation,
girls **boys**
24 32
30
2 : 6
1 : 3
So the number of girls will be $= \dfrac{1}{4} \times 100 = 25$

33. (b) Average speed = Total Distance / Total time
$$= \frac{9\frac{50}{60} + 8\frac{80}{60} + 7.5\frac{100}{60}}{\frac{50}{60} + \frac{80}{60} + \frac{100}{60}}$$
$= (45 + 64 + 75) / 23 = 184/23$
$= 8$ kmph

34. (b) Sum of n consecutive natural numbers = n (n + 1)/2
Average of n consecutive natural numbers
= (n + 1)/2
For first 50 average = 51/2 = x
When next 4 natural numbers are included.

Then, average of 54 continuons natural number $= \frac{55}{2}$

$= \frac{51}{2} + \frac{4}{2} = \boxed{x+2}$

35. (c) All such 2 digt numbers whose digits are same, 11, 22, 33, 44 .... upto 99

Average = sum/9 = $\frac{(11+22+33+.....+99)}{9} = 55$

36. (a) Radha's Age = A
Rani's Age = B

$\frac{A-5}{B-5} = 3 \Rightarrow A-5 = 3(B-5)$ ...(i)

$\frac{A-1}{B-1} = 2 \Rightarrow A-1 = 2(B-1)$ ...(ii)

from equation (i) and (ii)
$\Rightarrow -4 = B - 13 \quad \Rightarrow B = 9$
So $A - 1 = 18 - 2 \Rightarrow A = 17$
$A - B = 17 - 9 = 8$ years

37. (c) Total sunday = 5
other day = 25
$\therefore$ Average number of visitor in a month

$= \frac{5\times510+25\times240}{30}$

$= \frac{2550+6000}{30} = \frac{8550}{30} = 285$

38. (c) Let number of boys and girls in the class is 4x and 3x also Average height of girl is h
Now,
$\Rightarrow 7x \times 4.6 = 4x \times 4.8 + 3x \times h$
$\Rightarrow 32.2x = 19.2x + 3x \times h$

$\therefore h = \frac{13}{3} = 4.\bar{3}$

CHAPTER 8

Simple and Compound Interest

1. A man borrowed ₹ 40000 at 8% simple interest per year. At the end of second year, he paid back certain amount and at the end of fifth year, he paid back ₹ 35960 and cleared the debt. What is the amount did he pay back after the second year? **[2007-I]**
(a) ₹ 16200 (b) ₹ 17400
(c) ₹ 18600 (d) None of these

2. A sum of ₹ 24000 is borrowed for $1\frac{1}{2}$ years at the rate of interest 10% per annum compound semi-annually. What is the compound interest (x) ? **[2007-I]**
(a) $x <$ ₹ 3000 (b) ₹ 3000 $< x <$ ₹ 4000
(c) ₹ 4000 $< x <$ ₹ 5000 (d) $x >$ ₹ 5000

3. An amount at compound interest doubles itself in 4 years. In how many years will it become 8 times of itself? **[2007-I]**
(a) 8 years (b) 12 years
(c) 16 years (d) 24 years

4. At what rate per cent annum calculated in simple interest will a sum of money double in 10 years ? **[2007-I]**
(a) 10% (b) 2% (c) 12.5% (d) 13.5%

5. A man invested ₹ 1000 on a simple interest at a certain rate and ₹ 1500 at 2% higher rate. The total interest in three years is ₹ 390. What is the rate of interest for ₹ 1000? **[2007-II]**
(a) 4% (b) 5% (c) 6% (d) 8%

6. If P is principal amount and the rate of interest is R% per annum and the compound interest is calculated k times in a year, then what is the amount at the end of n years ? **[2007-II]**
(a) $P\left(1+\frac{r}{100k}\right)^{nk}$ (b) $P\left(1+\frac{kr}{100}\right)^{nk}$
(c) $P\left(1+\frac{kr}{100}\right)^{\frac{n}{k}}$ (d) $P\left(1+\frac{kr}{100k}\right)^{\frac{n}{k}}$

7. Ram had ₹ 2 lakh, part of which he lent at 15% per annum and rest at 12% per annum. Yearly interest accured was ₹ 27600. How much did he lent at 15%? **[2008-I]**
(a) ₹ 120000 (b) ₹ 100000
(c) ₹ 80000 (d) ₹ 60000

8. Out of a sum of ₹ 640, a part was lent at 6% simple interest and the other at 9% simple interest. If the interest on the first part after 3 years equal to the interest on the second part after 6 years, then what is the second part ? **[2008-I]**
(a) ₹ 120 (b) ₹ 140
(c) ₹ 160 (d) ₹ 180

9. An amount of ₹ x at compound interest at 20% per annum for 3 years becomes y. What is $y : x$?**[2008-I]**
(a) 3 : 1 (b) 36 : 25
(c) 216 : 125 (d) 125 : 216

10. At what rate per cent per annum simple interest, will a sum of money triple itself in 25 years ? **[2008-II]**
(a) 8% (b) 9% (c) 10% (d) 12%

11. The compound interest on a sum for 2 years is ₹ 832 and the simple interest on the same sum at the same rate for the same period is ₹ 800. What is the rate of interest? **[2009-I]**
(a) 6% (b) 8% (c) 10% (d) 12%

12. If the rate of interest is 10% per annum and is compound half-yearly, then the principle of ₹ 400 in 3/2 years will amount to **[2009-II]**
(a) ₹ 463.00 (b) ₹ 463.05
(c) ₹ 463.15 (d) ₹ 463.20

13. A person borrowed ₹ 7500 at 16% compound interest. How much does he have to pay at the end of 2 years to clear the loan ? **[2009-II]**
(a) ₹ 9900 (b) ₹ 10092
(c) ₹ 11000 (d) ₹ 11052

14. The simple interest on a certain sum of money for 3 years at 8% per annum is half the compound interest on ₹ 4000 for 2 years at 10% per annum. What is the sum placed on simple interest? **[2010-I]**
(a) ₹ 1550 (b) ₹ 1650
(c) ₹ 1750 (d) ₹ 2000

15. A person invested some amount at the rate of 12% simple interest and the remaining at 10%. He received yearly an interest of ₹ 130. Had he interchanged the amounts invested, he would have received an interest of ₹ 134. How much money did he invest at different rates? **[2010-I]**
(a) ₹ 500 at the rate of 10%, ₹ 800 at the rate of 12%
(b) ₹ 700 at the rate of 10%, ₹ 600 at the rate of 12%
(c) ₹ 800 at the rate of 10%, ₹ 400 at the rate of 12%
(d) ₹ 700 at the rate of 10%, ₹ 500 at the rate of 12%

16. A sum of money lent on simple interest triples itself in 15 years and 6 months. In how many year still it be doubled? **[2010-II]**
(a) 5 years and 3 months (b) 7 years and 9 months
(c) 8 years and 3 months (d) 9 years and 6 months

17. What is the least number of years in which a sum of money at 20% compound interest will be more than doubled ? **[2011-I]**
(a) 7 (b) 6 (c) 5 (d) 4

18. A sum of money on compound interest amount to ₹ 9680 in 2 years and to ₹ 10648 in 3 years. What is the rate of interest per annum ? **[2011-I]**
(a) 5% (b) 10%
(c) 15% (d) 20%

19. The sum which amounts to ₹ 364.80 in 8 years at 3.5% simple interest per annum is **[2011-II]**
(a) ₹ 285 (b) ₹ 280 (c) ₹ 275 (d) ₹ 270

20. A certain sum at simple interest amounts to ₹ 1350 in 5 years and to ₹ 1620 in 8 years. What is the sum? **[2011-II]**
(a) ₹ 700 (b) ₹ 800 (c) ₹ 900 (d) ₹ 1000

21. What is the compound interest on ₹ 1600 at 25% per annum of 2 years compounded annually? **[2012-I]**
(a) ₹ 700 (b) ₹ 750 (c) ₹ 800 (d) ₹ 900

22. A sum of money becomes 3 times in 5 years. In how many years will the same sum become 6 times at the same rate of simple interest ? **[2012-II]**
(a) 15 years (b) 12.5 years
(c) 10 years (d) 7.5 years

23. The principal on which a simple interest of ₹ 55 will be obtained after 9 months at the rate of $3\frac{2}{3}\%$ per annum is **[2013-I]**
(a) ₹ 1000 (b) ₹ 1500 (c) ₹ 2000 (d) ₹ 2500

24. When an article is sold at 20% discount, the selling price is ₹24. What will be the selling price when the discount is 30% ? **[2014-II]**
(a) ₹ 25 (b) ₹ 23 (a) ₹ 21 (d) ₹ 20

25. The difference between compound interest and simple interest for 2 yr at the rate of 10% over principal amount of ₹ X is ₹10. What is the value of X ? **[2014-II]**
(a) ₹ 100 (b) ₹ 1000 (c) ₹ 500 (d) ₹ 5000

26. A sum of money becomes 3 times in 5 yr at simple interest. In how many years, will the same sum become 6 times at the same rate of simple interest ? **[2014-II]**
(a) 10 *yr* (b) 12 *yr* (c) 12.5 *yr* (d) 10.5 *yr*

27. The difference between compound interest and simple interest at the same rate of interest R per cent per annum on ₹ 15,000 for 2 years is ₹ 96. What is the value of R? **[2015-I]**
(a) 8 (b) 10 (c) 12
(d) Cannot be determined due to insufficient data

28. There is 60% increase in an amount in 6 years at simple interest. What will be the compound interest on ₹ 12,000 after 3 years at the same rate of interest? **[2015-I]**
(a) ₹ 2,160 (b) ₹ 3,120
(c) ₹ 3,972 (d) ₹ 6,240

29. In how much time would the simple interest on a principal amount be 0.125 time the principal amount at 10% per annum? **[2015-I]**
(a) $1\frac{1}{4}$ years (b) $1\frac{3}{4}$ years
(c) $2\frac{1}{4}$ years (d) $2\frac{3}{4}$ years

30. An automobile financer claims to be lending money at simple interest, but he includes the interest every six months for calculating the principal. If he is charging an interest at the rate of 10%, the effective rate of interest becomes **[2015-I]**
(a) 10.25% (b) 10.5% (c) 10.75% (d) 11%

31. A sum of ₹10,000. is deposited for 1 year at the rate of interest 10% compounded half yearly. What will be the interest at the end of one year ? **[2015-II]**
(a) ₹ 1000 (b) ₹ 1025 (c) ₹1050 (d) ₹1100

32. If a sum of money at a certain rate of simple interest per year doubles in 5 years and at a different rate of simple interest per year becomes three times in 12 years, then the difference in the two rates of Simple interest per year is **[2016-I]**
(a) 2% (b) 3% (c) $3\frac{1}{3}\%$ (d) $4\frac{1}{3}\%$

33. A sum of ₹ 8,400 was taken as a loan. This is to be paid in two equal instalments. If the rate of interest is 10% per annum, compounded annually, then the value of each instalment is
(a) ₹ 4,200 (b) ₹ 4,480
(c) ₹ 4,840 (d) None of the above

34. The difference between the simple and the compound interest on a certain sum of money at 4% per annum in 2 years is ₹ 10. What is the sum ?
(a) ₹ 5,000 (b) ₹ 6,000
(c) ₹ 6,250 (d) ₹ 7,500

35. The difference between the compound interest (compounded annually) and simple interest on a sum of money deposited for 2 years at 5% per annum is ` 15. What is the sum of money deposited?
(a) ₹ 6,000 (b) ₹ 4,800 (c) ₹ 3,600 (d) ₹ 2,400

36. A person borrowed ₹5,000 at 5% rate of interest per annum and immediately lent it at 5.5%. After two years he collected the amount and settled his loan. What is the amount gained by him in this transaction? **[2018-I]**
(a) ₹25 (b) ₹50 (c) ₹100 (d) ₹200

37. A merchant commences with a certain capital and gains annually at the rate of 25%. At the end of 3 years he has ₹10,000. What is the original amount that the merchant invested? **[2018-I]**
(a) ₹ 5,120 (b) ₹ 5,210 (c) ₹ 5,350 (d) ₹ 5,500

38. The annual income of a person decreases by ₹64 if the annual rate of interest decreases from 4% to 3.75%. What is his original annual income? **[2018-I]**
(a) ₹ 24,000 (b) ₹ 24,000 (c) ₹ 25,600 (d) ₹ 24,600

39. What is the principal amount which earns ‘ 210 as compound interest for the second year at 5% per annum? **[2018-II]**
(a) ₹ 2000 (b) ₹ 3200 (c) ₹ 4000 (d) ₹ 4800

40. Two equal amounts were borrowed at 5% and 4% simple interest. The total interest after 4 years amounted to ₹405. What was the total amount borrowed? **[2018-II]**
(a) ₹1075 (b) ₹ 1100 (c) ₹1125 (d) ₹1150

41. What is the least number of complete years in which a sum of money put out at 40% annual compound interest will be more than trebled ? **[2019-I]**
(a) 3 (b) 4 (c) 5 (d) 6

42. A person divided a sum of ₹ 17, 200 into three parts and invested at 5%, 6% and 9% per annum simple interest. At the end of two years, he got the same interest on each part of money. What is the money invested at 9% ? **[2019-I]**
(a) ₹ 3, 200 (b) ₹ 4, 000 (c) ₹ 4, 800 (d) ₹ 5, 000

43. The rate of interest on two different schemes is the same and it is 20%. But in one of the schemes, the interest is compounded half yearly and in the other the interest is compounded annually. Equal amounts are invested in the schemes. If the difference of the returns after 2 years is ₹482, then what is the principal amount in each scheme ? **[2019-II]**

(a) ₹10,000 (b) ₹16,000
(c) ₹20,000 (d) ₹24,000

44. *A* lent ₹25000 to *B* and at the same time lent some amount to *C* at same 7% simple interest. After 4 years *A* received ₹11200 as interest from *B* and *C*. How much did *A* lend to *C*? **[2019-II]**

(a) ₹20000 (b) ₹25000 (c) ₹15000 (d) ₹10000

45. A sum was put at simple interest at certain rate for 2 years. Had it been put at 1% higher rate of interest, it would have fetched ₹24 more. What is the sum? **[2020-I]**

(a) ₹500 (b) ₹600 (c) ₹800 (d) ₹1200

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1. (b) Total borrowed money = ₹ 40000
Rate of interest = 8%

The interest for 2 years = $\frac{40000 \times 8 \times 2}{100}$ = ₹ 6400

Let he paid ₹ x at the end of second year. Interest will be calculated on ₹ $(40000 - x + 6400)$.

Interest for 3 years = $\frac{(46400 - x) \times 3 \times 8}{100}$

$= ₹\ \frac{6}{25}(46400 - x)$

$\therefore \frac{6}{25}(46400 - x) + 46400 - x = 35960$

$\Rightarrow 11136 - \frac{6x}{25} + 46400 - x = 35960$

$\Rightarrow \frac{31x}{25} = 21576$

$\therefore x = \frac{21576 \times 25}{31} = ₹\ 17400$

2. (b) Given, $P = ₹\ 24000$, $T = \frac{3}{2}$ years and $R\% = 10\%$ per annum

Semi-annual compounding,

$$CI = \left\{P\left(1 + \frac{R}{200}\right)^{2T} - P\right\}$$

$$= 24000\left\{\left(1 + \frac{10}{200}\right)^{2 \times \frac{3}{2}} - 1\right\}$$

$$= 24000\left\{\left(\frac{21}{20}\right)^3 - 1\right\} = 24000\left\{\frac{9261}{8000} - 1\right\}$$

$$= 24000\left\{\frac{1261}{8000}\right\} = 3 \times 1261 = ₹\ 3783$$

But CI = x

$\therefore ₹\ 3000 < x < ₹\ 4000$

3. (b) Let sum of money be P and required time be t years.

$$2P = P\left(1 + \frac{R}{100}\right)^4$$

$$\Rightarrow 2 = \left(1 + \frac{R}{100}\right)^4$$

$$\Rightarrow 2^{1/4} = 1 + \frac{R}{100} \quad ...(i)$$

Again, sum of money become 8 times.

$$\text{Then, } 8P = P\left(1 + \frac{R}{100}\right)^T$$

$$\Rightarrow 8 = \left(1 + \frac{R}{100}\right)^T \quad ...(ii)$$

On putting the value of $\left(1 + \frac{R}{100}\right)$ in eq. (ii),

$\Rightarrow 8 = (2^{1/4})^T$ [from Eq. (*i*)]

$\Rightarrow 8 = 2^{T/4} \Rightarrow 2^3 = 2^{T/4}$

On comparing, we get

$$3 = \frac{T}{4} \Rightarrow T = 12 \text{ years}$$

4. (a) Let principal be x, then amount = $2x$

$\therefore$ SI = $A - P = 2x - x = x$

$\because$ SI = $\frac{P \times R \times T}{100}$

$\Rightarrow x = \frac{x \times R \times 10}{100}$

$\therefore R = 10\%$

5. (a) Let a man invest ₹ 1000 at a rate $R\%$
According to question,

$$\frac{1000 \times R \times 3}{100} + \frac{1500 \times (R + 2) \times 3}{100} = 390$$

$\Rightarrow 30R + 45R + 90 = 390$

$\Rightarrow 75R = 300$

$\Rightarrow R = 4\%$

6. (a) Given, principal amount = ₹ P

Rate of interest, $r = \frac{r}{k}\%$

Time, $t = nk$

$\therefore \quad A = P\left(1+\frac{r}{100k}\right)^{nk}$

7. (a) Let first part be ₹ x, then second part be ₹ $(200000 - x)$

According to question,

$$\frac{x\times1\times15}{100}+\frac{(200000-x)\times1\times12}{100}=27600$$

$\Rightarrow \quad 15x - 12x + 2400000 = 2760000$

$\Rightarrow \quad 3x = 2760000 - 2400000$

$\Rightarrow \quad 3x = 360000$

$\therefore \quad x = 120000$

Therefore he lented ₹ 120000 at 15%.

8. (c) Let first part be ₹ x, then second part ₹ $(640 - x)$

According to question,

$$\frac{x\times3\times6}{100} = \frac{(640-x)\times6\times9}{100}$$

$\Rightarrow \quad x = 1920 - 3x$

$\Rightarrow \quad 4x = 1920$

$\Rightarrow \quad x =$ ₹ 480

$\therefore$ Second part = ₹ (640 – 480) = ₹ 160

9. (c) Given, $P =$ ₹ x, $R = 20\%$, $T = 3$ years, $A =$ ₹ y

$\because \quad A = P\left(1+\frac{R}{100}\right)^T$

$\Rightarrow \quad y = x\left(1+\frac{20}{100}\right)^3$

$\Rightarrow \quad y = x\left(\frac{6}{5}\right)^3$

$\Rightarrow \quad \frac{y}{x} = \frac{216}{125}$

$y : x = 216 : 125$

10. (a) Let principal amount = P

As amount = $3P$, $T = 25$ years

$\therefore \quad$ SI $= 3P - P = 2P$

$\because \quad \text{Rate} = \frac{100\times \text{SI}}{P\times T} = \frac{100\times 2P}{P\times 25} = 8\%$

11. (b) Given CI = ₹ 832, SI = ₹ 800 and $T = 2$ years

From formula,

$$\text{CI} = P\left\{\left(1+\frac{R}{100}\right)^T - 1\right\}$$

$$\therefore \quad 832 = P\left\{\left(1+\frac{R}{100}\right)^2 - 1\right\}$$

$$\Rightarrow \quad 832 = P\left\{1+\frac{R^2}{10000}+\frac{2R}{100}-1\right\}$$

$$\Rightarrow \quad 832 = P\left\{\frac{R^2}{10000}+\frac{2R}{100}\right\} \quad ...(i)$$

Using SI formula,

$$\text{SI} = \frac{P\times R\times T}{100}$$

$$\Rightarrow \quad 800 = \frac{P\times R\times 2}{100} \Rightarrow P = \frac{40000}{R} \quad ...(ii)$$

Now putting the value of P in Eq. (i) then,

$$832 = \frac{40000}{R}\left(\frac{R^2}{10000}+\frac{2R}{100}\right)$$

$\Rightarrow \quad 832 = 4R + 800 \Rightarrow 4R = 32$

$\therefore \quad R = \frac{32}{4} = 8\%$

12. (b) Given $R = 10\%$, $P =$ ₹ 400 and $T = \frac{3}{2}$ years

Compounding is half-yearly, then,

$T = \frac{3}{2} \times 2 = 3$ years

$P = \frac{10}{2} = 5\%$

Amount, $A = P\left(1+\frac{R}{100}\right)^T$

$A = 400\left(1+\frac{5}{100}\right)^3$

$= 400 \times \frac{21}{20}\times\frac{21}{20}\times\frac{21}{20} =$ ₹ 463.05

13. (b) **Note:** By Tricky Formula,

Equivalent rate of interest.

$R_1 + R_2 + \frac{R_1R_2}{100} = 16 + 16 + \frac{16\times16}{100}$

$= 34.56\%$

Interest $= \frac{7500\times34.56}{100} = 2592$

Total amount = 7500 + 2592 = ₹ 10092

14. (c) Let the principal amount be ₹ P.

Given, SI $= \frac{1}{2}$CI

$$\Rightarrow \quad \frac{P\times8\times3}{100} = \frac{1}{2}\left[4000\left(1+\frac{10}{100}\right)^2 - 4000\right]$$

$$\Rightarrow \quad \frac{24P}{100} = \frac{1}{2}\left[4000\times\frac{121}{100} - 4000\right]$$

$$= \frac{1}{2}[4840-4000]$$

$\Rightarrow \quad \frac{24P}{100} = 420 \Rightarrow P = \frac{420\times100}{24}$

$\Rightarrow \quad P =$ ₹ 1750

15. (d) Let the person invest ₹ x and y at two different rates 12% and 10% respectively.

$\therefore \quad \dfrac{x \times 12 \times 1}{100} + \dfrac{y \times 10 \times 1}{100} = 130$

$\left(\because \text{SI} = \dfrac{P \times R \times T}{100}\right)$

$\Rightarrow \quad 12x + 10y = 13000 \quad \ldots(i)$

After inter changing invested amount.

$\dfrac{y \times 12 \times 1}{100} + \dfrac{x \times 10 \times 1}{100} = 134$

$\Rightarrow \quad 12y + 10x = 13400 \quad \ldots(ii)$

On solving equations (i) and (ii), we get

x = ₹ 500 and y = ₹ 700

16. (b) Let initial sum of money be ₹ P, then A = ₹ $3P$

T = 15 years and 6 months = $\dfrac{31}{2}$ years

$\therefore$ SI = $A - P$ = ₹ $2P$

$\Rightarrow \quad P \times \dfrac{31}{2} \times \dfrac{R}{100} = 2P \Rightarrow R = \dfrac{2 \times 2 \times 100}{31}$

Let the sum of money doubled in T_1 years.

So, SI = $2P - P$ = ₹ P

$\therefore \quad T_1 = \dfrac{\text{SI} \times 100}{P \times R} \Rightarrow T_1 = \dfrac{P \times 100 \times 31}{P \times 400}$

$= \dfrac{31}{4}$ = 7 years and 9 months.

17. (d) Let the sum of money = ₹ P

$\therefore$ Amount = $2P$

$\Rightarrow \quad A = P\left(1 + \dfrac{R}{100}\right)^T \Rightarrow 2P = P\left(1 + \dfrac{20}{100}\right)^T$

$\Rightarrow \quad \dfrac{2P}{P} = \left(\dfrac{6}{5}\right)^T \Rightarrow 2 = \left(\dfrac{6}{5}\right)^T$

Putting $T = 4$, we get

$\left(\dfrac{6}{5}\right)^4 = \dfrac{1296}{625} = 2$ (approx.)

$\therefore$ Least number of years = 4

18. (b) Let the rate of interest per annum be R% and principal amount be ₹ P, then

Amount in 2 years,

$P\left(1 + \dfrac{R}{100}\right)^2 = 9680 \quad \ldots(i)$

Amount in 3 years,

$P\left(1 + \dfrac{R}{100}\right)^3 = 10648 \quad \ldots(ii)$

On dividing equation (ii) by equation (i),

Then, $1 + \dfrac{R}{100} = \dfrac{10648}{9680}$

$\Rightarrow \quad R = \left(\dfrac{10648}{9680} - 1\right) \times 100$

$\therefore \quad R = \dfrac{968}{9680} \times 100 = 10\%$

19. (a) Given that, $T = 8$ years, $R = 3.5\%$ and A = ₹ 364.80

Let the sum = ₹ P

$\therefore \quad A = P\left(1 + \dfrac{RT}{100}\right)$

$\Rightarrow \quad 364.80 = P\left(1 + 3.5 \times \dfrac{8}{100}\right)$

$\Rightarrow \quad 364.80 = P\left(1 + \dfrac{35 \times 8}{1000}\right)$

$\Rightarrow \quad \dfrac{3648}{10} = P\left(\dfrac{128}{100}\right)$

$\therefore \quad P = \dfrac{36480}{128}$ = ₹ 285

20. (c) Given, A_1 = ₹ 1350, A_2 = ₹ 1620

T_1 = 5 years and T_2 = 8 years

Let principal amount be ₹ x.

For 5 years:

$1350 - x = \dfrac{x \times 5 \times r}{100} \quad \ldots\ldots \text{(i)}$

For 8 years:

$1620 - x = \dfrac{x \times 8 \times r}{100} \quad \ldots\ldots \text{(ii)}$

Now, divide equation (ii) by equation (i),

$\Rightarrow \quad \dfrac{1620 - x}{1350 - x} = \dfrac{8}{5}$

Both sides subtracted (i), we get

$\Rightarrow \quad \dfrac{270}{1350 - x} = \dfrac{3}{5}$

$\Rightarrow \quad x$ = ₹ 900

21. (d) Given, P = ₹ 1600, R = 25% and n = 2 yr

$\therefore \quad A = P\left[1 + \dfrac{r}{100}\right]^n = 1600\left[1 + \dfrac{25}{100}\right]^2$

$= 1600 \times \dfrac{5}{4} \times \dfrac{5}{4} = 2500$

$\therefore$ Compound interest = 2500 – 1600 = ₹ 900.

22. (b) Using the formula, SI = $\dfrac{P \times R \times T}{100}$

$\Rightarrow \quad 2P = \dfrac{P \times R \times 5}{100}$

$\Rightarrow \quad r = 40\%$

Let required time be t_1 years and rate of interest is same.

$$5P = \frac{P \times R \times T_1}{100}$$

$$\Rightarrow T_1 = \frac{500}{R} = \frac{500}{40} = 12.5 \text{ years}$$

23. (c) Let P be the principal amount.

Then, SI = ₹ 55, time (T) = 9 months = $\frac{9}{12}$ years

Rate (R) = $3\frac{2}{3}\% = \frac{11}{3}\%$ per annum

By formula, SI = $\frac{P \times R \times T}{100}$

$$\Rightarrow P = \frac{\text{SI} \times 100}{R \times T} = \frac{55 \times 100 \times 3 \times 12}{11 \times 9}$$

$= 5 \times 100 \times 4 = 2000$

$\therefore$ Principal (P) = ₹ 2000

24. (c) Let CP of article be ₹ x

So. SP = $x \times \frac{80}{100} = 24$

x = ₹ 30

When given 30% discount

SP = $30 \times \frac{70}{100}$ = ₹21

25. (b) Rate of interest from CI

$= x + y + \frac{xy}{100}$

$= 10 + 10 + \frac{10 \times 10}{100} = 21\%$

Rate of interest for SI = 2 × 10 = 20%

Difference between rate of interest = 21–20 = 1%

$$Diff = \frac{p \times 1\%}{100}$$

$$\Rightarrow 10 = \frac{p \times 1\%}{100}$$

$p = x = 1000$

26. (c) Let principal amount P be ₹ x.

Amount, A = ₹ $3x$

Time, $T = 5$ yr

$\therefore$ SI = $3x - x = 2x$

Suppose rate of interest be R.

By formula, SI = $\frac{PRT}{100}$

$$\therefore 2x = \frac{x \times R \times 5}{100}$$

$$\Rightarrow 2x \times 100 = 5Rx \Rightarrow R = \frac{2x \times 100}{5x}$$

$\therefore R = 40\%$

Required amount = $6x$

SI = $6x - x = 5x$

$$\therefore \text{Requierd time} = \frac{100 \times \text{SI}}{P \times R} = \frac{100 \times 5x}{x \times 40} = 12.5\ yr$$

27. (a) Difference = $\frac{p \times r}{100}$ ($\because R$ rate of diff between CI and SI)

$$\Rightarrow 96 = \frac{15000 \times R}{100}$$

$$R = \frac{96}{150} = .64$$

By option (a)

Rate of Interest for 2 year (compounded)

$$= 8 + 8 + \frac{8 \times 8}{100} = 16.64$$

Rate of Interest for SI for two year = 2 × 8 = 16

Diff = 16.64 – 16 = .64

28. (c) For simple Interest

60% increase in amount so, that

$$\Rightarrow \text{SI} = \frac{PRT}{100}$$

$$\Rightarrow \frac{60p}{100} = \frac{P \times R \times 6}{100}$$

$R = 10\%$

Now, for compound interest

$$\text{A} = \text{P}\left(1 + \frac{r}{100}\right)^t$$

$$= 12000\left(1 + \frac{10}{100}\right)^3 = 12000 \times \frac{1331}{1000}$$

$= 12 \times 1331 = 15972$

Compound Interest = 15972 – 12000

= ₹ 3972

29. (a) Let Principal = P

$\therefore$ Interest = .125 × P

$$= \frac{125P}{1000} = \frac{P}{8}$$

Now, SI = $\frac{P \times R \times T}{100}$

$$\Rightarrow \frac{P}{8} = \frac{P \times 10 \times T}{100}$$

$$T = \frac{5}{4} = 1\frac{1}{4} \text{ years}$$

30. (a) Interest is calculating on every six month $r = \frac{10}{2} = 5\%$

So effective rate of interest

$$= r_1 + r_2 + \frac{r_1 r_2}{100}$$

$$= 5 + 5 + \frac{5 \times 5}{100} = 10.25\%$$

31. (b) Rate $= \frac{10}{2} = 5\%$

Time $= 1 \times 2 = 2$ years

$$A = P\left(1+\frac{r}{100}\right)^n$$

$$A = 10000\left(1+\frac{5}{100}\right)^2$$

$$= 10000 \times \frac{21}{20} \times \frac{21}{20}$$

$= 441 \times 25 = 11025$

Interest $= 11025 - 10000$

$=$ ₹ 1025

So, option (b) is correct

32. (c) Let Ist P = ₹ x

Ist Amount = ₹ 2x

Ist S.I. = A – P = 2x – x = ₹ x

T = 5 years

$$R = \frac{S.I.\times 100}{P\times T} = \frac{x\times 100}{x\times 5} = 20\%$$

Let 2nd P = ₹ x

2nd Amount = ₹ 3x

2nd S.I. = A – P = 3x – x = ₹ 2x

T = 12 years

$$R = \frac{S.I\times 100}{P\times T} = \frac{2x\times 100}{x\times 12} = \frac{50}{3}$$

Difference between two rates

$$= 20 - \frac{50}{3} = \frac{10}{3} = 3\frac{1}{3}\%$$

∴ Option (c) is correct.

33. (c) P = ₹ 8400 R = 10% n = 2

Let installment = x

$$P = \frac{x}{\left(1+\frac{R}{100}\right)^1} + \frac{x}{\left(1+\frac{R}{100}\right)^2}$$

$$8400 = \frac{x}{\left(1+\frac{10}{100}\right)} + \frac{x}{\left(1+\frac{10}{100}\right)^2}$$

$$8400 = \frac{x}{\left(\frac{110}{100}\right)} + \frac{x}{\left(\frac{110}{100}\right)^2}$$

$$8400 = x\left[\frac{10}{11} + \left(\frac{10}{11}\right)^2\right]$$

$$8400 = x\left[\frac{110+100}{121}\right]$$

$$x = \frac{8400\times 121}{210} = 40 \times 121 = ₹\ 4840$$

34. (c) Using formula when time (t) is 2 year

$$D = P\left[\frac{R}{100}\right]^2$$

$$10 = P\left(\frac{4}{100}\right)^2$$

$$\frac{10\times 100\times 100}{16} = P$$

$P = 6250$

35. (a) The difference between the compound interest (compounded annually) and simple interest on a sum of money deposited for 2 years at R% p.a. is $P\left(\frac{R}{100}\right)^2$.

$$P\left(\frac{R}{100}\right)^2 = 15 \Rightarrow P\left(\frac{5}{100}\right)^2 = 15 \Rightarrow P\left(\frac{1}{20}\right)^2$$

$$= 15 \Rightarrow \frac{P}{400} = 15 \Rightarrow P = 6000$$

36. (b) Total interest collected by person

$$= \frac{5000\times 2\times 5.5}{100} = 550$$

Total interest given by person

$$= \frac{5000\times 2\times 5}{100} = 500$$

Profit $= 550 - 500 = 50$

37. (a) Let original amount be x

$$x\left(1+\frac{25}{100}\right)^3 = 10000$$

$$x\left(\frac{5}{4}\right)^3 = 10000$$

$$x \times \frac{125}{64} = 10000 \qquad \therefore \quad x = \frac{10000\times 64}{125} = 5120$$

38. (c) Let original amount be x

Annual income at 4% rate of interest

$$\frac{x\times 4\times 1}{100} = \frac{x}{25}$$

Annual income at 3.75% rate of interest

$$= \frac{x\times 3.75\times 1}{100} = \frac{3x}{80}$$

According to the question

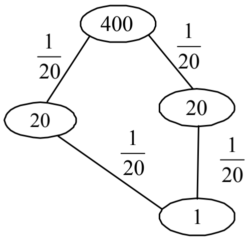
$$\frac{x}{25} - \frac{3x}{80} = 64$$

$$\frac{16x - 15x}{400} = 64$$

$\therefore \quad x = 64 \times 400 = 25600$

39. (c) Rate% = 5% = $\frac{5}{100} = \frac{1}{20}$

Let principal = $(20)^2 = 400$



Second year's compound interest = 21 unit

According to question, second year's compound interest = ₹210

1 unit = ₹10 $\Rightarrow$ Principal = 400 unit = ₹4000

40. (c) Let total amount borrowed = ₹ x

Then, ATQ,

$$\frac{x \times 4 \times 5}{100} + \frac{x \times 4 \times 4}{100} = 405$$

$$\frac{36x}{100} = 405$$

$\therefore$ Total amount = ₹ x = $\frac{405 \times 100}{36}$ = ₹1125

41. (b) A = P $(1 + R/100)^t$

3P < P $(1 + 40/100)^t$

$3 < (1.4)^t$

When $t = 3$; $1.4^3 = 2.744$

And when $t = 4$; $1.4^4 = 3.8416$

T = 4

42. (b) Let sum invested 5% be P1, 6% be P2 then 9%

= 17200 (P1 + P2)

So according to question

P1 × 5 × 2/100 = P2 × 6 ×2/100 or P1 = (6/5) P2

Also P2× 6 × 2/100 = [17200 – (P1 + P2)] × 9 × 2/100

or 2 P2 = [17200 – (11/5)P2] × 3

or (2 + 33/5) P2 = 17200 × 3

P2 = 17200 × 3 × 5 / 43 = 6000

So P1 = 6/5 P2 = 7200

So Sum invested at 9% = 17200 – (6000 + 7200)

= ₹4000

43. (c) If ₹1 is the principal then difference in amount

$= (1.1)^4 - (1.2)^2 = 1.4641 - 1.44 = 0.0241$

$\therefore$ p = $\frac{482}{0.0241} = 2 \times 10^4$ = ₹ 20,000

44. (c) Interest from B $\rightarrow \frac{25000 \times 4 \times 7}{100} = 7000$

Total interest $\rightarrow$ 11200

Interest received from c $\rightarrow$ 11200 – 7000 = 4200

Then money lent to c = $\frac{4200 \times 100}{7 \times 4} = 15000$

45. (d) 1% = $\frac{24}{2}$ = ₹ 12

$\therefore$ Principle = ₹1200

CHAPTER

9 Profit and Loss

1. A trader marked a watch 40% above the cost price and then gave a discount of 10%. He made a net profit of ₹ 468 after paying a tax of 10% on the gross profit. What is the cost price of the watch? **[2007-I]**
(a) ₹ 1200 (b) ₹ 1800 (c) ₹ 2000 (d) ₹ 2340

2. By giving 25% discount a trader earns 25% profit. If he sells the item at 10% discount, what is his profit? **[2007-I]**
(a) 10% (b) 40% (c) 45% (d) 50%

3. A man buys 4 tables and 5 chairs for ₹ 1000. If he sells the tables at 10% profit and chairs 20% profit, he earns a profit of ₹ 120. What is the cost of one table? **[2007-II]**
(a) ₹ 200 (b) ₹ 220 (c) ₹ 240 (d) ₹ 260

4. In respect of a bill of ₹ 10000, what is the difference between a discount of 40% and two successive discounts of 36% and 4%? **[2007-II]**
(a) ₹ 0 (b) ₹ 144 (c) ₹ 256 (d) ₹ 400

5. A refrigerator and a camera were sold for ₹ 12000 each. The refrigerator was sold at a loss of 20% of the cost and the camera at a gain of 20% of the cost. The entire transaction results in which one of the following? **[2008-I]**
(a) No loss or gain (b) Loss of ₹ 1000
(c) Gain of ₹ 1000 (d) Loss of ₹ 2000

6. Successive discounts of $12\frac{1}{2}\%$ and $7\frac{1}{2}\%$ are given on the marked price of a cupboard. If the customer pays ₹ 2590, then what is the marked price? **[2008-I]**
(a) ₹ 3108 (b) ₹ 3148 (c) ₹ 3200 (d) ₹ 3600

7. The marked price of a machine is ₹ 18000. By selling it at a discount of 20%, the loss is 4%. What is the cost price of the machine? **[2008-II]**
(a) ₹ 10000 (b) ₹ 12000 (c) ₹ 14000 (d) ₹ 15000

8. A dishonest dealer professes to sell his good at cost price but uses a false weight and thus gains 20%. For a kilogram he uses a weight of **[2009-I]**
(a) 700 g (b) 750 g (c) 800 g (d) 850 g

9. What the seller marked the printed price of a watch purchased at ₹ 380, so that giving 5% discount, there is 25% profit? **[2009-I]**
(a) ₹ 400 (b) ₹ 450 (c) ₹ 500 (d) ₹ 600

10. A person *A* sells a table costing ₹ 2000 to a person *B* and earns a profit of 6%. The person *B* sells it to another person *C* at a loss of 5%. At what price did *B* sell the table? **[2009-I]**
(a) ₹ 2054 (b) ₹ 2050 (c) ₹ 2024 (d) ₹ 2014

11. A trader sells two cycles at ₹ 1188 each and gains 10% on the first and loses 10% on the second. What is the profit or loss per cent on the whole? **[2009-II]**
(a) 1% loss (b) 1% gain
(c) No loss no gain (d) 2% loss

12. A milk vendor bought 28 *l* of milk at the cost of ₹ 8.50 per *l*. After adding some water, he sold the mixture at the same price. If he gains 12.5%, how much water did he add? **[2009-II]**
(a) 5.5 *l* (b) 4.5 *l* (c) 3.5 *l* (d) 2.5 *l*

13. One saree was purchased for ₹ 564 after getting a discount of 6% and another saree was purchased for ₹ 396 after getting a discount of 1%. Taking both the items as a single transaction, what is the percentage of discount? **[2010-I]**
(a) 3.5 (b) 4 (c) 7 (d) 7.5

14. A man bought a number of oranges at 3 for a rupee and an equal number at 2 for a rupee. At what price per dozen should he sell them to make a profit of 20%? **[2010-I]**
(a) ₹ 4 (b) ₹ 5 (c) ₹ 6 (d) ₹ 7

15. By selling 8 dozen pencils, a shopkeeper gains the selling price of 1 dozen pencils. What is the gain? **[2010-II]**
(a) $12\frac{1}{2}\%$ (b) $13\frac{1}{7}\%$ (c) $14\frac{2}{7}\%$ (d) $87\frac{1}{2}\%$

16. On selling an article for ₹ 240, a trader loses 4%. In order to gain 10%, he must sell the article for **[2010-II]**
(a) ₹ 275 (b) ₹ 280 (c) ₹ 285 (d) ₹ 300

17. A trader marks 10% higher than the cost price. He gives a discount of 10% on the marked price. In this kind of sales how much per cent does the trader gain or loss? **[2011-I]**
(a) 5% gain (b) 2% gain
(c) 1% loss (d) 3% loss

18. A fruit-seller buys lemons at 2 for a rupee and sells them at 5 for three rupees. What is his gain per cent? **[2011-II]**
(a) 10% (b) 15% (c) 20% (d) 25%

19. A man sold two watches, each for ₹ 495. If he gained 10% on one watch and suffered a loss of 10% on the other, then what is the loss or gain percentage in the transaction? **[2011-II]**
(a) 1% gain (b) 1% loss
(c) $\frac{100}{99}\%$ loss (d) No gain no loss

20. A person sold an article for ₹ 136 and got 15% loss. Had he sold it for ₹ x, he would have got a profit of 15%. Which one of the following is correct? **[2012-I]**
(a) $190 < x < 200$ (b) $180 < x < 190$
(c) $170 < x < 180$ (d) $160 < x < 170$

21. The cost of two articles are in the ratio 3 : 5. If there is 30% loss on the first article and 20% gain on the second article, what is overall percentage of loss or gain? **[2012-I]**
(a) 2.25% gain (b) 5.25% loss
(c) 2% loss (d) None of these

22. A person bought 8 quintal of rice for certain rupees. After a week, he sold 3 quintal of rice at 10% profit, 3 quintal of rice with neither profit nor loss and 2 quintal at 5% loss. In this transaction, what is the profit? **[2012-I]**
(a) 10% (b) 20%
(c) 25% (d) None of these

23. A man buys a television set which lists for ₹ 5000 at 10% discount. He gets an additional 2% discount (after the first discount) for paying cash. What does he actually pay for the set? **[2012-II]**
(a) ₹ 4410 (b) ₹ 4400 (c) ₹ 4000 (d) ₹ 4500

24. A merchant earns a profit of 20% by selling a basket containing 80 apples which cost is ₹ 240 but he gives one-fourth of it to his friend at cost price and sells the remaining apples. In order to earn the same profit, at what price must he sell each apple? **[2012-II]**
(a) ₹ 3.00 (b) ₹ 3.60 (c) ₹ 3.80 (d) ₹ 4.80

25. A cloth store is offering buy 3, get 1 free. What is the net percentage discount being offered by the store? **[2012-II]**
(a) 20% (b) 25% (c) 30% (d) $33\frac{1}{3}\%$

26. Two lots of onions with equal quantity, one costing ₹ 10 per kg and the other costing ₹15 per kg, are mixed together and whole lot is sold at ₹ 15 per kg. What is the profit or loss? **[2013-II]**
(a) 10% loss (b) 10% profit
(c) 20% profit (d) 20% loss

27. A person selling an article for ₹ 96 finds that his loss per cent is one — fourth of the amount of rupees that he paid for the article. What can be the cost price? **[2014-II]**
(a) Only ₹160 (b) Only ₹ 240
(c) Either ₹ 160 or ₹ 240 (d) Neither ₹ 160 nor ₹ 240

28. A shopkeeper sells his articles at their cost price but uses a faulty balance which reads 1000*g* for 800*g*. What is his actual profit percentage ? **[2014-II]**
(a) 25% (b) 20% (c) 40% (d) 30%

29. *A* man buys 200 oranges for ₹ 1000. How many oranges for ₹ 100 can be sold, so that his profit percentage is 25%? **[2014-II]**
(a) 10 (b) 14 (c) 16 (d) 20

30. The value of a single discount on some amount which is equivalent to a series of discounts of 10%, 20% and 40% on the same amount, is equal to **[2015-II]**
(a) 43.2% (b) 50% (c) 56.8% (d) 70%

31. A cloth merchant buys cloth from a weaver and cheats him by using a scale which is 10 cm longer than a normal metre scale. He claims to sell cloth at the cost price to his customers, but while selling uses a scale which is 10 cm shorter than a normal metre scale. What is his gains? **[2016-I]**
(a) 20% (b) 21% (c) $22\frac{2}{9}\%$ (d) $23\frac{1}{3}\%$

32. The cost of 2.5 kg rice is ₹125. The cost of 9 kg rice is equal to that of 4 kg pulses. The cost of 14 kg pulses is equal to that of 1.5 kg tea. The cost of 2 kg tea is equal to that of 5 kg nuts. What is the cost of 11 kg nuts ? **[2016-II]**
(a) ₹2310 (b) ₹3190 (c) ₹4070 (d) ₹4620

33. A shopkeeper increases the cost price of an item by 20% and offers a discount of 10% on this marked price. What is his percentage gain ? **[2016-II]**
(a) 15% (b) 12% (c) 10% (d) 8%

34. Rajendra bought a mobile with 25% discount on the selling price. If the mobile cost him ₹ 4,875, what is the original selling price of the mobile? **[2017-I]**
(a) ₹ 6,300 (b) ₹ 6,400 (c) ₹ 6,500 (d) ₹ 6,600

35. Sudhir purchased a chair with three consecutive discounts of 20%, 12.5% and 5%. The actual deduction will be **[2017-II]**
(a) 33.5% (b) 30% (c) 32% (d) 35%

36. Ram buys 4 chairs and 9 stools for ₹ 1,340. If he sells chairs at 10% profit and stools at 20% profit, he earns a total profit of ₹ 188. How much money did he have to pay for the chairs? **[2017-II]**
(a) ₹ 200 (b) ₹ 400 (c) ₹ 800 (d) ₹ 1,600

37. An article is sold at a profit at 32%. If the cost price is increased by 20% and the sale price remains the same, then the profit percentage becomes **[2018-I]**
(a) 10% (b) 12% (c) 15% (d) 20%

38. A milk vendor bought 28 litres of milk at the rate of ‘ 8.50 per litre. After adding some water he sold the mixture at the same price. If his gain is 12.5%, how much water did he add? **[2018-II]**
(a) 4.5 litres (b) 4 litres
(c) 3.5 litres (d) 3 litres

39. If an article is sold at a gain of 6% instead of a loss of 6%, the seller gets ₹ 6 more. What is the cost price of the article? **[2019-I]**
(a) ₹ 18 (b) ₹ 36 (c) ₹ 42 (d) ₹ 50

40. A person sells two items each at ₹ 990, one at a profit of 10% and another at a loss of 10%. What is the combined percentage of profit or loss for the two items ? **[2019-II]**
(a) 1% loss (b) 1% profit
(c) No profit no loss (d) 0.5% profit

41. A trader sells two computers at the same price, making a profit of 30% on one and a loss of 30% on the other. What is the net loss or profit percentage on the transaction ? **[2019-II]**
(a) 6% loss (b) 6% gain (c) 9% loss (d) 9% gain

HINTS & SOLUTIONS

1. (c) Let the cost price of the watch = ₹ x
After 40% marked price and 10% discount

$= x \times \frac{90}{100} \times \frac{140}{100} = \frac{126x}{100}$

Profit $= \frac{126x}{100} - x = \frac{26x}{100}$

According to question,
10% pay tax on profit

$= \frac{26x}{100} \times \frac{90}{100} = 468$

$x = \frac{468 \times 100 \times 100}{26 \times 90}$ = ₹ 2000

2. (d) Let cost price = ₹ x,
Marked price = ₹ y
and profit % at 10% discount = r %

$\therefore \frac{75y}{100} = \frac{125x}{100} \Rightarrow 3y = 5x$ ...(i)

And $\frac{9y}{10} = \frac{(100+r)x}{100}$

$\Rightarrow \frac{15x}{10} = \frac{(100+r)x}{100}$ $[\because 3y = 5x]$

$\Rightarrow 15 = \frac{100+r}{10}$

$\Rightarrow r = 50\%$

Note that discount is always given on marked price and profit is always occured on cost price.

3. (a) Let cost of 1 table be ₹ x and cost of 1 chair be ₹ y.
4x + 5y = 1000 ... (i)

	Table	Chair
CP	4x	5y
SP	$4x\left(1+\frac{1}{10}\right) = \frac{44x}{10}$	$5y\left(1+\frac{1}{5}\right) = 6y$

$\because$ SP – CP = Profit

$\therefore \left(\frac{44x}{10} - 4x\right) + 6y - 5y = 120$

$\Rightarrow \frac{4x}{10} + y = 120$ ... (ii)

From equations (i) and (ii),
x = ₹ 200

4. (b) Two successive discounts

$= 36 + 4 - \frac{36 \times 4}{100} = 38.56\%$

$\therefore$ Difference between discounts
= 40% – 38.56% = 1.44%
$\therefore$ Required difference = 10000 × 1.44%

$= \frac{10000 \times 1.44}{100}$ = ₹ 144

5. (b) $x + y + \frac{xy}{100}$

$= +20 - 20 - \frac{20 \times 20}{100} = -4\%$

Total selling price of a refrigerator and a camera
= 12000 + 12000 = ₹ 24000
Now, loss is 4%

$CP \times \frac{96}{100} = 24000$

CP = ₹ 25000
Loss amount = (25000 – 24000) = ₹ 1000

6. (c) Let the marked price of a cupboard = ₹ x
According to question,

$\therefore x \times \frac{(100-12.5)}{100} \times \left(\frac{100-7.5}{100}\right)$ = ₹ 2590

$\Rightarrow x = \frac{2590 \times 100 \times 100}{87.5 \times 92.5}$

x = ₹ 3200

7. (d) Given marked price of machine = ₹ 18000

$\therefore$ Discount $= \frac{20}{100} \times 18000$ = ₹ 3600

$\therefore$ SP = 18000 – 3600 = ₹ 14400
If loss of 4%, then

$CP = \frac{100 \times SP}{100 - r} = \frac{100 \times 14400}{100 - 4}$

$= \frac{100 \times 14400}{96}$ = ₹ 15000

8. (c) Let SP = CP = ₹ x

Gain percentage $= x \times \frac{20}{100} = \frac{x}{5}$

Gain weight $= \frac{x}{5} \times 1000 = 200$ g

Uses weight = 1000 – 200 = 800 g

9. (a) Let marked price be ₹ x.

$\therefore$ Discount = 5% of $x = \frac{95}{100} \times x = \frac{19x}{20}$

According to question,

$\Rightarrow 380 = \frac{19x}{20}$

$\therefore$ x = ₹ 400.

10. (d) The cost price of table for person *B*

$= 2000 + 6 \times \frac{2000}{100}$

$= 2000 + 120 = ₹\ 2120$

Selling price $= 2120 - \frac{2120 \times 5}{100}$

$= 2120 - 106 = ₹\ 2014$

11. (a) Profit/loss $= \left(x + y + \frac{xy}{100}\right)\%$

Given, $x = +10\%$, $y = -10\%$

$\therefore$ Profit/loss $= \left(10 - 10 - \frac{10 \times 10}{100}\right)\% = -1\%$

(– sign represent, there is a loss of 1%.)

12. (c) Total C.P. of milk $= 28 \times 8.50 = ₹\ 238$

Profit = 12.5% of 238

$= \frac{12.5}{100} \times 238 = 29.75$

Let he added x *l* of water.

$\therefore$ Profit $= x \times 8.5$

$\Rightarrow 29.75 = x \times 8.5$

$\therefore x = 3.5\ l$

13. (b) Let marked price of two sarees be ₹ x and ₹ y respectively

$\therefore x - \frac{6x}{100} = 564 \Rightarrow \frac{94x}{100} = 564$

$\Rightarrow x = ₹\ 600$ and $y - \frac{y}{100} = 396$

$\Rightarrow \frac{99y}{100} = 396 \Rightarrow y = ₹\ 400$

$\therefore$ Total M.P. amount $= 600 + 400 = ₹\ 1000$

Total amount after discount $= 564 + 396 = ₹\ 960$

$\therefore$ Discount per cent $= \frac{1000 - 960}{1000} \times 100 = \frac{40}{10}\% = 4\%$

14. (c) Cost of 1 orange of 1st variety $= ₹\ \frac{1}{3}$

Cost of 1 orange of 2nd variety $= ₹\ \frac{1}{2}$

Cost of 1 orange after mixing $= \frac{5}{12}$

Profit of 20% $= \frac{5}{12} \times \frac{120}{100} = \frac{1}{2}$

Selling price of 1 orange $= ₹\ \frac{1}{2}$

$\therefore$ Then, SP of 12 oranges $= \frac{1}{2} \times 12 = ₹\ 6$

15. (c) Let the cost price = ₹ x

Profit = ₹ x

Cost price of 8 dozen pencil = ₹ 7x

Gain per cent $= \frac{x}{7x} \times 100 = \frac{100}{7} = 14\frac{2}{7}\%$

16. (a) $\because$ Selling price of an article = ₹ 240

$\therefore$ Cost price of an article for the loss of 4%

$= ₹\ \frac{240 \times 100}{96}$

$\therefore$ Selling price of an article for a profit of 10%

$= \frac{240 \times 100}{96} \times \frac{110}{100} = \frac{240 \times 110}{96} = ₹\ 275$

17. (c) Let cost price = ₹ x

Marked price $= \frac{x \times 110}{100} = ₹\ \frac{11x}{10}$

$\therefore$ SP $= \frac{11x}{10} \times \frac{90}{100} = \frac{99x}{100}$

$\therefore$ Loss per cent $= \frac{\frac{99x}{100} - x}{x} \times 100 = -1\%$

(– sign show loss.)

18. (c) ₹ 1 = 2 lemons × CP

₹ 1 $= \frac{5}{3}$ lemons × SP

$\Rightarrow 2 \times CP = \frac{5}{3} \times SP$

$\Rightarrow 6\ CP = 5\ SP$

$\Rightarrow \frac{SP}{CP} = \frac{6}{5}$ (Here SP > CP, then profit)

Both sides substract 1

$\frac{SP - CP}{CP} = \frac{1}{5}$

Profit per cent $= \frac{1}{5} \times 100 = 20\%$

19. (b) Loss/gain per cent $= \left(10 - 10 - \frac{10 \times 10}{100}\right)\% = -1\%$

(– sign indicate that there is a loss of 1%)

20. (b) Cost Price $= \frac{\text{Selling price}}{1 - \frac{\text{Loss\%}}{100}} = \frac{136}{1 - \frac{15}{100}}$

$= \frac{136 \times 100}{85} = ₹\ 160$

Selling price (x) $= \frac{160 \times (100 + 15)}{100} = \frac{160 \times 115}{100}$

$= ₹\ 184$

$\therefore$ Hence, option (b) is correct because $180 < x < 190$.

21. (d) Let the CP of two articles be 3x and 5x, respectively.

$\therefore$ SP of first article $= \frac{3x \times 70}{100} = \frac{21x}{10}$

SP of second article $= \frac{5x \times 120}{100} = 6x$

$\therefore$ Total SP $= 6x + \frac{21x}{10} = \frac{60x + 21x}{10} = \frac{81x}{10}$

$\therefore$ Total CP $= 3x + 5x = 8x$

$\therefore$ Profit $= \frac{81x}{10} - 8x = \frac{81x - 80x}{10} = \frac{x}{10}$

$\therefore$ Overall percentage of gain $= \frac{\frac{x}{10} \times 100}{8x}$

$= \frac{x \times 100}{10 \times 8x} = 1.25\%$

22. (d) Let CP of 8 quintal rice = ₹ x

$\therefore$ CP of 1 quintal rice = ₹ $\frac{x}{8}$

$\therefore$ SP of rice 3 quintal of rice at 10% profit

$= \frac{3x}{8} + \frac{3x}{8} \times \frac{1}{10} = \frac{3x}{8} + \frac{3x}{80} = \frac{33x}{80}$

SP of 3 quital rice without profit or loss = ₹ $\frac{3x}{8}$

SP of 2 quintal rice at 5% loss $= \frac{2x}{8} - \frac{2x}{8} \times \frac{5}{100}$

$= \frac{x}{4} - \frac{x}{4 \times 20} = \frac{19x}{4 \times 20} = \frac{19x}{80}$

$\therefore$ Total SP $= \frac{33x}{80} + \frac{3x}{8} + \frac{19x}{80}$

$= \frac{33x + 30x + 19x}{80} = \frac{82x}{80}$

$\therefore$ Profit $= \frac{SP - CP}{CP} \times 100 = \frac{\frac{82x}{80} - x}{x} \times 100$

$= \frac{(82 - 80)x}{80x} \times 100 = \frac{2}{80} \times 100 = 2.5\%$

23. (a) Actual payment for the television set
= 98% of 90% of 5000 = ₹ 4410

24. (c) Cost price of 1 apple $= \frac{240}{80}$ = ₹ 3

No. of apples give to his friend $= \frac{1}{4} \times 80 = 20$

Remaining apples = 80 – 20 = 60
Cost of apples to his friend = 20 × 3 = ₹ 60

Total SP at a profit of 20% = 240 × $\frac{120}{100}$ = ₹ 288

SP of remaining 60 apples = ₹ (288 – 60) = ₹ 228

SP of 1 apple = $\frac{228}{60}$ = ₹ 3.80

25. (b) We know that,

Net percentage discount $= \frac{\text{Discount}}{\text{Cost price}} \times 100$

$= \frac{1}{4} \times 100 = 25\%$

26. (c) Let each lot of onion contains x kg onion, then total cost price of these two lots together

= 10x + 15x = 25x

Selling price of whole lot = 15 × (x + x)
= 15 × 2x = 30x

Profit percentage $= \frac{30x - 25x}{25x} \times 100$

$= \frac{5x}{25x} \times 100 = 20\%$

27. (c) Let the cost price of an article = ₹ x
Selling price of an article = ₹ 96
According to the question,

$$\frac{x - 96}{x} \times 100 = \frac{1}{4} \times x \Rightarrow 400x - 96 \times 400 = x^2$$

$\Rightarrow x^2 - 400x + 38400 = 0$

$\Rightarrow x^2 - 160x - 240x + 38400 = 0$

$\Rightarrow x(x - 160x) - 240(x - 160) = 0$

$\Rightarrow (x - 160)(x - 240) = 0$

$\therefore x = 160$ or 240

Hence, the cost price of an article is ₹ 160 or ₹ 240.

28. (a) Actual profit percentage

$= \frac{\text{Fair weight} - \text{Unfair weight}}{\text{Unfair weight}} \times 100$

$= \frac{1000 - 800}{800} \times 100 = \frac{200}{800} \times 100 = 25\%$

29. (c) Cost price of 200 oranges = ₹1000.

Cost price of 1 orange $= \frac{1000}{200}$ = ₹ 5.

Selling price of 1 orange $= 5 \times \frac{125}{100}$ = ₹ 6.25

So, in ₹ 6.25, number of oranges can be sold = 1
In ₹ 100, number of oranges can be sold

$= \frac{1}{6.25} \times 100 = 16$

Hence, 16 oranges can be sold in ₹ 100 for profit 25%

30. (c) Discount equivalent to 10% and 20%

$= 10 + 20 - \frac{10 \times 20}{100} = 28\%$

Discount equivalent to 28% and 40%

$= 28 + 40 - \frac{28 \times 40}{100}$

$= 68 - 11.2 = 56.8\%$

So, option (c) is correct.

31. (c) Let the actual C.P. of cloth = ₹1

Then the effective C.P. $= \frac{100}{110} = \frac{10}{11}$

(Since he purchases 110 articles by paying ₹ 100)

Again S.P. $= \frac{100}{90} = \frac{10}{9}$

(Since he sells only 90 articles charging the CP of 100 articles).

$\therefore$ Gain% $= \dfrac{S.P - C.P}{C.P} \times 100 = \dfrac{\frac{10}{9} - \frac{10}{11}}{\frac{10}{11}} \times 100$

$= \dfrac{20}{99} \times \dfrac{11}{10} \times 100 = \dfrac{200}{9} = 22\dfrac{2}{9}\%$

$\therefore$ Option (c) is correct.

32. (d) cost of 2.5 kg rice = ₹ 125

cost 9 kg rice = ₹ $\dfrac{125}{2.5} \times 9$ = cost of 4 kg pulse

cost of 14 kg pulses = $\dfrac{125 \times 9}{2.5 \times 4} \times 14$ = cost of 1.5 kg Tea

cost of 2 kg tea = $\dfrac{125 \times 9 \times 14 \times 2}{2.5 \times 4 \times 1.5}$ = cost of 5 kg nuts

cost of 11 kg nuts = ₹ $\dfrac{125 \times 9 \times 14 \times 2 \times 11}{2.5 \times 4 \times 1.5 \times 5}$

$\Rightarrow \dfrac{125 \times 9 \times 14 \times 2 \times 11 \times 100}{25 \times 4 \times 15 \times 5} \Rightarrow$ ₹4620.

33. (d) Let CP = 100
MP = 120
SP after giving discount = 108
Profit = 8%

34. (c) Let SP of Mobile = x
discount = 25%

$\dfrac{75x}{100} = 4875$

$x = \dfrac{4875 \times 100}{75}$ = ₹ 6500

35. (a) Considerthe two discounts (taking two at a time) 20% and 12.5%
Single equivalent discount

$= \left(x + y - \dfrac{xy}{100}\right)\% = \left(20 + 12.5 - \dfrac{20 \times 12.5}{100}\right)\%$

$= (32.5 - 2.5)\% = 30\%$

Considering 30% and 5% as the two discounts
Final reduction

$= \left(x + y - \dfrac{xy}{100}\right)\% = \left(30 + 5 - \dfrac{30 \times 5}{100}\right)\%$

$= (35 - 1.5)\% = 33.5\%$

36. (c) Let the number of chairs be x and the number of stools be y.
Therefore, according to the question,

$4x + 9y = 1340$ ...(1)

10% of $4x$ + 20% of $9y = 188$

$\dfrac{40x}{100} + \dfrac{180y}{100} = 188 \Rightarrow \dfrac{4x}{10} + \dfrac{18y}{10} = 188$

$\Rightarrow 4x + 18y = 1880$ ...(2)

Solving the equations (1) and (2) by elimination method, we get

$-9y = -540 \Rightarrow y = 60$

Therefore, equation (1) gives

$4x = 1340 - 9y \Rightarrow 4x = 1340 - 9 \times 60$

$\Rightarrow 4x = 1340 - 540 \Rightarrow 4x = 800$

Thus, the money to be paid for the chairs is ₹800.

37. (a) Let original cost price of artical be 100
Original selling price of artical

$= 100 + 100 \times \dfrac{32}{100} = 132$

Now
New cost price after increase

$= 100 + 100 \times \dfrac{20}{100} = 120$

New selling price = 132
Profit = 132 – 120 = 12

Required % $= \dfrac{12}{120} \times 100 = 10\%$

38. (c) Cost price of milk = 28 × 8.5 = ₹238
Let x litre of water is added
So, selling price = (28 + x) × 8.5 = 238 + 8.5x
Profit = S. P – C. P = (238 + 8.5x) – (238) = 8.5x
According to question: Profit% = 12.5%
i.e. 8.5x/238 × 100% = 12.5
⇒ x = 12.5 × 238/8.5×100
⇒ x = 3.5 litres

39. (d) Let C.P be ₹$100x$

$\therefore 106x - 94x = 6$

$\Rightarrow 12x = 6$

$x = \dfrac{1}{2}$

$\therefore 100x = 100 \times \dfrac{1}{2}$ = ₹50

40. (a) Combined % loss = $\dfrac{10 \times 10}{100}\% = 1\%$(loss)

41. (c) Let selling price of two computers is ₹100 each.
then cost price of first computer

$= 100 \times \dfrac{100}{(100 + 30)} = 76.92$

and cost price of second computer

$= 100 \times \dfrac{100}{(100 - 30)} = 142.86$

total cost price = 142.86 + 76.92 = 219.78
Loss = 219.78 – 200 = 19.78

Loss % $= \dfrac{19.78}{219.78} \times 100 \approx 9$ % Loss

CHAPTER 10

Time, Speed and Distance

1. A scooterist completes a certain journey in 10 h. He covers half the distance at 30 km/h and the rest at 70 km/h. What is total distance of the journey ? **[2007-I]**
(a) 210 km (b) 400 km (c) 420 km (d) 500 km
2. A train 200 m long fully passes a platform of 200 m length in 15 s. What is the speed of the train ? **[2007-II]**
(a) 36 km/h (b) 48 km/h (c) 72 km/h (d) 96 km/h
3. A person goes from a place A to another place B at the speed of 4 km/h and returns at a speed of 3 km/h. If he takes 7 h in all, then what is the distance between the two places ? **[2007-II]**
(a) 12 km (b) 8 km (c) 6 km (d) 5 km
4. A bike travels a distance of 200 km at a constant speed. If the speed of the bike is increased by 5 km/h, the journey would have taken 2 h less. What is the speed of the bike ? **[2008-I]**
(a) 30 km/h (b) 25 km/h
(c) 20 km/h (d) 15 km/h
5. A train crosses a telegraph post in 8s and a bridge 200 m long in 24 s. What is the length of the train? **[2008-II]**
(a) 100 m (b) 120 m (c) 140 m (d) 160 m
6. A train 110 m long is running with a speed of 60 km/h. What is the time in which it will pass a man who starts from the engine running at the speed of 6 km/h in the direction opposite to that of the train ? **[2008-II]**
(a) 5 s (b) 6 s (c) 10 s (d) 15 s
7. A train of length 150 m takes 10 s to cross another train 100 m long coming from the opposite direction. If the speed of first train is 30 km/h. What is the speed of second train ? **[2008-II]**
(a) 72 km/h (b) 60 km/h
(c) 54 km/h (d) 48 km/h
8. A boy walks from his house to school at 2.5 km/h and arrives 12 min late. The next day he walks at 4 km/h and reaches the school 15 min earlier. What is the distance from his house to school ? **[2008-II]**
(a) 2 km (b) 2.5 km (c) 3 km (d) 3.5 km
9. Two persons P and Q start at the same time from city A for city B, 60 km away. P travels 4 km/h slower than Q. Q reaches city B and at once turns back meeting P, 12 km from city B. What is the speed of P ? **[2008-II]**
(a) 8 km/h (b) 12 km/h
(c) 16 km/h (d) 20 km/h
10. Two men P and Q start from a place walking at 5 km/h and 6.5 km/h, respectively. What is the time they will take to be 92 km apart, if they walk in opposite directions? **[2009-II]**
(a) 2 h (b) 4 h (c) 6 h (d) 8 h
11. Two trains travel in the same direction at 50 km/h and 32 km/h, respectively. A man in the slower train observes that 15 s elapse before the faster train completely passes him. What is the length of the faster train ? **[2009-II]**
(a) 75 m (b) 125 m
(c) 150 m (d) $\frac{625}{3}$ m
12. The speed of a boat in still water is 11 km/h. It can go 12 km upstream and return downstream to the initial point in 2 h 45 min. What is the speed of stream? **[2010-I]**
(a) 5 km/h (b) 4 km/h (c) 3 km/h (d) 2 km/h
13. A father and his son start a point A with speeds of 12 km/h and 18 km/h respectively and reach another point B. If his son starts 60 min after his father at A and reaches B, 60 min before his father, what is the distance between A and B ? **[2010-II]**
(a) 90 km (b) 72 km
(c) 36 km (d) None of these
14. Two trains of lengths 100 m and 150 m are travelling in opposite directions at speeds of 75 km/h and 50 km/h, respectively. What is the time taken by them to cross each other ? **[2010-II]**
(a) 7.4 s (b) 7.2 s (c) 7 s (d) 6.8 s
15. A motorboat takes 2 h to travel a distance of 9 km down the current and it takes 6 h to travel the same distance against the current. What is the speed of the boat in still water ? **[2010-II]**
(a) 3 (b) 2 (c) 1.5 (d) 1
16. *AB* is a straight line. *C* is a point whose perpendicular distance from *AB* is 3 cm. What are the number of points which are at a perpendicular distance of 1 cm from *AB* and at a distance 4 cm from *C*? **[2011-I]**
(a) 1 (b) 2 (c) 3 (d) 4
17. Two trains each 200 m long move towards each other on parallel lines with velocities 20 km/h and 30 km/h, respectively. What is the time that elapses when they first meet until they have cleared each other? **[2011-I]**
(a) 20 s (b) 24.8 s (c) 28.8 s (d) 30 s

18. A man can walk uphill at the rate of 2.5 km/h and downhill at the rate of 3.25 km/h. If the total time required to walk a certain distance up the hill and return to the starting position is 4 h 36 min, what is the distance he walked up the hill ? **[2011-I]**
(a) 3.5 km (b) 4.5 km (c) 5.5 km (d) 6.5 km

19. A train 280 m long is moving at a speed of 60 km/h. What is the time taken by the train to cross a platform 220 m long? **[2012-I]**
(a) 45 s (b) 40 s (c) 35 s (d) 30 s

20. A student moves $\sqrt{2}x$ km East from his residence and then moves x km North. He, then goes x km North-East and finally he takes a turn of 90° towards right and moves a distance x km and reaches his school. What is the shortest distance of the school from his residence? **[2012-I]**
(a) $(2\sqrt{2}+1)x$ km (b) $3x$ km
(c) $2\sqrt{2}x$ km (d) $3\sqrt{2}x$ km

21. A car travels along the four sides of a square at speeds v, 2v, 3v and 4v, respectively. If u is the average speed of the car in its travel around the square, then which one of the following is correct ? **[2012-I]**
(a) $u = 2.25\ v$ (b) $u = 3\ v$
(c) $v < u < 2\ v$ (d) $3\ v < u < 4v$

22. A wheel of radius 2.1 m of a vehicle makes 75 revolutions in 1 min. What is the speed of the vehicle? **[2012-I]**
(a) 78 km/h (b) 59.4 km/h
(c) 37.4 km/h (d) 35.4 km/h

23. A car is travelling at a constant rate of 45 km/h. The distance travelled by car from 10 : 40 am to 1 : 00 pm is **[2012-II]**
(a) 165 km (b) 150 km (c) 120 km (d) 105 km

24. A train takes 10 s to cross a pole and 20 s to cross a platform of length 200 m. What is the length of the train? **[2012-II]**
(a) 400 m (b) 300 m (c) 200 m (d) 100 m

25. A person travels a certain distance at 3 km/h and reaches 15 min late. If he travels at 4 km/h, he reaches 15 min earlier. The distance he has to travel is **[2013-I]**
(a) 4.5 km (b) 6 km (c) 7.2 km (d) 12 km

26. A sailor sails a distance of 48 km along the flow of a river in 8 h. If it takes 12 h return the same distance, then the speed of the flow of the river is **[2013-I]**
(a) 0.5 km/h (b) 1 km/h
(c) 1.5 km/h (d) 2 km/h

27. If a body cover a distance at the rate of x km/h and another equal distance at the rate of y km/h, then the average speed (in km/h) is **[2013-I]**
(a) $\frac{x+y}{2}$ (b) $\sqrt{xy}$ (c) $\frac{2xy}{x+y}$ (d) $\frac{x+y}{xy}$

28. A man cycles with a speed of 10 km/h and reaches his office at 1 p.m. However, when he cycles with a speed of 15 km/h, he reaches his office at 11 am. At what speed sould he cycle, so that he reaces his office at 12 noon? **[2013-II]**
(a) 12.5 km/h (b) 12 km/h
(c) 13 km/h (d) 13.5 km/h

29. Two cars A and B start simultaneously from a certain place at the speed of 30 km/h and 45 km/hr, respectively. The car B reaches the distination 2 h earlier than A. What is the distance between the starting point and destination? **[2013-II]**
(a) 90 km (b) 180 km (c) 270 km (d) 360 km

30. A train running at the speed of 72 km/h goes past a pole in 15 s. What is the length of the train ? **[2013-II]**
(a) 150 m (b) 200 m (c) 300 m (d) 350 m

31. A train takes 9 s to cross a pole. If the speed of the train is 48 km/h, the length of the train is **[2014-I]**
(a) 150 m (b) 120 m (c) 90 m (d) 80 m

32. The distance between two points (A and B) is 110 km. X starts running from point A at a speed of 60 km/h and Y starts running from point B at a speed of 40 km/h at the same time. They meet at a point C, somewhere on the line AB. What is the ratio of AC to BC? **[2014-II]**
(a) 3 : 2 (b) 2 : 3 (c) 3 : 4 (d) 4 : 3

33. A man rides one–third of the distance from A to B at the rate of x km/h and the remainder at the rate of $2y$ km/h. If he had travelled at a uniform rate of $6z$ km/h, then he could have ridden from A to B and back again in the same time. Which one of the following is correct? **[2014-II]**
(a) $z = x + y$ (b) $3z = x + y$
(c) $\frac{1}{z}=\frac{1}{x}+\frac{1}{y}$ (d) $\frac{1}{2z}=\frac{1}{x}+\frac{1}{y}$

34. A train travels at a speed of 40 km/h and another train at a speed of 20 m/s. What is the ratio of speed of the first train to that of the second train ? **[2014-II]**
(a) 2 : 1 (b) 5 : 9 (c) 5 : 3 (d) 9 : 5

35. A man rows downstream 32 km and 14 km upstream, and he takes 6 hours to cover each distance. What is the speed of the current? **[2015-I]**
(a) 0.5 km/hr (b) 1 km/hr
(c) 1.5 km/hr (d) 2 km/hr

36. A car travels the first one-third of a certain distance with a speed of 10 km/hr, the next one-third distance with a speed of 20 km/hr and the last one-third distance with a speed of 60 km/hr. The average speed of the car for the whole journey is **[2015-I]**
(a) 18 km/hr (b) 24 km/hr
(c) 30 km/hr (d) 36 km/hr

37. Two persons A and B start simultaneously from two places c km apart, and walk in the same direction. If A travels at the rate of p km/hr and B travels at the rate of q km/hr, then A has travelled before he overtakes B a distance of **[2015-I]**

(a) $\frac{qc}{p+q}km$ (b) $\frac{pc}{p-q}km$

(c) $\frac{qc}{p-q}km$ (d) $\frac{pc}{p+q}km$

38. In a flight of 600 km, an aircraft was slowed down due to bad weather. Its average speed for the trip was reduced by 200 km/hr and the time of flight increased by 30 minutes. The duration of the flight is **[2015-I]**
(a) 1 hour (b) 2 hours
(c) 3 hours (d) 4 hours

39. With a uniform speed, a car covers a distance in 8 hours. Had the speed been increased by 4 km/hr, the same distance could have been covered in 7 hours and 30 minutes. What is the distance covered? **[2015-I]**
(a) 420 km (b) 480 km (c) 520 km (d) 640 km

40. *A* runs $1\frac{2}{3}$ times as fast as *B*. If *A* gives *B* a start of 80 m, how far must the winning post from the starting point be so that *A* and *B* might reach it at the same time? **[2015-I]**
(a) 200 m (b) 300 m (c) 270 m (d) 160 m

41. A thief is noticed by a policeman from a distance of 200 m. The thief starts running and the policeman chases him. The thief and the policeman run at the speed of 10 km/hr and 11 km/hr respectively. What is the distance between them after 6 minutes ? **[2015-I]**
(a) 100 m (b) 120 m (c) 150 m (d) 160 m

42. Two trains are moving in the same direction at 1.5 km/minute and 60 km/ hour respectively. A man in the faster train observes that it takes 27 seconds to cross the slower train. The length of the slower train is **[2015-II]**
(a) 225 m (b) 230m (c) 240m (d) 250 m

43. In a race A, B and C take part. A beats B by 30 m, B beats C by 20 m and A beats C by 48 m. Which of the following is/ are correct ? **[2015-II]**
1. The length of the race is 300 m.
2. The speeds of A, B and C are in the ratio 50 :45 :42.

Select the correct answer using the code given below :
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

44. A motor boat, whose speed is 15 km/ hour in still water goes 30 km down–stream and comes back in a total of 4 hour and 30 minutes. The speed of the stream is **[2015-II]**
(a) 4 km/ hour (b) 5 km/hour
(c) 6 km/ hour (d) 10 km/hour

45. By increasing the speed of his car by 15 km/hour, a person covers 300 km distance by taking an hour less than before. The original speed of the car was **[2015-II]**
(a) 45 km/ hour (b) 50 km/hour
(c) 60 km/hour (d) 75 km/ hour

46. Two trains, one is of 121 m in length at the speed of 40 km/hour and the other is of 99 m in length at the speed of 32 km/hour are running in opposite directions. In how much time will they be completely clear from each other from the moment they meet? **[2015-II]**
(a) 10 s (b) 11 s (c) 16 s (d) 21 s

47. Three athletes run a 4 km race. Their speeds are in the ratio 16 : 15 : 11. When the winner wins the race, then the distance between the athlete in the second position to the athlete in the third position is **[2015-II]**
(a) 1000m (b) 800m (c) 750 m (d) 600 m

48. In a race of 100 m, A beats B by 4m and A beats C by 2 m. By how many metres (approximately) would C beat B in another 100 m race assuming C and B run with their respective speeds as in the earlier race ? **[2015-II]**
(a) 2 (b) 2.04 (c) 2.08 (d) 3.2

49. The speeds of three buses are in the ratio 2 : 3 : 4. The time taken by these buses to travel the same distance will be in the ratio **[2015-II]**
(a) 2 : 3 : 4 (b) 4 : 3 : 2
(c) 4 : 3 : 6 (d) 6 : 4 : 3

50. A man walking at 5 km/hour noticed that a 225 m long train coming in the opposite direction crossed him in 9 seconds. The speed of the train is **[2016-I]**
(a) 75 km/hour (b) 80 km/hour
(c) 85 km/hour (d) 90 km/hour

51. A cyclist moves non-stop from A to B, a distance of 14 km, at a certain average speed. If his average speed reduces by 1 km per hour, he takes 20 minutes more to cover the same distance. The original average speed of the cyclist is **[2016-I]**
(a) 5 km/hour (b) 6 km/hour
(c) 7 km/hour (d) None of the above

52. A bike consumes 20 mL of petrol per kilometre, if it is diriven at a speed in the range of 25 – 50 km/hour and consume 40 mL of petrol per kilometer at any other speed. How much petrol is consumed by the bike in travelling distance of 50 km, if the bike is driven at a speed of 40 km/hour for the first 10 km, at a speed of 60 km/hour for the next 30 km and at a speed of 30 km/hour for the last 10 km? **[2016-I]**
(a) 1 L (b) 1.2 L (c) 1.4 L (d) 1.6 L

53. A passenger train takes 1 hour less for a journey of 120 km, if its speed is increased by 10 km/hour from its usual speed. What is its usual speed? **[2016-I]**
(a) 50 km/hour (b) 40 km/hour
(c) 35 km/hour (d) 30 km/hour

54. In a race of 1000 m. A beats B by 100 m or 10 seconds. If they start a race of 1000 m simultaneously from the same point and if *B* gets injured after running 50 m less than half the race length and due to which his speed gets haived, then by how much time will *A* beat *B*? **[2016-I]**
(a) 65 seconds (b) 60 seconds
(c) 50 seconds (d) 45 seconds

55. Two men, A and B run a 4 km race on a course 0.25 km round. If their speeds are in the ratio 5:4, how often does the winner pass the other ? **[2016-II]**
(a) Once (b) Twice
(c) Thrice (d) Four times

56. When the Speed of a train is increased by 20%, it takes 20 minutes less to cover the same distance. What is the time taken to cover the same distance with the original speed ? **[2016-II]**

(a) 140 minutes (b) 120 minutes
(c) 100 minutes (d) 80 minutes

57. A person can row downstream 20 km in 2 hours and upstream 4 km in 2 hours. What is the speed of the current? **[2016-II]**

(a) 2 km/hour (b) 2.5 km/hour
(c) 3 km/hour (d) 4 km/hour

58. A train is travelling at 48 km/hour completely crosses another train having half its length and travelling in opposite direction at 42 km/hour in 12 s. It also passes a railway platform in 45 s. What is the length of the platform? **[2016-II]**

(a) 600 m (b) 400 m (c) 300 m (d) 200 m

59. The speeds of three cars are in the ratio 2: 3 :4. What is the ratio between the times taken by these cars to travel the same distance ? **[2016-II]**

(a) 4 : 3 : 2 (b) 2 : 3 : 4
(c) 4 : 3 : 6 (d) 6 : 4 : 3

60. The wheels of a car are of diameter 80cm each. The car is travelling at a speed of 66 Km/hour. What is the number of complete revolutions each wheel makes in 10 minutes ? **[2016-II]**

(a) 4275 (b) 4350 (c) 4375 (d) 4450

61. A motorist travels to a place 150 km away at an average speed of 50 km/hour and returns at 30 km/hour. What is the average speed for the whole journey ? **[2016-II]**

(a) 35 km/hour (b) 37 km/hour
(c) 37.5 km/hour (d) 40 km/hour

62. In a 100 m race, A runs at a speed of $\frac{5}{3}$ m/s. If A gives a start of 4 m to B and still beats him by 12 seconds, What is the speed of B ? **[2017-I]**

(a) $\frac{5}{4}$ m/s (b) $\frac{7}{5}$ m/s (c) $\frac{4}{3}$ m/s (d) $\frac{6}{5}$ m/s

63. A passenger train departs from Delhi at 6 pm, for Mumbai. At 9 p.m., an express train, whose average speed exceeds that of the passenger train by 15 km/hour leaves Mumbai for Delhi. Two trains meet each other mid-route. At what time do they meet, given that the distance between the cities is 1080 km ? **[2017-I]**

(a) 4 pm. (b) 2 am.
(c) 12 midnight (d) 6 am

64. A 225 m long train is running at a speed of 30 km/hour. How much time does it take to cross a man running at 3 km/hour in the same direction ? **[2017-I]**

(a) 40 seconds (b) 30 seconds
(c) 25 seconds (d) 15 seconds

65. A thief is spotted by a policeman from a distance of 100 m. When the policeman starts the chase, the thief also starts running. If the speed of the thief is 8 km/hour and that of the policeman is 10 km/hour, then how far will the thief have to run before he is overtaken ? **[2017-I]**

(a) 200 m (b) 300 m (c) 400 m (d) 500 m

66. If a train crosses a km-stone in 12 seconds, how long will it take to cross 91 km-stones completely if its speed in 60 km/hr? **[2017-II]**

(a) 1 hr 30 min (b) 1 hr 30 min 12 sec
(c) 1 hr 51 min (d) 1 hr 1 min 3 sec

67. In a 100 m race, A runs at 6 km/hr. If A gives B a start of 8 m and still beats him by 9 seconds, what is the speed of B? **[2017-II]**

(a) 4.6 km/hr (b) 4.8 km/hr
(c) 5.2 km/hr (d) 5.4 km/hr

68. A boy went to his school at a speed of 12 km/hr and returned to his house at a speed of 8 km/hr. If he has taken 50 minutes for the whole journey, what was the total distance walked? **[2017-II]**

(a) 4 km (b) 8 km (c) 16 km (d) 20 km

69. A man rows down a river 18 km in 4 hours with the stream and returns in 10 hours. **[2017-II]**

Consider the following statements :

1. The speed of the man against the stream is 1.8 km/hr.
2. The speed of the man is still water is 3.15 km/hr.
3. The speed of the stream is 1.35 km/hr.

Which of the above statements are correct?

(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

70. A man travelled 12 km at a speed of 4 km/hr and further 10 km at a speed of 5 km/hr. What was his average speed? **[2017-II]**

(a) 4.4 km/hr (b) 4.5 km/hr
(c) 5.0 km/hr (d) 2.5 km/hr

71. A train moving with a speed of 60 km per hour crosses an electric pole in 30 seconds. What is the length of the train in metres? **[2018-I]**

(a) 300 (b) 400 (c) 500 (d) 600

72. A passenger train and a goods train are running in the same direction on parallel railway tracks. If the passenger train now takes three times as long to pass the goods train, as when they are running in opposite directions, then what is the ratio of the speed of the passenger train to that of the goods train? **[2018-I]**

(Assume that the trains run at uniform speeds)

(a) 2 : 1 (b) 3 : 2 (c) 4 : 3 (d) 1 : 1

73. A man can row at a speed of x km/hr in still water. If in a stream which is flowing at a speed of y km/hr it takes him z hours to row to a place and back, then what is the distance between the two places? **[2018-I]**

(a) $\frac{z(x^2-y^2)}{2y}$ (b) $\frac{z(x^2-y^2)}{2x}$

(c) $\frac{(x^2-y^2)}{2zx}$ (d) $\frac{z(x^2-y^2)}{x}$

74. A car has an average speed of 60 km per hour while going from Delhi to Agra and has an average speed of y km per hour while returning to Delhi from Agra (by travelling the same distance). If the average speed of the car for the whole journey is 48 km per hour, then what is the value of y? **[2018-I]**
(a) 30 km per hour (b) 35 km per hour
(c) 40 km per hour (d) 45 km per hour

75. A train 100 m long passes a platform 100 m long in 10 seconds. The speed of the train is **[2018-II]**
(a) 36 kmph (b) 45 kmph
(c) 54 kmph (d) 72 kmph

76. A cyclist covers his first 20 km at an average speed of 40 kmph, another 10 km at an average speed of 10 kmph and the last 30 km at an average speed of 40 kmph. Then the average speed of the entire journey is **[2018-II]**
(a) 20 kmph (b) 26.67 kmph
(c) 28.24 kmph (d) 30 kmph

77. In a race of 1000 m, A beats B by 150 m, while in another race of 3000 m, C beats D by 400 m. Speed of B is equal to that of D. (Assume that A, B, C and D run with uniform speed in all the events). If A and C participate in a race of 6000 m, then which one of the following is correct? **[2018-II]**
(a) A beats C by 250 m (b) C beats A by 250 m
(c) A beats C by 115.38 m (d) C beats A by 115.38 m

78. The minute hand of a clock overtakes the hour hand after every 72 minutes of correct time. How much time does the clock lose or gain in a day of normal time? **[2018-II]**
(a) Lose $121\frac{9}{11}$ minutes (b) Lose $157\frac{1}{11}$ minutes
(c) Gain $121\frac{9}{11}$ minutes (d) Gain $157\frac{1}{11}$ minutes

79. A thief steals a car parked in a house and goes away with a speed of 40 kmph. The theft was discovered after half an hour and immediately the owner sets off in another car with a speed of 60 mph. When will the owner meet the thief? **[2018-II]**
(a) 55 km from the owner's house and one hour after the theft
(b) 60 km from the owner's house and 1.5 hours after the theft
(c) 60 km from the owner's house and one hours after the discovery of the theft
(d) 55 km from the owner's house and 1.5 hours after the theft

80. Three cars A, B and C started from a point at 5 p.m., 6 p.m. and 7 p.m. respectively and travelled at uniform speeds of 60 km/hr, 80 km/hr and x km/hr respectively in the same direction. If all the three met at another point at the same instant during their journey, then what is the value of x ? **[2019-I]**
(a) 120 (b) 110 (c) 105 (d) 100

81. Radha and Hema are neighbours and study in the same school. Both of them use bicycles to go to the school. Radha's speed is 8 km/hr whereas Hema's speed is 10 km/hr. Hema takes 9 minutes less than Radha to reach the school. How far is the school from the locality of Radha and Hema ? **[2019-I]**
(a) 5 km (b) 5.5 km (c) 6 km (d) 6.5 km

82. A plane is going in circles around an airport. The plane takes 3 minutes to complete one round. The angle of elevation of the plane from a point P on the ground at time t seconds is equal to that at time $(t + 30)$ seconds. At time $(t + x)$ seconds, the plane flies vertically above the point P. What is x equal to ? **[2019-I]**
(a) 75 seconds (b) 90 seconds
(c) 105 seconds (d) 135 seconds

83. It takes 11 hours for a 600 km journey if 120 km is done by train and the rest by car. It takes 40 minutes more If 200 km are covered by train and the rest by car. What is the ratio of speed of the car to that of the train? **[2019-II]**
(a) 3 : 2 (b) 2 : 3 (c) 3 : 4 (d) 4 : 3

84. The ratio of speeds of X and Y is 5 : 6. If Y allows X a start of 70 m in a 1.2 km race, then who will win the race and by what distance ? **[2020-I]**
(a) X wins the race by 30 m
(b) Y wins the race by 90 m
(c) Y wins the race by 130 m
(d) The race finishes in a dead heat

85. A train takes two hours less for a journey of 300 km if its speed is increased by 5 km/hr from its usual speed. What is its usual speed? **[2020-I]**
(a) 50 km/hr (b) 40 km/hr
(c) 35 km/hr (d) 25 km/hr

86. In covering certain distance, the average speeds of X and Y are in the ratio 4 : 5. If X takes 45 minutes more than Y to reach the destination, then what is the time taken by Y to reach the destination? **[2020-I]**
(a) 135 mmminutes (b) 150 minutes
(c) 180 minutes (d) 225 minutes

HINTS & SOLUTIONS

1. (c) Let total distance of the journey be x km.

first part of journey = $\frac{x}{2}$ km

Then second part of journey = $\frac{x}{2}$ km

According to question,

$$\frac{\frac{x}{2}}{30}+\frac{\frac{x}{2}}{70} = 10 \Rightarrow \frac{7x+3x}{210} = 20$$

$\therefore$ x = 2 × 210 = 420 km

2. (d) Total distance travel by train
= length of platform + length of train
= 200 + 200 = 400 m

$$\therefore \text{ Required speed } = \frac{\text{Total distance travel}}{\text{Time taken}}$$

$$= \frac{400}{15} \text{ m/s } = \frac{400}{15}\times\frac{18}{5} \text{ km/h} = 96 \text{ km/h}$$

3. (a) Time = $\frac{\text{Distance}}{\text{Speed}}$

Let distance travel from A to B be x km.
According to question

$\therefore\ 7 = \frac{x}{4}+\frac{x}{3} = \frac{x(3+4)}{12} = \frac{7x}{12} \quad \Rightarrow\ x = 12$ km

4. (c) Let the speed of bike = v km/h

$\therefore$ Time taken to cover 200 km at a speed of v km/ $h = \frac{200}{v}h$

New speed of bike = $(v + 5)$ km/h

$\therefore$ Time taken to cover 200 km at a speed of (v + 5) km/h = $\frac{200}{v+5}$

According to question, $\frac{200}{v}-\frac{200}{v+5} = 2$

$\Rightarrow \frac{(v+5-v)200}{v^2+5v} = 2 \Rightarrow 500\ v^2 + 5v$

$\Rightarrow v^2 + 5v - 500 = 0 \ \Rightarrow v^2 + 25v - 20v - 500 = 0$

$\Rightarrow v\,(v + 25) - 20\,(v + 25) = 0$

$\Rightarrow (v - 20)\,(v + 25) = 0 \qquad (\because v \neq -25)$

$\therefore \quad v = 20\text{km}/\text{h}$

So the original speed of a bike is 20 km/h

5. (a) Let the speed of a train be v m/s and length of the train be x m.

when crosses telegraph $t = 8s$

Time = $\frac{\text{Distance}}{\text{Speed}} \Rightarrow 8 = \frac{x}{v}$

$\Rightarrow$ x = 8 v ...(i)

When crosses bridge

t = 24s, Speed = v

Distance = x + 200 $\quad \because$ Time = $\frac{\text{Distance}}{\text{Speed}}$

$\therefore\ 24 = \frac{x+200}{v} \Rightarrow 24v = x + 200$

$\Rightarrow 24v = 8v + 200$ [from eq. (i)]

$\Rightarrow 16v = 200 \Rightarrow v = \frac{25}{2}$

From eq. (i) $x = 8v = 8 \times \frac{25}{2} = 100$ m

6. (b) Train and man running opposite to each other.

$\therefore$ Relative speed = 60 + 6 = 66 km/h

$= \frac{66\times 5}{18}$ m/s

$\therefore$ Required time = $\frac{\text{Distance}}{\text{Speed}}$

$= \frac{110}{\frac{66\times 5}{18}} = \frac{110\times 18}{66\times 5} = 6\text{s}$

7. (b) Time taken to cross the trains = $\frac{150+100}{\frac{25}{3}+v}$

where v is the speed of second train.

$\Rightarrow 10 = \frac{250\times 3}{25+3v} \quad \Rightarrow\ 250 + 30v = 750$

$\Rightarrow 30v = 500 \quad \Rightarrow\ v = \frac{50}{3}$ m/s

$\therefore\ v = \frac{50}{3}\times\frac{18}{5} = 60$ km/h

8. (c) According to the question,

$t_1 - t_2 = 12 - (-15)$

$\Rightarrow t_1 - t_2 = 27$ min $\quad \Rightarrow \frac{x}{V_1}-\frac{x}{V_2} = \frac{27}{60}$ h

$\Rightarrow \frac{x}{2.5}-\frac{x}{4} = \frac{27}{60} \quad \Rightarrow \frac{2x}{5}-\frac{x}{4} = \frac{9}{20}$

$\Rightarrow \frac{3x}{20} = \frac{9}{20} \quad \Rightarrow\ 3x = 9$

$\therefore$ x = 3 km

9. (a) P $\longleftrightarrow$ Q

city A 60 km city B

Distance between P and Q = 60 km

Let the speed of Q = x km/h

Then the speed of P = (x – 4) km/h

Distance travelled by Q till they meet P = 60 + 12 = 72 km

Time taken by Q till they meet = $\frac{72}{x}$

Distance travelled by P till they meet = 60 – 12 = 48 km

Time taken by P till they meet = $\frac{48}{x-4}$

since time taken by both will be equal,

$\therefore\ \frac{72}{x} = \frac{48}{x-4} \quad \Rightarrow 72x - 288 = 48x$

$\Rightarrow 24x = 288 \Rightarrow x = 12$ km/h

$\therefore$ Speed of P = x – 4

= 12 – 4 = 8 km/h

10. (d) $\because$ Time = $\frac{\text{Distance}}{\text{Speed}} = \frac{92}{6.5+5} = \frac{92}{11.5} = 8$ h

11. (a) Relative velocity = (50 – 32) km/h = 18 km/h

Elapse time = 15s = $15\times\frac{1}{60}\times\frac{1}{60}h = \frac{15}{3600}h$

Time = $\frac{\text{Distance}}{\text{Speed}} \Rightarrow \frac{15}{3600} = \frac{x}{18}$

$\Rightarrow x \times 3600 = 18 \times 15$

$\therefore\ x = \frac{18\times 15}{3600}$ km $= \frac{18\times 15}{3600}\times 1000\,\text{m} = 75$ m.

12. (a) Let speed of stream = D km/h
Speed of boat in still water = B km/h = 11 km/h.
According to the question

$$B + D = \frac{12}{t_1}$$

$$t_1 = \frac{12}{B+D} \quad ...(i)$$

$$B - D = \frac{12}{t_2}$$

$$t_2 = \frac{12}{B-D} \quad ...(ii)$$

Now,

$$t_1 + t_2 = 2 + \frac{45}{60} = \frac{11}{4}$$

$$\Rightarrow \frac{12}{B+D} + \frac{12}{B-D} = \frac{11}{4} \quad (\text{Given } B = 11 \text{km/h})$$

$$\Rightarrow \frac{2B}{B^2 - D^2} = \frac{11}{4 \times 12}$$

$$\Rightarrow \frac{2 \times 11}{(11)^2 - D^2} = \frac{11}{4 \times 12}$$

$\Rightarrow D^2 = 25 \quad \therefore D = 5$ km/h.

13. (b) Let distance between A and B = x km
According to question

$$\frac{x}{12} - \frac{x}{18} = 2 \Rightarrow 6x = 2 \times 18 \times 12$$

$$\Rightarrow x = \frac{2 \times 18 \times 12}{6} = 72 \text{ km}$$

required distance = 72 km

14. (b) Relative speed = 75 + 50 = 125 km/h

$$= \frac{125 \times 5}{18} \text{ m/s}$$

Total covered distance = 100 + 150 = 250 m
$\therefore$ Time taken to cross each other

$$= \frac{\text{Total covered distance}}{\text{Relative speed}}$$

$$= \frac{250 \times 18}{125 \times 5} = 7.2 \text{ s}$$

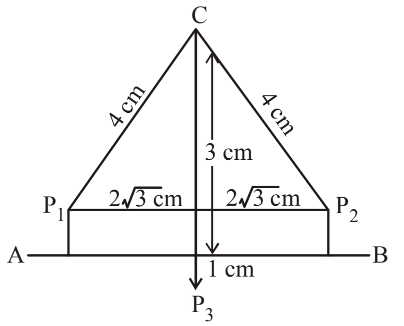
15. (a) Let speed of motorboat be B km/h.
Speed of water = D km/h.
According to question

$$B + D = \frac{9}{2} = 4.5 \quad ....(i)$$

$$B - D = \frac{9}{6} = \frac{3}{2} = 1.5 \quad .... (ii)$$

Now, on solving eqs. (i) and (ii), we get
B = 3 km/h and D =1.5 km/h
speed of boat = 3km/h.

16. (c)



$\therefore$ Required number of points = 3
($\because$ P_1, P_2 and P_3)

17. (c) Relative speed of trains = (20 + 30) km/h

$$= 50 \text{ km/h} = 50 \times \frac{5}{18} \text{ m/s}$$

Total relative distance = 200 + 200 = 400 m

$$\therefore \text{Required time} = \frac{400 \times 18}{50 \times 5} = 28.8 \text{ s}$$

18. (d) Let a distance walked be x km.
According to question

$$\frac{x}{2.5} + \frac{x}{3.25} = 4\frac{36}{60} \Rightarrow x\left(\frac{1}{2.5} + \frac{1}{3.25}\right) = \frac{276}{60}$$

$$\Rightarrow x\left(\frac{2}{5} + \frac{4}{13}\right) = \frac{276}{60}$$

$$\therefore x = \frac{276}{60} \times \frac{65}{46} = 6.5 \text{ km}$$

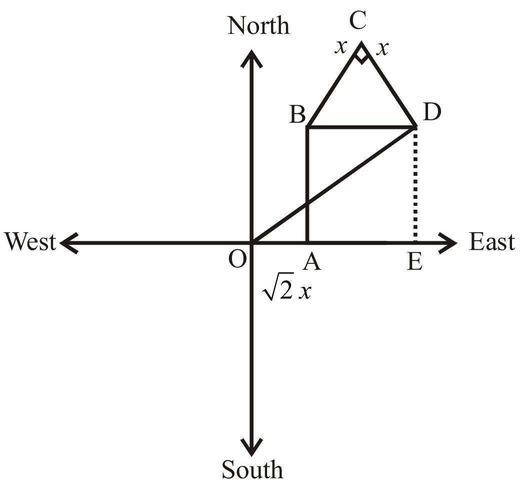
19. (d) To cover a distance by train = 280 + 220 = 500 m

$$\text{Speed of train} = 60 \text{ km/h} = \frac{60 \times 1000}{60 \times 60} = \frac{50}{3} \text{ m/s}$$

$$\therefore \text{Time taken by train} = \frac{\text{Total distance}}{\text{Speed}}$$

$$= \frac{500}{\frac{50}{3}} = 30\text{s}$$

20. (b) In fig, O is residence and D is school.
In ΔBCD,



$BD^2 = BC^2 + CD^2 = x^2 + x^2$

$\Rightarrow BD = \sqrt{2}x \Rightarrow BD = AE = \sqrt{2}x$

$\therefore OE = OA + AE = \sqrt{2}.x + \sqrt{2}.x = 2\sqrt{2}x$

$\because BA = DE = x$

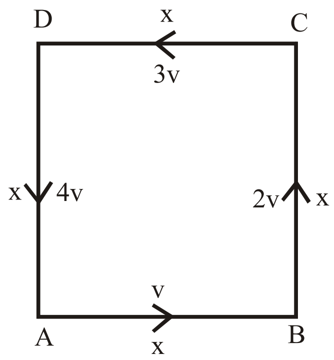
in ΔOED,

$OD^2 = OE^2 + DE^2$

$\therefore$ Minimum distance,

$OD = \sqrt{(2\sqrt{2}.x)^2 + x^2} = \sqrt{8x^2 + x^2} = 3x$ km

21. (c) Let side of a square be x.



$\because$ Average speed $(u) = \dfrac{\text{Total Distance}}{\text{Total Time}}$

$= \dfrac{(x+x+x+x)}{\frac{x}{v}+\frac{x}{2v}+\frac{x}{3v}+\frac{x}{4v}} = \dfrac{4 \times v}{1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}} = \dfrac{48v}{25} = 1.92v$

Average speed lies between $v < u < 2\,v$

22. (b) Radius of the wheel = 2.1 m

Distance covered in 1 revolution = $2\pi r$

$= 2 \times \frac{22}{7} \times 2.1$

$\therefore$ Distance covered in 75 revolutions

$= 2\pi r \times 75 = 2 \times \frac{22}{7} \times 2.1 \times 75$

$= 990$ m $= 0.99$ km

Time $= \frac{1}{60}h$

$\therefore$ Required speed $= \dfrac{\text{Distance}}{\text{Time}} = \dfrac{0.99}{\frac{1}{60}}$ km/h

$= 59.4$ km/h

23. (d) Speed of a car = 45 km/hr

Time taken by the car = 140 min $= \frac{140}{60}$h

So, required distance travelled by car = Speed × Time

$= 45 \times \frac{140}{60} = 105$ km

24. (c) Let the speed of a train be v m/s and length of train be x m.

when cross the pole in time $t = 10$ s

Time $= \dfrac{\text{Distance}}{\text{Speed}}$

$\Rightarrow 10 = \frac{x}{v} \Rightarrow x = 10\,v$ ... (i)

When cross the platform in time, t = 20 s

$20 = \frac{x+200}{v} \Rightarrow 20v = x + 200$

$\Rightarrow 20v = 10v + 200$ [from Eq. (i)]

$\Rightarrow 10v = 200$

$\therefore v = 20$

From Eq. (i),

$x = 10v = 10 \times 20 = 200$ m

25. (b) According to question,

$t_1 - t_2 = 15 - (-15) \Rightarrow 15 + 15$

$\Rightarrow 30$ min $\Rightarrow \frac{x}{v_1} - \frac{x}{v_2} = \frac{30}{60}h$

Here, $v_1 = 3$ km/h, $v_2 = 4$ km/h

$\Rightarrow \frac{x}{3} - \frac{x}{4} = \frac{1}{2} \Rightarrow \frac{4x-3x}{12} = \frac{1}{2} \Rightarrow \frac{x}{12} = \frac{1}{2}$

$\therefore x = \frac{12}{2} = 6$ km.

26. (b) Let speed of the flow of water be v km/h and rate of sailing of sailer be u km/h.

Then, $u + v = \frac{48}{8} \Rightarrow u + v = 6$ ...(i)

and $u - v = \frac{48}{12} \Rightarrow u - v = 4$ ...(ii)

On solving eqs. (i) and (ii), we get

v = 1 km/hr

27. (c) Average speed of two object when distance is same

Average speed $= \dfrac{2v_1v_2}{v_1 + v_2}$

Here $v_1 = x$ km/h. $v_2 =$ y km/h $= \dfrac{2 \times xy}{x+y}$.

28. (b) $t_2 - t_1 = 2h \Rightarrow \frac{x}{10} - \frac{x}{15} = 2$

$\Rightarrow \frac{3x-2x}{30} = 2$ Relative speed

$x = 60$ km.

$= 15 - 10 = 5$km 1h.

speed $= \dfrac{\text{distance}}{\text{relative speed}} = \dfrac{60}{5} = 12$ km/h.

29. (b) Let distance between starting and their destination be x km.

$t_2 - t_1 = 2h$

$\Rightarrow \frac{x}{30} - \frac{x}{45} = 2 \Rightarrow \left(\frac{3x-2x}{90}\right) = 2$

$\Rightarrow \frac{x}{90} = 2$

$\therefore x = 180$ km,

30. (c) Speed of train

$= \dfrac{\text{Length of train}}{\text{Time taken to cross the stationary object}}$

$\therefore$ Length of train = Speed of train × Time taken to cross the stationary object

$= \dfrac{72 \times 1000 \times 15}{3600} = 300$ m

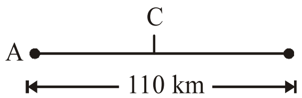
31. (b) Let the length of the train = x m.

speed = 48 km/h = $\dfrac{48 \times 1000}{60 \times 60}$ m/s

$\therefore$ Length of train, (x) = Speed × Time to cross the pole = $\dfrac{48 \times 1000}{60 \times 60} \times 9 = 120$ m

32. (a) Distance between two points = 110 km
Relative speed = 60 + 40 = 100 km/h
Time after which they meet

$= \dfrac{\text{Total distance}}{\text{Relative speed}} = \dfrac{110}{100} = 1.10$ h



Distance corvered by A in 1.10 h = AC = 60 × 1.10 = 66 km
Remaining distance = BC = 110 – 66 = 44 km
Required ratio = AC : BC = 66 : 44 = 3 : 2

33. (c) Let the total distance be d km

Then, time taken to cover $\frac{1}{3}d$ distance

$t_1 = \dfrac{\frac{1}{3}d}{x} = \dfrac{d}{3x}$

Remaining distance = $d - \frac{1}{3}d = \frac{2}{3}d$ km.

Now, time taken to cover $\frac{2}{3}rd$ distance,

$t_2 = \dfrac{\frac{2}{3}d}{2y} = \dfrac{2d}{6y}$

Time taken to cover distance from A to B and B to A.

$t = \dfrac{2d}{6z}$

According to the question, $t_1 + t_2 = t$

$\Rightarrow \dfrac{d}{3x} + \dfrac{2d}{6y} = \dfrac{2d}{6z} \Rightarrow \dfrac{1}{3x} + \dfrac{1}{3y} = \dfrac{1}{3z}$

$\Rightarrow \dfrac{1}{x} + \dfrac{1}{y} = \dfrac{1}{z}$

34. (b) speed of a train = 40 km/h

$= 40 \times \dfrac{5}{18}$ m/s

Speed of another train = 20 m/s

$\therefore$ Required ratio $= \dfrac{\text{Speed of first train}}{\text{Speed of second train}}$

$= \dfrac{40 \times \frac{5}{18}}{20} = \dfrac{2 \times 5}{18} = \dfrac{10}{18} = \dfrac{5}{9}$ or 5 : 9

35. (c) Let speed of boat be B km/h and speed of stream be D km/h.
Downstream

$B + D = \dfrac{32}{6}$ ...(i)

Upstream

$B - D = \dfrac{14}{6}$ ...(ii)

Now on solving equations (i) and (ii)

we get $2B = \dfrac{46}{6}$

$B = \dfrac{23}{6}$ km/h

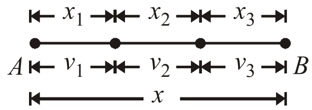
Put value of B in equation ...(i)

$D = \dfrac{32}{6} - \dfrac{23}{6}$

$= \dfrac{9}{6} = \dfrac{3}{2} = 1.5$ km/h

36. (a) Let total distance = x km.

Average speed = $\dfrac{\text{total distance}}{\text{total time}}$

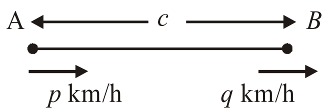


$\dfrac{x}{t_1 + t_2 + t_3} = \dfrac{x}{\frac{x_1}{v_1} + \frac{x_2}{v_2} + \frac{x_3}{v_3}}$

$x_1 = \dfrac{x}{3}, x_2 = \dfrac{x}{3}, x_3 = \dfrac{x}{3} \quad = \dfrac{x}{\frac{x}{3}\left(\frac{1}{10} + \frac{1}{20} + \frac{1}{60}\right)}$

$= \dfrac{3}{\left(\frac{6+3+1}{60}\right)} = \dfrac{3 \times 60}{10} = 18$ km/h

37. (c)



Relative speed = ($p - q$) km/hr.

Time taken to cover the c km = $\dfrac{c}{p-q}$ hr

Distance travelled by A before he overtakes

$B = \dfrac{qc}{p-q}$ km

38. (a) Let average speed of flight = v

Time taken by flight (t) = $\dfrac{600}{v}$ ... (i)

Now, flight speed is reduced by 200 km/hr

$= \frac{600}{v-200} = t + \frac{30}{60}$ ... (ii)

Now, put value of t in eqn (ii)

$\Rightarrow \frac{600}{\frac{600}{t} - 200} = t + \frac{1}{2}$

$\Rightarrow \frac{600t}{600 - 200t} = t + \frac{1}{2}$

$\Rightarrow 600\,t - 200t^2 + 300 - 100t = 600\,t$

$\Rightarrow 2t^2 + t - 3 = 0$

$t = \frac{-1 \pm \sqrt{1+24}}{2 \times 2} = \frac{-1 \pm 5}{4}, \frac{-6}{4}, \frac{4}{4}$; $t = 1$ hour

Duration of flight = 1 hour

39. (b) Let Initial speed = v and total distance = x km.

Time taken by car = $\frac{x}{v} = 8$

$x = 8v$ ... (i)

Now, speed is increased by 4 km/hr

$\Rightarrow \frac{x}{v+4} = 7 + \frac{30}{60}$ $\Rightarrow \frac{x}{v+4} = 7 + \frac{1}{2} = \frac{15}{2}$

$\Rightarrow 2x = 15v + 60$

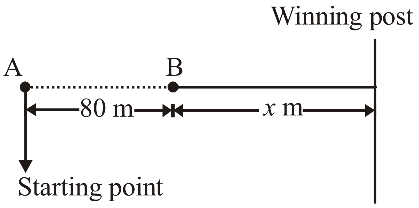
Put value of x from equation (i)

$\Rightarrow 16v = 15v + 60$; $v = 60$

Distance covered = $8 \times 60 = 480$ km

40. (a) Let speed of B = V

Then speed of A = $\frac{5}{3}$V



Time taken by B to reach winning post = $\frac{x}{v}$ ...(i)

Time taken by A to reach winning post = $\frac{x+80}{v}$ ...(ii)

Now, from eqs (i) and (ii)

$\Rightarrow \frac{x}{v} = \frac{x+80}{\frac{5v}{3}}$ $\Rightarrow 5x = 3x + 240$; $x = 120$ km

Distance from starting point = $x + 80$

= 120 + 80 = 200 m

41. (a) Relative speed = $(11 - 10)\,\frac{5}{18}$ m/s $= \frac{5}{18}$ m/s

Distance travelled by police in 6 min

$= \frac{5}{18} \times 60 \times 6 = 100$ m

Distance between them = 200 – 100 = 100 m

42. (a) Speed of 1st train = 1.5 km/minute = 1.5×60 km/h = 90 km/h

Relative speed of trains = 90 – 60 = 30 km/h

$= 30 \times \frac{5}{18} = \frac{25}{3}$ m/sec

Length of slower train = $\frac{25}{3} \times 27 = 25 \times 9 = 225$ m

So, option (a) is correct.

43. (c) Let the length of race be 'x' meters and velocity of A, B, C be respectively v_A, v_B and v_C.

According to question-

$\Rightarrow \frac{x}{v_A} = \frac{x-30}{v_B}$

$\Rightarrow \frac{v_A}{v_B} = \frac{x-30}{x}$ ---(i)

$\Rightarrow \frac{x}{v_B} = \frac{x-20}{v_C}$

$\Rightarrow \frac{v_C}{v_B} = \frac{x-20}{x}$ ---(ii)

$\Rightarrow \frac{x}{v_A} = \frac{x-48}{v_C}$

$\Rightarrow \frac{v_C}{v_A} = \frac{x-48}{x}$ ---(iii)

Multiplying eq (i) & (ii)-

$\frac{v_B}{v_A} \times \frac{v_C}{v_B} = \left(\frac{x-30}{x}\right)\left(\frac{x-20}{x}\right)$

$\frac{v_C}{v_A} = \left(\frac{x-30}{x}\right)\left(\frac{x-20}{x}\right)$ ---(iv)

From eq (iii) & eq (iv)-

$\left(\frac{x-48}{x}\right) = \left(\frac{x-30}{x}\right)\left(\frac{x-20}{x}\right)$

$\Rightarrow x^2 - 48x = x^2 + 600 - 50x$

$\Rightarrow 2x = 600$

$\Rightarrow \boxed{x = 300}$

Putting x = 300 in equation (i) & eq (ii)

$\Rightarrow \frac{v_B}{v_A} = \frac{300-20}{300} = \frac{270}{300} = \frac{9}{10}$

$\Rightarrow v_B = \frac{9}{10}\,v_A$ ---(v)

$\Rightarrow \frac{v_C}{v_B} = \frac{300-20}{300} = \frac{280}{300} = \frac{28}{30}$

$\Rightarrow v_C = \frac{28}{30}\,v_B = \frac{28}{30} \times \frac{9}{10}\,v_A = \frac{84}{100}\,v_A$ ---(vi)

From eq (v) 7 eq (vi)-

$$v_A : v_B : v_C = v_A : \frac{9}{10} v_A : \frac{84}{100} v_A$$

$= 100 : 90 : 84 = 50 : 45 : 42$

so, option (c) is correct.

44. (b) Let the speed of stream be 'x' km/hour.

$$\Rightarrow \frac{30}{(15+x)} + \frac{30}{(15-x)} = 4.5$$

$$\Rightarrow \frac{15-x+15+x}{(15+x)(15-x)} = \frac{4.5}{30}$$

$$\Rightarrow \frac{30}{225-x^2} = \frac{45}{300}$$

$\Rightarrow 225 - x^2 = 200$

$\Rightarrow x^2 = 25$

$\Rightarrow x = 5$

So, option (b) is correct

45. (c) Let the original speed of car be = x km/hour

$$\frac{300}{x} - \frac{300}{(x+15)} = 1$$

$$\Rightarrow \frac{x+15-x}{(x+15)x} = \frac{1}{300}$$

$\Rightarrow x^2 + 15x - 4500 = 0$

$\Rightarrow (x+75)(x-60) = 0$

$\Rightarrow x \neq -75$

$\boxed{x = 60}$

So, option (c)is correct.

46. (b) Speed of 1st train $= 40 \times \frac{5}{18} = \frac{200}{18}$ m/sec

Speed of 2nd train $= 32 \times \frac{5}{18} = \frac{160}{18}$ m/sec

Relative speed of trains

$$= \frac{200}{18} + \frac{160}{18} = \frac{360}{18} = 20 \text{ m/sec}$$

Total length of both the trains = 121 + 99 = 220 m

Time required $= \frac{220}{20} = 11$ sec

So, option (b) is correct.

47. (a) Time taken by the 1st athlete

$$= \frac{4000}{16} = 250 \text{ unit}$$

Distance covered by 2nd Athlete

$= 15 \times 250 = 3750$ m

Distance covered by 3rd Athlete

$= 11 \times 250 = 2750$ m

Distance between 2nd and 3rd Athlete

$= 3750 - 2750 = 1000$ m

So, option (a) is correct.

48. (b) In a race of 100 m-

when A covers total 100 m.

B will cover = 100 – 4 = 96 m

C will cover = 100 – 2 = 98 m

When C covers 98 m, B covers = 96 m

When C covers 100 m, B will cover

$$= \frac{96}{98} \times 100 = \frac{4800}{49} \text{ m}$$

So C will beat B by $= 100 - \frac{4800}{49}$

$$\Rightarrow \frac{4900-4800}{49} = \frac{100}{49} = 2.04 \text{ m}$$

So, option (b) is correct

49. (d) Ratio of speed = 2 : 3 : 4

Ratio of time $= \frac{1}{2} : \frac{1}{3} : \frac{1}{4} = \frac{12}{2} : \frac{12}{3} : \frac{12}{4} = 6 : 4 : 3$

So, option (d) is correct.

50. (c) Let the speed of train = x km/hr

Distance of train = 225 m $= \frac{225}{1000}$ km

Time taken = 9 sec $= \frac{9}{3600}$ hr

Speed of man = 5 km/hr

Relative speed = Speed of train + speed of man

$= (x + 5)$ km/hr

But relative speed $= \dfrac{\frac{225}{1000}}{\frac{9}{3600}}$ km / hr

$$= \frac{225}{9 \times 1000} \times 3600 = 90 \text{km / hr}$$

$\Rightarrow x + 5 = 90 \quad \Rightarrow x = 85$ km/hr

$\therefore$ Option (c) is correct

51. (c) Let Average speed = x km/hr

Time taken = t hr

Distance = 14 km

$$\Rightarrow \quad x = \frac{14}{t} \quad \text{...(1)}$$

According to question,

Now Average speed = (x – 1) km/hr

Time taken $= \left(t + \frac{20}{60}\right)$hr $= \left(t + \frac{1}{3}\right)$hr

Distance = 14 km

$$\Rightarrow \quad x - 1 = \frac{14}{t + \frac{1}{3}}$$

$$\Rightarrow \quad \frac{14}{t} - 1 = \frac{14}{t + \frac{1}{3}} \qquad \left(\because x = \frac{14}{t}\right)$$

$$\Rightarrow \quad \frac{14-t}{t} = \frac{42}{3t+1}$$

$\Rightarrow (14 - t)(3t + 1) = 42t$

$\Rightarrow 42t - 3t^2 + 14 - t = 42t$

$\Rightarrow -3t^2 + 14 - t = 0$

$\Rightarrow \quad (3t+7)(t-2)=0$

$\Rightarrow \quad t = 2\text{ hr} \quad \left(t \neq -\frac{7}{3}\right)$

$\Rightarrow \quad x = \frac{14}{t} = \frac{14}{2}$ Stat

$x = 7$ km/hr

$\therefore$ Option (c) is correct

52. (d) Speed of bike = 25 – 50 km/h
Consumption of petrol at 25-50 hm/hr = 20 ml/km and consumes petrol = 40 ml/km at other speed
Given Distance

$50\text{ km} \longrightarrow \boxed{10\text{ km}} + \boxed{30\text{ km}} + \boxed{10\text{ km}}$

Speed $\longrightarrow$ 40 km/hr 60 km/hr 30 km/hr

Petrol consumed $= (10+10)\times 20\text{ ml} + 30\times 40\text{ ml}$
$= 400\text{ ml} + 1200\text{ ml}$
$= 1600\text{ ml}$

$= \frac{1600}{100}L = 1.6L$

$\therefore$ Option (d) is correct.

53. (d) Let usual speed of train = 4 km/hr
Time taken by train = t hr
Distance travelled = 120 km

So, $u = \frac{120}{t} \Rightarrow t = \frac{120}{4}\text{hr}$ ...(1)

According to question
New speed = (u + 10) km/hr.
Distance = 120 km
Time taken = (t – 1) hr

So $u + 10 = \frac{120}{t-1}$

(using (1))

$u + 10 = \frac{120}{\frac{120}{4} - 1} = \frac{120u}{120-u}$

$\Rightarrow \quad (u+10)(120-4) = 1204 \Rightarrow 1204 + 1200 - u^2 - 104 = 1204$

$\Rightarrow \quad u^2 + 10u - 1200 = 0$

$\Rightarrow \quad (u+40)(u-30) = 0 \Rightarrow u = -40$ is not possible

$\Rightarrow \quad u = 30$ km/hr

$\therefore$ Option (d) is correct.

54. (a) $V_B = \frac{100\text{ m}}{10\text{ sec}} = 10\text{ m/sec}$

$\frac{V_A}{V_B} = \frac{1000}{900} = \frac{10}{9}$

$\Rightarrow \quad \frac{V_A}{10} = \frac{10}{9} \Rightarrow V_A = \frac{100}{9}$

$\frac{450}{10} + \frac{x}{5} = \frac{1000}{\frac{100}{9}}$

$\Rightarrow \quad 45 + \frac{x}{5} = 90$

$\Rightarrow \quad x = 45 \times 5 = 225$

Now $450 + 225 = 675$

$1000 - 675 = 325$

$\frac{325}{5} = 65$

Hence A will beat B by 65 sec.

55. (c) A runs 5 rounds, B runs 4 rounds (ratio of speed)
A passes B each time A has run 5 rounds or

$5x\frac{1}{4} = \frac{5}{4}\text{ km} = 1\frac{1}{4}\text{ km}$

$1\frac{1}{4}$ km is contained in 4 km 3 times
hence A passes B thrice

56. (b) By the relation $\frac{S_1}{S_2} = \frac{T_2}{T_1}$

Time ratio

$= \frac{x}{x-20} = \frac{120}{100} = 5x = 6x - 120$

$x = 120$ minutes.

57. (d) Let speed of boat and current be x and y

$x + y = \frac{20}{2} = 10\ ;\ x - y = \frac{4}{2} = 2$

by adding both equation

$2x = 12$

$x = 6$

$y = 10 - 6 = 4$

58. (b) Let length of A train = 2l
length of B train = l

effective speed $= (48+42) \times \frac{5}{18}$ m/s

Case I

$\frac{3l}{90 \times \frac{5}{18}} = 12$

$l = 100$ m

Length of A train = 200 m

Case II let length of plat for m = P

$\frac{200+P}{48 \times \frac{5}{18}} = 45$

$200 + P = 45 \times \frac{48\times 5}{18}$

$\ell = 600$ m

$P = 400$ m

59. (d) When speed ratio is $S_1 : S_2 : S_3 : S_4$

than time ratio will be $\frac{1}{S_1} : \frac{1}{S_2} : \frac{1}{S_3} : \frac{1}{S_4}$ ...

i.e. time ratio $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$

$\Rightarrow 6 : 4 : 3$

60. (c) Radius of wheels = $\frac{80}{2}$ cm = 40 cm

Wheel covers in 1 revolution = $2\Pi r = 2 \times \frac{22}{7} \times 40$

Speed of wheel = 66 km/hr

or

$\frac{66 \times 1000 \times 100 \text{cm}}{60 \text{min}}$

Distance covered in 10 minutes

$= \frac{66 \times 1000 \times 100 \times 10}{60}$

No. of revolution = $\frac{66 \times 1000 \times 100 \times 10}{60 \times 2 \times 22 \times 40} \times 7 = 4375$

61. (c) Since distance is same in both ride than the average speed = $\frac{2xy}{x+y} = \frac{2 \times 50 \times 30}{30+50} = \frac{3000}{80} = 37.5$ km/hour

62. (c) Time taken for A to complete the race

$= 100 \times \frac{6}{10} = 60$ sec.

B took 60 + 12 sec to complete 96 m

So speed is $\frac{96}{72} = 1.33$ i.e. $\frac{4}{3}$ m/s

63. (d) Let time taken by passenger train = t

time taken by express train = t + 3

when distance = 540

ATQ

$\frac{540}{t} - \frac{540}{t+3} = 15 \qquad \because S = \frac{D}{T}$

$540\left[\frac{t+3-t}{t^2+3t}\right] = 15$

$108 = t^2 + 3t$

$t^2 + 3t - 108 = 0$

$t^2 + 12t - 9t - 108$

$(t-9)(t+12)$

t = 9 hr

Express train takes 9 hr

9pm + 9 hr = 6 am

64. (b) Length of train = 225 m

Speed of train = 30 km/h

Speed of Man = 3 km/hr

$\because$ the man is going in same direction than relatives speed = (30 – 3) = 27 km/h

Then time taken

$\frac{225}{27 \times \frac{5}{18}}$ sec $= \frac{225 \times 2}{3 \times 5} = 30$ seconds.

65. (c) Difference of distance covered by Police and thief is 100 m

Speed of Police = 10 km/h Speed of theif = 8 km/hr

let time be t

$10 \times \frac{5}{18} t - 8 \frac{5}{18} t = 100 \qquad [\because D = 8T]$

$\frac{5t}{9} = 100$

t = 180 S

Theif ran = $8 \times \frac{5}{18} \times 180 = 400$ m

66. (b) Here, we need to find the time that will take to cross 91 km stones completely.

Given that, in 1 hr. train travels 60 km i.e. 60 km is travelled in 60 min.

This means in 1 minute 1 km is travelled.

Therefore, 90 km is travelled in 90 minutes i.e. 1 hour 30 minutes and the remaining 1 km in 12 seconds.

Thus, The total time taken is 1 hr. 30 min. 12 sec.

67. (b)

100 m

A 8 m B 92 m B(A)

We are given that *A* gives *B* a start of 8 m. This means *B* starts from the point where *A* finishes its 8 m. therefore *B* covers 92 m.

Given that *A* runs at 6 km/hr.

i.e. *A* runs at $\frac{6 \times 1000 \text{m}}{60 \text{min}} = 100$ m/ min

It is also given that even after giving *B* a start of 8 m, *A* reaches early than *B* by 9 seconds.

Therefore, if *A* takes 60 sec to complete 100 m race, then *B* takes (60 + 9) seconds i.e. 69 seconds to complete 92 m.

$\frac{D}{T} = \frac{92}{69} \text{m/s} = \frac{92}{69} \times \frac{18}{5} = \frac{24}{5} \text{km/hr} = 4.8 \text{km/hr}$

68. (b) Distance travelled by the boy from house to school in 1 hr. i.e. 60 minutes = 12 km

Distance travelled by the boy from house to school in 1 minute = $\frac{12}{60} = \frac{1}{5}$ km

Similarly, distance travelled by the boy from school to house in 60 minutes = 8 km

Distance travelled by the boy from school to house in 1 minute = $\frac{8}{60} = \frac{2}{15}$ km

This means, total distance travelled in 2 minutes

$= \frac{1}{5} + \frac{2}{15} = \frac{3+2}{15} = \frac{5}{15} = \frac{1}{3}$ km

Therefore, total distance travelled in 1 minute

$= \frac{1}{3 \times 2} = \frac{1}{6}$ km

Thus, total distance travelled in 50 minutes

$= \frac{1}{6} \times 50 = 8.333... \approx 8$ km

69. (d) Let the speed of the man in still water be x km/hr and let the speed of the stream be y km/hr.

Speed of the man downstream = $x + y$ km/hr

Speed of the man upstream = $x - y$ km/hr

Therefore,

$x + y = \frac{18}{4}$ ...(1)

$x - y = \frac{18}{10} = 1.8$ km/hr ...(2)

Solving these equations by elimination method, we get

$2x = \frac{18}{4} + 1.8 = 4.5 + 1.8 = 6.3 \Rightarrow x = 3.15$ km/hr ...(3)

$3.15 - y = 3.15 - y = 1.8 \Rightarrow y = 3.15 - 1.8$

$\Rightarrow y = 1.35 \Rightarrow y = 1.35$ km/hr ...(4)

Therefore, equations (2), (3) and (4) implies that all the given statements are correct.

70. (a) Time taken to travel 12 km at the speed of 4 km/hr

$= \frac{12}{4} = 3$ hrs.

Time taken to travel 10 km at a speed of 5 km/hr

$= \frac{10}{5} = 2$ hrs.

Average speed = total distance ÷ total time taken

$= \frac{10+12}{3+2} = \frac{22}{5} = 4.4$ km/hr.

71. (c) Speed of train = 60 km/h $= 60 \times \frac{5}{18} = \frac{50}{3}$ m / s

time = 30 sec

length of train $= \frac{50}{3} \times 30 = 500$ m

72. (a) Let speed of passenger train be x km/h and speed of goods train be y km/h

Speed in same direction = x – y km/h

Speed in opposite direction = (x + y) km/h

Let total length of trains be 100 m

According to the question

$$\frac{100}{x-y} = \left(\frac{100}{x+y}\right)3$$

$$\frac{100}{x-y} = \frac{300}{x+y}$$

$\Rightarrow 100x + 100y = 300x - 300y$

$\Rightarrow 200x = 400y$ $\therefore$ x : y = 2 : 1

73. (b) Speed in still water = x km/h

Speed of stream = y km/h

Speed in down stream = (x + y) km/h

Speed in up stream = (x – y) km/h

Total time = Z hrs

Let distance be D km from each side

According to the question

$$\frac{D}{x+y} + \frac{D}{x-y} = Z \; ; \; \frac{Dx - Dy + Dx + Dy}{x^2 - y^2} = Z$$

$2x \times D = Z(x^2 - y^2)$

$$\therefore \; D = \frac{Z\left(x^2 - y^2\right)}{2x}$$

74. (c) V_1 = speed from Delhi to Agra = 60 km/h

V_2 = Speed from Agra to Delhi = y km/h

Average speed = 48 km/h

Average speed $= \frac{2V_1V_2}{V_1 + V_2}$

According to the question

$$\frac{2 \times 60 \times y}{60 + y} = 48$$

$\Rightarrow 120y = 48 \times 60 + 48y$

$\Rightarrow 120y - 48y = 48 \times 60$

$\Rightarrow 72y = 48 \times 60$

$\therefore \; y = \frac{48 \times 60}{72} = 40$ km / h

75. (d) Speed of the train (m/sec)

$$= \frac{\text{Length of (train + plateform)}}{\text{Time to cross the plateform}}$$

$$= \frac{(100+100)}{10} = \frac{200}{10} = 20 \, {}^{m}\!/_{sec}$$

Speed (in Kmph) $= 20 \times \frac{18}{5} = 72$ Kmph

76. (b) Time taken to cover 20 km$(t_1) = \frac{20}{40} = \frac{1}{2}$h

Time taken to cover 10 km $(t_2) = \frac{10}{10} = 1$h

Time taken to cover 30 km $(t_3) = \frac{30}{40} = \frac{3}{4}$h

Total time $t = t_1 + t_2 + t_3$

$\frac{1}{2} + 1 + \frac{3}{4} = \frac{9}{4}$

Speed $= \frac{\text{distance}}{\text{time}} = \frac{20+10+30}{9/4} = \frac{60 \times 4}{9}$

$= \frac{80}{3} = 26.67$ kmph

77. (c) A : B = 1000 : 850 = 20 : 17

B : C = 2600 : 3000 = 13 : 15

$\Rightarrow$ A : C = 20 × 13 : 15 × 17 = 52 : 51

Given, A runs 6000 m. = C runs $= \frac{6000}{52} \times 51 = 5884.61$

Hence A beats C by (60000 – 5884.61) = 115.39m

78. (a) Given that, time to overtake N = 72 min

The required result $= \left[\frac{720}{11} - N\right] \times \left[\left(\frac{60 \times 24}{N}\right)\right]$ min

$= \left[\frac{720}{11} - 72\right] \times \left[\frac{60 \times 24}{72}\right]$

$= \frac{-72}{11} \times \frac{60 \times 24}{72} = 121\frac{9}{11}$ min loss.

79. (b) Let thief covered the distance = x km
and x km covered by thief in t hours.
Owner sets off in another car after half an hour.
∴ x km covered by owner in (t – ½) hours.
∵ distance = Speed × time
$= 40 \times t = 60\,(t - ½) = 40t = 60\,t - 30 \Rightarrow 20t = 30$
t = 1.5 hours

distance $= 40 \times t = 40 \times \frac{3}{2} = 60\text{km}$

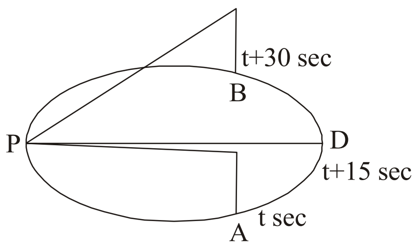
80. (a) Distance covered by A till 6pm = 60 km
Distance covered by A till 7pm = 120 km
Time taken by B to catch A = 60/(80 – 60) = 3 hrs
So A and B will meet at 6pm + 3 hrs = 9pm
Since we know that all three met at the some time
The time taken by C to cover 120 km difference will be = 9pm – 7pm = 2hrs
Therefore, $(x - 60) \times 2 = 120$
$\Rightarrow 2x - 120 = 120$
$2x = 240$
$\therefore x = 120$ km/hr.

81. (c) Let the distance between be D km
Time taken by Radha – Time taken by Hema = 9 mins
So D/8 – D/10 = 9/60 hrs

$$\frac{10D - 8D}{80} = \frac{9}{60} \Rightarrow \frac{2D}{80} = \frac{9}{60}$$

$$D = \frac{9 \times 80}{2 \times 60} = 6\text{km}$$

82. (c)



Let the plane be at point A at t seconds and at point B after t + 30 seconds
Since the motion is uniform, we can say that at time t + 15 seconds, the plane is above the point is diametrically opposite to the point P from where the angle is same.
Now since the time taken to cover the full circle is 3 minutes (180 seconds), the time taken by the plane to reach the diametrically opposite point will be 90 seconds.
So the time after which the plane reaches the point P will be = t + 15 + 90 seconds = (t + 105) seconds

83. (b) Let speed of train and Car are x and y km/hr respectively.

$$\frac{120}{x} + \frac{480}{y} = 11 \qquad ...(i)$$

$$\frac{200}{x} + \frac{400}{y} = \frac{35}{3} \qquad ...(ii)$$

from (i) and (ii)

$$\frac{1200}{y} = 20 \Rightarrow y = 60 \quad \text{and} \quad x = 40.$$

Now, x : y = 40 : 60
= 2 : 3

84. (c) Let us assume that speed of X = 5m/s
∴ speed of Y = 6m/s
Time taken by Y to cover 1.2 km race

$$t_Y = \frac{1200}{6} = 200\,\text{sec}$$

Time taken by X to cover 1.2 km race

$$t_X = \frac{1200 - 70}{5} = \frac{1130}{5} = 226\,\text{sec}$$

So Y wins the race by 26 sec.
Distance travelled by X in
26 sec = 26 × 5 = 130 m

85. (d) Let usual speed = v
increased speed = v + 5

$$\text{Now, } \frac{300}{v} - \frac{300}{v+5} = 2$$

$$\frac{300(v+5) - 300v}{v(v+5)} = 2$$

$\Rightarrow 2v^2 + 10v - 1500 = 0$
$\Rightarrow (v + 30)(2v - 50) = 0$
$\therefore v = -30, 25$
$\therefore$ v = 25 km/hr {since V is always + ve value}

86. (c) Time taken by X and Y to covered distance is 5x and 4x respectively
∴ 5x – 4x = 45 min
x = 45 min
∴ Time = 4x = 45 × 4 = 180 min

CHAPTER

11 Time & Work

1. 42 men take 25 days to dig a pond. If the pond would have to be dug in 14 days, then what is the number of men to be employed? **[2007-I]**
(a) 67 (b) 75 (c) 81 (d) 84

2. A person can do a job as fast as his two sons working together. If one son does the job in 6 days and the other in 12 days, how many days does it take the father to do the job? **[2007-I]**
(a) 9 days (b) 6 days (c) 4 days (d) 3 days

3. The ratio of the radii of the taps T_1 and T_2 is 2:1. Water is flowing through them with the same velocity. What is ratio of the time required to completely fill two identical drums kept under T_1 and T_2? **[2007-I]**
(a) 2 : 1 (b) 1 : 2 (c) 4 : 1 (d) 1 : 4

4. *A* can finish a work in 8 days and *B* can do it in 12 days. After *A* had worked for 3 days, *B* also joins *A* to finish the remaining work. In how many days will the remaining work be finished? **[2007-II]**
(a) 2 days (b) 3 days (c) 4 days (d) 5 days

5. If 18 men earn ₹ 1440 in 5 days, how many men can earn ₹ 1920 in 8 days? **[2007-II]**
(a) 10 (b) 12 (c) 15 (d) 18

6. Ram can do a piece of work in 6 days and Shyam can finish the same work in 12 days. How much work will be finished, if both work together for 2 days? **[2008-I]**
(a) One-fourth of the work
(b) One-third of the work
(c) Half of the work
(d) Whole of the work

7. If one man or two women or three boys can do a peice of work in 55 days, then one man, one woman and one boy will do it how many days? **[2008-II]**
(a) 20 days (b) 30 days
(c) 40 days (d) 50 days

8. If *m* men can do a job in *p* days, then (*m*+*r*) men can do the job in how many days? **[2008-II]**
(a) $(p+r)$ days (b) $\frac{mp}{m+r}$ days
(c) $\frac{p}{m+r}$ days (d) $\frac{m+r}{p}$ days

9. *A* and *B* can do a piece of work in 8 days, *B* and *C* can do the same work in 12 days. If *A,B* and *C* can complete the same work in 6 days, in how many days can *A* and *C* complete the same work? **[2009-I]**
(a) 8 days (b) 10 days
(c) 12 days (d) 16 days

10. Two taps can fill a tub in 5 min and 7 min respectively. A pipe can empty it in 3 min. If all the three are kept open simultaneously, when will the tub be full? **[2009-I]**
(a) 60 min (b) 85 min (c) 90 min (d) 105 min

11. *P* and *Q* can do a job in 2 days; *Q* and *R* can do it in 4 days and *P* and *R* in 12/5 days. What is the number of days required for *P* alone to do the job? **[2009-II]**
(a) 5/2 (b) 3 (c) 14/5 (d) 6

12. The mess charges for 35 students for 24 days in ₹6300. In how many days will the mess charges be ₹ 3375 for 25 students? **[2009-II]**
(a) 12 (b) 15 (c) 18 (d) 21

13. A person walks a distance in 114 days, when he rests 9h a day. How long will he take to walk twice the distance, if he walks twice as fast and rests twice as long each day as before? **[2009-II]**
(a) 57 days (b) 228 days
(c) 285 days (d) 324 days

14. In an army camp ration is available for 100 soldiers for 10 days. After 2 days, 60 soldiers joined. Then, for how many more days will the remaining ration last? **[2009-II]**
(a) 7 days (b) 6 days (c) 5 days (d) 4 days

15. $\frac{1}{48}$ of a work is completed in half a day by 5 persons. Then, $\frac{1}{40}$ of the work can be completed by 6 persons in how many days? **[2010-I]**
(a) 1 (b) 2 (c) 3 (d) $\frac{1}{2}$

16. If 6 men and 8 boys can do a peice of work in 10 days while 26 men and 48 boys can do the same in 2 days, what is the time taken by 15 men and 20 boys in doing the same type of work? **[2010-I]**
(a) 4 days (b) 5 days
(c) 6 days (d) 7 days

17. A can do a piece of work in 24 days. If *B* is 60% more efficient then *A,* then how many days does *B* require to do the same work? **[2010-II]**
(a) 12 (b) 15 (c) 16 (d) 18

18. Four taps can individually fill a cistern of water in 1h, 2h, 3h and 6h , respectively. If all the four taps are opened simultaneously, the cistern can be filled in how many minutes? **[2011-I]**
(a) 20 (b) 30 (c) 35 (d) 40

19. *X* completes a job in 2 days and *Y* completes it in 3 days and *Z* takes 4 days to complete it. If they work together and get ₹3900 for the job, then how much amount does *Y* get? **[2011-I]**
(a) ₹ 1800 (b) ₹ 1200 (c) ₹ 900 (d) ₹ 800

20. A garrison of 'n' men had enough food to last for 30 days. After 10 days, 50 more men joined them. If the food now lasted for 16 days, what is the value of n? **[2011-I]**
(a) 200 (b) 240 (c) 280 (d) 320

21. Consider the following statements :
I. If 18 men can earn ₹ 1440 in 5 days, then 10 men can earn ₹1280 in 6 days.
II. If 16 men can earn ₹1120 in 7 days, then 21 men can earn ₹ 800 in 4 days.
Which of the above statements is/are correct? **[2011-I]**
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

22. 15 men complete a work in 16 days. If 24 men are employed, then the time required complete that work will be **[2014-I]**
(a) 7 days (b) 8 days
(c) 10 days (d) 12 days

23. A, B and C can do a piece of work individually in 8, 12 and 15 days, respectively. A and B start working but A quits after working for 2 days. After this, C joins B till the completion of work. In how many days will the work be completed ? **[2014-I]**
(a) $5\frac{8}{9}$ days (b) $4\frac{6}{7}$ days
(c) $6\frac{7}{13}$ days (d) $3\frac{3}{4}$ days

24. A can do a piece of work in 4 days and B can complete the same work in 12 days. What is the number of days required to do the same work together ? **[2014-I]**
(a) 2 days (b) 3 days (c) 4 days (d) 5 days

25. A bus starts with some passengers. At the first stop, one–fifth of the passengers gets down and 40 passengers get in. At the second stop, half of the passengers gets down and 30 get in. The number of passengers now is 70. The number of passengers with which the bus started was **[2014-I]**
(a) 40 (b) 50 (c) 60 (d) 70

26. X can do a work in 16 days. In how many days will the work be completed by Y, if the efficiency of Y is 60% more than of X? **[2014-I]**
(a) 10 days (b) 12 days
(c) 25 days (d) 30 days

27. 2 men and 1 woman can complete a piece of work in 14 days, while 4 women and 2 men can do the same work in 8 days. If a man gets ₹ 90 per day, what should be the wages per day of a woman ? **[2014-I]**
(a) ₹ 48 (b) ₹ 60 (c) ₹ 72 (d) ₹ 135

28. 18 men can earn ₹ 360 in 5 days. How much money will 15 men earn in 9 days ? **[2014-I]**
(a) ₹ 600 (b) ₹ 540 (c) ₹ 480 (d) ₹ 360

29. 20 workers working for 5 h per day complete a work in 10 days. if 25 workers are employed to work 10 h per day, what is the time required to complete the work ? **[2014-I]**
(a) 4 days (b) 5 days (c) 6 days (d) 8 days

30. A can finish a work in 15 days, B in 20 days and C in 25 days. All these three worked together and earned ₹ 4700. The share of C is **[2014-I]**
(a) ₹ 1200 (b) ₹ 1500 (c) ₹ 1800 (d) ₹ 2000

31. 4 goats or 6 sheeps can graze a field in 50 days. 2 goats and 9 sheeps can graze the field in **[2014-I]**
(a) 100 days (b) 75 days
(c) 50 days (d) 25 days

32. Pipe A can fill a tank in 10 min and pipe B can empty it in 15 min. If both the pipes are opened in an empty tank, the time taken to make it full is **[2014-I]**
(a) 20 min (b) 25 min
(c) 30 min (d) None of these

33. X can complete a job in 12 days. If X and Y work together, they can complete the job in $6\frac{2}{3}$ days. Y alone can complete the job in **[2014-I]**
(a) 10 days (b) 12 days
(c) 15 days (d) 18 days

34. A mason can build a tank in 12 h. After working for 6 h, hetook the help of a boy and finished the work in another 5h. The time that the boy will take alone to complete the work is **[2014-I]**
(a) 30h (b) 45h (c) 60h (d) 64h

35. A swimming pool 70m long, 44m wide and 3 m deep is filled by water flowing from a pipe at the rate of 30800 cm^3/s. The time taken to fill the swimming pool is **[2014-I]**
(a) 71/2 h (b) 80 h
(c) 250/3 h (d) None of these

36. If 3 men and 4 boys can do a piece of work in 8 days, then 6 men and 8 boys can do the same work in **[2014-I]**
(a) 2 days (b) 4 days (c) 6 days (d) 16 days

37. X can do a piece of work in 25 days. Y is 25% more efficient than X. The number of days taken by Y is **[2014-I]**
(a) 15 days (b) 20 days
(c) 21 days (d) 30 days

38. 45 people take 18 days to dig a pond. If the pond would have to be dug in 15 days, then the number of people to be employed will be **[2014-I]**
(a) 50 (b) 54 (c) 60 (d) 72

39. A and B can do a piece of work in 10 h. B and C can do it in 15 h, while A and C take 12 h to complete the work. B independently can complete the work in **[2014-I]**
(a) 12h (b) 16h (c) 20h (d) 24h

40. There are two taps A and B to fill up a water tank. The tank can be filled in 40 min, if both taps are on. The same tank can be filled in 60 min, if tap A alone is on. How much time will tap B alone take, to fill up the same tank? **[2014-I]**
(a) 64 min (b) 80 min (c) 96 min (d) 120 min

41. A stock of food is enough for 240 men for 48 days. How long will the same stock last for 160 men? **[2014-I]**
(a) 54 days (b) 60 days
(c) 64 days (d) 72 days

42. *A* can do a piece of work in '*x*' days and *B* can do the same work $3x$ days, To fininsh the work together they take 12 days. What is the value of '*x*' **[2014-I]**

(a) 8 (b) 10 (c) 12 (d) 16

43. *A,B* and *C* can do a peice of work individually in 8, 10 and 15 days, respectively. *A* and *B* start working but *A* quits after working for 2 days. After this, *C* joins *B* till the completion of work. In how many days will the work be completed? **[2014-I]**

(a) 53/9 days (b) 34/7 days
(c) 85/13 days (d) 53/10 days

44. 76 ladies complete a job in 33 days. Due to some reason some ladies did not join the work and therefore, it was completed in 44 days. The number of ladies who did not report for the work is **[2014-I]**

(a) 17 (b) 18
(c) 19 (d) 20

45. How many men will be required to plough 100 acres of land in 10 days, if 10 men require 8 days to plough 20 acres of land? **[2014-I]**

(a) 30 (b) 40
(c) 50 (d) 60

46. Pipe *A* can fill a tank in 3 *h* but there is a leakage also, due to which it takes 3.5 *h* for the tank to be filled. How much time will the leakage take in emptying the tank, if the tank is filled initially? **[2014-II]**

(a) 21 *h* (b) 20 *h*
(c) 18 *h* (d) 10.5 *h*

47. *A*, *B* and *C* can do a piece of work individually in 8, 12 and 15 days, respectively. *A* and *B* start working together but *A* quits after working for 2 days. After this, *C* joins and works till completion of the work. In how many days, will the work be completed ? **[2014-II]**

(a) $3\frac{8}{9}$ days (b) $5\frac{8}{9}$ days

(c) $5\frac{2}{3}$ days (d) $6\frac{1}{18}$ days

48. *A* is thrice as efficient as *B* and hence completes a work in 40 days less than the number of days taken by *B*. What will be the number of days taken by both of them when working together ? **[2014-II]**

(a) 22.5 days (b) 15 days
(c) 20 days (d) 18 days

49. If 10 persons can dig 8 ft trench in 12 days, then how many days will 8 persons take to dig 6 ft trench ? **[2014-II]**

(a) 10 days (b) 10.25 days
(c) 11 days (d) 11.25 days

50. The efficiency of *P* is twice that of *Q*, whereas the efficiency of *P* and *Q* together is three times that of *R*. If *P*, *Q* and *R* work together on a job, in what ratio should they share their earnings? **[2015-I]**

(a) 2 : 1 : 1 (b) 4 : 2 : 1
(c) 4 : 3 : 2 (d) 4 : 2 : 3

51. A and B are two taps which can fill a tank individually in 10 minutes and 20 minutes respectively. However, there is a leakage at the bottom, which can empty a filled tank in 40 minutes. If the tank is empty initially, how much time will both the taps take to fill the tank with leakage ? **[2015-II]**

(a) 2 minutes (b) 4 minutes
(c) 5 minutes (d) 8 minutes

52. If 4 men working 4 hours per day for 4 days complete 4 units of work, then how many units of work will be completed by 2 men working for 2 hours per day in 2 days ? **[2015-II]**

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{8}$.

53. If m persons can paint a house in d days, how many days will it take for (m +2) persons to paint the same house? **[2015-II]**

(a) $md+2$ (b) $md-2$

(c) $\frac{m+2}{md}$ (d) $\frac{md}{m+2}$

54. Two pipes A and B can fill a tank in 60 minutes and 75 minutes respectively. There is also an outlet C. If *A*, *B* and *C* are opened together, the tank is full in 50 minutes. How much time will be taken by *C* to empty the full tank? **[2016-I]**

(a) 100 minutes (b) 110 minutes
(c) 120 minutes (d) 125 minutes

55. A can do 50% more work than B in the same time. B alone can do a piece of work in 30 hours. B starts working and had already worked for 12 hours when A joins him. How many hours should B and A work together to complete the remaining work ? **[2016-II]**

(a) 6 hours (b) 12 hours
(c) 4.8 hours (d) 7.2 hours

56. A tank can be filled by pipe X in 2 hours and pipe Y in 6 hours. At 10 am. pipe X was opened. At what time will the tank be filled if pipe Y is opened at 11 am. ? **[2016-II]**

(a) 12:45 hours (b) 5:00 pm.
(c) 11:45 a.m. (d) 11:50 am.

57. If 15 men take 21 days of 8 hours each to do a piece of work, then what is the number of days of 6 hours each that 21 women would take, if 3 women would do as much work as 2 men ? **[2017-I]**

(a) 18 (b) 20 (c) 25 (d) 30

58. A and B working together can finish a piece of work in 12 days while B alone can finish it in 30 days. In how many days can A alone finish the work? **[2017-I]**

(a) 18 days (b) 20 days
(c) 24 days (d) 25 days

59. If 5 men can do a piece of work in 10 days and 12 women can do the same work in 15 days, the number of days required to complete the work by 5 men and 6 women is **[2017-I]**

(a) $7\frac{1}{2}$ days (b) 8 days

(c) $9\frac{1}{2}$ days (d) 12 days

60. 30 men can complete a job in 40 days. However, after 24 days some men out of the assigned 30 left the job. The remaining people took another 40 days to complete the job. The number of men who left the job is **[2017-II]**
(a) 24 (b) 18 (c) 12 (d) 6

61. 4 goats or 6 sheep can graze a field in 50 days. 2 goats and 3 sheep will graze it in **[2017-II]**
(a) 200 days (b) 150 days
(c) 100 days (d) 50 days

62. A tap can fill a tub in 10 hours. After opening the tap for 5 hours it was found that a small outlet at the bottom of the tub was open and water was leaking through it. It was then immediately closed. It took 7 hours to fill the tub after closing the outlet. What time will be taken by the outlet to empty the full tub of water? **[2017-II]**
(a) 35 hours (b) 25 hours
(c) 20 hours (d) 17 hours

63. A work when done by 10 women is completed in 12 days. The same work can be completed in 8 days when done by 5 men. How many days will it take to completed when 6 women and 3 men are employed to perform the same job? **[2018-I]**
(a) 12 (b) 10 (c) 8 (d) 5

64. A man undertakes to do a certain work in 150 days. He employs 200 men. He finds that only a quarter of the work is done in 50 days. How many additional men should he employ so that the whole work is finished in time? **[2018-I]**
(a) 75 (b) 85 (c) 100 (d) 120

65. If 5 tractors can plough 5 hectares of land in 5 days, then what is the number of tractors required to plough 100 hectares in 50 days? **[2018-I]**
(a) 100 (b) 20 (c) 10 (d) 5

66. A water tank has been fitted with two taps P and Q and a drain pipe R. Taps P and Q fill at the rate of 12 litres per minute and 10 litres per minute respectively.
Consider the following statements S1, S2 and S3 :
S1 : Pipe R drains out at the rate of 6 litres per minute.
S2 : If both the taps and the drain pipe are opened simultaneously, then the tank is filled in 5 hours 45 minutes.
S3 : Pipe R drains out (fully) the filled tank in 15 hours 20 minutes.
To know what is the capacity of the tank, which one of the following is correct? **[2018-I]**
(a) S2 is only sufficient
(b) S1, S2 and S3 are necessary
(c) Any two out of S1, S2 and S3 are sufficient
(d) None of the above

67. Two workers 'A' and 'B' working together completed a job in 5 days. Had 'A' worked twice as efficienctly as he actually did and 'B' worked one-third as efficiently as he actually did, the work would have completed in 3 days. In how many days could 'A' alone complete the job? **[2018-II]**
(a) $3\frac{1}{2}$ days (b) $4\frac{1}{6}$ days
(c) $5\frac{1}{2}$ days (d) $6\frac{1}{4}$ days

68. 'A' is thrice as good a workman as 'B' and takes 10 days less to do a piece of work than 'B' takes. The number of days taken by 'B' alone to finish the work is **[2018-II]**
(a) 12 (b) 15 (c) 20 (d) 30

69. Twelve (12) men work 8 hours per day and require 10 days to build a wall. If 8 men are available, how many hours per day must they work to finish the work in 8 days? **[2018-II]**
(a) 10 hours (b) 12 hours
(c) 15 hours (d) 18 hours

70. X and Y together can finish a job in 6 days. X can alone do the same job in 12 days. How long will Y alone take to do the same job? **[2018-II]**
(a) 16 days (b) 12 days
(c) 10 days (d) 8 days

71. Twelve (12) persons can paint 10 identical rooms in 16 days. In how many days can 8 persons paint 20 such rooms? **[2018-II]**
(a) 12 (b) 24 (c) 36 (d) 48

72. A field can be reaped by 12 men or 18 women in 14 days. In how many days can 8 men and 16 women reap it ? **[2019-I]**
(a) 26 days (b) 24 days (c) 9 days (d) 8 days

73. Tushar takes 6 hours to complete a piece of work, while Amar completes the same work in 10 hours. If both of them work together, then what is the time required to complete the work ? **[2019-I]**
(a) 3 hours (b) 3 hours 15 minutes
(c) 3 hours 30 minutes (d) 3 hours 45 minutes

74. If the ratio of the work done by $(x + 2)$ workers in $(x - 3)$ days to the work done by $(x + 4)$ workers in $(x - 2)$ days is 3 : 4, then what is the value of x ? **[2019-II]**
(a) 8 (b) 10 (c) 12 (d) 15

75. Two taps X and Y are fixed to a water tank. If only X is opened, it drains out the full tank of water in 20 minutes. It both X and Y are opened, then they drain out the full tank of water in 15 minutes. If only Y is opened, how long does it take to drain out the full tank of water ? **[2019-II]**
(a) 30 minutes (b) 45 minutes
(c) 60 minutes (d) 90 minutes

76. A stock of food grains is enough for 240 men for 48 days. How long will the same stock last for 160 men ? **[2019-II]**
(a) 72 days (b) 64 days (c) 60 days (d) 54 days

77. If 6 men and 8 women can do a piece of work in 10 days; and 13 men and 24 women can do the same work in 4 days, then what is the ratio of daily work done by a man to that of a woman ? **[2020-I]**
(a) 2 : 1 (b) 1 : 2 (c) 4 : 3 (d) 3 : 4

78. In a water tank there are two outlets. It takes 20 minutes to empty the tank if both the outlets are opened. If the first outlet is opened, the tank is emptied in 30 minutes. What is the time taken to empty the tank by second outlet ? **[2020-I]**
(a) 30 minutes (b) 40 minutes
(c) 50 minutes (d) 60 minutes

HINTS & SOLUTIONS

1. (b) Let the number of men be n?

Men	Days
$42 \downarrow$	$25 \uparrow$
n	14

$\therefore \frac{n}{42} = \frac{25}{14} \Rightarrow n = 75$

2. (c) One day's work of first son = $\frac{1}{6}$

One day's work of second son = $\frac{1}{12}$

$\therefore$ One day's work of them working together

$= \frac{1}{6} + \frac{1}{12} = \frac{2+1}{12} = \frac{1}{4}$

father will finish the work in 4 days.

3. (d) Given that, $\frac{T_1}{T_2} = \frac{2}{1}$

$\therefore \frac{\text{Area of first drum}}{\text{Area of second drum}} = \frac{\pi r_1^2}{\pi r_2^2} = \frac{4}{1}$

time taken will be inversely proportional to the area

then required time ratio $= \frac{1}{4} = 1:4$

4. (b) Work of A in one day = $\frac{1}{8}$th part

Work of B in one day = $\frac{1}{12}$th part

3 days work of A = $\frac{3}{8}$th part

Remaining work of A = $1 - \frac{3}{8} = \frac{5}{8}$th part

Together one day's work of A and B = $\frac{1}{8} + \frac{1}{12}$

$= \frac{3+2}{24} = \frac{5}{24}$

$\therefore$ Number of days to finish the work

$= \frac{5}{8} \div \frac{5}{24} = 3$ days

5. (c) According to the formula

$\frac{M_1 D_1}{W_{a_1}} = \frac{M_2 D_2}{W_{a_2}}$

$\Rightarrow \frac{18 \times 5}{1440} = \frac{m_2 \times 8}{1920}$

$\therefore m_2 = \frac{1920 \times 18 \times 5}{8 \times 1440} = 15$

6. (c) $\therefore$ One day's work of Ram = $\frac{1}{6}$

$\therefore$ One day's work of Shyam = $\frac{1}{12}$

One day's work together, Ram and Shyam

$= \frac{1}{6} + \frac{1}{12} = \frac{2+1}{12} = \frac{3}{12} = \frac{1}{4}$

$\therefore$ Two day's work together = $\frac{1}{2}$ (half of the work)

7. (b) 1 man = 2 women = 3 boys

$\therefore$ 1 man + 1 woman + 1 boy

= 3 boys + $\frac{3}{2}$ boys + 1 boy

$\therefore$

Boys	Days
$3 \uparrow$	$55 \downarrow$
$\frac{11}{2}$	x

$M_1 D_1 = M_2 D_2$

$\Rightarrow 3 \times 55 = \frac{11}{2} \times D_2$

$D_2 = \frac{3 \times 55 \times 2}{11} = 30$ days

8. (b)

Men	Days
m	p
$(m+r) \uparrow$	$x \downarrow$

$\therefore \frac{x}{p} = \frac{m}{m+r}$

$\Rightarrow x = \frac{mp}{m+r}$ days

9. (a) $A's$ and $B's$ one day work = $\frac{1}{8}$

$B's$ and $C's$ one day work = $\frac{1}{12}$

$A's,\ B's$ and $C's$ one day work $= \frac{1}{6}$

$B's$ one day work $= \frac{1}{8}+\frac{1}{12}-\frac{1}{6}=\frac{1}{24}$

$A's$ and $C's$ one day work $= \frac{1}{6}-\frac{1}{24}=\frac{3}{24}=\frac{1}{8}$

A and C can do the work in 8 days.

10. (d) Net filling in 1 min $= \frac{1}{5}+\frac{1}{7}-\frac{1}{3}$

$= \frac{21+15-35}{105}=\frac{1}{105}$

the tub will be full in 105 min.

11. (b) $P's$ and $Q's$ 1 day work $= \frac{1}{2}$

$Q's$ and $R's$ 1 day work $= \frac{1}{4}$

$P's$ and $R's$ 1 day work $= \frac{5}{12}$

$\Rightarrow$ Now, $(P+Q)+(Q+R)+(P+R)=\frac{1}{2}+\frac{1}{4}+\frac{5}{12}$

$\Rightarrow 2P+2Q+2R=\frac{6+3+5}{12} \Rightarrow 2(P+Q+R)=\frac{14}{12}$

$P+Q+R$ work in one day $= \frac{14}{24}=\frac{7}{12}$

$Q+R$ work in one day $= \frac{1}{4}$

P alone work in one day $= \frac{7}{12}-\frac{1}{4}=\frac{7-3}{12}=\frac{4}{12}=\frac{1}{3}$

$P's$ can do alone in 3 days.

12. (c) From formula

$\frac{M_1D_1}{W_1}=\frac{M_2D_2}{W_2} \Rightarrow \frac{35\times 24}{6300}=\frac{M_2\times 25}{3375}$

$\therefore M_2=\frac{35\times 24\times 3375}{6300\times 25}=18$

13. (c) **Distance** **Speed** **Hour/Day** **Speed**

$\begin{matrix}1\\2\end{matrix}\downarrow$ $\begin{matrix}1\\2\end{matrix}\uparrow$ $\begin{matrix}15\\6\end{matrix}\uparrow$ $\begin{matrix}114\\x\end{matrix}\downarrow$

$$\left.\begin{matrix}1:2\\2:1\\6:15\end{matrix}\right\}114:x$$

$\therefore 1\times 2\times 6\times x=2\times 1\times 15\times 114$

$\Rightarrow x=\frac{2\times 15\times 114}{2\times 6}$ = 285 days

14. (c) **Soldiers** **Days**

$\begin{matrix}100\\160\end{matrix}\uparrow$ $\begin{matrix}8\\x\end{matrix}\downarrow$

$\Rightarrow \frac{160}{100}=\frac{8}{x}$

$\therefore x=\frac{8\times 100}{160}$ = 5 days

15. (d) $\because \frac{M_1D_1}{W_1}=\frac{M_2D_2}{W_2} \Rightarrow \frac{5\times\frac{1}{2}}{\frac{1}{48}}=\frac{6\times D_2}{\frac{1}{40}}$

$\Rightarrow D_2=\frac{\frac{1}{2}\times 5\times 48}{40\times 6}=\frac{1}{2}$

16. (a) $6M + 8B$ = 10 days ...(i)

26M + 48B = 2 days ... (ii)

15M + 20 B = ?

By to formula $\Rightarrow$ 20B = 10 days

$M_1 D_1 = M_2 D_2 \Rightarrow (6M+8B)\times 10=(26M+48B)\times 2$

$\Rightarrow$ 60M + 80B = 52M + 96B $\Rightarrow$ 8M = 16B

M = 2B

Now in eq. (i), put M = 2B

6 × 2B + 8B = 10 days

12B + 8B = 10 days

Again

15M + 20B = 15 × 2B + 20B = 30B + 20B = 50B

From formula, $M_1 D_1 = M_2 D_2$

$\Rightarrow 20\times 10=50\times D_2$

$D_2=\frac{20\times 10}{50}=D_2=4$ days.

17. (b) $\underset{100}{A}\ \xrightarrow{(+\ 60\%)}\ \underset{160}{B}$

efficiency is proportional to the number of days

$\Rightarrow n_1D_1=n_2D_2 \Rightarrow 100\times 24=160\times D_2$

$\therefore D_2=\frac{100\times 24}{160}=15$ days

18. (b) Required time $= \frac{1}{\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{6}}=\frac{6}{6+3+2+1}$

$= \frac{6}{12}\text{h}=\frac{6}{12}\times 60$ min = 30 min

19. (b) Ratio of work done by X, Y and $Z = \frac{1}{2}:\frac{1}{3}:\frac{1}{4}$

= 6 : 4 : 3

ratio in their amount = 6 : 4 : 3

$\therefore$ Part of $Y = \frac{4}{6+4+3}\times 3900 =$ ₹ 1200

20. (a) $M_1 = n$, $d_1 = 30 - 10 = 20$, $M_2 = (50 + n)$ $d_2 = 16$

By formula,

$M_1 d_1 = M_2 d_2$

$\Rightarrow \quad n \times 20 = (n + 50) \times 16$

$\Rightarrow 20n = 16n + 800$

$\Rightarrow 4n = 800$

$\Rightarrow n = 200$

21. (d) **From statement I**

$$\frac{M_1 d_1}{W_{a_1}} = \frac{M_2 d_2}{W_{a_2}}$$

Here $M_1 = 18$, $d_1 = 5$ $W_{a_1} = 1440$

$M_2 = 10$, $d_2 = 6$, $W_{a_2} = ?$

$$\Rightarrow \quad \frac{18 \times 5}{1440} = \frac{10 \times 6}{W_{a_2}}$$

$$\Rightarrow \quad W_{a_2} = \frac{10 \times 6 \times 1440}{18 \times 5} = ₹\ 960$$

But Wa_2 is given as 1280 so it is not correct.

From statement II

$$\frac{M_1 d_1}{W_{a_1}} = \frac{M_2 d_2}{W_{a_2}}$$

Here $M_1 = 16$, $d_1 = 7$, $W_{a1} = 1120$

$M_2 = 21$, $d_2 = 4$, $W_{a2} = ?$

$$\Rightarrow \quad \frac{16 \times 7}{1120}$$

$$= \frac{21 \times 4}{W_{a_2}}$$

$$Wa_2 = \frac{21 \times 4 \times 1120}{16 \times 7} = 3 \times 280$$

$= ₹\ 840$

Now, $Wa_2 = ₹\ 800$

So both statements are not correct.

22. (c) According to the formula,

$$\frac{M_1 D_1}{W_1} = \frac{M_2 D_2}{W_2}$$

Here, $M_1 = 15$, $D_1 = 16$, $W_1 = W_2 = 1$

$M_2 = 24$ and $D_2 = ?$

$$\Rightarrow \quad \frac{15 \times 16}{1} = \frac{24 \times D_2}{1}$$

$$D_2 = \frac{15 \times 16}{24} = 10$$

10 days are required to complete the work.

23. (c) Work done by A and B in 1 day $= \frac{1}{8} + \frac{1}{12} = \frac{5}{24}$

2 days work of A and B $= \frac{10}{24}$

After 2 days A left the work.

$\therefore$ Remaining work $= 1 - \frac{10}{24} = \frac{14}{24}$

work of B and C together $= \frac{1}{12} + \frac{1}{15} = \frac{9}{60}$

the number of days required by B and C to finish work

$$= \frac{\frac{14}{24}}{\frac{9}{60}} = \frac{14}{24} \times \frac{60}{9} = \frac{35}{9}$$

$\therefore$ Total days to complete the work

$= 2 + \frac{35}{9} = \frac{53}{9} = 5\frac{8}{9}$ days

24. (b) A's one day work $= \frac{1}{4}$

B's one day work $= \frac{1}{12}$

One day work of A and B together $= \frac{1}{4} + \frac{1}{12}$

$$= \frac{3+1}{12} = \frac{4}{12} = \frac{1}{3}$$

No. of days together = 3 days.

25. (b) Let bus starts with x number of passengers.

After 1st stoppage, number of passengers

$$= x - \frac{x}{5} + 40 = \frac{5x - x + 200}{5}$$

$$= \frac{4x + 200}{5}$$

After 2nd stoppage, number of passengers

$$= \frac{4x + 200}{5} - \frac{4x + 200}{5 \times 2} + 30$$

$$\Rightarrow \frac{4x + 200}{5} - \frac{4x + 200}{10} + 30 = 70$$

$$\Rightarrow \frac{4x + 200}{5}\left(1 - \frac{1}{2}\right) + 30 = 70$$

$$\Rightarrow \frac{4x + 200}{10}\left(\frac{2-1}{2}\right) = 40$$

$\Rightarrow \frac{4x+200}{5} \times \frac{1}{2} = 40$

$\Rightarrow \frac{4x+200}{10} = 40$

$\Rightarrow$ 4x + 200 = 400 $\Rightarrow$ 4x = 200

$\therefore$ x = $\frac{200}{4}$ = 50

26. (a) Efficiency is proportional to days

$\underset{100}{X} \xrightarrow{(+60\%)} \underset{160}{Y}$

$\Rightarrow$ 100 × 16 = 160 × D

$\therefore$ D = $\frac{100 \times 16}{160}$ = 10 days

27. (b) $M_1D_1 = M_2D_2$

$\Rightarrow (2M+1W) \times 14 = (4W+2M) \times 8$

$\Rightarrow 28M+14W = 32W+16M$

$\Rightarrow 12M = 18W$

$\frac{M}{W} = \frac{18}{12} = \frac{3}{2}$

Now, a man gets ₹ 90 per day

$\therefore$ 1 woman wages = $\frac{2}{3}$ × (wages of 1 man)

= $\frac{2}{3}$ × 90 = ₹ 60.

28. (b) $\frac{M_1D_1}{W_1} = \frac{M_2D_2}{W_2}$

$\Rightarrow$ Here M_1 = 18, D_1 = 5, W_1 = ₹ 360

M_2 = 15, D_2 = 9, W_2 = ?

$\Rightarrow$ 18 × 5 × W_2 = 15 × 9 × 360

$\therefore$ $W_2 = \frac{15 \times 9 \times 360}{18 \times 5}$ = ₹ 540

29. (a) $M_1D_1T_1 = M_2D_2T_2$

Here $M_1 = 20_1$, $D_1 = 10_1 T_1$ = 5h, M_2 = 25, D_2 = ?, T_2 = 10 h

$\Rightarrow$ 20 × 10 × 5 = 25 × D_2 × 10

$\therefore$ $D_2 = \frac{20 \times 10 \times 5}{25 \times 10}$ = 4 days

30. (a) A's one day work = $\frac{1}{15}$

B's one day work = $\frac{1}{20}$

C's one day work = $\frac{1}{25}$

A, B and C worked together one day work

$= \frac{1}{15} + \frac{1}{20} + \frac{1}{25}$

$= \frac{20+15+12}{300} = \frac{47}{300}$

time taken to complete work by A, B and C working together = $\frac{300}{47}$

$\therefore$ Share of C = $\frac{1}{25} \times \frac{300}{47} \times 4700$ = ₹ 1200

31. (d) Given that

1 Goats = $\frac{3}{2}$ sheeps.

Now, 2 goats + 9 sheeps

$= 2 \times \frac{3}{2}$ sheeps + 9 sheeps = 12 sheeps

Here $M_1D_1 = M_2 D_2$

$\Rightarrow 6 \times 50 = 12 \times D_2$

D_2 = 25 days

32. (c) Part filled by pipe A in 1 min = $\frac{1}{10}$

Part empty by pipe B in 1 min = $\frac{1}{15}$

$\therefore$ Total tank filled in minutes

$= \frac{1}{10} - \frac{1}{15} - \frac{3-2}{30} = \frac{1}{30}$

Hence, the tank will be filled in 30 min.

33. (c) $X's$ one day's work = $\frac{1}{12}$

$(X+Y)'s$ one day's work = $\frac{3}{20}$

$\therefore$ Y's one day's work = $\frac{3}{20} - \frac{1}{12} = \frac{4}{60} = \frac{1}{15}$

$\therefore$ Number of day's taken by Y = 15 days

34. (c) Mason work for 1h=$\frac{1}{12}$

Mason work for 6h = $\frac{6}{12} = \frac{1}{2}$

Work left = $1 - \frac{1}{2} = \frac{1}{2}$

Let the boy can finish the work in x h

Then, thier 1 h work = $\frac{1}{12} + \frac{1}{x} = \frac{x+12}{12x}$

$\therefore \frac{x+12}{12x} \times 5 = \frac{1}{2}$

$\Rightarrow \dfrac{5x+60}{12x}=\dfrac{1}{2}$

$\Rightarrow 10x+120=12x$

$\Rightarrow 120=2x$

$\Rightarrow x=60\,\text{h}$

35. (c) Required time taken to fill the pool

$= \dfrac{\text{Volume of the pool}}{\text{Part of pool filled in 1 s}}$

$= \dfrac{70\times 44\times 3\times 100\times 100\times 100}{30800}\,\text{s}$

$= 300000\,\text{s} = \dfrac{250}{3}\,\text{h}$

36. (b) 3 men + 4 boys = 8 days, 6 men + 8 boys = ?

By formula, $M_1D_1 = M_2D_2$

$\Rightarrow (3M+4B)\times 8 = (6M+8B)\times D_2$

$\Rightarrow (3M+4B)\times 8 = 2(3M+4B)\times D_2$

$\Rightarrow D_2 = \dfrac{(3M+4B)\times 8}{2\times(3M+4B)}$ = 4 days

37. (b) X Y

$100\underline{(+\ 25\%)}125$ efficiency is proportional to days

$\Rightarrow 100\times 25 = 125\times \text{Days}$

$\therefore$ Days of $Y = \dfrac{100\times 25}{125}$ = 20 days.

38. (b) $M_1 = 45, D_1 = 18$

Here, $M_2 = x, D_2 = 15$

By the formula, $M_1D_1 = M_2D_2$

$\therefore M_2 = \dfrac{M_1D_1}{D_2}$

$\Rightarrow x = \dfrac{45\times 18}{15} = 3\times 18$ = 54

39. (d) $A's$ and $B's$ 1 h work = $\dfrac{1}{10}$

$B's$ and $C's$ 1 h work = $\dfrac{1}{15}$

$A's$ and $C's$ 1 h work = $\dfrac{1}{12}$

$\therefore A's, B's\ and\ C's$ 1 h work

$= \dfrac{1}{2}\left(\dfrac{1}{10}+\dfrac{1}{15}+\dfrac{1}{12}\right)$

$= \dfrac{1}{2}\times\dfrac{1}{4}=\dfrac{1}{8}$

$B's$ work in 1 h = $\dfrac{1}{8}-\dfrac{1}{12}=\dfrac{1}{24}$

B can complete the work in 24 h

40. (d) Work done by tap B in 1 min

$= \dfrac{1}{40}-\dfrac{1}{60}=\dfrac{3-2}{120}=\dfrac{1}{120}$

Total time taken by the tap B to fill the tank is 120 min.

41. (d) Required days = $\dfrac{240\times 48}{160}$

= 72 days

42. (d) 1 day work of A = $\dfrac{1}{x}$

1 day work of B = $\dfrac{1}{3x}$

Both A and B days work = $\dfrac{1}{x}+\dfrac{1}{3x}=\dfrac{4}{3x}$

Work of both A and B = $\dfrac{1}{12}$

$\Rightarrow \dfrac{4}{3x}=\dfrac{1}{12} \Rightarrow 3x=48$

$\therefore x=16$

43. (d) A work in one day = $\dfrac{1}{8}$

B work in one day = $\dfrac{1}{10}$

C work in one day = $\dfrac{1}{15}$

$A+B$ work in one day = $\dfrac{1}{8}+\dfrac{1}{10}=\dfrac{5+4}{40}=\dfrac{9}{40}$

$A+B$ work in two day = $\dfrac{2\times 9}{40}=\dfrac{9}{20}$

Remaining work $1-\dfrac{9}{20}=\dfrac{11}{20}$

$B+C$ work in one day $=\dfrac{1}{10}+\dfrac{1}{15}=\dfrac{3+2}{30}=\dfrac{5}{30}=\dfrac{1}{6}$

So, $(B+C)$ a work together in 6 days.

$\therefore \dfrac{11}{20}$ work together = $6\times\dfrac{11}{20}=\dfrac{11\times 3}{10}=\dfrac{33}{10}$ days

$\because$ Total number of days =

$2+\dfrac{33}{10}=\dfrac{20+33}{10}=\dfrac{53}{10}$ days

44. (c) Here $M_1D_1 = M_2D_2$
M_1 = 76 Ladies, M_2 = x Let
D_1 = 33 days D_2 = 44 days
$\Rightarrow\ 76 \times 33 = x \times 44$

$\therefore x = 57$ days

no. of ladies who did not Report for work = 76 – 57 = 19.

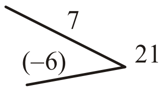
45. (b) Here, $M_1 = 10, D_1 = 8, W_1 = 20$

$M_2 = x(\text{let}), D_2 = 10, W_2 = 100$

$\because \frac{M_1D_1}{W_1} = \frac{M_2D_2}{W_2}$

$\therefore \frac{10 \times 8}{20} = \frac{x \times 10}{100} \Rightarrow x = 8 \times 5 = 40$

46. (a) Time taken by pipe $A = 3h$



Due to leakage, time taken = 3.5 = $\frac{35}{10} = \left(-\frac{7}{2}\right)$

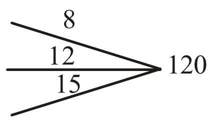
Here LCM of 3 and $\frac{7}{2}$ is 21

Here 21 is full capacity of tank

then due to leakage in 1 h tank filled by A = (7–6) = 1l.

leakage takes 21 h

47. (b) $A = 8$
$B = 12$
$C = 15$



Here, 120 is total work

(A + B) starts together in 1 day work = $\frac{1}{8} + \frac{1}{12} = \frac{5}{24}$

2 days work of (A + B) $= \frac{5}{24} \times 2 = \frac{5}{12}$

Remaining work = $1 - \frac{5}{12} = \frac{7}{12}$

Now, (B + C) work together in 1 day

$= \frac{1}{12} + \frac{1}{15} = \frac{5+4}{60} = \frac{3}{20}$

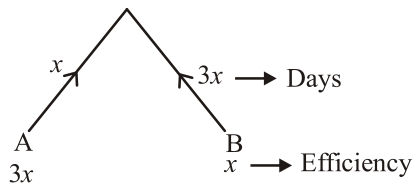
$(B + C)$ Complete in days $= \frac{7}{12} \times \frac{20}{3} = \frac{35}{9} = 3\frac{8}{9}$ days

A worked for 2 days.

So A, B, C all 3 completed work in $2 + 3\frac{8}{9}$

$= 5\frac{8}{9}$ days

48. (b) Let efficiency of B is x



Days are inversaly proportional to efficiency.
According to question
$\Rightarrow\ 3x - x = 40$
$\Rightarrow\ 2x = 40$
$\therefore\ x = 20$
Time taken by $A = x$ days = 20 days
Time taken by $B = 3x$ days = 3 × 20 = 60 days

Time taken by A and B together $= \frac{20 \times 60}{80}$

= 15 days

49. (d) We know that

$\frac{M_1D_1}{W_1} = \frac{M_2D_2}{W_2}$

Here given $M_1 = 10$, $W_1 = 8$, $D_1 = 12$
$M_2 = 8$, $W_2 = 6$, $D_2 = ?$

$\Rightarrow \frac{10 \times 12}{8} = \frac{8 \times D_2}{6}$

$\Rightarrow D_2 = 11.25$ days

50. (a) Earning of man is directly proportinonal to their income

P		Q	$P + Q$		R
2	:	1	3	:	1
		Q	:	R	
		1	1		

Ratio of efficiency = 2 : 1 : 1

51. (d) In 1 minute all the taps will fill the part of the tank

$= \frac{1}{10} + \frac{1}{20} - \frac{1}{40}$

$= \frac{4+2-1}{40} = \frac{5}{40} = \frac{1}{8}$

$\frac{1}{8}$ th part of tank will fill in 1 minute

Full tank will fill in $1 \times 8 = 8$ minutes

So, option (d) is correct.

52. (c) $\frac{M_1D_1H_1}{W_1} = \frac{M_2D_2H_2}{W_2}$

$\frac{4 \times 4 \times 4}{9} = \frac{2 \times 2 \times 2}{W} \Rightarrow W = \frac{1}{2}$ unit

So, option (c) is correct.

53. (d) $M_1D_1 = M_2D_2$

$(m)(d) = (m+2)D_2$

$D_2 = \frac{md}{m+2}$

So, option (d) is correct.

54. (a) Let pipes A, B, C fill a tank in x, y and z hrs. respectively.

Then $x = 60$ min, $y = 75$ min, $z = z$ min

Required time to fill the tank = 50 min.

But Required time $= \frac{xyz}{yz + zx - xy}$

$50 = \frac{60 \times 75 \times z}{75z + 60z - 60 \times 75}$

$\Rightarrow \ 5 = \frac{450z}{135z - 4500}$

$\Rightarrow \ 5(135z - 4500) = 450z$

$\Rightarrow \ 675z - 4500 \times 5 = 450z$

$\Rightarrow \ 675z - 450z = 4500 \times 5$

$\Rightarrow \ z = \frac{4500 \times 5}{225} = 100$

$\therefore$ Option (a) is correct.

55. (d) $\frac{\text{efficiency of A}}{\text{efficiency of B}} = \frac{150}{100}$

$\frac{\text{Time taken by A}}{\text{Time taken by B}} = \frac{100}{150}$

$\frac{A}{30} = \frac{100}{150}$

$A = 20$ days

ATQ

$\frac{x}{A} + \frac{12}{B} + \frac{x}{B} = 1$

$x\left[\frac{1}{20} + \frac{1}{30}\right] = 1 - \frac{12}{30}$

$x\left[\frac{5}{60}\right] = \frac{18}{30}$

$x = 7.2$ hrs

56. (c) X fill in 2 hr, y in 6 hr

ATQ

$\frac{x}{2} + \frac{x-1}{6} = 1$

$3x + x - 1 = 6$

$4x = 7$

$x = \frac{7}{4}$ or 1 hr $\frac{3}{4} \times 60$ min

$= 1$ hr 45 min

tank will be filled by 11 : 45 am

57. (d) Given $3w = 2M$

$w = \frac{2M}{3}$

$M_1d_1h_1 = M_2d_2h_2$

$15 \times 21 \times 8 = 21\left(\frac{2}{3}\right) \times 6 \times d$

$d = \frac{15 \times 21 \times 8 \times 3}{21 \times 2 \times 6}$

$d = 30$ days

58. (b) A and B together can do in = 12 days

B alone can do in = 30 days

A can do in

$= \frac{1}{12} - \frac{1}{30} = \frac{3}{60} = \frac{1}{20}$ i.e. 20 days

59. (a) 5 Men can do work in 10 days, 1 man will do in 10×5. 12 women can do work in 15 days, 1 women will do in 12×15

ATQ. 5 men and 6 women will do in

$\Rightarrow \frac{5}{10 \times 5} + \frac{6}{12 \times 15}$

$\Rightarrow \frac{1}{10} + \frac{1}{30} = \frac{4}{30}$

$\frac{2}{15}$ i.e. $\frac{15}{2}$ or $7\frac{1}{2}$ days

60. (b) If M1 men can do W1 work in D1 days and M2 men can do W2 work in D2 days (where all men work at the same rate), then $\frac{M_1D_1}{W_1} = \frac{M_2D_2}{W_2}$.

Therefore, here, according to the question, $M_1 = 30$,

$D_1 = 40, W_1 = 1, M_2 = x, D_2 = 40, W_2 = \frac{16}{40}$

Thus,

$$\frac{M_1 D_1}{W_1} = \frac{M_2 D_2}{W_2} \Rightarrow \frac{30 \times 40}{1} = \frac{x \times 40}{\frac{16}{40}} \Rightarrow 1200$$

$$= \frac{40x \times 40}{16} \Rightarrow x = \frac{1200 \times 16}{40 \times 40} = 12$$

Thus, the number of men who left the job are (30 – 12) i.e. 18 men.

61. (d) Using the formula, $\left(\frac{\text{AND}}{\text{OR}}\right) x$ = given number of days, where x is the number of days we have to divide.

Thus,

$$\left(\frac{\text{AND}}{\text{OR}}\right) x = 50 \Rightarrow \left(\frac{2}{4} + \frac{3}{6}\right) x = 50$$

$$\Rightarrow \left(\frac{1}{2} + \frac{1}{2}\right) x = 50 \Rightarrow x = 50$$

62. (b) Let us assume the capacity of the tub is 100L.

It is given that a tap can fill 100L in 10 hrs.

This means, in 1 hr. a tap can fill only 10L.

Therefore, in 7 hrs. a tap can fill only 70L.

This means in 5 hrs. a tap fills only 30L but actually the tap should fill 50L in 5 hrs.

This means, there is a leakage of 20L which has a duration of 5 hrs.

If 20L of water is leaked in 5 hrs., then 1L water is leaked in $\frac{5}{20} = \frac{1}{4}$ hrs.

This means 100L water is leaked in $\frac{1}{4} \times 100 = 25$ hrs.

63. (c) $M_1 = 10$ $D_1 = 12$

$M_2 = 5$ $D_2 = 8$

10 women × 12 = 5 men × 8

1 man = $\frac{120}{40}$ women = 3 women

Now

6 women + 3 men = 6 women + 3 × 3 women = 15 women

∴ 15 women × No. of days = 10 women × 12 days

∴ Required no. of days $= \frac{10 \text{ women} \times 12 \text{ days}}{15 \text{ women}}$

= 8 days

64. (c) $M_1 = 200$ $D_1 = 50$ $w_1 = \frac{1}{4}$

$M_2 = 200 + x$ $D_2 = 100$ $w_2 = 1 - \frac{1}{4} = \frac{3}{4}$

$$\frac{M_1 D_1}{M_2 D_2} = \frac{w_1}{w_2}$$

$$\frac{200 \times 50}{(200 + x)100} = \frac{\frac{1}{4}}{\frac{3}{4}}$$

$$\frac{100}{200 + x} = \frac{1}{3} \quad \therefore 200 + x = 300$$

$\therefore x = 300 - 200 = 100$

Hence, additional number of men = 100

65. (c) $M_1 = 5$ $D_1 = 5$ $W_1 = 5$

$M_2 = x$ $D_2 = 50$ $W_2 = 100$

$$\frac{M_1 D_1}{M_2 D_2} = \frac{W_1}{W_2}$$

$$\Rightarrow \frac{5 \times 5}{x \times 50} = \frac{5}{100} \Rightarrow x = 10$$

Hence, required number of tractors is 10.

66. (c) From S_1 and S_2

volume of water filled in 1 minute by all three pipes together = 12 + 10 – 6 = 16 ltrs

Total time = 5 hrs 45 minutes = 5 × 60 + 45 = 345 minutes

Capacity of tank = 345 × 16 = 5520 ltrs

From S_1 and S_3

Water drained by pipe in 1 minute = 6 ltrs

Total time = 15 hrs 20 minutes = 15 × 60 + 20 = 920 minutes

Capacity of tank = 920 × 6 = 5520 ltrs

From S_2 and S_3

Time required for all three pipes = 5 hrs 45 minutes = 345 minutes

Time required for drain pipe = 15 hrs 20 minutes = 920 minutes

Capacity of tank = 920 × 6 = 5520 ltrs.

From S_2 and S_3

Time required for all there pipees = 5hrs 45 minutes = 345 min.

Time required for drain pipe = 15 hrs 20 minutes = 920 minutes

Time required for first and second pipe

$$= \frac{1}{\frac{1}{345}+\frac{1}{920}} = \frac{\frac{1}{8+3}}{2760} = \frac{2760}{11} \text{ minutes}$$

Capacity of the tank $= \frac{2760}{11} \times 22 = 5520$ ltrs

Hence, any two out of S_1, S_2 and S_3 are sufficient.

67. (d) Let A and B complete the work in x and y days, then,

$$\frac{1}{x}+\frac{1}{y}=\frac{1}{5} \quad \text{...(i)}$$

and, $\frac{2}{x}+\frac{1}{3y}=\frac{1}{3}$ ...(ii)

From equation (i) and (ii), we have

$$\frac{1}{3x}-\frac{2}{x}=\frac{1}{15}-\frac{1}{3} \Rightarrow \frac{1-6}{3x}=\frac{1-5}{15} \quad ; x = \frac{25}{4}=6\frac{1}{4} \text{ days}$$

68. (b) Efficiency 3 : 1

Time taken 1 : 3 $\xrightarrow{\times 5}$ 15 days

$\times 5$ 3 – 1 = 2 $\longrightarrow$ 10 days

5 days

Thus, time taken by B is 15 days

69. (c) Using formula,

$$\frac{M_1D_1H_1}{W_1}=\frac{M_2D_2H_2}{W_2}$$

Here work done is same in both cases.

$12 \times 10 \times 8 = 8 \times 8 \times H_2$; $H_2 = \frac{12\times10\times8}{8\times8}$

$\Rightarrow H_2 = 15$ hours

70. (b) Let x and y finish the job in A and B days

$\frac{1}{A}+\frac{1}{B}=\frac{1}{6}$ and $\frac{1}{A}=\frac{1}{12}$ (given)

$\therefore \frac{1}{12}+\frac{1}{B}=\frac{1}{6} \Rightarrow \frac{1}{B}=\frac{1}{6}-\frac{1}{12}=\frac{2-1}{2}=\frac{1}{12}$

So, B = 12

Hence, y alone can finish the job in 12 days

71. (d) $\frac{P_1 \times t_2}{W_1}=\frac{P_2\times t_2}{W_2}$; $\frac{12\times16}{10}=\frac{8\times t_2}{20} = t_2 = 48$ days

72. (c) 12 men = 18 women

$\therefore$ 1men = 18/12 = 1.5 women

8 men + 16 women = 12 women + 16 women = 28 women 18 women completes in 14 days

1 woman completes in 14 × 18 days

28 women completes in (14 × 18)/28 days = 9 days

73. (d) Let both of then take x hours working together

$1/x = 1/10 + 1/6 = 8/30$

$x = 30/8$ hours = 3 hours 45 minutes

74. (b) $\frac{(x+2)(x-3)}{(x+4)(x-2)}=\frac{x^2-x-6}{x^2+2x-8}=\frac{3}{4}$

$\Rightarrow 4x^2-4x-24=3x^2+6x-24$

$\Rightarrow x^2-10x=0 \quad \Rightarrow x=0, 10$

75. (c) x drain full tank of water in 20 minutes

x and y drain full tank of water 15 minutes

L.C.M 20 and 15 is 60

x — 3 — $\boxed{60}$

xy — 4 — $\boxed{60}$

total capacity of tank is 60 unit

y drain full tank of water in one minutes

= (4 –3) unit = 1 unit.

$\therefore$ time taken to drain full tank when only y is opened.

= 60 × 1 = 60 minutes

76. (a) Required number of days.

$$=\frac{240\times48}{160}=72 \text{ days}$$

77. (a) (6M + 8W) 10 = (13M +24W)4

60M + 80W = 52M + 96W

8M = 16W

$\therefore$ M : W = 2 : 1

78. (d) Time taken to empty the tank by second oulet

$$=\frac{1}{\left(\frac{1}{20}-\frac{1}{30}\right)}=\frac{1}{\frac{1}{60}}=60 \text{ minutes.}$$

CHAPTER 12

Logarithm

1. What is the value of

$$\left(\log_{\frac{1}{2}} 2\right)\left(\log_{\frac{1}{3}} 3\right)\left(\log_{\frac{1}{4}} 4\right).....\left(\log_{\frac{1}{1000}} 1000\right)?$$

[2007-I]

(a) 1 (b) –1 (c) 1 or –1 (d) 0

2. What is the value of $\log_{100} 0.1$? **[2008-II]**

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) –2 (d) 2

3. What is the value of

$$2\log\left(\frac{5}{8}\right)+\log\left(\frac{128}{125}\right)+\log\left(\frac{5}{2}\right)?$$ **[2009-I]**

(a) 0 (b) 1 (c) 2 (d) 5

4. If $\log_r 6 = m$ and $\log_r 3 = n$, then what is $\log_r\left(\frac{r}{2}\right)$ equal to? **[2009-I]**

(a) $m - n + 1$ (b) $m + n - 1$
(c) $1 - m - n$ (d) $1 - m + n$

5. What is the value of $\frac{[\log_{13}(10)]}{[\log_{169}(10)]}$? **[2009-II]**

(a) $\frac{1}{2}$ (b) 2 (c) 1 (d) $\log_{10} 13$

6. What is the value of

$$\left(\frac{1}{3}\log_{10}125-2\log_{10}4+\log_{10}32+\log_{10}1\right)?$$ **[2010-II]**

(a) 0 (b) $\frac{1}{5}$ (c) 1 (d) $\frac{2}{5}$

7. What is the value of $\frac{1}{2}\log_{10}25 - 2\log_{10}3 + \log_{10}18$ **[2012-I]**

(a) 2 (b) 3 (c) 1 (d) 0

8. What is the value of $[\log_{10}(5\log_{10}100)]^2$? **[2011-I]**

(a) 4 (b) 3 (c) 2 (d) 1

9. What is $\log_{10}\left(\frac{3}{2}\right)+\log_{10}\left(\frac{4}{3}\right)+\log_{10}\left(\frac{5}{4}\right)+$ ..... upto 8 terms equal to? **[2011-II]**

(a) 0 (b) 1
(c) $\log_{10} 5$ (d) None of these

10. What is the logarithm of 0.0001 with respect to base 10? **[2012-I]**

(a) 4 (b) 3 (c) –4 (d) –3

11. If $\log_{10} a = p$ and $\log_{10} b = q$, then what is the value of $\log_{10}(a^p p^q)$? **[2012-II]**

(a) $p^2 + q^2$ (b) $p^2 - q^2$ (c) $p^2 q^2$ (d) $\frac{p^2}{q^2}$

12. The value of $\frac{1}{5}\log_{10}3125 - 4\log_{10}2 + \log_{10}32$ is **[2016-I]**

(a) 0 (b) 1 (c) 2 (d) 3

13. What is the solution of

$$\log_{10}\left[1-\left\{1-\left(1-x^2\right)^{-1}\right\}^{-1}\right]^{-\frac{1}{2}} = 1\ ?$$ **[2016-II]**

(a) x=100 (b) x=10 (c) x=1 (d) x = 0

14. What are the roots of the equation $2^{x+2}.27^{\frac{x}{x-1}} = 9$? **[2016-II]**

(a) $2, 1-\log\left(\frac{3}{2}\right)$ (b) $2, 1-\log\left(\frac{2}{3}\right)$

(c) $-2, 1-\log\left(\frac{3}{2}\right)$ (d) $-2, 1\left(\frac{\log 2}{\log 3}\right)$

15. The number of digits in 3^{30} is n and it is given that $\log_{10} 3 = 0.4771$. What is the value of n ? **[2016-II]**

(a) 13 (b) 14 (c) 15 (d) 16

16. What is the number of digits in 2^{40} ?
(Given that $\log_{10} 2 = 0.301$) **[2017-I]**

(a) 14 (b) 13 (c) 12 (d) 11

17. What is the solution of the equation $x\log_{10}\left(\frac{10}{3}\right)+\log_{10}3 = \log_{10}(2+3^x)+x$? **[2017-I]**

(a) 10 (b) 3 (c) 1 (d) 0

18. Let P, Q, R be the mid-points of sides AB, BC, CA respectively of a triangle ABC. If the area of the triangle ABC is 5 square units, then the area of the triangle PQR is **[2017-I]**

(a) $\frac{5}{3}$ square units (b) $\frac{5}{2\sqrt{2}}$ square units

(c) $\frac{5}{4}$ square units (d) 1 square unit

19. If $\log_{10} 6 = 0.7782$ and $\log_{10} 8 = 0.9031$, then what is the value of $\log_{10} 8000 + \log_{10} 600$? **[2017-II]**
(a) 4.6813 (b) 5.5813 (c) 1.5813 (d) 6.6813

20. For $0 < m < 1$, which one of the following is correct? **[2018-I]**
(a) $\log_{10} m < m^2 < m < m^{-1}$
(b) $m < m^{-1} < m^2 < \log_{10} m$
(c) $\log_{10} m < m < m^{-1} < m^2$
(d) $\log_{10} m < m^{-1} < m < m^2$

21. If $\log_{10}2 = 0.3010$ and $\log_{10}3 = 0.4771$, then the vlaue of $\log_{100}(0.72)$ is equal to **[2018-II]**
(a) 0.9286 (b) $\bar{1}.9286$
(c) 1.8572 (d) $\bar{1}.8572$

22. There are n zeroes appearing immediately after the decimal point in the value of $(0.2)^{25}$. It is given that the value of $\log_{10}2 = 0.30103$. The value of n is **[2018-II]**
(a) 25 (b) 19 (c) 18 (d) 17

23. It is given that $\log_{10} 2 = 0.301$ and $\log_{10} 3 = 0.477$. How many digits are there in $(108)^{10}$? **[2019-I]**
(a) 19 (b) 20 (c) 21 (d) 22

24. What is the number of digits in 7^{25}, 8^{23} and 9^{20} respectively? [Given $\log_{10}2 = 0.301$, $\log_{10}3 = 0.477$, $\log_{10}7 = 0.845$] **[2019-II]**
(a) 21, 20, 19 (b) 20, 19, 18
(c) 22, 21, 20 (d) 22, 20, 21

25. The sides of a triangle are 30 cm, 28 cm and 16 cm respectively. In order to determine its area, the logarithm of which of the quantities are required ? **[2019-II]**
(a) 37, 11, 28, 16 (b) 21, 30, 28, 7
(c) 37, 21, 11, 9 (d) 37, 21, 9, 7

26. If $\log_{10}1995 = 3.3000$, then what is the value of $(0.001995)^{\frac{1}{8}}$? **[2019-II]**
(a) $\dfrac{1}{10^{0.3475}}$ (b) $\dfrac{1}{10^{0.3375}}$
(c) $\dfrac{1}{10^{0.3275}}$ (d) $\dfrac{1}{10^{0.3735}}$

27. Let XYZ be an equilateral triangle in which $XY = 7$ cm. If A denotes the area of the triangle, then what is the value of $\log_{10}A^4$? (Given that $\log_{10}1050 = 3.0212$ and $\log_{10}35 = 1.5441$) **[2019-II]**
(a) 5.3070 (b) 5.3700 (c) 5.5635 (d) 5.6535

28. What is the value of $\log_{10}(\cos\theta) + \log_{10}(\sin\theta) + \log_{10}(\tan\theta) + \log_{10}(\cot\theta) + \log_{10}(\sec\theta) + \log_{10}(\text{cosec}\,\theta)$? **[2019-II]**
(a) –1 (b) 0 (c) 0.5 (d) 1 1

29. How many digits are there in $(54)^{10}$? **[2020-I]**
(Given that $\log_{10}2 = 0.301$ and $\log_{10}3 = 0.477$)
(a) 16 (b) 18 (c) 19 (d) 27

30. If $\log x = 1.2500$ and $y = x^{\log x}$, then what is $\log y$ equal to? **[2020-I]**
(a) 4.2500 (b) 2.5625 (c) 1.5625 (d) 1.2500

HINTS & SOLUTIONS

1. (b) $\left(\log_{\frac{1}{2}} 2\right)\left(\log_{\frac{1}{3}} 3\right)\left(\log_{\frac{1}{4}} 4\right).....\left(\log_{\frac{1}{1000}} 1000\right)$

$$= \left(\frac{\log 2}{\log \frac{1}{2}}\right)\left(\frac{\log 3}{\log \frac{1}{3}}\right)\left(\frac{\log 4}{\log \frac{1}{4}}\right).....\left(\frac{\log 1000}{\log\left(\frac{1}{1000}\right)}\right)$$

$$\left(\because \log_b a = \frac{\log a}{\log b}\right)$$

$$= \left(\frac{\log 2}{-\log 2}\right)\left(\frac{\log 3}{-\log 3}\right)\left(\frac{\log 4}{-\log 4}\right).....\left(\frac{\log 1000}{-\log 1000}\right)$$

$= (-1) \times (-1) \times (-1) \times ..... \times (-1)$
($\because$ number of factors is odd)
$= -1$

2. (b) $\log_{100} 0.1 = \log_{100} \dfrac{1}{10}$
$= \log_{100} 1 - \log_{10^2} 10$
$= 0 - \dfrac{1}{2}\log_{10} 10 = -\dfrac{1}{2} \times 1 = -\dfrac{1}{2}$

3. (a) $2\log\left(\frac{5}{8}\right) + \log\left(\frac{128}{125}\right) + \log\left(\frac{5}{2}\right)$

$$= \log\left(\frac{5}{8}\right)^2 + \log\left(\frac{128}{125}\right) + \log\left(\frac{5}{2}\right)$$

$$= \log\frac{5^2 \times 128 \times 5}{8^2 \times 125 \times 2} = \log\frac{5^2 \times 2^7 \times 5}{\left(2^3\right)^2 \times 5^3 \times 2}$$

$$= \log\frac{2^7 \times 5^3}{2^6 \times 5^3 \times 2} = \log\frac{2^7 \times 5^3}{2^7 \times 5^3} = \log 1 = 0$$

4. (d) Given, $\log_r 6 = m$ and $\log_r 3 = n$
$\because \log_r 6 = \log_r (2 \times 3)$
$= \log_r 2 + \log_r 3$
$\therefore \log_r 3 + \log_r 2 = m$
$\Rightarrow n + \log_r 2 = m$
$\Rightarrow \log_r 2 = m - n$

$\therefore \quad \log_r\left(\frac{r}{2}\right) = \log_r r - \log_r 2$

$= 1 - m + n$

5. (b) $\dfrac{\log_{13}(10)}{\log_{169}(10)} = \dfrac{\log_{13}(10)}{\log_{13^2}(10)}$

$\left(\because \log_{a^b} c = \frac{1}{b}\log_a c\right)$

$= \dfrac{\log_{13} 10}{\frac{1}{2}\log_{13} 10} = \dfrac{1}{\frac{1}{2}} = 2$

6. (c) $\frac{1}{3}\log_{10} 125 - 2\log_{10} 4 + \log_{10} 32 + \log_{10} 1$

$= \frac{1}{3}\log_{10}(5)^3 - 2\log_{10}(2)^2 + \log_{10}(2)^5 + 0$

$= \log_{10} 5 - 4\log_{10} 2 + 5\log_{10} 2$

$= \log_{10} 5 + \log_{10} 2 = \log_{10} 5 \times 2 = \log_{10} 10 = 1$

7. (c) $\frac{1}{2}\log_{10} 25 - 2\log_{10} 3 + \log_{10} 18$

$= \log_{10} 25^{1/2} - \log_{10} 3^2 + \log_{10} 18$

$= \log_{10} 5 - \log_{10} 9 + \log_{10} 18$

$= \log_{10}\dfrac{5\times 18}{9} = \log_{10}\dfrac{90}{9} = \log_{10} 10 = 1$

8. (d) $[\log_{10}(5\log_{10} 100)]^2 = [\log_{10}(5\log_{10} 10^2)]^2$

$= [\log_{10}(10\log_{10} 10)]^2 \qquad (\because \log_{10} 10 = 1)$

$= [\log_{10} 10]^2$

$= 1^2 = 1$

9. (c) $\log_{10}\left(\frac{3}{2}\right) + \log_{10}\left(\frac{4}{3}\right) + \log_{10}\left(\frac{5}{4}\right) + \ldots.. + 8^{th}$ term

$= \log_{10}\left(\frac{3}{2}\right) + \log_{10}\left(\frac{4}{3}\right) + \log_{10}\left(\frac{5}{4}\right) + \ldots.. + \log_{10}\left(\frac{10}{9}\right)$

$= \log_{10}\left(\frac{3}{2}\times\frac{4}{3}\times\frac{5}{4}\times\ldots..\times\frac{10}{9}\right)$

$= \log_{10}\left(\frac{10}{2}\right)\log_{10} 5.$

10. (c) Let $\log_{10} 0.0001 = a$

$a = \log_{10}\dfrac{1}{(10)^4}$

$= \log_{10} 1 - \log_{10}(10)^4 = 0 - 4 = -4$

11. (a) Given that,

$\log_{10} a = p$ and $\log_{10} b = q$

$\log_{10}(a^p b^q) = \log_{10} a^p + \log_{10} b^q$

$= p\log_{10} a + q\log_{10} b$

$= p \times p + q \times q = p^2 + q^2$

12. (b) $\frac{1}{5}\log_{10} 3125 - 4\log_{10} 2 + \log_{10} 32$

$= \frac{1}{5}\log_{10}(5)^5 - 4\log_{10} 2 + \log_{10}(2)^5$

$= \frac{5}{5}\log_{10} 5 - 4\log_{10} 2 + 5\log_{10} 2 = \log_{10} 5 + \log_{10} 2$

$= \log_{10}(5\times 2)$

$[\log m + \log n = \log mn]$

$= \log_{10} 10 = 1$

$\therefore$ Option (b) is correct.

13. (b) $\log_{10}\left[1-\left\{1-(1-x^2)^{-1}\right\}^{-1}\right]^{-\frac{1}{2}} = 1$

$\Rightarrow \log_{10}\left[1-\left\{1-\frac{1}{1-x^2}\right\}^{-1}\right]^{-\frac{1}{2}} = 1$

$\Rightarrow \log_{10}\left[1-\left\{\frac{-x^2}{1-x^2}\right\}^{-1}\right]^{-\frac{1}{2}} = 1$

$\Rightarrow \log_{10}\left[1-\frac{(1-x^2)}{-x^2}\right]^{-\frac{1}{2}} = 1$

$\Rightarrow \log_{10}\left[\frac{-x^2-(1-x^2)}{-x^2}\right]^{-\frac{1}{2}} = 1$

$\Rightarrow \log_{10}\left[\frac{1}{x^2}\right]^{-\frac{1}{2}} = 1$

$\Rightarrow \log_{10} x = \log_{10}{}^{10}$

$x = 10$

$\Rightarrow \log_{10}$

14. (c) $2^{x+2}\, 27^{\left(\frac{x}{x-1}\right)} = 9$

$2^{x+2}\, 27^{\left(\frac{x}{x-1}\right)} = 2^0\, 3^2$

$2^{x+2} = 2^0$

$x = -2$

now taking $2^{x+2}, \dfrac{x}{27^{x-1}} = 9$

$\log\left[2^{x+2}\, 27^{\left(\frac{x}{x-1}\right)}\right] = \log 3^2$

$(x+2)\log(2) + \frac{3x}{x-1}\log 3 = 2\log 3$

$(x+2)\log 2 = \left[2 - \frac{3x}{x-1}\right]\log 3$

$(x+2)\log 2 = \left[\frac{-x-2}{x-1}\right]\log 3$

$1 - x = \frac{\log 3}{\log 2}$

$x = 1 - \log\left(\frac{3}{2}\right)$

$x = -2$ and $1 - \log\left(\frac{3}{2}\right)$

15. (c) let $3^{30} = 10^x$

$30\log[3] = x\log(10)$

$30 \times 0.4771 = x$

$14.313 = x$

$x > 14$

i.e. 15

16. (b) let $2^{40} = 10^x$

$\log 2^{40} = \log 10^x$

$40\log^2 = x\log^{10}$

$40(.301) = x$

$12.04 = x$

i.e. $x > 12$

$\therefore x = 13$

No. of digit in 2^{40} is 13

17. (d) $x\log_{10}\left(\frac{10}{3}\right) + \log_{10} 3 = \log_{10}(2+3^x) + x$

$x\log_{10} 10 - x\log_{10} 3 + \log_{10} 3 = \log_{10}(2+3^x) + x$

$x - \log_{10^{3^x}} + \log_{10^3} = \log_{10}(2+3^x) + x$

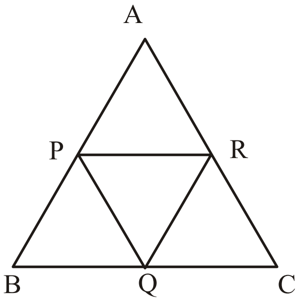
$\log_{10}\left[\frac{3}{3^x}\right] \log_{10}(2+3^x)$

$\Rightarrow 3^{1-x} = 2 + 3^x$

$\Rightarrow 3^{1-x} - 3^x = 3^1 - 3^0$

$x = 0$

18. (c)



$PQ = \frac{1}{2}AC, QR = \frac{1}{2}AB, PR = \frac{1}{2}BC$

$\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ} = \frac{2}{1}$

$\frac{\text{area of ABC}}{\text{area of PQR}} = \left(\frac{2}{1}\right)^2 = \frac{4}{1}$

$= \text{in } \Delta ABC = 5\text{ cm}^2$

as $\Delta PQR = \frac{5}{4}\text{ cm}^2$

19. (d) We are given $\log_{10} 6 = 0.7782$ and $\log_{10} 8 = 0.9031$

Therefore,

$\log_{10} 8000 + \log_{10} 600 = \log_{10} 6 \times 10^2 + \log_{10} 8 \times 10^3$

$= \log_{10} 6 + \log_{10} 10^2 + \log_{10} 8 + \log_{10} 10^3$

$= 0.7782 + 2\log_{10} 10 + 0.9031 + 3\log_{10} 10$

$= 1.6813 + 2 + 3$

$= 6.6813$

20. (a)

21. (b) $\log_{100}(0.72) = \log_{100}\left(\frac{72}{100}\right)$

$= \log_{100} 72 - \log_{100} 100 = \log_{100}(8 \times 9) - 1$

$= \log_{100}(2^3 \times 3^2) - 1 = \log_{(10)^2} 2^3 + \log_{(10)^2}(3^2) - 1$

$= \frac{3}{2}\log_{10} 2 + \frac{2}{2}\log_{10} 3 - 1 = \bar{1}.9286$

22. (d) Let $y = (0.2)^{25}$

Talking log on both sides, we got

$\log(y) = (0.2)^{25} = 25.\log(0.2)$

$= 25\left(\log\left(\frac{2}{10}\right)\right) = 25(\log 2 - \log 10)$

$= 25(0.330103 - 1) = -17.47475$

Hence, n = 17

23. (c) $\log(108)^{10} = 10\log 108 = 10\log(2^2 \times 3^3) = 10(2\log 2 + 3\log 3)$

$= 10(2 \times 0.301 + 3 \times 0.477) = 10(.602 + 1.431)$

$= 10 \times 2.033 = 20.33$

integral part = 20

No. of digits = 20 + 1 = 21

24. (d) $\log 7^{25} = 25\log 7 = 25 \times 0.845$

$= 21.125$

Its characterlstic is 21 $\Rightarrow$ No. of digits = 22

$\log 8^{23} = \log 2^{69} = 69\log 2$

$= 69 \times 0.301 = 20.769$

Its characterishe is 20 $\Rightarrow$ No. of digits = 21

$\log 9^{20} = \log 3^{40}$

$= 40 \times \log 3$

$= 40 \times 0.477 = 19.08 \Rightarrow$ No. of digts = 20

Near about is 22, 21, 20 so option (d) satisfy

25. (d) Semiperimeter of triangle $= \dfrac{30+16+28}{2} = 37$

$P - a = 7$
$P - b = 9$
$P - c = 21$
$P = 37$

$\therefore$ quantities that are required $= 37, 21, 9, 7$

26. (b) Let $z = (0.001995)^{1/8}$
$= (1995 \times 10^{-6})^{1/8}$

$\Rightarrow \log_{10} z = \dfrac{1}{8}[\log_{10} 1995 + \log_{10} 10^{-6}]$

$= \dfrac{1}{8}[3.3 - 6] = \dfrac{-2.7}{8} = -0.3375$

$\log_{10} z = -0.3375$

$\Rightarrow z = 10^{-0.3375} = \dfrac{1}{10^{0.3375}}$

27. (a) $\log_{10} 1050 = \log_{10}(3 \times 10 \times 35)$

$= \log_{10} 3 + 1 + \log_{10} 35$

$\Rightarrow 3.0212 = \log_{10} 3 + 1 + 1.5441$

$\Rightarrow \log_{10} 3 = 0.4771$

Now, $\log_{10} 35 = \log \dfrac{7 \times 10}{2}$

$= \log_{10} 7 + \log_{10} - \log_{10} 2$

$\Rightarrow \log_{10} 7 = 0.8451$

Now, $A = \dfrac{\sqrt{3}}{4} \times (7)^2$

$\log 10^{A^4} = 4 \log_{10} A$

$= 4 \log \dfrac{\sqrt{3} \times 7^2}{4}$

$= 4\left[\dfrac{1}{2}\log_3 + 2\log_{10} 7 - 2\log 2\right] = 5.3070$

28. (b) $\log_{10}{}^{(\cos\theta)} + \log_{10}{}^{(\sin\theta)} + \log_{10}{}^{(\tan\theta)} + \log_{10}{}^{(\cot\theta)}$

$+ \log_{10}{}^{(\sec\theta)} + \log_{10}{}^{(\operatorname{cosec}\theta)}$

we known

$\log_{10} a + \log_{10} b = \log_{10}(a.b)$

$\therefore \log_{10}{}^{(\cos\theta + \sin\theta \times \tan\theta \times \cot\theta + \sec\theta \times \operatorname{cosec}\theta)}$

$\therefore \log_{10}{}^{1} = 0$

29. (b) $\log(54)^{10} = 10 \log 54 = 10[3\log 3 + \log 2]$
$= 10[1.431 + 0.301] = 17.32$

$\therefore$ Number of digits = 17 + 1 = 18

30. (c) We have, $y = x^{\log x}$

Taking log on both sides

$\log y = \log(x^{\log x})$

$= \log x . \log x = (1.25)^2 = 1.5625$

CHAPTER

13 Basic Operation and Factorisation

1. For what value of k, will the expression $3x^3 - kx^2 + 4x + 16$ be divisible by $\left(x - \frac{k}{2}\right)$? **[2007-I]**
(a) 4 (b) −4 (c) 2 (d) 0

2. Which one of the following is the factor of $x^4 + xy^3 + xz^3 + x^3y + y^4 + yz^3$? **[2007-I]**
(a) $x + y + z$ (b) $x^2 + y^2 + z^2$
(c) $x^3 + y^3 + z^3$ (d) $x^2 + y^2$

3. If pqr = 1, what is the value of the expression $\frac{1}{1+p+q^{-1}} + \frac{1}{1+q+r^{-1}} + \frac{1}{1+r+p^{-1}}$? **[2007-I]**
(a) 1 (b) −1 (c) 0 (d) 1/3

4. If x + y + z = 2s, then what is $(s - x)^3 + (s - y)^3 + 3(s - x)(s - y)z$ equal to? **[2007-I]**
(a) z^3 (b) $-z^3$ (c) x^3 (d) y^3

5. If $x^2 = y + z$, $y^2 = z + x$, $z^2 = x + y$, then what is the value of $\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1}$? **[2007-I]**
(a) 1 (b) 0 (c) −1 (d) 2

6. If p, q and r be such that p + q = r and pqr = 30, then what is the value of $p^3 + q^3 - r^3$? **[2007-I]**
(a) 0 (b) 90
(c) −90 (d) Cannot be determined

7. What is $\frac{x^8+4}{x^4+2x^2+2}$ equal to? **[2007-I]**
(a) $x^4 + 2x^2 - 2$ (b) $x^4 - 2x^2 + 2$
(c) $x^4 - 2x^2 - 2$ (d) Cannot be determined

8. If $2^x - 2^{x-1} = 4$, then what is the value of $2^x + 2^{x-1}$? **[2007-I]**
(a) 8 (b) 10 (c) 12 (d) 16

9. If $x + \left(\frac{1}{x}\right) = p$, then what is $x^6 + \left(\frac{1}{x^6}\right)$ equal to? **[2007-I]**
(a) $p^6 + 6p$
(b) $p^6 - 6p$
(c) $p^6 + 6p^4 + 9p^2 + 2$
(d) $p^6 - 6p^4 + 9p^2 - 2$

10. If x + y + z = 0, then what is
$$\left[\frac{(y-z-x)}{2}\right]^3 + \left[\frac{(z-x-y)}{2}\right]^3 + \left[\frac{(x-y-z)}{2}\right]^3$$
equal to? **[2007-I]**
(a) 24 xyz (b) −24 xyz (c) 3 xyz (d) xyz

11. If m and n are odd integers with n < m, then which is the largest integer among the following which divides $m^2 - n^2$? **[2007-II]**
(a) 2 (b) 4 (c) 6 (d) 8

12. What is the value of $2 + \sqrt{2} + \frac{1}{2+\sqrt{2}} - \frac{1}{2-\sqrt{2}}$? **[2007-II]**
(a) 2 (b) $2-\sqrt{2}$ (c) $4+\sqrt{2}$ (d) $2\sqrt{2}$

13. Under constant pressure conditions, the temperature varies directly as volume. At a given instant, volume = 60 m^3, temperature = 300 K. If the volume is increased to 100 m^3, then what will be the temperature? **[2007-II]**
(a) 200 K (b) 300 K (c) 400 K (d) 500 K

14. If one of the two factors of an expression which is the difference of two cubes, is $(x^4 + x^2y + y^2)$, then what is the other factor? **[2007-II]**
(a) x + y (b) x − y (c) $x^2 + y$ (d) $x^2 - y$

15. Which one of the following is one of the factors of $x^2(y - z) + y^2(z - x) - z(xy - yz - zx)$? **[2007-II]**
(a) x−y (b) x + y − z
(c) x − y − z (d) x + y + z

16. If $(3x^3 - 2x^2y - 13xy^2 + 10y^3)$ is divided by (x − 2y), then what is the remainder? **[2008-I]**
(a) 0 (b) y + 5 (c) y + 1 (d) $y^2 + 3$

17. What is the value of x in $1 + \frac{1}{1+\left\{\frac{1}{\left(1+\frac{1}{x}\right)}\right\}} = \frac{11}{7}$? **[2008-I]**
(a) 1 (b) 3 (c) $\frac{1}{2}$ (d) $\frac{7}{11}$

18. If (x + y + z = 0), then what is (x + y) (y + z) (z + x) equal to? **[2008-I]**
(a) –xyz (b) $x^2 + y^2 + z^3$
(c) $x^3 + y^3 + z^3 + 3xyz$ (d) xyz

19. If (a + b = 3), then what is the value of $(a^3 + b^3 + 9ab)$? **[2008-I]**
(a) 18 (b) 27
(c) 81 (d) Cannot be determined

20. If $(x^3 + 5x^2 + 10k)$ leaves remainder –2x when divided by $(x^2 + 2)$, then what is the value of k? **[2008-I]**
(a) –2 (b) –1
(c) 1 (d) 2

21. If $(5x^2 + 14x + 2)^2 - (4x^2 - 5x + 7)^2$ is divided by $(x^2 + x + 1)$, what is the remainder? **[2008-I]**
(a) –1 (b) 0 (c) 1 (d) 2

22. What is/are the factor(s) of $(x^{29} - x^{24} + x^{13} - 1)$? **[2008-I]**
(a) Only (x – 1)
(b) Only (x + 1)
(c) (x – 1) and (x + 1)
(d) Neither (x – 1) nor (x + 1)

23. What is the expression
$(x + y)^{-1} (x^{-1} + y^{-1}) (xy^{-1} + x^{-1}y)^{-1}$ equal to? **[2008-I]**
(a) x + y (b) $(x^2 + y^2)^{-1}$
(c) xy (d) $(x^2 + y^2)$

24. If (ab – b + 1 = 0) and (bc – c + 1 = 0), then what is (a – ac) equal to? **[2008-I]**
(a) –1 (b) 0 (c) 1 (d) 2

25. Let S be a set of all even integers. If the operations
I. addition II. subtraction
III. multiplication IV. division
are applied to any pair of numbers from S, then for which operations is the resulting number in S? **[2008-I]**
(a) I, II, III and IV (b) I, II and III
(c) I and III (d) II and IV

26. What is the value of the polynomial $r(x)$, so that $f(x) = g(x)\,q(x) + r(x)$ and deg $r(x) <$ deg $g(x)$, where $f(x) = x^2 + 1$ and $g(x) = x + 1$? **[2008-II]**
(a) 1 (b) –1 (c) 2 (d) –2

27 If $x^2 - 4x + 1 = 0$, then what is the value of $x^3 + \frac{1}{x^3}$? **[2008-II]**
(a) 44 (b) 48 (c) 52 (d) 64

28. If $x + y + z = 6$ and $xy + yz + zx = 11$, then what is the value of $x^3 + y^3 + z^3 - 3xyz$? **[2008-II]**
(a) 18 (b) 36 (c) 54 (d) 66

29. If a is a rational number such that $(x - a)$ is a factor of the polynomial $x^3 - 3x^2 - 3x + 9$, then **[2008-II]**
(a) a can be any integer
(b) a is an integer dividing 9
(c) a cannot be an integer
(d) a can take three values

30. If $x^2 - 11x + a$ and $x^2 - 14x + 2a$ have a common factor, then what are the values of a? **[2008-II]**
(a) 0, 7 (b) 5, 20 (c) 0, 24 (d) 1, 3

31. Which one of the following is a factor of $2x^3 - 3x^2 - 11x + 6$? **[2008-II]**
(a) $x + 1$ (b) $x - 1$ (c) $x + 2$ (d) $x - 2$

32. What should be subtracted from $27x^3 - 9x^2 - 6x - 5$ to make it exactly divisible by $(3x - 1)$? **[2008-II]**
(a) –5 (b) –7 (c) 5 (d) 7

33. Suppose p ★ q = 2P + 2q – pq, where p, q are natural numbers. If 8 ★ x = 4, then what is the value of x? **[2008-II]**
(a) 1 (b) 2 (c) 3 (d) 4

34. What is the simplified form of
$$\left(\frac{x^2-3x+2}{x^3-8}\right) \div \left(\frac{x^2-9}{x^2+7x+12}\right) \times \left(\frac{x^3+2x^2+4x}{x^2+3x-4}\right)?$$ **[2008-II]**
(a) $\frac{x}{x-3}$ (b) $\frac{x-2}{x-3}$ (c) $\frac{x}{x+3}$ (d) $\frac{x+3}{x+4}$

35. What is the value of $\left[\frac{(2.3)^3 - 0.027}{(2.3)^2 + 0.69 + 0.09}\right]$? **[2008-II]**
(a) 2.6 (b) 2 (c) 1.3 (d) 1

36. What is the simplified form of $9\sqrt{2} - \sqrt{8} - 4\sqrt{2}$? **[2008-II]**
(a) $4\sqrt{2}$ (b) $3\sqrt{2}$ (c) $2\sqrt{2}$ (d) $\sqrt{2}$

37. What is
$x(y - z)(y + z) + y(z - x)(z + x) + z(x - y)(x + y)$
equal to? **[2009-I]**
(a) $(x + y)(y + z)(z + x)$
(b) $(x - y)(x - z)(z - y)$
(c) $(x + y)(z - y)(x - z)$
(d) $(y - x)(z - y)(x - z)$

38. If the remainder of the polynomial $a_0 + a_1 x + a_2 x^2 + ... + a_n x^n$ when divided by $(x - 1)$ is 1, then which one of the following is correct? **[2009-I]**
(a) $a_0 + a_2 + ... = a_1 + a_3 + ...$
(b) $a_0 + a_2 + ... = 1 + a_1 + a_3 + ...$
(c) $1 + a_0 + a_2 + ... = -(a_1 + a_3 + ...)$
(d) $1 - a_0 - a_2 - ..... = a_1 + a_3 + ...$

39. When $(x^3 - 2x^2 + px - q)$ is divided by $(x^2 - 2x - 3)$, the remainder is $(x - 6)$. What are the values of p, q respectively? **[2009-I]**
(a) –2, –6 (b) 2, –6
(c) –2, 6 (d) 2, 6

40. If $x = 1 + \sqrt{2}$, then what is the value of $x^4 - 4x^3 + 4x^2$? **[2009-I]**

(a) –1 (b) 0 (c) 1 (d) 2

41. If $\frac{x}{y} = \frac{z}{w}$, then what is $(xy + zw)^2$ equal to? **[2009-I]**

(a) $(x^2 + z^2)(y^2 + w^2)$ (b) $x^2 y^2 + z^2 w^2$
(c) $x^2 w^2 + y^2 z^2$ (d) $(x^2 + w^2)(y^2 + z^2)$

42. If $\frac{1}{x+1} + \frac{2}{y+2} + \frac{1009}{z+1009} = 1$, then what is the value of $\frac{x}{x+1} + \frac{y}{y+2} + \frac{z}{z+1009}$? **[2009-I]**

(a) 0 (b) 2 (c) 3 (d) 4

43. If the unit of weight is $\frac{15}{4}$ kg, then what number will $\frac{3}{2}$ quintal represent? **[2009-I]**

(a) 25 (b) 6
(c) $\frac{1}{9}$ (d) None of these

44. If m and n are two integers such that $m = n^2 - n$, then $(m^2 - 2m)$ is always divible by **[2009-II]**

(a) 9 (b) 16 (c) 24 (d) 48

45. When $x^{40} + 2$ is divided by $x^4 + 1$, what is the remainder? **[2009-II]**

(a) 1 (b) 2 (c) 3 (d) 4

46. Which one of the following statements is correct? **[2009-II]**

(a) Remainder theorem is a special case of factor theorem
(b) Factor theorem is a special case of remainder theorem
(c) Factor theorem and remainder theorem are two independent results
(d) None of the above

47. If $x = 2 + \sqrt{3}$, then what is $(x^2 + x^{-2})$ equal to? **[2009-II]**

(a) 12 (b) 13 (c) 14 (d) 15

48. If $a = \frac{1+x}{2-x}$, then what is $\frac{1}{a+1} + \frac{2a+1}{a^2-1}$ equal to? **[2009-II]**

(a) $\frac{(1+x)(2+x)}{2x-1}$ (b) $\frac{(1-x)(2-x)}{x-2}$

(c) $\frac{(1+x)(2-x)}{2x-1}$ (d) $\frac{(1-x)(2-x)}{2x+1}$

49. If $pq + qr + rp = 0$, then what is the value of $\frac{p^2}{p^2 - qr} + \frac{q^2}{q^2 - rp} + \frac{r^2}{r^2 - pq}$? **[2009-II]**

(a) 0 (b) 1 (c) –1 (d) 3

50. If $x + y + z = 0$, then what is the value of $\frac{1}{x^2 + y^2 - z^2} + \frac{1}{y^2 + z^2 - x^2} + \frac{1}{z^2 + x^2 - y^2}$? **[2009-II]**

(a) $\frac{1}{x^2 + y^2 + z^2}$ (b) 1
(c) –1 (d) 0

51. What is $\frac{(x-y)^3 + (y-z)^3 + (z-x)^3}{4(x-y)(y-z)(z-x)}$ equal to? **[2009-II]**

(a) $-\frac{3}{4}$ (b) $\frac{1}{4}$ (c) $\frac{3}{4}$ (d) 0

52. If the expression $px^3 + 3x^2 - 3$ and $2x^3 - 5x + p$ when divided by $x - 4$ leave the same remainder, then what is the value of p? **[2010-I]**

(a) –1 (b) 1 (c) –2 (d) 2

53. If $x(x + y + z) = 9$, $y(x + y + z) = 16$ and $z(x + y + z) = 144$, then what is x equal to? **[2010-I]**

(a) $\frac{9}{5}$ (b) $\frac{9}{7}$ (c) $\frac{9}{13}$ (d) $\frac{16}{13}$

54. If u, v and w are real numbers such that $u^3 - 8v^3 - 27w^3 = 18uvw$, then which one of the following is correct? **[2010-I]**

(a) $u - v + w = 0$ (b) $u = -v = -w$
(c) $u - 2v = 3w$ (d) $u + 2v = -3w$

55. $x^4 + 4y^4$ is divisible by which one of the following? **[2010-I]**

(a) $(x^2 + 2xy + 2y^2)$
(b) $(x^2 + 2y^2)$
(c) $(x^2 - 2y^2)$
(d) None of these

56. The shadow of a pole 6 m high is 15 m long and at the same time the shadow of a tree is 25 m long. What is the height of the tree? **[2010-I]**

(a) 21 m (b) 10 m
(c) 35 m (d) None of these

57. If $a + b + c = 0$, then what is the value of $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}$? **[2010-II]**

(a) –3 (b) 0 (c) 1 (d) 3

58. If $(x^4 + x^{-4}) = 322$, then what is one of the value of $(x - x^{-1})$? **[2010-II]**

(a) 18 (b) 16 (c) 8 (d) 4

59. If $x = (b - c)(a - d)$, $y = (c - a)(b - d)$ and $z = (a - b)(c - d)$, then what is $x^3 + y^3 + z^3$ equal to? **[2010-II]**

(a) xyz (b) $2xyz$ (c) $3xyz$ (d) $-3xyz$

60. What are the factors of $x^2 + 4y^2 + 4y - 4xy - 2x - 8$? **[2010-II]**
(a) $(x - 2y - 4)$ and $(x - 2y + 2)$
(b) $(x - y + 2)$ and $(x - 4y + 4)$
(c) $(x - y + 2)$ and $(x - 4y - 4)$
(d) $(x + 2y - 4)$ and $(x + 2y + 2)$

61. If the expression $(px^3 + x^2 - 2x - q)$ is divisible by $(x - 1)$ and $(x + 1)$, what are the values of p and q, respectively? **[2010-II]**
(a) 2, –1 (b) –2, 1 (c) –2, –1 (d) 2, 1

62. If $x^5 - 9x^2 + 12x - 14$ is divisible by $(x - 3)$, then what is the remainder? **[2011-I]**
(a) 0 (b) 1 (c) 56 (d) 184

63. If $x - \frac{1}{x} = \frac{1}{3}$, then what is $9x^2 + \frac{9}{x^2}$ is equal to? **[2011-I]**
(a) 18 (b) 19 (c) 20 (d) 21

64. What is the value of $\frac{1}{1+\sqrt{2}+\sqrt{3}} + \frac{1}{1-\sqrt{2}+\sqrt{3}}$? **[2011-I]**
(a) 1 (b) $\sqrt{2}$ (c) $\sqrt{3}$ (d) 2

65. If $a + b + c = 6$ and $a^2 + b^2 + c^2 = 26$, then what is $ab + bc + ca$ equal to? **[2011-II]**
(a) 0 (b) 2 (c) 4 (d) 5

66. If $3x^3 - 2x^2 y - 13xy^2 + 10y^3$ is divided by $x - 2y$, then what is the remainder? **[2011-II]**
(a) 0 (b) x (c) $y + 5$ (d) $x - 3$

67. What is the remainder when $(x^{11} + 1)$ is divided by $(x + 1)$? **[2011-II]**
(a) 0 (b) 2 (c) 11 (d) 12

68. If $(x - 3)$ is a factor of $(x^2 + 4px - 11p)$, then what is the value of p? **[2011-II]**
(a) –9 (b) –3 (c) –1 (d) 1

69. What is $\frac{x^2-3x+2}{x^2-5x+6} \div \frac{x^2-5x+4}{x^2-7x+12}$ equal to? **[2011-II]**
(a) $\frac{x+3}{x-3}$ (b) 1 (c) $\frac{x+1}{x-1}$ (d) 2

70. What is the value of $\frac{(2.3)^3-0.027}{(2.3)^2+0.69+0.09}$? **[2011-II]**
(a) 0.3 (b) 2.3 (c) 1 (d) 2

71. If $x + y + z = 0$, then what is $\frac{xyz}{(x+y)(y+z)(z+x)}$ equal to (where, $x \neq -y, y \neq -z, z \neq -x$)? **[2011-II]**
(a) –1 (b) 1
(c) $xy + yz + zx$ (d) None of these

72. What is the value of
$$\frac{1.001\times1.001\times1.001+0.999\times0.999\times0.999}{1.001\times1.001-1.001\times0.999+0.999\times0.999}?$$
[2012-I]
(a) 1 (b) 2 (c) 3 (d) 4

73. AB is a straight line. C is a point whose distance from AB is 3cm. What is the number of points which are at a distance of 1cm from AB and 5cm from C? **[2012-I]**
(a) 1 (b) 2 (c) 3 (d) 4

74. What is the value of k that $(2x - 1)$ may be a factor of $4x^4 - (k - 1)x^3 + kx^2 - 6x + 1$? **[2012-I]**
(a) 8 (b) 9 (c) 12 (d) 13

75. If $p(x)$ is a common multiple of degree 6 of the polynomials $f(x) = x^3 + x^2 - x - 1$ and $g(x) = x^3 - x^2 + x - 1$, then which one of the following is correct? **[2012-I]**
(a) $p(x) = (x - 1)^2 (x + 1)^2 (x^2 + 1)$
(b) $p(x) = (x - 1) (x + 1)^2 (x^2 + 1)^2$
(c) $p(x) = (x - 1)^3 (x + 1)^2 (x^2 + 1)$
(d) $p(x) = (x - 1)^2 (x^4 + 1)$

76. If $x^3 + 5x^2 + 10k$ leaves remainder $-2x$ when divided by $x^2 + 2$, then what is the value of k? **[2012-I]**
(a) –2 (b) –1 (c) 1 (d) 2

77. If $x + \frac{1}{x} = a$, then what is the value of $x^3 + x^2 + \frac{1}{x^3} + \frac{1}{x^2}$? **[2012-I]**
(a) $a^3 + a^2$ (b) $a^3 + a^2 - 5a$
(c) $a^3 + a^2 - 3a - 2$ (d) $a^3 + a^2 - 4a - 2$

78. If $a^x = b$, $b^y = c$ and $xyz = 1$, then what is the value of c^z? **[2012-I]**
(a) a (b) b (c) ab (d) $\frac{a}{b}$

79. If $3^{x+y} = 81$ and $81^{x-y} = 3$, then what is the value of x? **[2012-I]**
(a) $\frac{17}{16}$ (b) $\frac{17}{8}$ (c) $\frac{17}{4}$ (d) $\frac{15}{4}$

80. $x^4 + xy^3 + x^3 y + xz^3 + y^4 + yz^3$ is divisible by **[2012-I]**
(a) Only $(x - y)$
(b) Only $(x^3 + y^3 + z^3)$
(c) Both $(x + y)$ and $(x^3 + y^3 + z^3)$
(d) None of the above

81. What is the value of
$3 \times 0.3 \times 0.03 \times 0.003 \times 0.0003 \times 30$? **[2012-II]**
(a) $(0.09)^3$ (b) $(0.009)^3$
(c) $(0.0009)^3$ (d) None of these

82. What is the value of
$$\frac{\left[(2.247)^3+(1.730)^3+(1.023)^3-3\times2.247\times1.730\times1.023\right]}{\left[(2.247)^2+(1.730)^2+(1.023)^2-2.247\times1.730-1.730\times1.023-2.247\times1.023\right]}?$$
[2012-II]
(a) 1.730 (b) 4 (c) 5 (d) 5.247

83. If two factors of $a^4 - 2a^3 - 9a^2 + 2a + 8$ are $(a + 1)$ and $(a - 1)$, then what are the other two factors? **[2012-II]**
(a) $(a - 2)$ and $(a + 4)$ (b) $(a + 2)$ and $(a + 4)$
(c) $(a + 2)$ and $(a - 4)$ (d) $(a - 2)$ and $(a - 4)$

84. If $0.764\, y = 1.236\, x$, then what is the value of $\left(\frac{y-x}{y+x}\right)$? **[2012-II]**
(a) 0.764 (b) 0.236 (c) 2 (d) 0.472

85. If the expression $x^3 + 4x^2 + 4x + k$ has $(x + 4)$ as a factor, then what is the value of k? **[2012-II]**
(a) –16 (b) 16 (c) 32 (d) –32

86. What is the value of k which will make the expression $4x^2 + 12x + k$ a perfect square? **[2012-II]**
(a) 5 (b) 7 (c) 8 (d) 9

87. For what value of k, $(x + 5)$ is a factor of $6x^2 + kx + 10$? **[2013-I]**
(a) 5 (b) 32 (c) 36 (d) 40

88. One of the factors of the polynomial $x^4 - 7x^3 + 5x^2 - 6x + 81$ is: **[2013-I]**
(a) $x + 2$ (b) $x - 2$ (c) $x + 3$ (d) $x - 3$

89. $(a + 1)^4 - a^4$ is divisible by **[2013-I]**
(a) $-2a^2 + 2a - 1$ (b) $2a^3 - 2a - 1$
(c) $2a^3 - 2a + 1$ (d) $2a^2 + 2a + 1$

90. The factor(s) of $5px - 10qy + 2rpx = 4qry$ is/are **[2013-I]**
(a) Only $(5 + 2r)$
(b) Only $(px - 2qy)$
(c) Both $(5 + 2r)$ and $(px - 2qy)$
(d) Neither $(5 + 2r)$ nor $(px - 2qy)$

91. If the expression $x^3 + 3x^2 + 4x + p$ contains $(x + 6)$ as a factor, then the value of p is **[2013-I]**
(a) 132 (b) 141 (c) 144 (d) 151

92. What is the value of
$$\frac{725\times725\times725+371\times371\times371}{725\times725-725\times371+371\times371}?$$
[2013-I]
(a) 9610 (b) 1960 (c) 1096 (d) 1016

93. Consider the following statements :
I. $x + 3$ is the factor of $x^3 + 2x^2 + 3x + 8$.
II. $x - 2$ is the factor of $x^3 + 2x^2 + 3x + 8$.
Which of the statements given above is/are correct? **[2013-II]**
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

94. What is $\dfrac{(x^2+y^2)(x-y)-(x-y)^3}{x^2y-xy^2}$ equal to? **[2013-II]**
(a) 1 (b) 2 (c) 4 (d) –2

95. If the expression $x^3 + 3x^2 + 4x + k$ has a factor $x + 5$, then what is the value of k? **[2013-II]**
(a) –70 (b) 70 (c) 48 (d) –48

96. The quantity which must be added to $(1 - x)(1 + x^2)$ to obtain x^3 is **[2013-II]**
(a) $2x^3 + 3x^2 + x + 1$ (b) $2x^3 + x^2 + x - 1$
(c) $2x^3 - x^2 + x - 1$ (d) $-x^2 + x - 1$

97. If $(49)^2 - (25)^2 = 37x$, then what is x equal to? **[2013-II]**
(a) 64 (b) 74 (c) 48 (d) 42

98. For what values of k will
$$4x^5 + 9x^4 - 7x^3 - 5x^2 - 4kx + 3k^2$$
contain $x - 1$ as a factor? **[2013-II]**
(a) $3, -\frac{1}{2}$ (b) $3, -1$ (c) $0, \frac{1}{3}$ (d) $1, \frac{1}{3}$

99. $x(y^2 - z^2) + y(z^2 - x^2) + z(x^2 - y^2)$ is divisible by **[2013-II]**
(a) Only $(y - z)$
(b) Only $(z - x)$
(c) Both $(y - z)$ and $(z - x)$
(d) Neither $(y - z)$ nor $(z - x)$

100. $x^3 + 6x^2 + 11x + 6$ is divisible by **[2014-I]**
(a) Only $(x + 1)$ (b) Only $(x + 2)$
(c) Only $(x + 3)$ (d) All of these

101. What should be added to be $x(x + a)(x + 2a)(x + 3a)$, so that the sum be a perfect square? **[2014-I]**
(a) $9a^2$ (b) $4a^2$
(c) a^4 (d) None of these

102. If $3x^4 - 2x^3 + 3x^2 - 2x + 3$ is divided by $(3x + 2)$, then the remainder is **[2014-I]**
(a) 0 (b) $\frac{185}{27}$ (c) $\frac{181}{25}$ (d) $\frac{3}{4}$

103. The expression $2x^3 + x^2 - 2x + 1$ is divisible by **[2014-I]**
(a) $x + 2$ (b) $2x + 1$
(c) $x - 2$ (d) $2x - 1$

104. If $\left(x^2+\frac{1}{x^2}\right)=\frac{17}{4}$, then what is $\left(x^3-\frac{1}{x^3}\right)$ equal to? **[2014-I]**
(a) $\frac{75}{16}$ (b) $\frac{63}{8}$
(c) $\frac{95}{8}$ (d) None of these

105. If $(x + k)$ is the common factor of $x^2 + ax + b$ and $x^2 + cx + d$. of and then what is k equal to ? **[2014-II]**
(a) $(d - b) / (c - a)$ (b) $(d - b) / (a - c)$
(c) $(d + b) / (c + a)$ (d) $(d - b) / (c + a)$

106. What is the remainder when $x^5 - 5x^2 + 125$ is divided by $x + 5$? **[2014-II]**
(a) 0 (b) 125 (c) –3125 (d) 3125

107. What is the lowest common multiple of $ab(x^2 + 1) + x(a^2 + b^2)$ and $ab(x^2 - 1) + x(a^2 - b^2)$? **[2014-II]**
(a) $(a^2x^2 - b^2)(a + bx)$
(b) $(a^2x^2 - b^2)(a + bx)^2$
(c) $(a^2x^2 - b^2)(a - bx)$
(d) $(a^2x^2 - b^2)(a - bx)^2$

108. Consider the following statements : **[2014-II]**
1. $(a - b - c)$ is one of the factors of $3abc + b^3 + c^3 - a^3$.
2. $(b + c - 1)$ is one of the factors of $3bc + b^3 + c^3 - 1$

Which of the above statements is / are correct?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

109. If the equation $x^2 + 2(1 + k)x + k^2 = 0$ has equal roots, then what is the value of k ? **[2014-II]**

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) 1 (d) – 1

110. What is the highest common factor of $2x^3 + x^2 - x - 2$ and $3x^3 - 2x^2 + x - 2$? **[2014-II]**

(a) $x - 1$ (b) $x + 1$
(c) $2x + 1$ (d) $2x - 1$

111. What is the remainder when (1235 × 4523 × 2451) is divided by 12 ? **[2014-II]**

(a) 1 (b) 3 (c) 5 (d) 7

112. For what value of k is $(x - 5)$ a factor of $x^3 - 3x^2 + kx - 10$? **[2015-I]**

(a) – 8 (b) 4 (c) 2 (d) 1

113. The expression $x^3q^2 - x^3pt + 4x^2pt - 4x^2q^2 + 3xq^2 - 3xpt$ is divisible by **[2015-I]**

(a) $(x - 1)$ only
(b) $(x - 3)$ only
(c) both $(x - 1)$ and $(x-3)$
(d) neither $(x - 1)$ nor $(x - 3)$

114. The value of $\frac{1}{1\times 4}+\frac{1}{4\times 7}+\frac{1}{7\times 10}+....+\frac{1}{16\times 19}$ **[2015-II]**

(a) $\frac{5}{19}$ (b) $\frac{6}{19}$ (c) $\frac{8}{19}$ (d) $\frac{9}{19}$

115. Consider the following in respect of the equation **[2015-II]**

$$y = \frac{\sqrt{(x-1)^2}}{x-1}$$

1. $y = 1$ if $x > 1$
2. $y = -1$ if $x < 1$
3. y exists for all values of x

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) 1 and 2 only (d) 1, 2 and 3

116. If a, b and c satisfy the equation $x^3 - 3x^2 + 2x + 1 = 0$ then what is the value of $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$? **[2015-II]**

(a) $-\frac{1}{2}$ (b) 2 (c) –2 (d) $\frac{1}{2}$

117. Which one of the following is correct? **[2015-II]**

(a) $(x + 2)$ is a factor of $x^4 - 6x^3 + 12x^2 - 24x + 32$
(b) $(x + 2)$ is a factor of $x^4 + 6x^3 - 12x^2 + 24x - 32$
(c) $(x - 2)$ is a factor of $x^4 - 6x^3 + 12x^2 - 24x + 32$
(d) $(x - 2)$ is a factor of $x^4 + 6x^3 - 12x^2 + 24x - 32$

118. If $x = \frac{91}{216}$, then the value of $3-\frac{1}{(1-x)1/3}$ is **[2015-II]**

(a) $\frac{9}{5}$ (b) $\frac{5}{9}$ (c) $\frac{4}{9}$ (d) $\frac{4}{5}$

119. If the equations $x^2 - px + q = 0$ and $x^2 + qx - p = 0$ have a common root, then which one of the following is correct? **[2016-I]**

(a) $p - q = 0$ (b) $p + q - 2 = 0$
(c) $p + q - 1 = 0$ (d) $p - q - 1 = 0$

120. If $(s - a) + (s - b) + (s - c) = s$, then the value of

$\frac{(s-a)^2+(s-b)^2+(s-c)+s^2,}{a^2+b^2+c^2}$ will be **[2016-I]**

(a) 3 (b) 1 (c) 0 (d) –1

121. If the polynomial $x^6 + px^5 + qx^4 - x^2 - x - 3$ is divisible by $(x^4 - 1)$, then the value of $p^2 + q^2$ is **[2016-I]**

(a) 1 (b) 9 (c) 10 (d) 13

122. Let m be a non-zero integer and n be a positive integer. Let R be the remainder obtained on dividing the polynomial $x^2 + m^2$ by $(x - m)$. Then **[2016-I]**

(a) R is a non zero even integer
(b) R is odd, if m is odd
(c) $R + t^3$ for some integer s, if n is even
(d) $R = t^3$ for some integer t, if 3 divides n

123. If $x = \frac{\sqrt{a+2b}+\sqrt{a-2b}}{\sqrt{a+2b}-\sqrt{a-2b}}$

then $bx^2 - ax + b$ is equal to (given that $b \neq 0$) **[2016-I]**

(a) 0 (b) 1 (c) ab (d) $2ab$

124. If $a^3 = 117 + b^3$ and $a = 3 + b$, then the value of $a + b$ is (given that $a > 0$ and $b > 0$) **[2016-I]**

(a) 7 (b) 9 (c) 11 (d) 13

125. If $\frac{a}{b}=\frac{b}{c}=\frac{c}{d}$, then which of the following is/are correct?

1. $\frac{b^3+c^3+d^3}{a^3+b^3+c^3}=\frac{d}{a}$ 2. $\frac{a^2+b^2+c^2}{b^2+c^2+d^2}=\frac{a}{d}$

Select the correct answer using the code given below. **[2016-I]**

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

126. For what value of k is $(x + 1)$ a factor of $x^3 + kx^2 - x + 2$? **[2016-I]**

(a) 4 (b) 3 (c) 1 (d) –2

127. Which of the points $P(5, -1)$, $Q(3, -2)$ and $R(1, 1)$ lie in the solution of the system of inequations $x + y \leq 4$ and $x - y \geq 2$? **[2016-I]**

(a) Q and R only (b) P and R only
(c) P and Q only (d) P Q and R

128. If $x^2 = y + z$, $y^2 = z + x$ and $z^2 = x + y$, then what is the value of $\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1}$? **[2016-II]**

(a) –1 (b) 1
(c) 2 (d) 4

129. $2^{122} + 4^{62} + 8^{42} + 4^{64} + 2^{130}$ is divisible by which one of the following integers ? **[2016-II]**

(a) 3 (b) 5 (c) 7 (d) 11

130. If $4x + 3a = 0$, then. what is the value of

$$\frac{x^2 + ax + a^2}{x^3 - a^3} - \frac{x^2 - ax + a^2}{x^3 + a^3}?$$ **[2016-II]**

(a) $-\frac{4}{7a}$ (b) $\frac{7}{a}$ (c) $-\frac{32}{7a}$ (d) $\frac{24}{7a}$

131. If $x + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{3}}} = 2$, then what is x equal to ? **[2016-II]**

(a) $\frac{7}{10}$ (b) $\frac{13}{10}$ (c) $\frac{11}{10}$ (d) $\frac{17}{10}$

132. $(x + 4)$ is a factor of which one of the following expressions ? **[2017-I]**

(a) $x^2 - 7x + 44$ (b) $x^2 + 7x - 44$
(c) $x^2 - 7x - 44$ (d) $x^2 + 7x + 44$

133. If $x = t^{\frac{1}{t-1}}$ and $y = t^{\frac{t}{t-1}}$, $t > 0$, $t \neq$, then what is the relation between x and y ? **[2017-I]**

(a) $y^x = x^{1/y}$ (b) $x^{1/y} = y^{1/x}$
(c) $x^y = y^x$ (d) $x^y = y^{1/x}$

134. If A : B = 3 : 4, then what is the value of the expression $\left(\frac{3A^2 + 4B}{3A - 4B^2}\right)$? **[2017-I]**

(a) $\frac{43}{55}$ (b) $-\frac{43}{55}$
(c) $\frac{47}{55}$ (d) Cannot be determined

135. If $x = 2 + 2^{2/3} + 2^{1/3}$, then what is the value of $x^3 - 6x^2 + 6x$? **[2017-I]**

(a) 3 (b) 2 (c) 1 (d) 0

136. If $\sqrt{\frac{x}{y}} = \frac{24}{5} + \sqrt{\frac{y}{x}}$ and $x + y = 26$, then what is the value of xy? **[2017-I]**

(a) 5 (b) 15 (c) 25 (d) 30

137. If $a + b = 5$ and $ab = 6$, then What is the value of $a^3 + b^3$?

(a) 35 (b) 40 (c) 90 (d) 125

138. What are the factors of $x^3 + 4x^2 - 11x - 30$? **[2017-II]**

(a) $(x - 2)$, $(x + 3)$ and $(x + 5)$
(b) $(x + 2)$, $(x + 3)$ and $(x - 5)$
(c) $(x + 2)$, $(x - 3)$ and $(x + 5)$
(d) $(x + 2)$, $(x - 3)$ and $(x - 5)$

139. If x = 111 .... 1 (20 digits), y = 333 .... 3 (10 digits) and z = 222 .... 2 (10 digits), then what is $\frac{x - y^2}{z}$ equal to? **[2017-II]**

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) 3

140. If $5x^3 + 5x^2 - 6x + 9$ is divided by $(x + 3)$, then the remainder is **[2017-II]**

(a) 135 (b) –135 (c) 63 (d) –63

141. The quotient of $8x^3 - y^3$ when divided by $2xy + 4x^2 + y^2$ is **[2017-II]**

(a) $2x + y$ (b) $x + 2y$
(c) $2x - y$ (d) $4x - y$

142. If $(x + 2)$ is a common factor of $x^2 + ax + b$ and $x^2 + bx + a$, then the ratio a : b is equal to **[2017-II]**

(a) 1 (b) 2 (c) 3 (d) 4

143. Let
$f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + ... + a_{n-1}x + a_n$, where $a_0, a_1, a_2, ..., a_n$ are real numbers.
If f(x) is divided by $(ax - b)$, then the remainder is **[2017-II]**

(a) $f\left(\frac{b}{a}\right)$ (b) $f\left(-\frac{b}{a}\right)$ (c) $f\left(\frac{a}{b}\right)$ (d) $f\left(-\frac{a}{b}\right)$

144. The product of the polynomials $(x + 2)$, $(x - 2)$, $(x^3 - 2x^2 + 4x - 8)$ and $(x^3 + 2x^2 + 4x + 8)$ is **[2017-II]**

(a) $x^8 - 256$ (b) $(x^4 - 16)^2$
(c) $(x^4 + 16)^2$ (d) $(x^2 - 4)^4$

145. The factors of $x(x + 2)(x + 3)(x + 5) - 72$ are

(a) x, $(x + 3)$, $(x + 4)$ and $(x - 6)$
(b) $(x - 1)$, $(x + 6)$ and $(x^2 - 2x - 12)$
(c) $(x - 1)$, $(x + 6)$ and $(x^2 + 5x + 12)$
(d) $(x + 1)$, $(x - 6)$ and $(x^2 - 5x - 12)$

146. a, b, c, b are non-zero integers such that (ab) divides (cd). If a and c are coprime, then which one of the following is correct? **[2017-II]**

(a) a is a factor of c (b) a is a factor of b
(c) a is a factor of d (d) d is a factor of a

147. If $ab + bc + ca = 0$, then what is the value of $\frac{a^2}{a^2 - bc} + \frac{b^2}{b^2 - ca} + \frac{c^2}{c^2 - ab}$? **[2017-II]**

(a) 3 (b) 0 (c) 1 (d) –1

148. What is $\frac{(x-y)(y-z)(z-x)}{(x-y)^3+(y-z)^3+(z-x)^3}$ equal to? **[2017-II]**

(a) $-\frac{1}{3}$ (b) $\frac{1}{3}$ (c) 3 (d) –3

149. $5^{17}+5^{18}+5^{19}+5^{20}$ is divisible by **[2018-I]**

(a) 7 (b) 9 (c) 11 (d) 13

150. If $\frac{b}{y}+\frac{z}{c}=1$ and $\frac{c}{z}+\frac{x}{a}=1$, then what is $\frac{ab+xy}{bx}$ equal to? **[2018-I]**

(a) 1 (b) 2 (c) 0 (d) –1

151. If $\frac{a^2-1}{a}=5$, then what is the value of $\frac{a^6-1}{a^3}$? **[2018-I]**

(a) 125 (b) –125 (c) 140 (d) –140

152. If $(x+3)$ is a factor of x^3+3x^2+4x+k, then what is the value of k? **[2018-I]**

(a) 12 (b) 24 (c) 36 (d) 72

153. Which one of the following is a zero of the polynomial $3x^3$ $4x^2-7$? **[2018-I]**

(a) 0 (b) 1 (c) 2 (d) –1

154. The remainder when $3x^3+kx^2+5x-6$ is divided by $(x+1)$ is –7. What is the value of k? **[2018-I]**

(a) –14 (b) 14 (c) –7 (d) 7

155. If f(x) and g(x) are polynomials of degree p and q respectively, then the degree of $\{f(x) \pm g(x)\}$ (if it is non-zero) is **[2018-I]**

(a) Greater than min (p, q)
(b) Greater than max (p, q)
(c) Less than or equal to max (p, q)
(d) Equal to min (p, q)

156. If α and β are two real numbers such that $\alpha+\beta=-\frac{q}{p}$ and $\alpha\beta=\frac{r}{p}$, where $1<p<q<r$, then which one of the following is the greatest? **[2018-II]**

(a) $\frac{1}{\alpha+\beta}$ (b) $\frac{1}{\alpha}+\frac{1}{\beta}$ (c) $-\frac{1}{\alpha\beta}$ (d) $\frac{\alpha\beta}{\alpha+\beta}$

157. If the sum of the squares of three consecutive natural numbers is 110, then the sum of their cubes is **[2018-II]**

(a) 625 (b) 654 (c) 684 (d) 725

158. The number of all pairs (m, n), where m and n are positive integers, such that $\frac{1}{m}+\frac{1}{n}-\frac{1}{mn}=\frac{2}{5}$ is **[2018-II]**

(a) 6 (b) 5 (c) 4 (d) 2

159. If H is the harmonic mean of P and Q, then the value of $\frac{H}{P}+\frac{H}{Q}$ is **[2018-II]**

(a) 1 (b) 2 (c) $\frac{P+Q}{PQ}$ (d) $\frac{PQ}{P+Q}$

160. The remainder when $3x^3-2x^2y-13xy^2+10y^3$ is divided by $(x-2y)$ is equal to **[2018-II]**

(a) zero (b) y (c) $y-5$ (d) $y+3$

161. If $ab+bc+ca=0$, then the value of **[2018-II]**

$\frac{(b^2-ca)(c^2-ab)+(a^2-bc)(c^2-ab)+(a^2-bc)(b^2-ca)}{(a^2-bc)(b^2-ca)(c^2-ab)}$ is

(a) –1 (b) 0 (c) 1 (d) 2

162. For $x>0$, what is the minimum value of $x+\frac{x+2}{2x}$? **[2019-I]**

(a) 1 (b) 2
(c) $2\frac{1}{2}$ (d) Cannot be determined

163. If $\frac{1+px}{1-px}\sqrt{\frac{1-qx}{1+qx}}=1$, then what are the non-zero solutions of x? **[2019-I]**

(a) $\pm\frac{1}{p}\sqrt{\frac{2p-q}{q}}$, $2p\neq q$ (b) $\pm\frac{1}{pq}\sqrt{p-q}$, $p\neq q$

(c) $\pm\frac{p}{q}\sqrt{p-q}$, $p\neq q$ (d) $\pm\frac{q}{p}\sqrt{2p-q}$, $2p\neq q$

164. If a, b and c are positive integers such that

$\frac{1}{a+\frac{1}{b+\frac{1}{c+\frac{1}{2}}}}=\frac{16}{23}$, then what is the mean of a, b and c? **[2019-I]**

(a) 1 (b) 2 (c) 1.33 (d) 2.33

165. For any two real numbers a and b,

$\sqrt{(a-b)^2}+\sqrt{(b-a)^2}$ is **[2019-I]**

(a) always zero (b) never zero
(c) positive only if $a\neq b$ (d) positive if and only if $a>b$

166. If $\frac{a}{b+d}=\frac{b}{c+a}=\frac{c}{a+b}$, then which one of the following statements is correct? **[2019-I]**

(a) Each fraction is equal to 1 or –1,
(b) Each fraction is equal to $\frac{1}{2}$ or 1.
(c) Each fraction is equal to $\frac{1}{2}$ or –1.
(d) Each fraction is equal to $\frac{1}{2}$ only.

167. Given that the polynomial $(x^2 + ax + b)$ leaves the same remainder when divided by $(x - 1)$ or $(x + 1)$. What are the values of a and b respectively ? **[2019-I]**

(a) 4 and 0 (b) 0 and 3
(c) 3 and 0 (d) 0 and any integer

168. What is $\dfrac{(x-y)^3+(y-z)^3+(z-x)^3}{3(x-y)(y-z)(z-x)}$ equal to? **[2019-I]**

(a) 1 (b) 0 (c) $\dfrac{1}{3}$ (d) 3

169. If $a^x = b^y = c^z$ and $b^2 = ac$, then what is $\dfrac{1}{x}+\dfrac{1}{z}$ equal to ? **[2019-I]**

(a) $\dfrac{1}{y}$ (b) $-\dfrac{1}{y}$ (c) $\dfrac{2}{y}$ (d) $-\dfrac{2}{y}$

170. What is the maximum value of the expression $\dfrac{1}{x^2+5x+10}$? **[2019-II]**

(a) $\dfrac{15}{4}$ (b) $\dfrac{15}{2}$ (c) 1 (d) $\dfrac{4}{15}$

171. What is the value of $\dfrac{(x-y)^3+(y-z)^3+(z-x)^3}{9(x-y)(y-z)(z-x)}$? **[2019-II]**

(a) 0 (b) $\dfrac{1}{3}$ (c) $\dfrac{1}{9}$ (d) 1

172. If the sum of a real number and its reciprocal is $\dfrac{26}{5}$, then how many such numbers are possible ? **[2019-II]**

(a) None (b) One (c) Two (d) Four

173. If $\dfrac{36}{11}=3+\cfrac{1}{x+\cfrac{1}{y+\cfrac{1}{z}}}$, where x, y and z are natural numbers, then what is $(x + y + z)$ equal to ? **[2019-II]**

(a) 6 (b) 7 (c) 8 (d) 9

174. A real number x, such that $(x - x^2)$ is maximum. What is x equal to ? **[2019-II]**

(a) -1.5 (b) -0.5 (c) 0.5 (d) 1.5

175. For what value of k can the expression $x^3 + kx^2 - 7x + 6$ be resolved into three linear factors ? **[2019-II]**

(a) 0 (b) 1 (c) 2 (d) 3

176. The quotient when $x^4 - x^2 + 7x + 5$ is divided by $(x + 2)$ is $ax^3 + bx^2 + cx + d$. What are the values of a, b, c and d respectively ? **[2019-II]**

(a) $1, -2, 3, 1$ (b) $-1, 2, 3, 1$
(c) $1, -2, -3, -1$ (d) $-1, 2, -3, -1$

177. What is $(x - a)(x - b)(x - c)$ equal to ? **[2019-II]**

(a) $x^3 - (a + b + c)x^2 + (bc + ca + ab)x - abc$
(b) $x^3 + (a + b + c)x^2 + (bc + ca + ab)x + abc$
(c) $x^3 - (bc + ca + ab)x^2 + (a + b + c)x - abc$
(d) $x^3 + (bc + ca + ab)x^2 - (a + b + c)x - abc$

178. What are the values of p and q respectively, if $(x - 1)$ and $(x + 2)$ divide the polynomial x^3+4x^2+px+q ? **[2020-I]**

(a) $1, -6$ (b) $2, -6$ (c) $1, 6$ (d) $2, 6$

179. What is $\dfrac{1}{a^{m-n}-1}+\dfrac{1}{a^{n-m}-1}$ equal to ? **[2020-I]**

(a) 1 (b) -1 (c) 0 (d) $2a^{m-n}$

180. If $f(x)$ is divided by $(x - \alpha)(x - \beta)$ where $\alpha \neq \beta$, then what is the remainder ? **[2020-I]**

(a) $\dfrac{(x-\alpha)f(\alpha)-(x-\beta)f(\beta)}{\alpha-\beta}$

(b) $\dfrac{(x-\alpha)f(\beta)-(x-\beta)f(\alpha)}{\alpha-\beta}$

(c) $\dfrac{(x-\beta)f(\alpha)-(x-\alpha)f(\beta)}{\alpha-\beta}$

(d) $\dfrac{(x-\beta)f(\beta)-(x-\alpha)f(\alpha)}{\alpha-\beta}$

181. If $(x^2 - 1)$ is a factor of $ax^4 + bx^3 + cx^2 + dx + e$, then which one of the following is correct ? **[2020-I]**

(a) $a+b+c=d+e$ (b) $a+b+e=c+d$
(c) $b+c+d=a+e$ (d) $a+c+e=b+d$

182. If $\left(x^8+\dfrac{1}{x^8}\right)=47$, what is the value of $\left(x^6+\dfrac{1}{x^6}\right)$? **[2020-I]**

(a) 36 (b) 27 (c) 18 (d) 9

HINTS & SOLUTIONS

1. (b) The expression $3x^3 - kx^2 + 4x + 16$ is divisible by $x - \frac{k}{2}$.

Then, $x = \frac{k}{2}$ satisfy the equation

$\Rightarrow 3\left(\frac{k}{2}\right)^3 - k\left(\frac{k}{2}\right)^2 + 4\left(\frac{k}{2}\right) + 16 = 0$

$\Rightarrow \frac{3k^3 - 2k^3 + 16k + 128}{8} = 0$

$\Rightarrow k^3 + 16k + 128 = 0$

$\Rightarrow (k + 4)(k^2 - 4k + 32) = 0$

$\Rightarrow k + 4 = 0$

$\Rightarrow k = -4$

2. (c) $x^4 + xy^3 + xz^3 + x^3y + y^4 + yz^3$

$= x(x^3 + y^3 + z^3) + y(x^3 + y^3 + z^3)$

$= (x + y)(x^3 + y^3 + z^3)$

Hence, $(x^3 + y^3 + z^3)$ is a factor of $x^4 + xy^3 + xz^3 + x^3y + y^4 + yz^3$

3. (a) $\frac{1}{1+p+q^{-1}} + \frac{1}{1+q+r^{-1}} + \frac{1}{1+r+p^{-1}}$

$= \frac{1}{1+p+\frac{1}{q}} + \frac{1}{1+q+\frac{1}{r}} + \frac{1}{1+r+\frac{1}{p}}$

$= \frac{q}{1+pq+q} + \frac{r}{r+rq+1} + \frac{p}{p+rp+1}$

$= \frac{q}{1+pq+q} + \frac{r}{\frac{1}{pq}+\frac{1}{p}+1} + \frac{p}{p+\frac{1}{q}+1}$

$(\because pqr = 1)$

$= \frac{q}{1+pq+q} + \frac{rpq}{1+q+pq} + \frac{pq}{pq+1+q}$

$= \frac{q+rpq+pq}{1+pq+q}$ $(\because pqr = 1)$

$= \frac{q+1+pq}{1+pq+q} = 1$

4. (a) Given, $x + y + z = 2s$

Also, $(s - x) + (s - y) - z = 2s - (x + y + z)$

$= 2s - 2s = 0$

$\therefore (s - x)^3 + (s - y)^3 - z^3 + 3(s - x)(s - y)(z) = 0$

$(\because a + b + c = 0 \Rightarrow a^3 + b^3 + c^3 = 3abc)$

$\Rightarrow (s - x)^3 + (s - y)^3 + 3(s - x)(s - y)(z) = z^3$

5. (a) Given, $x^2 = y + z$

$\Rightarrow x^2 + x = x + y + z$

$\Rightarrow \frac{x}{x+y+z} = \frac{1}{x+1}$

Similarly, $\frac{1}{y+1} = \frac{y}{x+y+z}$

and $\frac{1}{z+1} = \frac{z}{x+y+z}$

$\therefore \frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{z+1}$

$= \frac{x}{x+y+z} + \frac{y}{x+y+z} + \frac{z}{x+y+z}$

$= \frac{x+y+z}{x+y+z} = 1$

6. (c) Here, $p^3 + q^3 - r^3$

$= (p + q)(p^2 + q^2 - pq) - r^3$

$= (p + q)[(p + q)^2 - 3pq] - r^3$

$= r\left[r^2 - 3\cdot\frac{30}{r}\right] - r^3$

$(\because p + q = r$ given and $pqr = 30)$

$= r^3 - 90 - r^3 = -90$

7. (b) $\frac{x^8+4}{x^4+2x^2+2} = \frac{x^8+4x^4+4-4x^4}{x^4+2x^2+2}$

$= \frac{(x^4+2)^2-(2x^2)^2}{x^4+2x^2+2}$

$= \frac{(x^4+2x^2+2)(x^4-2x^2+2)}{x^4+2x^2+2}$

$= x^4 - 2x^2 + 2$

8. (c) Given, $2^x - 2^{x-1} = 4$

$\Rightarrow 2^x\left(1-\frac{1}{2}\right) = 4$

$\Rightarrow 2^x = 8$

$\Rightarrow 2^x = 2^3$

$\Rightarrow x = 3$

$\therefore 2^x + 2^{x-1} = 2^3 + 2^{3-1} = 8 + 4 = 12$

9. (d) Given, $x + \frac{1}{x} = p$

$\Rightarrow \left(x+\frac{1}{x}\right)^2 = p^2$

$\Rightarrow\ x^2 + \frac{1}{x^2} + 2 = p^2$

$\Rightarrow\ x^2 + \frac{1}{x^2} = p^2 - 2 \quad \ldots (i)$

$\Rightarrow\ \left(x^2 + \frac{1}{x^2}\right)^3 = (p^2 - 2)^3$

$\Rightarrow\ x^6 + \frac{1}{x^6} + 3\left(x^2 + \frac{1}{x^2}\right) = p^6 - 8 - 6p^2 (p^2 - 2)$

$\Rightarrow\ x^6 + \frac{1}{x^6} + 3(p^2 - 2) = p^6 - 8 - 6p^4 + 12p^2$

[from equation (i)]

$\Rightarrow\ x^6 + \frac{1}{x^6} = p^6 - 6p^4 + 9p^2 - 2$

10. (c) Given, $x + y + z = 0$

$\therefore\ \left(\frac{y-z-x}{2}\right)^3 + \left(\frac{z-x-y}{2}\right)^3 + \left(\frac{x-y-z}{2}\right)^3$

$= \left(\frac{y-(z+x)}{2}\right)^3 + \left(\frac{z-(x+y)}{2}\right)^3 + \left(\frac{x-(y+z)}{2}\right)^3$

$= \left(\frac{y-(-y)}{2}\right)^3 + \left(\frac{z-(-z)}{2}\right)^3 + \left(\frac{x-(-x)}{2}\right)^3$

$= \left(\frac{2y}{2}\right)^3 + \left(\frac{2z}{2}\right)^3 + \left(\frac{2x}{2}\right)^3 = y^3 + z^3 + x^3$

$= 3xyz$

(since, $a^3 + b^3 + c^3 = 3abc$, if $a + b + c = 0$)

11. (d) m and n are odd and $m > n$.
Let $m = 3$, $n = 1$
$\therefore\ m^2 - n^2 = 3^2 - 1 = 8$
Again, let $m = 5$, $n = 3$
$\therefore\ m^2 - n^2 = 5^2 - 3^2 = 16$
So, it is clear that $m^2 - n^2$ is always divisible by 8.

12. (a) $2 + \sqrt{2} + \frac{1}{2+\sqrt{2}} - \frac{1}{2-\sqrt{2}}$

$= 2 + \sqrt{2} + \frac{2-\sqrt{2}-2-\sqrt{2}}{4-2}$

$= 2 + \sqrt{2} + \frac{(-2\sqrt{2})}{2} = 2 + \sqrt{2} - \sqrt{2} = 2$

13. (d) When the volume increases upto 60 cm^3, then temperature 300 K.
$\therefore$ Volume is increased to 100 m^3, temperature

$= \frac{300 \times 100}{60} = 500$ K

14. (d) Now, $(x^2)^3 - y^3 = (x^2 - y)(x^4 + y^2 + x^2 y)$
Hence, the other factor is $x^2 - y$.

15. (c) $x^2 (y - z) + y^2 (z - x) - z(xy - yz - zx)$
$= x^2 y - x^2 z + y^2 z - y^2 x - zxy + yz^2 + z^2 x$
$= xy(x - y - z) - z(x^2 - y^2) + z^2 (x + y)$
$= xy(x - y - z) - z(x + y)(x - y - z)$
$= (x - y - z)(xy - yz - zx)$

16. (a) Required remainder
$= 3(2y)^3 - 2(2y)^2 y - 13(2y)y^2 + 10y^3$
$= 24y^3 - 8y^3 - 26y^3 + 10y^3$
$= 34y^3 - 34y^3 = 0$

17. (b) Given that,

$1 + \frac{1}{1 + \left\{\frac{1}{\left(1 + \frac{1}{x}\right)}\right\}} = \frac{11}{7}$

$\Rightarrow\ 1 + \frac{1}{\left\{1 + \frac{x}{1+x}\right\}} = \frac{11}{7}$

$\Rightarrow\ 1 + \frac{1+x}{1+2x} = \frac{11}{7}$

$\Rightarrow\ \frac{1+2x+1+x}{1+2x} = \frac{11}{7}$

$\Rightarrow\ \frac{2+3x}{1+2x} = \frac{11}{7}$

$\Rightarrow\ 14 + 21x = 11 + 22x$

$\therefore\ x = 3$

18. (a) Given, $x + y + z = 0$
$\therefore\ (x + y)(y + z)(z + x) = (-z)(-x)(-y) = -xyz$

19. (b) Given, $a + b = 3$
$\therefore\ (a + b)^3 = 3^3$
$\Rightarrow\ a^3 + b^3 + 3ab(a + b) = 27$
$\Rightarrow\ a^3 + b^3 + 9ab = 27$
($\because\ a + b = 3$)

20. (c)

$$\begin{array}{r|l} & x + 5 \\ x^2 + 2 & x^3 + 5x^2 + 10k \\ & \underline{\underset{-}{x^3} \quad \underset{-}{+2x}} \\ & 5x^2 - 2x + 10k \\ & \underline{\underset{-}{5x^2} \quad \underset{-}{+10}} \\ & -2x + 10k - 10 \end{array}$$

But remainder is $-2x$.
$\therefore\ -2x = -2x + 10k - 10 \Rightarrow k = 1$.

21. (b) $\because (5x^2 + 14x + 2)^2 - (4x^2 - 5x + 7)^2$
$= (5x^2 + 14x + 2 - 4x^2 + 5x - 7)(5x^2 + 14x + 2 + 4x^2 - 5x + 7)$
$= (x^2 + 19x - 5)(9x^2 + 9x + 9)$
$= 9(x^2 + 19x - 5)(x^2 + x + 1)$
From above it is clear that $(x^2 + x + 1)$ is a factor of $\{(5x^2 + 14x + 2)^2 - (4x^2 - 5x + 7)^2\}$, then remainder is zero.

22. (a) For $x = 1$,
$(1)^{29} - (1)^{24} + (1)^{13} - 1 = 1 - 1 + 1 - 1 = 0$
So, $(x - 1)$ is a factor of $x^{29} - x^{24} + x^{13} - 1$.
For $x = -1$,
$(-1)^{29} - 1(-1)^{24} + (-1)^{13} - 1 = -1 - 1 - 1 - 1 = -4$
So, $(x + 1)$ is not a factor of $x^{29} - x^{24} + x^{13} - 1$.

23. (b) $(x + y)^{-1} (x^{-1} + y^{-1}) (xy^{-1} + x^{-1}y)^{-1}$

$$= \left(\frac{1}{x+y}\right)\left(\frac{1}{x}+\frac{1}{y}\right)\left(\frac{x}{y}+\frac{y}{x}\right)^{-1}$$

$$= \left(\frac{1}{x+y}\right)\left(\frac{x+y}{xy}\right)\left(\frac{x^2+y^2}{xy}\right)^{-1}$$

$$= \frac{1}{xy}\cdot\frac{xy}{x^2+y^2} = \frac{1}{x^2+y^2} = (x^2 + y^2)^{-1}.$$

24. (c) Given, $ab - b + 1 = 0$
$\Rightarrow b(a - 1) = -1$

$$\Rightarrow b = \frac{1}{1-a} \qquad \text{... (i)}$$

Also, $bc - c + 1 = 0$

$$\Rightarrow b = \frac{-1+c}{c} \qquad \text{... (ii)}$$

From equations (i) and (ii),

$$\frac{1}{1-a} = \frac{-1+c}{c} \Rightarrow c = (1 - a)(-1 + c)$$

$\Rightarrow c = -1 + c + a - ac \Rightarrow a - ac = 1$

25. (b) Let $S = \{... -6, -4, -2, 0, 2, 4, 6, ...\}$
I. Now, $2 + (-2) = 0 \in S$, it is applied.
II. Now, $-2 -2 = -4 \in S$, it is applied.
III. Now, $-4 \times (4) = -16 \in S$, it is applied.
IV. Now, $-4 \div 4 = -1 \notin S$, it is not applied.

26. (c)

$$\begin{array}{r|l} & x-1 \\ \hline x+1 & x^2+1 \\ & \underline{\overset{-}{x^2} \overset{-}{+x}} \\ & -x+1 \\ & \underline{\overset{+}{-x} \overset{+}{-1}} \\ & 2 \end{array}$$

Hence, $r(x) = 2$.

27. (c) Given, equation is $x^2 - 4x + 1 = 0$

$$\therefore \quad x = \frac{4 \pm \sqrt{16 - 4\times1\times1}}{2\times1} = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$$

If $x = 2 + \sqrt{3}$, then

$$x^3 + \frac{1}{x^3} = \left(2+\sqrt{3}\right)^3 + \left(\frac{1}{2+\sqrt{3}}\right)^3$$

$$= \left(2+\sqrt{3}\right)^3 + \left(2-\sqrt{3}\right)^3$$

$$= 2^3 + \left(\sqrt{3}\right)^3 + 3\cdot2\cdot\sqrt{3}\left(2+\sqrt{3}\right) + (2)^3 - \left(\sqrt{3}\right)^3 - 3\cdot2\cdot\sqrt{3}\left(2-\sqrt{3}\right)$$

$$= 8 + \left(\sqrt{3}\right)^3 + 12\sqrt{3} + 18 + 18 - \left(\sqrt{3}\right)^3 - 12\sqrt{3} + 8$$

$= 8 + 18 + 8 + 18 = 52$

If $x = 2 - \sqrt{3}$,

then $x^3 + \dfrac{1}{x^3} = 52$

28. (a) Given, $x + y + z = 6$ and $xy + yz + zx = 11$
$\therefore$ $x^3 + y^3 + z^3 - 3xyz$
$= (x + y + z)\,[(x + y + z)^2 - 3\,(xy + yz + zx)]$
$= 6\,[6^2 - 3\,(11)] = 6 \times 3 = 18$

29. (d) $x^3 - 3x^2 - 3x + 9 = (x^2 - 3)(x - 3)$
So, the factors are $\sqrt{3}$, $-\sqrt{3}$ and 3.

30. (c) Let α be the common factors of the given equations.
$\therefore$ $\alpha^2 - 11\alpha + a = 0$ and $\alpha^2 - 14\alpha + 2a = 0$
Thus, it represents the same equation

$$\therefore \quad \frac{\alpha^2}{-22a+14a} = \frac{\alpha}{a-2a} = \frac{1}{-14+11}$$

(cross multiplication method)

$$\Rightarrow \frac{\alpha^2}{-8a} = \frac{\alpha}{-a} = \frac{1}{-3}$$

$$\Rightarrow \frac{\alpha}{-a} = -\frac{1}{3}$$

$$\Rightarrow \alpha = \frac{a}{3} \quad \text{and} \quad \frac{\alpha^2}{-8a} = -\frac{1}{3}$$

$$\Rightarrow \frac{\frac{a^2}{9}}{-8a} = -\frac{1}{3}$$

$\Rightarrow a = 0, a = 24.$

31. (c) Let $f(x) = 2x^3 - 3x^2 - 11x + 6$
Put $x = -2$, we get

$f(-2) = 2(-2)^3 - 3(-2)^2 - 11(-2) + 6$
$= -16 - 12 + 22 + 6 = 0$
Hence, $(x + 2)$ is a factor of $f(x)$.

32. (b)

$$\begin{array}{r|l} & 9x^2 - 2 \\ \hline 3x - 1 & 27x^3 - 9x^2 - 6x - 5 \\ & \underset{-}{27x^3} \underset{+}{-\ 9x^2} \\ \hline & \qquad -6x - 5 \\ & \qquad \underset{+}{-6x} \underset{-}{+\ 2} \\ \hline & \qquad\qquad -7 \end{array}$$

33. (b) Given, p ★ q = 2p + 2q – pq
$\because$ 8 ★ x = 4
$\Rightarrow 2(8) + 2(x) - 8x = 4$
$\Rightarrow -6x = -12$
$\therefore x = 2$

34. (a)

$$\left(\frac{x^2-3x+2}{x^3-8}\right) \div \left(\frac{x^2-9}{x^2+7x+12}\right) \times \left(\frac{x^3+2x^2+4x}{x^2+3x-4}\right)$$

$$= \left(\frac{x^2-3x+2}{x^3-8} \times \frac{x^2+7x+12}{x^2-9}\right) \times \frac{x^3+2x^2+4x}{x^2+3x-4}$$

$$= \frac{(x-1)\,(x-2)}{(x-2)\,(x^2+4+2x)} \times \frac{(x+4)\,(x+3)}{(x-3)\,(x+3)}$$

$$\times \frac{x\,(x^2+2x+4)}{(x-1)\,(x+4)}$$

$$= \frac{x}{x-3}$$

35. (b) $\left[\dfrac{(2.3)^3 - 0.027}{(2.3)^2 + 0.69 + 0.09}\right]$

$$= \frac{(2.3)^3 - (0.3)^3}{(2.3)^2 + 2.3\,(0.3) + (0.3)^2}$$

$= 2.3 - 0.3 = 2$
$[\because a^3 - b^3 = (a - b)(a^2 + ab + b^2)]$

36. (b) $9\sqrt{2} - \sqrt{8} - 4\sqrt{2}$

$= 9\sqrt{2} - 2\sqrt{2} - 4\sqrt{2} = 3\sqrt{2}$

37. (b) $x(y - z)(y + z) + y(z - x)(z + x) + z(x - y)(x + y)$
$= x(y^2 - z^2) + y(z^2 - x^2) + z(x^2 - y^2)$
$= x(y^2 - z^2) + yz^2 - yx^2 + zx^2 - zy^2$
$= x(y - z)(y + z) + x^2(z - y) + yz(z - y)$
$= (y - z)(xy + xz - x^2 - yz)$
$= (y - z)[y(x - z) + x(z - x)]$
$= (y - z)(z - x)(x - y)$
$= (x - y)(x - z)(z - y)$

38. (d) Let $f(x) = a_0 + a_1 x + a_2 x^2 + ... + a_n x^n$
$\therefore$ $f(1) = a_0 + a_1 + a_2 + ... + a_n$
$\Rightarrow$ $1 = a_0 + a_1 + a_2 + ... + a_n$
$\Rightarrow$ $1 - a_0 - a_2 - ..... = a_1 + a_3 + ...$

39. (c)

$$\begin{array}{r|l} & x \\ \hline x^2 - 2x - 3 & x^3 - 2x^2 - px - q \\ & \underset{-}{x^3} \underset{+}{-\ 2x^2} \underset{+}{-\ 3x} \\ \hline & \qquad (p+3)x - q \end{array}$$

According to question,
$(p + 3)x - q = x - 6$
$\Rightarrow$ $p + 3 = 1,\ q = 6$
$\Rightarrow$ $p = -2,\ q = 6$

40. (c) Given, $x = 1 + \sqrt{2}$
$\therefore$ $x^4 - 4x^3 + 4x^2 = x^2(x^2 - 4x + 4) = x^2(x - 2)^2$

$= \left(1+\sqrt{2}\right)^2 \left(1+\sqrt{2}-2\right)^2$

$= \left(1+\sqrt{2}\right)^2 \left(\sqrt{2}-1\right)^2 = (2-1)^2 = 1$

41. (a) Given, $\dfrac{x}{y} = \dfrac{z}{w}$

$\Rightarrow xw = yz$
Now, $(xy + zw)^2 = x^2y^2 + z^2w^2 + 2(xy \cdot zw)$
$= x^2y^2 + z^2w^2 + 2(yz \times yz)$
$= x^2y^2 + y^2z^2 + z^2w^2 + y^2z^2$
$= y^2(x^2 + z^2) + z^2w^2 + x^2w^2$
$= y^2(x^2 + z^2) + w^2(x^2 + z^2)$
$= (x^2 + z^2)(y^2 + w^2)$

42. (b) Given, $\dfrac{1}{x+1} + \dfrac{2}{y+2} + \dfrac{1009}{z+1009} = 1$

$$\Rightarrow \quad \frac{1}{x+1} - 1 + \frac{2}{y+2} - 1 + \frac{1009}{z+1009} - 1 = 1 - 3$$

$$\Rightarrow \quad -\frac{x}{x+1} - \frac{y}{y+2} - \frac{z}{z+1009} = -2$$

$$\Rightarrow \quad \frac{x}{x+1} + \frac{y}{y+2} + \frac{z}{z+1009} = 2$$

43. (d) $\because$ 1 quintal = 100 kg

$\therefore$ $\dfrac{3}{2}$ quintal $= 100 \times \dfrac{3}{2} = 150$ kg

Given, $\dfrac{15}{4}$ kg = 1 unit

1 kg $= \dfrac{4}{15}$ unit

$\therefore$ 150 kg $= \dfrac{4}{15} \times 150 = 40$ units

44. (c) Given, $m = n^2 - n$

$\therefore$ $m^2 - 2m = (n^2 - n)^2 - 2(n^2 - n)$

$= n(n - 1)(n^2 - n - 2)$

$= (n + 1)\, n(n - 1)(n - 2)$

So, it is a product of consecutive number. Therefore, it is divisible by 24.

45. (c) Let $f(x) = x^{40} + 2$

Put $x^4 = -1$,

$f(x) = (-1)^{10} + 2 = 3$

46. (b) Factor theorem is a special case of remainder theorem.

47. (c) $\therefore\ x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$

$$= \left(2 + \sqrt{3} + \frac{1}{2 + \sqrt{3}}\right)^2 - 2$$

$$= \left(2 + \sqrt{3} + \frac{2 - \sqrt{3}}{1}\right)^2 - 2 = 16 - 2 = 14$$

48. (c) Given, $a = \frac{1 + x}{2 - x}$

$$\therefore\ \frac{1}{a+1} + \frac{2a+1}{a^2-1} = \frac{3a}{a^2-1} = \frac{3\left(\frac{1+x}{2-x}\right)}{\left(\frac{1+x}{2-x}\right)^2 - 1}$$

$$= \frac{3(1+x)(2-x)}{1 + x^2 + 2x - (4 + x^2 - 4x)}$$

$$= \frac{3(1+x)(2-x)}{6x-3} = \frac{(1+x)(2-x)}{(2x-1)}$$

49. (b) Given, $pq + qr + rp = 0$

$$\therefore\ \frac{p^2}{p^2 - qr} + \frac{q^2}{q^2 - rp} + \frac{r^2}{r^2 - pq}$$

$$= \frac{p^2}{p^2 + rp + pq} + \frac{q^2}{q^2 + pq + qr} + \frac{r^2}{r^2 + qr + rp}$$

$$= \frac{p}{p + r + q} + \frac{q}{q + p + r} + \frac{r}{r + q + p}$$

$$= \frac{p + q + r}{p + q + r} = 1$$

50. (d) Given, $x + y + z = 0$

Now, $\frac{1}{x^2 + y^2 - z^2} = \frac{1}{z^2 - 2xy - z^2}$ .. (i)

$$= -\frac{1}{2xy}$$

$$\therefore\ \frac{1}{x^2 + y^2 - z^2} + \frac{1}{y^2 + z^2 - x^2} + \frac{1}{z^2 + x^2 - y^2}$$

$$= \frac{1}{-2xy} + \frac{1}{-2yz} + \frac{1}{-2zx}$$

$$= \frac{1}{2}\left(\frac{z + x + y}{xyz}\right) = 0$$

[from equation (i)]

51. (c) We know that, if $a + b + c = 0$

$\therefore$ $a^3 + b^3 + c^3 = 3abc$

Here, $x - y + y - z + z - x = 0$

$$\therefore\ \frac{(x-y)^3 + (y-z)^3 + (z-x)^3}{4(x-y)(y-z)(z-x)}$$

$$= \frac{3(x-y)(y-z)(z-x)}{4(x-y)(y-z)(z-x)} = \frac{3}{4}$$

52. (b) Let $f(x) = px^3 + 3x^2 - 3$ and $g(x) = 2x^3 - 5x + p$

At $x = 4$,

$f(4) = p(4)^3 + 3(4)^2 - 3$

$= 64p + 48 - 3 = 64p + 45$

and $g(4) = 2(4)^3 - 5(4) + p$

$= 128 - 20 + p = 108 + p$

But $f(4) = g(4)$

(given)

$\therefore$ $64p + 45 = 108 + p$

$\Rightarrow$ $63p = 63 \Rightarrow p = 1$

53. (c) Given, $x(x + y + z) = 9$ ... (i)

$y(x + y + z) = 16$ ... (ii)

and $z(x + y + z) = 144$ ... (iii)

On adding (i), (ii) and (iii), we get

$(x + y + z)(x + y + z) = 9 + 16 + 144$

$\Rightarrow$ $(x + y + z)^2 = 169$

$\Rightarrow$ $x + y + z = 13$

[$\because$ $x(x + y + z) = 9$]

$\Rightarrow$ $x(13) = 9 \Rightarrow x = \frac{9}{13}$

54. (c) Given, $(u)^3 + (-2v)^3 + (-3w)^3 = 3 \times (-2)(-3)uvw$

$\therefore$ $u + (-2v) + (-3w) = 0$

$\Rightarrow$ $u - 2v - 3w = 0$

$\Rightarrow$ $u - 2v = 3w$

55. (a): $x^4 + 4y^4 = x^4 + 4y^4 + 4x^2y^2 - 4x^2y^2$

$= (x^2 + 2y^2)^2 - (2xy)^2$

$= (x^2 + 2y^2 - 2xy)(x^2 + 2y^2 + 2xy)$

From above it is clear that $x^4 + 4y^4$ is divisible by $x^2 + 2y^2 + 2xy$.

56. (b) At the same time, ratio of height and length of shadow of an object are same.

$$\therefore \frac{6}{15} = \frac{\text{Height of tree}}{25}$$

$\Rightarrow$ Height of tree = 10m

57. (d) Given, a + b + c = 0

$\Rightarrow a^3 + b^3 + c^3 = 3abc$

$$\Rightarrow \frac{a^3}{abc}+\frac{b^3}{abc}+\frac{c^3}{abc}=3$$

$$\Rightarrow \frac{a^2}{bc}+\frac{b^2}{ac}+\frac{c^2}{ab}=3$$

58. (d) Given, $x^4+\frac{1}{x^4}=322$

$$\Rightarrow \left(x^2+\frac{1}{x^2}\right)^2-2=322$$

$$\Rightarrow \left(x^2+\frac{1}{x^2}\right)^2=324=18^2$$

$$\Rightarrow x^2+\frac{1}{x^2}=18$$

$$\Rightarrow \left(x-\frac{1}{x}\right)^2+2=18$$

$$\Rightarrow x-\frac{1}{x}=4$$

59. (c) Given, $x = (b - c)(a - d)$, $y = (c - a)(b - d)$ and $z = (a - b)(c - d)$

$\therefore\ x + y + z = (b - c)(a - d) + (c - a)(b - d) + (a - b)(c - d) = 0$

$\therefore\ x^3 + y^3 + z^3 = 3xyz$

60. (a) $x^2 + 4y^2 + 4y - 4xy - 2x - 8$

$= (x - 2y)^2 - 2(x - 2y) - 8$

$= (x - 2y)^2 - 4(x - 2y) + 2(x - 2y) - 8$

$= (x - 2y - 4)(x - 2y + 2)$

61. (d) Here, $px^3 + x^2 - 2x - q$ is divisible by $(x - 1)$ and $(x + 1)$

$\therefore\ p(1)^3 + (1)^2 - 2(1) - q = 0$

$\Rightarrow p - q = 1$ ... (i)

and $p(-1)^3 + (-1)^2 - 2(-1) - q = 0$

$\Rightarrow p + q = 3$ ... (ii)

From equations (i) and (ii),

$p = 2$ and $q = 1$

62. (d) On putting = 3 in $x^5 - 9x^2 + 12x - 14$, we get

Remainder $= (3)^5 - 9(3)^2 + 12 \times 3 - 14$

$= 243 - 81 + 36 - 14 = 184$

63. (b) Given, $x-\frac{1}{x}=\frac{1}{3}$

$$\Rightarrow 3x-\frac{3}{x}=1$$

Squaring both sides,

$$9x^2+\frac{9}{x^2}-2\times 9=1$$

$$\Rightarrow 9x^2+\frac{9}{x^2}=19$$

64. (d) $\frac{1}{1+\sqrt{2}+\sqrt{3}}+\frac{1}{1-\sqrt{2}+\sqrt{3}}$

$$=\frac{1}{(1+\sqrt{3})+\sqrt{2}}+\frac{1}{(1+\sqrt{3})-\sqrt{2}}$$

$$=\frac{1+\sqrt{3}-\sqrt{2}+1+\sqrt{3}+\sqrt{2}}{(1+\sqrt{3})^2-2}=\frac{2(1+\sqrt{3})}{2(1+\sqrt{3})}=1$$

65. (d) $\because\ (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$(6)^2 = 26 + 2(ab + bc + ca)$

$\Rightarrow 2(ab + bc + ca) = 10$

$\Rightarrow ab + bc + ca = 5$

66. (a)

$$\begin{array}{r|l} & 3x^2+4xy-5y^2 \\ \hline x-2y & 3x^3-2x^2y-13xy^2+10y^3 \\ & \underset{-}{3x^3}\ \underset{+}{-6x^2y} \\ \hline & 4x^2y-13xy^2 \\ & \underset{-}{4x^2y}\ \underset{+}{-8xy^2} \\ \hline & -5xy^2+10y^3 \\ & \underset{+}{-5xy^2}\ \underset{-}{+10y^3} \\ \hline & \times \end{array}$$

Thus, the remainder is zero.

67. (a) Let $f(x) = x^{11} + 1$

Put $x = -1$, we get

$f(-1) = (-1)^{11} + 1 = -1 + 1 = 0$

$(a^n + b^n)$, where n is odd number then it is divisible by $(a + b)$. Then remainder is zero.

68. (a) Let $f(x) = x^2 + 4px - 11p$

Since, $(x - 3)$ is a factor of $f(x)$.

$\therefore\ f(3) = 0$

$\Rightarrow (3)^2 + 4p(3) - 11p = 0 \Rightarrow p = -9$

69. (b) $\frac{x^2-3x+2}{x^2-5x+6}\div\frac{x^2-5x+4}{x^2-7x+12}$

$$=\frac{x^2-3x+2}{x^2-5x+6}\times\frac{(x^2-7x+12)}{(x^2-5x+4)}$$

$$=\frac{(x-1)(x-2)}{(x-3)(x-2)}\times\frac{(x-4)(x-3)}{(x-4)(x-1)}=1$$

70. (d) $\frac{(2.3)^3-0.027}{(2.3)^2+0.69+0.09}=\frac{(2.3)^3-(0.3)^3}{(2.3)^2+0.69+0.09}$

$$=\frac{(2.3-0.3)[(2.3)^2+(0.3)^2+2.3\times 0.3)]}{(2.3)^2+0.69+0.09}$$

$$=\frac{2[(2.3)^2+0.09+0.69]}{(2.3)^2+0.69+0.09}=2$$

71. (a) Given, $x + y + z = 0$

$\therefore \quad \dfrac{xyz}{(x+y)(y+z)(z+x)} = \dfrac{xyz}{(-z)(-x)(-y)}$

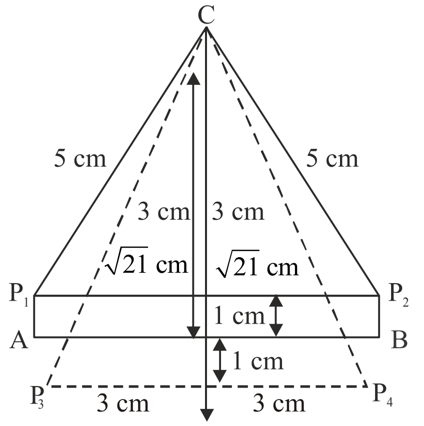
$= \dfrac{xyz}{-xyz} = -1$

72. (b) Here, a = 1.001 and b = 0.999 (assume)
Now,

$\dfrac{a^3+b^3}{a^2-ab+b^2} = \dfrac{(a+b)(a^2-ab+b^2)}{a^2-ab+b^2} = a+b$

$= 1.001 + 0.999 = 2$

73. (d) Required number of points = $4(P_1 P_2 P_3 P_4)$



74. (d) According to question $(2x - 1)$ is a factor of $4x^4 - (k-1)x^3 + kx^2 - 6x + 1$ ... (i)

On putting $x = \dfrac{1}{2}$ in Eq. (i), we get

$4\times\left(\dfrac{1}{2}\right)^4 - (k-1)\times\left(\dfrac{1}{2}\right)^3 + k\times\left(\dfrac{1}{2}\right)^2 - 6\times\dfrac{1}{2} + 1 = 0$

$\Rightarrow \dfrac{1}{4} - \dfrac{(k-1)}{8} + \dfrac{k}{4} - 2 = 0$

$\Rightarrow \dfrac{k}{4} - \dfrac{(k-1)}{8} = 2 - \dfrac{1}{4}$

$\Rightarrow \dfrac{2k-(k-1)}{8} = \dfrac{8-1}{4} = \dfrac{7}{4}$

$\Rightarrow 8k - 4k + 4 = 56$

$\Rightarrow 4k = 52$

$\therefore k = 13$

75. (a) $f(x) = x^3 + x^2 - x - 1$

$g(x) = x^3 - x^2 + x - 1$

$p(x) = f\{g(x)\} = p(x) \times f(x)$

$f(x) \cdot g(x) = (x^3 + x^2 - x - 1)(x^3 - x^2 + x - 1)$

$= x^6 - x^5 + x^4 - x^3 + x^5 - x^4 + x^3 - x^2 - x^4 + x^3 - x^2 + x - x^3 + x^2 - x + 1$

$p(x) = f(x) \cdot g(x) = x^6 - x^4 - x^2 + 1$

$p(x) = x^4(x^2 - 1) - 1(x^2 - 1)$

$= (x^2 - 1)(x^4 - 1) = (x - 1)(x + 1)[(x^2)^2 - 1]$

$= (x - 1)(x + 1)(x^2 - 1)(x^2 + 1)$

$= (x - 1)(x + 1)(x - 1)(x + 1)(x^2 + 1)$

$= (x - 1)^2 (x + 1)^2 (x^2 + 1)$

76. (c)

$$\begin{array}{r|l} & x+5 \\ \hline x^2+2 & x^3+5x^2+10k \\ & \underline{\underset{-}{x^3}\ \underset{-}{+2x}} \\ & 5x^2-2x+10k \\ & \underline{\underset{-}{5x^2}\ \underset{-}{+10}} \\ & -2x-10+10k = \text{Remainder} \end{array}$$

Given, remainder $= -2x$

$\therefore -2x - 10 + 10k = -2x$

$\Rightarrow 10k = 10 \Rightarrow k = 1$

77. (c) Given that, $x + \dfrac{1}{x} = a$

Then, $x^3 + x^2 + \dfrac{1}{x^3} + \dfrac{1}{x^2} = \left(x^3 + \dfrac{1}{x^3}\right) + \left(x^2 + \dfrac{1}{x^2}\right)$

$= \left(x + \dfrac{1}{x}\right)^3 - 3\left(x + \dfrac{1}{x}\right) + \left(x + \dfrac{1}{x}\right)^2 - 2$

$= a^3 - 3a + a^2 - 2 = a^3 + a^2 - 3a - 2$

78. (a) Given, $xyz = 1$, $a^x = b$, $b^y = c$

$b = a^x \Rightarrow b^y = a^{xy}$

$\Rightarrow b^{yz} = a^{xyz} \Rightarrow c^z = a$

79. (b) Given $3^{x+y} = 81$

$\Rightarrow 3^{x+y} = 3^4 \Rightarrow x + y = 4$ ... (i)

$81^{x-y} = 3 \Rightarrow (3^4)^{x-y} = 3^1$

$\Rightarrow x - y = \dfrac{1}{4}$ ... (ii)

From equations (i) and (ii),

then, $x + y = 4 \Rightarrow x - y = \dfrac{1}{4} \Rightarrow 2x = \dfrac{17}{4}$

$\therefore x = \dfrac{17}{8}$

80. (c) Given, $x^4 + xy^3 + x^3y + xz^3 + y^4 + yz^3$

$= (x^4 + xy^3 + xz^3) + (x^3y + y^4 + yz^3)$

$= x(x^3 + y^3 + z^3) + y(x^3 + y^3 + z^3)$

$= (x + y)(x^3 + y^3 + z^3)$

Thus, the equation is divisible by both $(x + y)$ and $(x^3 + y^3 + z^3)$.

81. (b) Given, $3 \times 0.3 \times 0.03 \times 0.003 \times 0.0003 \times 30$

$= 3 \times 3 \times 10^{-1} \times 3 \times 10^{-2} \times 3 \times 10^{-3} \times 3 \times 10^{-4} \times 3 \times 10$

$= (3)^6 \times (10)^{-9} = (3^2)^3 (10^{-3})^3$

$= (9)^3 \times \dfrac{1}{(1000)^3} = \left(\dfrac{9}{1000}\right)^3 = (0.009)^3.$

82. (c) We know that,

$a^3 + b^3 + c^3 - 3abc$

$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$$\Rightarrow (a+b+c) = \left(\frac{a^3+b^3+c^3-3abc}{a^2+b^2+c^2-ab-bc-ca}\right) \quad \text{... (i)}$$

Given that,

$$\frac{\left[(2.247)^3+(1.730)^3+(1.023)^3-3\times 2.247 \times 1.730\times 1.023\right]}{\left[(2.247)^2+(1.730)^2+(1.023)^2-(2.247\times 1.730) - (1.730\times 1.023)-(2.247\times 1.023)\right]}$$

$= (2.247 + 1.730 + 1.023)$

[from Eq. (i)]

$= 5.000 = 5$

83. (c)

$$\begin{array}{r|l} & a^2-2a-8 \\ \hline a^2-1 & a^4-2a^3-9a^2+2a+8 \\ & \underset{-}{a^4} \quad \underset{+}{-a^2} \\ \hline & -2a^3-8a^2+2a+8 \\ & \underset{+}{-2a^3} \quad \underset{-}{+2a} \\ \hline & -8a^2 \quad +8 \\ & \underset{+}{-8a^2} \quad \underset{-}{+8} \\ \hline & \times \end{array}$$

Required factor is $(a^2 - 2a - 8)$

Further its factorize by spilt the middle term,

$= a^2 - 4a + 2a - 8$

$= a(a - 4) + 2(a - 4)$

$= (a - 4)(a + 2)$

Other two factors are $(a + 2)$ and $(a - 4)$.

84. (b) Given, $0.764\,y = 1.236\,x$

$$\Rightarrow \frac{y}{x} = \frac{1.236}{0.764} \quad \text{... (i)}$$

Now,

$$\frac{y-x}{y+x} = \frac{\frac{y}{x}-1}{\frac{y}{x}+1}$$

$$\Rightarrow \frac{\frac{1.236}{0.764}-1}{\frac{1.236}{0.764}+1} = \frac{1.236-0.764}{1.236+0.764}$$

$$= \frac{0.472}{2.000} = 0.236$$

85. (b) Given that, expression $x^3 + 4x^2 + 4x + k$ has $(x + 4)$ as a factor, so it will satisfy that expression.

Let $f(x) = x^3 + 4x^2 + 4x + k$

$\therefore f(-4) = 0$ (by condition)

$\Rightarrow (-4)^3 + 4(-4)^2 + 4(-4) + k = 0$

$\Rightarrow -64 + 64 - 16 + k = 0$

$\therefore k = 16$

86. (d) Given expression is $f(x) = 4x^2 + 12x + k$

$= (2x)^2 + 2.3 \cdot (2x) + k$

For the value of k

$$= \left[\frac{\text{Coefficient of } (2x)}{2}\right]^2 = \left(\frac{6}{2}\right)^2 = (3)^2 = 9$$

Hence, 9 is the value of k which will make the expression $f(x)$ a perfect square.

87. (b) Let $f(x) = 6x^2 + kx + 10$

If $(x + 5)$ is a factor of $f(x)$, then

$f(-5) = 0$

So, $6(-5)^2 + k(-5) + 10 = 0$

$\Rightarrow 6 \times 25 - 5k + 10 = 0$

$\Rightarrow 5k = 150 + 10 = 160$

$\therefore k = 32$

88. (d) Let $f(x) = x^4 - 7x^3 + 5x^2 - 6x + 81$

By hit and trial method,

$x^4 - 7x^3 + 5x^2 - 6x + 81$

$= x^3(x - 3) - 4x^2(x - 3) - 7x(x - 3) - 27(x - 3)$

$= (x - 3)(x^3 - 4x^2 - 7x - 27)$

$(x - 3)$ is one of the factor of the given polynomial.

89. (d) Given, $(a + 1)^4 - a^4$

$= \{(a + 1)^2 - a^2\}\{(a + 1)^2 + a^2\}$

$= \{(a + 1) + a\}\{(a + 1) - a\}\{a^2 + 1 + 2a + a^2\}$

$= (2a + 1)(1)(2a^2 + 2a + 1)$

$= (2a + 1)(2a^2 + 2a + 1)$

Hence, $(2a^2 + 2a + 1)$ is a factor of $\{(a + 1)^4 - a^4\}$.

90. (c) Given expression $= 5px - 10qy + 2rpx - 4qry$

$= (5px + 2rpx) - (10qy + 4qry)$

$= px(5 + 2r) - 2qy(5 + 2r)$

$= (5 + 2r)(px - 2qy)$

91. (a) Zero of $(x + 6)$ is -6, i.e., $x + 6$

$\Rightarrow x = -6$

If $(x + 6)$ is a factor of the expression

$f(x) = x^3 + 3x^2 + 4x + p$

Then, $f(-6) = 0$

$\Rightarrow (-6)^3 + 3(-6)^2 + 4(-6) + p = 0$

$\Rightarrow -216 + 108 - 24 + p = 0$

$\therefore p = 240 - 108 = 132$

92. (c) $$\frac{725\times 725\times 725+371\times 371\times 371}{725\times 725-725\times 371+371\times 371} = 725 + 371$$

$$\left(\because \frac{a^3+b^3}{a^2-ab+b^2} = \frac{(a+b)(a^2-ab+b^2)}{(a^2-ab+b^2)} = a+b\right)$$

$= 1096$

93. (d) **Statement I :**
When $x = -3$ then $x^3 + 2x^2 + 3x + 8$
$= (-3)^3 + 2(-3)^2 + 3(-3) + 8$
$= -10 \neq 0$
Hence, $(x - 3)$ is not the factor of $x^3 + 2x^2 + 3x + 8$
Statement II :
Similarly,
When $x = 2$, then $x^3 + 2x^2 + 3x + 8$
$= (2)^3 + 2(2)^2 + 3(2) + 8$
$= 30 \neq 0$
Hence, $x - 2$ is also not the factor of $x^3 + 2x^2 + 3x + 8$.

94. (b) $\dfrac{(x^2+y^2)(x-y)-(x-y)^3}{x^2y-xy^2}$

$$= \frac{x^3+xy^2-x^2y-y^3-(x^3-y^3-3x^2y+3xy^2)}{x^2y-xy^2}$$

$$= \frac{x^3+xy^2-x^2y-y^3-x^3+y^3+3x^2y-3xy^2}{x^2y-xy^2}$$

$$= \frac{2x^2y-2xy^2}{x^2y-xy^2} = \frac{2(x^2y-xy^2)}{x^2y-xy^2} = 2$$

95. (b) Here, $x + 5$ is a factor.
So,
$x + 5 = 0$
$\Rightarrow x = -5$
Now, when $x = -5$, then
$x^3 + 3x^2 + 4x + k = (-5)^3 + 3 \times (-5)^2 + 4 \times (-5) + k$
$= -125 + 75 - 20 + k = -70 + k$
since $(x - 5)$ is a factor of $x^3 + 3x^2 + 4x + k$
$\therefore -70 + k = 0$
$\therefore k = 70$

96. (c) $(1 - x)(1 + x^2) = 1 - x + x^2 - x^3$
$2x^3 - x^2 + x - 1$ is added to $1 - x + x^2 - x^3$ to obtain x^3.

97. (c) $(49)^2 - (25)^2 = 37x$
$\Rightarrow 2401 - 625 = 37x$
$\Rightarrow 1776 = 37x$

$$x = \frac{1776}{37} = 48$$

98. (d) If $x - 1$ is a factor, then $x = 1$, then
$4x^5 + 9x^4 - 7x^3 - 5x^2 - 4kx + 3k^2 = 0$
$\Rightarrow 4 \times (1)^5 + 9 \times (1)^4 - 7 \times (1)^3 - 5 \times (1)^2$
$-4 \times k \times (1) + 3 \times k^2 = 0$
$\Rightarrow 4 + 9 - 7 - 5 - 4k + 3k^2 = 0$
$\Rightarrow 3k^2 - 4k + 1 = 0$
$\Rightarrow 3k^2 - 3k - k + 1 = 0$
$\Rightarrow 3k(k - 1) - 1(k - 1) = 0$
$\Rightarrow (3k - 1)(k - 1) = 0$

$$k = \frac{1}{3}, 1$$

99. (c) $x(y^2 - z^2) + y(z^2 - x^2) + z(x^2 - y^2)$
If divisible by $(y - z)$, then $y - z = 0 \Rightarrow y = z$
On putting $y = z$, we get
$x(z^2 - z^2) + z(z^2 - x^2) + z(x^2 - z^2)$
$= z^3 - zx^2 + zx^2 - z^3 = 0$
Hence, $y - z$ is a factor, so it is divisible by $(y - z)$.
Also, if $z - x$ is a factor, then
$z - x = 0 \Rightarrow z = x$
On putting $z = x$, we get
$x(y^2 - x^2) + y(x^2 - x^2) + x(x^2 - y^2)$
$= xy^2 - x^3 + x^3 - xy^2 = 0$
Hence, $(z - x)$ is also a factor, so it is also divisible by $(z - x)$.

100. (d) Let $f(x) = x^3 + 6x^2 + 11x + 6$
$f(-1) = 0$
$\therefore (x + 1)$ is a factor of $f(x)$.
Also $f(-2) = f(-3) = 0$
Hence $(x + 2)$ and $(x + 3)$ are also factors of $f(x)$.

101. (c) $x(x + a)(x + 2a)(x + 3a)$
$= (x^2 + ax)(x^2 + 5ax + 6a^2)$
$= x^4 + ax^3 + 5ax^3 + 5a^2x^2 + 6a^2x^2 + 6a^3x$
$= x^4 + ax(x^2 + 5x^2 + 5ax + 6ax + 6a^2)$
$= x^4 + ax(6x^2 + 11ax + 6a^2)$ ... (i)
So, for terms to be perfect square,
$= (x + y)^2 (x + y)^2 = (x^2 + 2xy + y^2)(x^2 + y^2 + 2xy)$
$= x^4 + 2x^3y + x^2y^2 + x^2y^2 + 2xy^3 + y^4 + 2x^3y + 4x^2y^2 + 2xy^3$
$= x^4 + xy(4x^2 + 6xy + 4y^2) + y^4$
On comparing equations. (i) and (ii), as $y = a$
a^4 must be added to make it a perfect square.

102. (b) Let $f(x) = 3x^4 - 2x^3 + 3x^2 - 2x + 3$

$$\text{Remainder} = f\left(-\frac{2}{3}\right)$$

$$= 3\left(-\frac{2}{3}\right)^4 - 2\left(-\frac{2}{3}\right)^3 + 3\left(-\frac{2}{3}\right)^2 - 2\left(-\frac{2}{3}\right) + 3$$

$$= \frac{185}{27}$$

103. (b) Given the $f(x) = 2x^3 + x^2 - 2x + 1$
$= x^2(2x + 1) - 1(2x + 1) = (2x + 1)(x^2 - 1)$
$= (2x + 1)(x + 1)(x - 1)$
Hence, expression is divisible by $(2x + 1)$.

104. (b) $\left(x^2 + \dfrac{1}{x^2}\right) = \dfrac{17}{4}$

$$\Rightarrow x^2 + \frac{1}{x^2} + 2 - 2 = \frac{17}{4}$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 + 2 = \frac{17}{4}$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 = \frac{17}{4} - 2$$

$$\Rightarrow \left(x - \frac{1}{x}\right)^2 = \frac{9}{4}$$

$\Rightarrow \left(x-\frac{1}{x}\right)=\frac{3}{2}$

On cubing both sides, we get

$\left(x-\frac{1}{x}\right)^3=\left(\frac{3}{2}\right)^3$

$\Rightarrow x^3-\frac{1}{x^3}-3\times\frac{1}{x}.x\left(x-\frac{1}{x}\right)=\frac{27}{8}$

$\Rightarrow x^3-\frac{1}{x^3}=\frac{27}{8}+3\times\left(\frac{3}{2}\right)$

$\Rightarrow x^3-\frac{1}{x^3}=\frac{27}{8}+\frac{9}{2}$

$\therefore x^3-\frac{1}{x^3}=\frac{63}{8}$

105. (a) Given, $(x + k)$ is the common factor of $x^2 + ax + b$ and $x^2 + cx + d$.
$\therefore \quad k^2 - ka + b = 0$ ...(i)
$k^2 - kc + d = 0$ ...(ii)
Now, from equation (i) and equation (ii)
$k^2 - ka + b = k^2 - kc + d$
$\Rightarrow k(c - a) = d - b$
$\therefore k = \frac{d-b}{c-a}$

106. (c) Let $f(x) = x^5 - 5x^2 + 125$
$\therefore$ Required remainder $= f(-5) = (-5)^2 - 5(-5)^2 + 125$
$= -3125 - 125 + 125 = -3125$

107. (a) We have, $ab(x^2 + 1) + x(a^2 + b^2)$.
$= abx^2 + ab + a^2x + b^2x = ax(a + bx) + b(a + bx)$
$= (a + bx)(ax + b)$
and $ab(x^2 - 1) + x(a^2 - b^2)$
$= abx^2 - ab + a^2x - b^2x = ax(a + bx) + b(a + bx)$
$= (a + bx)(ax - b)$
Lowest common multiple
$= (a + bx)(ax + b)(ax - b)$
$= (a + bx)(a^2x^2 - b^2)$

108. (c) 1. Given, $3abc + b^3 + c^3 - a^3$
$= -(a^3 - b^3 - c^3 - 3abc)$
$= -[a^3 + (-b)^3 + (-c)^3 - 3(a)(-b)(-c)]$
$= -(a - b - c)(a^2 + b^2 + c^2 + ab - bc + ac)$
Hence, $(a - b - c)$ is a factor of $3abc + b^3 + c^3 - a^3$
Therefore, Statement 1 is correct.
Given, $3bc + b^3 + c^3 - 1$
$= b^3 + c^3 - (1)^3 - 3bc(-1)$
$= (b + c - 1)[b^2 + c^2 + 1^2 - bc + c + b]$
Hence, $(b+c-1)$ is a factor of $3bc + b^3 + c^3 - 1$.
Therefore, Statement 2 is also correct.

109. (b) $x^2 + 2(1 + k)x + k^2 = 0$.
If it has equal roots, then $D = 0$
$\Rightarrow \{2(1 + k)\}^2 - 4k^2 = 0$
$\Rightarrow 4(1 + k^2 + 2k) - 4k^2 = 0$
$\Rightarrow 4 + 4k^2 + 8k - 4k^2 = 0 \Rightarrow 4 + 8k = 0$
$\Rightarrow k = -\frac{4}{8}$
$\therefore k = -\frac{1}{2}$

110. (a) $f(x) = 2x^3 + x^2 - x - 2$
$= (x - 1)(2x^2 + 3x + 2)$
and $g(x) = 3x^3 - 2x^2 + x - 2$
$= (x - 1)(3x^2 + x + 2)$
Therefore, the highest common factor of $f(x)$ and $g(x)$ is $(x-1)$.

111. (b) $\frac{1235\times4523\times2451}{12}$
1235, 4523 and 2451 is divided by 12 then its remainder are 11, 11 and 3 respectively
$=\frac{11\times11\times3}{12}=\left(\frac{11}{12}\right)\times\left(\frac{11}{12}\right)\times\left(\frac{3}{12}\right)$
$= (-1) \times (-1) \times 3$ (By Negative Concept of Remainder)
$= 1 \times 3 = 3$ (Remainder)

112. (a) If $(x-5)$ is a factor of $x^3 - 3x^2 + hx - 10$
then $x - 5 = 0 \Rightarrow x = 5$
So, $\Rightarrow (5)^3 - 3 \times (5)^2 + k \times 5 - 10 = 0$
$\Rightarrow 125 - 75 + 5k - 10 = 0$
$\Rightarrow 5k = -40$
$\boxed{\therefore k = -8}$

113. (c) $x^3q^2 - x^3pt + 4x^2pt - 4x^2q^2 + 3xq^2 - 3xpt$
If we put $x = 1$
$= q^2 - pt + 4pt - 4q^2 + 3q^2 - 3pt = 0$
So, $(x-1)$ is a factor of this function of we put $x = 3$
Checking $x = 3$ also,
$= 27q^2 - 27pt + 36pt - 36q^2 + 9q^2 - 9pt = 0$
So, $(x - 1)$ and $(x - 3)$ both are factors.

114. (b) $\frac{1}{1\times4}+\frac{1}{4\times7}+\frac{1}{7\times10}......+\frac{1}{16\times19}$

$\Rightarrow \frac{1}{3}\left[\frac{3}{1\times4}+\frac{3}{4\times7}+\frac{3}{7\times10}+......+\frac{3}{16\times19}\right]$

$\Rightarrow \frac{1}{3}\left[1-\frac{1}{4}+\left(\frac{1}{4}-\frac{1}{7}\right)+\left(\frac{1}{7}-\frac{1}{10}\right)+\cdots\cdots+\left(\frac{1}{16}-\frac{1}{19}\right)\right]$

$\Rightarrow \frac{1}{3}\left(1-\frac{1}{19}\right)$

$\Rightarrow \frac{1}{3}\times\frac{18}{19}=\frac{6}{19}$

So, option (b) is correct

115. (c) $4 = \frac{\sqrt{(x-1)^2}}{(x-1)}$

when $x > 1^-$

$4 = \frac{(x-1)}{x-1} = 1$

when $x < 1^-$

$y = \frac{0}{0} = \text{undefined}$

So, option (c) is correct.

116. (c) $x^3 - 3x^2 + 2x + 1 = 0$

as a, b and c are the roots-

$\Rightarrow ab + bc + ca = 2$ ----- (i)

$abc = -1$ ----- (ii)

Dividing eq (i) by eq (ii)-

$$\frac{(ab+bc+ca)}{abc} = \frac{2}{-1}$$

$$\frac{ab}{abc} + \frac{bc}{abc} + \frac{ca}{abc} = -2$$

$$\frac{1}{c} + \frac{1}{a} + \frac{1}{b} = -2$$

$$\boxed{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = -2}$$

So, option (c) is correct.

117. (c) If $(x + 2)$ is a factor, $x = -2$ will satisfy the expression

$\Rightarrow x^4 - 6x^3 + 12x^2 - 24x + 32$

$\Rightarrow (-2)^4 - 6(-2)^3 + 12(-2)^2 - 24(-2) + 32$

$\Rightarrow 16 + 48 + 48 + 48 + 32 \neq 0$

Again,

$\Rightarrow x^4 + 6x^3 - 12x^2 + 24x - 32$

$\Rightarrow (-2)^4 + 6(-2)^3 - 12(-2)^2 + 24(-2) - 32$

$\Rightarrow 16 - 48 - 48 - 48 - 32 \neq 0$

Again if $(x - 2)$ is a factor, $x = 2$ will satisfy the expression-

$\Rightarrow x^4 - 6x^3 + 12x^2 - 24x + 32$

$\Rightarrow (21^4 - 6(21^3 + 12(2)^2 - 24(2) + 32$

$\Rightarrow 16 - 48 + 48 - 48 + 32 = 0$

Again

$\Rightarrow x^4 + 6x^3 - 12x^2 + 24x - 32$

$\Rightarrow (2)^4 + 6(2)^3 - 12(2)^2 + 24(2) - 32$

$\Rightarrow 16 + 48 - 48 + 48 - 32 \neq 0$

So, option (c) is correct.

118. (a) $x = \frac{91}{216}$

$$\Rightarrow 3 - \frac{1}{(1-x)^{\frac{1}{3}}}$$

$$\Rightarrow 3 - \frac{1}{\left(1 - \frac{91}{216}\right)^{\frac{1}{3}}}$$

$$\Rightarrow 3 - \frac{1}{\left(\frac{125}{216}\right)^{\frac{1}{3}}}$$

$$\Rightarrow 3 - \frac{1}{\left(\frac{5}{6}\right)} = 3 - \frac{6}{5} = \frac{15-6}{5} = \frac{9}{5}$$

So, option (a) is correct.

119. (a) Let α be common root of both the equations

$x^2 - px + q = 0$ and $x^2 + qx - p = 0$

so $\alpha^2 - p\alpha + q = 0$ ...(i)

and $\alpha^2 + q\alpha - p = 0$ ...(ii)

From (i) we get $\alpha^2 = p\alpha - q$

Putting in (ii), we get

$\alpha^2 + q\alpha - p = 0$

$\Rightarrow p\alpha - q + q\alpha - p = 0$

$\Rightarrow (p + q)\alpha - (p + q) = 0$

$\Rightarrow (p + q)\alpha = p + q$

$\Rightarrow \alpha = 1$

From (i) we get $1^2 - p.1 + q = 0$

$\Rightarrow -p + q = 0$

$\Rightarrow p - q = 0$

$\therefore$ option (a) is correct.

120. (b) 121. (c) 122. (c)

123. (a) $\frac{x}{1} = \frac{\sqrt{a+2b} + \sqrt{a-2b}}{\sqrt{a+2b} - \sqrt{a-2b}}$

Applying componendo and dividendo, we get

$$\frac{x+1}{x-1} = \frac{2\sqrt{a+2b}}{2\sqrt{a-2b}}$$

$$\Rightarrow \frac{x+1}{x-1} = \frac{\sqrt{a+2b}}{\sqrt{a-2b}}$$

Squaring on both sides, we get

$$\frac{(x+1)^2}{(x-1)^2} = \frac{a+2b}{a-2b}$$

Again applying componendo and dividendo, we get

$$\frac{x^2+1+2x+x^2+1-2x}{x^2+1+2x-x^2-1+2x} = \frac{2a}{4ab} = \frac{a}{2b}$$

$$\Rightarrow \frac{2(x^2+1)}{4x} = \frac{a}{2b}$$

$\Rightarrow \frac{x^2+1}{x} = \frac{a}{b}$

$\Rightarrow bx^2 + b = ax$

$\Rightarrow bx^2 - ax + b = 0$

$\therefore$ Option (a) is correct.

124. (a) Given $a^3 = 117 + b^3$ ...(1)

and $a = 3 + b$

Putting the value of a in (1), we get

$(3 + b)^3 = 117 + b^3$

$27 + b^3 + 9b^2 + 27b = 117 + b^3$

$9b^2 + 27b - 90 = 0$

$b^2 + 3b - 10 = 0$

$(b + 5)(b - 2) = 0$

$b \neq -5 \ (\because b > 0)$

$\Rightarrow b = 2$

$\therefore a = 3 + b = 3 + 2$

$\Rightarrow a = 5$

Thus $a + b = 5 + 2 = 7$

$\therefore$ Option (a) is correct.

125. (a) Let $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = k$

$\Rightarrow a = bk, c = ck, c = dk$

$\Rightarrow abc = bcdk^3$

$\Rightarrow a = dk^3$

(1) $\frac{b^3+c^3+d^3}{a^3+b^3+c^3} = \frac{d}{a}$

(2) $\frac{a^2+b^2+c^2}{b^2+c^2+d^2} = \frac{a}{d}$

$\Rightarrow \frac{b^3+c^3+d^3}{b^3k^3+c^3k^3+d^3k^3} = \frac{d}{a}$

$\Rightarrow \frac{b^2k^2+c^2k^2+d^2k^2}{b^2+c^2+d^2} = \frac{a}{d}$

$\Rightarrow \frac{b^3+c^3+d^3}{\left(b^3+c^3+d^3\right)k^3} = \frac{d}{a}$

$\Rightarrow \frac{(b^2+c^2+d^2)k^2}{b^2+c^2+a^2} = \frac{a}{d}$

$\Rightarrow \frac{1}{k^3} = \frac{d}{a}$

$\Rightarrow k^2 = \frac{a}{d}$

$\Rightarrow k^2 - k^3$

$\Rightarrow \frac{1}{\frac{a}{d}} = \frac{d}{a}$

which is not possible

$\Rightarrow \frac{d}{a} = \frac{d}{a}$

$\Rightarrow$ only (1) is true.

$\therefore$ Option (a) is correct.

126. (d) Given $(x + 1)$ is a factor of $x^3 + kx^2 - x + 2$ ...(1)

$\therefore x = -1$ satisfies the equation (1), we get

$(-1)^3 + k(-1)^2 - (-1) + 2 = 0$

$-1 + k + 1 + 2 = 0$

$k = -2$

$\therefore$ Option (d) is correct.

127. (c) Inequations are

$x + y \le 4$ and $x - y \ge 2$

To check point P(5, – 1), we get

$5 - 1 \le 4$ and $5 + 1 \ge 2$

$4 \le 4$ and $6 \le 2$

P is true.

To check Q(3, – 2), we get

$3 - 2 \le 4$ and $3 + 2 \ge 2$

$1 \le 4$ and $5 \ge 2$

Q is true.

To check R(1, 1) we get

$1 + 1 \le 4$ and $1 - 1 \ge 2$

$2 \le 4$ and $0 \ge 2$

R is not true.

$\therefore$ Option (c) is correct.

128. (b) $x^2 = y + z, y^2 = z + x, z^2 = x + y$

$x^2 + x = x + y + z; \ \frac{1}{x+1} = \frac{x}{x+y+z}$

$y^2 + y = x + y + z; \ \frac{1}{y+1} = \frac{y}{x+y+z}$

$z^2 + z = x + y + z; \ \frac{1}{z+1} = \frac{z}{x+y+z}$

$\Rightarrow \frac{x}{x+y+z} + \frac{y}{x+y+z} + \frac{z}{x+y+z} = \frac{x+y+z}{x+y+z} = 1$

129. (d) $2^{122} + 4^{62} + 8^{42} + 4^{64} + 2^{130}$

$2^{122} + 2^{124} + 2^{126} + 2^{128} + 2^{130}$

$2^{122}[1 + 2^2 + 2^4 + 2^6 + 2^8]$

$2^{122}[1 + 4 + 16 + 64 + 256]$

$2^{122}[341]$

130. (c) $\frac{x^2+ax+a^2}{x^3-a^3}-\frac{x^2-ax+a^2}{x^3+a^3}$

$\Rightarrow \frac{x^2+ax+a^2}{(x-a)(x^2+ax+a^2)}-\frac{x^2-ax+1a^2}{(x+a)(x^2-ax+a^2)}$

$\Rightarrow \frac{1}{x-a}-\frac{1}{x+a}$

$\Rightarrow \frac{x+a-x+a}{x^2-a^2} \Rightarrow \frac{2a}{x^2-a^2}$

$\because 4x+3a=0$

$x=\frac{-3a}{4}$

putting $x=\frac{-3a}{4}$

$\Rightarrow \frac{2a}{\left(\frac{-3a}{4}\right)^2-a^2}=\frac{2a\times 16}{9a^2-16a^2} \Rightarrow -\frac{32}{7a}$

131. (b) $x+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{3}}}=2 \Rightarrow x+\cfrac{1}{1+\cfrac{3}{7}}=2$

$\Rightarrow x+\frac{7}{10}=2$

$x=2-\frac{7}{10}=\frac{13}{10}$

132. (c) Putting x = -4 in equations we find option c satisfy the equation $x^2-7x-44=0$

$(-4)^2-7(-4)-44=0$

$44-44=0$

133. (c) $x=t^{\frac{1}{t-1}}, y=t^{\frac{t}{t-1}}$

$y=\left((t)^t\right)^{\frac{1}{t-1}}$

$y=t^x, t=(y)^{\frac{1}{x}}$ ...(i)

$x=t^{\frac{1}{t-1}}$

$x^t=t^{\frac{t}{t-1}}$ ...(ii)

on comparing equation (i) and (ii)

$x^{\frac{y}{x}}=y$

$x^y=y^x$

134. (d)

135. (b) $x=2+2^{\frac{2}{3}}+2^{\frac{1}{3}}$

$(x-2)=2^{\frac{2}{3}}+2^{\frac{1}{3}}$

$(x-2)^3=\left(2^{\frac{2}{3}}+2^{\frac{1}{3}}\right)^3$

$x^3-8-6x^2+12x=4+2+3\ (2)^{\frac{2}{3}}(2)^{\frac{1}{3}}$

$x^3-8-6x^2+12x=6+6(x-2)$

$x^3-8-6x^2+12x=6+6x-12$

$x^3-6x^2+6x=2$

136. (c) $\sqrt{\frac{x}{y}}-\sqrt{\frac{y}{x}}=\frac{24}{5}, x+y=26$

$\frac{x-y}{\sqrt{xy}}=\frac{24}{5}$

$x-y=\frac{24\sqrt{xy}}{5}$ ....(i)

$x+y=26$ ...(ii)

Squaring and subtracting (i) from (ii)

$(x+y)^2-(x-y)^2=(26)^2-\left(\frac{24\sqrt{xy}}{5}\right)^2$

$4xy=676-\frac{576xy}{25}$

137. (a) $a+b=5 \quad ab=6$

$(a+b)^3=125$

$a^3+b^3+3ab\,(a+b)=125$

$a^3+b^3+3\,(6)\,(5)=125$

$a^3+b^3=125-90=35$

$4xy+\frac{576xy}{25}=676$

$676xy=25\times 676$

$xy=25$

138. (c) The given polynomial is of the form ax^3+bx^2+cx+d
Let α, β and γ be three zeroes of the given polynomial.

Then, sum of the zeroes i.e. $\alpha+\beta+\gamma=\frac{-b}{a}=-4$

Product of the zeroes (taken two at a time) i.e.

$=\alpha\beta+\beta\gamma+\gamma\alpha=\frac{c}{a}=-11$

Product of the zeroes (individual)

i.e. $\alpha\beta\gamma=\frac{-d}{a}=30$

Now, we will check each option for the correct answer.

In option (a), we have 2, –3 and –5 as three zeroes.

Sum of these zeroes is – 6 and product of these zeroes is 30.

In option (b), we have –2, –3 and 5 as three zeroes. Sum of these zeroes is 0 and product of these zeroes is 30.

In option (c), we have –2, 3 and –5 as three zeroes. Sum of these zeroes is – 4 and product of these zeroes is 30.

In option (d), we have –2, 3 and 5 as three zeroes. Sum of these zeroes is 6 and product of these zeroes is 30.

Out of these options, only results of option (c) matches with the results calculated above.

Thus, our correct answer is (c).

139. (b) We are given $x = 111\ \ ...1$(20 *digits*), $y = 333\ \ ...3$(10 *digits*) and $z = 222\ \ ...2$(10 *digits*)

Therefore,

$$\frac{x-y^2}{z}=\frac{111\ldots1(20digits)-(333\ldots3)^2(10digits)}{222\ldots2(10digits)}$$

$$=\frac{111\ldots1(20digits)-(3)^2(111\ldots1)^2(10digits)}{222\ldots2(10digits)}$$

$$=\frac{111\ldots1(20digits)}{2(111\ldots1)(10digits)}-\frac{9(111\ldots1)(10digits)}{2}$$

Since $111111 \div 111 = 1001$, therefore

$$\frac{111\ldots1(20digits)}{2(111\ldots1)(10digits)}-\frac{9(111\ldots1)(10digits)}{2}$$

$$=\frac{10000000001-999\ldots9(10digits)}{2}$$

Now, since $1001 - 999 = 2$ and $10001 - 9999 = 2$,

therefore $\dfrac{10000000001-999\ldots9(10digits)}{2}=\dfrac{2}{2}=1$

140. (d) Let $f(x)=5x^3+5x^2-6x+9$ and $g(x)=x+3$

To find the remainder, $g(x)$ should be equal to zero. Therefore, $g(x) = x + 3 = 0 \Rightarrow x = -3$

Putting this value of $f(x)$ in, we get

$$f(x) = 5.(-3)^3+5.(-3)^2-6.(-3)+9$$

$$f(x)=5.-27+5.9+18+9$$

$$=-135+45+27=-63$$

Thus, the remainder is – 63.

141. (c) $8x^3-y^3=(2x)^3-(y)^3=(2x-y)(4x^2+y^2+2xy)$

Quotient $=\dfrac{(2x-y)(4x^2+y^2+2xy)}{2xy+4x^2+y^2}=2x-y$

142. (a) We are given that $(x + 2)$ is a common factor of $x^2 + ax + b$ and $x^2 + bx + a$.

Let $f(x) = x^2 + ax + b$ and $g(x) = x^2 + bx + a$. Let $p(x) = x + 2$.

This means $x + 2 = 0 \Rightarrow x = -2$ and so, –2 is a zero of $f(x)$ and $g(x)$.

Therefore,

$x^2+ax+b=(-2)^2-2a+b=4-2a+b$ and

$x^2+bx+a=(-2)^2-2b+a=4-2b+a$

Both polynomials are same.

Thus,

$$4-2a+b=4-2b+a \Rightarrow -2a+b=-2b+a$$

$$\Rightarrow b+2b=a+2a \Rightarrow 3a=3b \Rightarrow \frac{3a}{3b}=1$$

$$\Rightarrow a:b=1:1$$

143. (a) According to remainder theorem, when a polynomial say $p(x)$ is divided by $(x - a)$, the remainder is $p(a)$. Therefore, when the given polynomial $f(x)$ is divided by $(ax - b)$, then the remainder is

$$f(ax-b=0) \Rightarrow f(ax=b) \Rightarrow f\left(x=\frac{b}{a}\right).$$

Hence, the remainder is $f\left(\dfrac{b}{a}\right)$.

144. (b)

$$(x+2)(x-2)(x^3-2x^2+4x-8)(x^3+2x^2+4x+8)$$

$$=(x^2-4)[x^2(x-2)+4(x-2)][x^2(x+2)+4(x+2)]$$

$$=(x^2-4)(x-2)(x^2+4)(x+2)(x^2+4)$$

$$=(x^2-4)(x^2-4)(x^2+4)^2$$

$$=(x^2-4)^2(x^2+4)^2$$

$$=\{(x^2-4)(x^2+4)\}^2$$

$$=(x^4-16)^2$$

145. (c) We can simply check the product of the constants in each option whether it yields the constant term or not i.e. –72.

Therefore, in option (a), there would be no constant term as each term will contain a variable x.

In option (b), we get the product of the constants as $-1 \times 6 \times -12$ i.e. 72.

In option (c), we get the product of the constants as $-1 \times 6 \times 12$ i.e. –72.

In option (d), we get the product of the constants as $1 \times -6 \times -12$ i.e. 72.

Thus, option (c) is correct.

146. (c) Since we are given that a and c are co-prime i.e. HCF of a and c is 1, therefore we can say that a definitely divides d exactly. So, a is a factor of d.

147. (c) We are given that $ab + bc + ca = 0$

Therefore, $ab+bc+ca=0 \Rightarrow ab+ca=-bc$

Similarly, $bc + ca = -ab$ and $ab + bc = -ca$

Now,

$$\frac{a^2}{a^2-bc}+\frac{b^2}{b^2-ca}+\frac{c^2}{c^2-ab}$$

$$=\frac{a^2}{a^2+ab+ca}+\frac{b^2}{b^2+ab+bc}+\frac{c^2}{c^2+bc+ca}$$

$$\Rightarrow\frac{a^2}{a(a+b+c)}+\frac{b^2}{b(a+b+c)}+\frac{c^2}{c(a+b+c)}$$

$$\Rightarrow\frac{a}{a+b+c}+\frac{b}{a+b+c}+\frac{c}{a+b+c}$$

$$\Rightarrow\frac{a+b+c}{a+b+c}=1$$

148. (b) We know that if $a+b+c=0$ then $a^3+b^3+c^3=3abc$

Now, we have $\dfrac{(x-y)(y-z)(z-x)}{(x-y)^3+(y-z)^3+(z-x)^3}$

Since, here, $(x-y)+(y-z)+(z-x)=0$ therefore, $(x-y)^3+(y-z)^3+(z-x)^3$

$=3(x-y)(y-z)(z-x)$

Thus,

$$\frac{(x-y)(y-z)(z-x)}{(x-y)^3+(y-z)^3+(z-x)^3}$$

$$=\frac{(x-y)(y-z)(z-x)}{3(x-y)(y-z)(z-x)}=\frac{1}{3}$$

149. (d) $5^{17}+5^{18}+5^{19}+5^{20}$

$=5^{17}(1+5^1+5^2+5^3)$

$=5^{17}(1+5+25+125)$

$=5^{17}\times 156=5^{17}\times 2\times 2\times 3\times 13$

Hence, this number is completely divisible by 13.

150. (a) $\dfrac{b}{y}+\dfrac{z}{c}=1$ (given)

$\dfrac{b}{y}=1-\dfrac{z}{c}=\dfrac{c-z}{c}$ $\quad\therefore \dfrac{y}{b}=\dfrac{c}{c-z}=\dfrac{-c}{z-c}$

$\dfrac{c}{z}+\dfrac{x}{a}=1$ (given)

$\dfrac{x}{a}=1-\dfrac{c}{z}=\dfrac{z-c}{z}$ $\quad\therefore \dfrac{a}{x}=\dfrac{z}{z-c}$

$$\frac{ab+xy}{bx}=\frac{ab}{bx}+\frac{xy}{bx}=\frac{a}{x}+\frac{y}{b}$$

$$=\frac{z}{z-c}+\left(\frac{-c}{z-c}\right)=\frac{z}{z-c}-\frac{c}{z-c}\quad=\frac{z-c}{z-c}=1$$

151. (c) $\dfrac{a^2-1}{a}=5$

$\Rightarrow \dfrac{a^2}{a}-\dfrac{1}{a}=5$

$\Rightarrow a-\dfrac{1}{a}=5$

$$\frac{a^6-1}{a^3}=\frac{a^6}{a^3}-\frac{1}{a^3}=a^3-\frac{1}{a^3}$$

$$a^3-\frac{1}{a^3}=\left(a-\frac{1}{a}\right)^3+3.a.\frac{1}{a}\left(a-\frac{1}{a}\right)$$

$=(5)^3+3\times 5=125+15=140$

152. (a) $x+3$ is a factor of x^3+3x^2+4x+k

$x+3=0 \qquad \therefore x=-3$

Now

$x^3+3x^2+4x+k=0$

$(-3)^3+3x(-3)^2+4\times(-3)+k=0$

$-27+27-12+k=0$

$k-12=0$

$\therefore\ k=12$

153. (b) $3x^3+4x^2-7=0$

according to the option

checking option (a)

$x=0$

$3\times 0^3+4\times 0^2-7=0+0-7=-7$

checking option (b)

$x=1$

$3\times(1)^3+4\times(1)^2-7=3+4-7=0$

Hence, 1 is the zero of polynomial $3x^3+4x^2-7$.

154. (d) According to the options

If we take $k=7$ then remainder is -7

Hence, check the options.

155. (a)

156. (c) From option {let $P=2$, $q=4$ and $r=8$}

(a) $\dfrac{1}{\alpha+\beta}=\dfrac{-P}{q}=-\dfrac{2}{4}=-0.5$

(b) $\dfrac{1}{\alpha}+\dfrac{1}{\beta}=\dfrac{\alpha+\beta}{\alpha\beta}=\dfrac{-q}{r}=-\dfrac{4}{8}=-0.5$

(c) $-\dfrac{1}{\alpha\beta}=-\dfrac{P}{r}=-\dfrac{2}{8}=-0.25$

(d) $\dfrac{\alpha\beta}{\alpha+\beta}=-\dfrac{r}{q}=-\dfrac{8}{4}=-2$

$\therefore\ -\dfrac{1}{\alpha\beta}$ is greatest

157. (c) Let three consecutive natural numbers are (n –1), n and (n + 1)
Then,
$(n-1)^2 + n^2 + (n+1)^2 = 110$
$3n^2 + 2 = 110$
$\Rightarrow\ 3n^2 = 108 \quad \Rightarrow n = 6$
Hence, three consecutive natural number are 5, 6, 7.
Now, $5^3 + 6^3 + 7^3 = 125 + 216 + 343 = 684$.

158. (c) $\frac{1}{m}+\frac{1}{n}-\frac{1}{mn}=\frac{2}{5}$

$\frac{n+m-1}{mn}=\frac{2}{5}\Rightarrow 2m=5(m+n-1)$

$\Rightarrow (2mn-5n)=5(m-1)\quad ;\ n(2m-5)=5(m-1)$

or $n=\frac{5(m-1)}{(2m-5)}$

As m, n > 0 and integer, then.

$\frac{5(m-1)}{(2m-5)}>0 \Rightarrow \quad m>1$ and $m\neq\frac{5}{2}$

But for $m = 2$, $\quad n < 0$, $\quad m \geq 3$

For $m = 3$, $\quad n=\frac{5(3-1)}{(6-5)}=10$.

For $m = 4$, $\quad n=\frac{5\times(4-1)}{(8-5)}=5$

For $m = 5$, $\quad n=\frac{5(5-1)}{5}=4$

For $m = 6$, $\quad n=\frac{5(6-1)}{(12-5)}=\frac{5\times 5}{7}$

(Not a integer)

For $m = 10$, $\quad n=\frac{5(10-1)}{(20-5)}=3$

For other value of m, we will not get positive integer value of n.
There, pair (m, n) = (3, 10), (4, 5) (5, 4) and (10, 3)

159. (b) As, H is H.M. of P and Q.

$\therefore\quad H=\frac{2P.Q}{P+Q}$

Now, $\frac{H}{P}+\frac{H}{Q}=\frac{(2P+2Q)}{P+Q}=\frac{2(P+Q)}{P+Q}=2$

160. (a) Given expression are
$G(x, y) = 3x^3 - 2x^2y - 13xy^2 + 10y^3$
Factor is $x - 2y = 0 \quad \Rightarrow \quad x = 2y$
$\therefore\quad G(2y) = 3(2y)^3 - 2(2y)^2.y - 13(2y).y^2 + 10y^3$
$= 24y^3 - 8y^3 - 26y^3 + 10y^3 = 0$

161. (b) $ab + bc + ca = 0$

$$\left.\begin{array}{l}-ab=c(b+a)\\-bc=a(b+c)\\-ca=b(a+c)\end{array}\right\}.........(i)$$

ATQ,

$$\frac{(b^2-ca)(c^2-ab)+(a^2-bc)(c^2-ab)+(a^2-bc)(b^2-ca)}{(a^2-bc)(b^2-ca)(c^2-ab)}$$

$$\frac{1}{a^2-bc}+\frac{1}{b^2-ca}+\frac{1}{c^2-ab}$$

$$\frac{1}{a(a+b+c)}+\frac{1}{b(a+b+c)}+\frac{1}{c(a+b+c)}$$

From (i)

$$\frac{bc+ac+ab}{(a+b+c)abc}=0$$

162. (c) $x + (x + 2)/2x = x + ½ + 1/x$
So we have to find the minimum of $x + 1/x$ and add ½ to it
As AM > GM, So $(x + 1/x)/2 > \sqrt{(x \times 1/x)}$
Or $x + 1/x > 2$
So min of $x + (x + 2)/2x = 2 + 1/2 = 5/2$

163. (a) $\frac{1+px}{1-px}\sqrt{\frac{1-qx}{1+qx}}=1$

On squaring and cross multiplying we get

$$\left(\frac{1+px}{1-px}\right)^2=\left(\sqrt{\frac{1+qx}{1-qx}}\right)^2$$

$$\frac{1+p^2x^2+2px}{1+p^2x^2-2px}=\frac{1+qx}{1-qx}$$

On applying componendo and dividendo

$$\frac{1+p^2x^2+2px+1+p^2x^2-2px}{1+p^2x^2+2px-1-p^2x^2+2px}=\frac{1+qx+1-qx}{1+qx-1+qx}$$

$$\frac{2(1+p^2x^2)}{4px}=\frac{2}{2qx}$$

$$\frac{1+p^2x^2}{2p}=\frac{1}{q}\Rightarrow q+p^2x^2q=2p$$

$$\Rightarrow\therefore x=\pm\frac{1}{p}\sqrt{\frac{2p-q}{q}}$$

164. (b) $$\frac{16}{23}=\frac{1}{\frac{23}{16}}=\frac{1}{1+\frac{7}{16}}=\frac{1}{1+\left(\frac{1}{\frac{16}{7}}\right)}=\frac{1}{1+\frac{1}{2+\left(\frac{2}{7}\right)}}$$

$$=\frac{1}{1+\left(\frac{1}{2+\left(\frac{1}{\frac{7}{2}}\right)}\right)}=\frac{1}{1+\left(\frac{1}{2+\left(\frac{1}{3+\frac{1}{2}}\right)}\right)}$$

On comparing equations we get a = 1, b = 2 and c = 3

Mean = a + b + c/3 = 6/3 = 2

165. (c) For the equation, $\sqrt{(a-b)}\,2 + \sqrt{(b-a)}\,2$

Where a and b are real numbers,

The roots of any square number is always positive and hence it can be zero only at $a = b$

So the above equation is positive only when $a \neq b$

166. (c) $a/(b + c) = b/(c + a) = c/(a + b)$

Taking reciprocal and adding 1 to each ratio we get;

$(b + c)/a + 1 = b/(c + a) + 1 = c/(a + b) + 1$

or $(a + b + c)/a = (a + b + c)/b = (a + b + c)/c$

So this can only be equal when $a = b = c$ or $a + b + c = 0$

When $a = b = c$ we get $a/(b + c) = ½$

When $a + b + c = 0$ we get $b + c = – a$

So $a/(b + c) = – 1$

So the ratios are ½ or – 1

167. (d) Since $x^2 + ax + b$ when divided by $x – 1$ or $x + 1$ leaves the same remainder

So on putting $x = 1$ and $x = –1$ we get the same value

$1 + a + b = 1 – a + b$

$2a = 0$

$a = 0$

Hence b can take any integer value

168. (a) We know that when $a + b + c = 0$, then

$a^3 + b^3 + c^3 = 3abc$

In the above question,

$(x – y) + (y – z) + (z – x) = 0$

Therefore,

$(x – y)^3 + (y – z)^3 + (z – x)^3 = 3\,(x – y)\,(y – z)\,(z – x)$

$$\frac{(x-y)^3+(y-z)^3+(z-x)^3}{3(x-y)(y-z)(z-x)}$$

$$= \frac{3(x-y)(y-z)(z-x)}{3(x-y)(y-z)(z-x)} = 1$$

169. (c) $a^x = b^y = c^z = k$

$a = k^{1/x}$

$b = k^{1/y}$

$c = k^{1/z}$

given $b^2 = ac$, putting the above values of a, b, c in the equation we get

$k^{2/y} = k^{1/x} \times k^{1/z}$

$2/y = 1/x + 1/z$

170. (d) $\dfrac{1}{x^2+5x+10}$ will be maximum

when $x^2 + 5x + 10$ will be minimum

Minimum value of $x^2 + 5x + 10$

$= -\dfrac{D}{4a}$ {where D = Discriminant = –15}

$= \dfrac{15}{4}$ Maximum value of expression $= \dfrac{4}{15}$

171. (b) We know if a + b + c = 0,

then $a^3 + b^3 + c^3 = 3abc$

Here

$(x – y) + (y – z) + (z – x) = 0$

$\Rightarrow (x – y)^3 (y – z)^3 + (z – x)^3$

$= 3(x – y)(y – z)(z – x)$

$$\therefore \frac{(x-y)^3+(y-z)^3+(z-x)^3}{9(x-y)(y-z)(z-x)}$$

$$= \frac{3(x-y)(y-z)(z-x)}{9(x-y)(y-z)(z-x)} = \frac{1}{3}$$

172. (c) $x+\dfrac{1}{x}=\dfrac{x^2+1}{x}=\dfrac{26}{5}$

$5x^2 – 26x + 5 = 0$

The qadridic eq. will have two different solution.

Required real number = 5 and $\dfrac{1}{5}$

173. (a) $\dfrac{36}{11}=3+\cfrac{1}{x+\cfrac{1}{y+\cfrac{1}{z}}}$ $\quad \dfrac{3}{11}=\cfrac{1}{x+\cfrac{1}{y+\cfrac{1}{z}}}$

$\Rightarrow x+\cfrac{1}{y+\cfrac{1}{z}} = \dfrac{11}{3}=3+\dfrac{2}{3}$

$\Rightarrow x = 3, \cfrac{1}{y+\cfrac{1}{z}}=\dfrac{2}{3} \Rightarrow y+\dfrac{1}{z}=\dfrac{3}{2} \Rightarrow y+\dfrac{1}{z}=1+\dfrac{1}{2}$

$\Rightarrow y = 1, z = 2$

$\Rightarrow x + y + z = 3 + 1 + 2 = 6$

174. (c) Let $y = x – x^2$

Differentiationy both, side w.r.t.x $\dfrac{dy}{dx} = 1 – 2x$

At max. $\dfrac{dy}{dx} = 0 \Rightarrow 1 – 2x = 0 \Rightarrow x = \dfrac{1}{2} = 0.5$

175. (a) For k = 0, $x^3 – 7x + 6 = 0$

$x^2 (x – 1) + x(x – 1) – 6(x – 1) = 0$

$(x – 1)(x^2 + x – 6) = 0$

$(x – 1)(x + 3)(x – 2) = 0$

$\therefore x = 1, 2,$ and $–3$

Hence, for k = 0, given expression can be resolved into three linear factors.

176. (a) $x^4 – x^2 + 7x + 5$

$= (x + 2)\,(ax^3 + bx^2 + cx + d) + k$

$= ax^4 + (b + 2a)\,x^3 + (c + 2b)x^2 + (d + 2c)x + 2d + k$

On Equating the coefficient, we get

$a = 1, b + 2a = 0, c + 2b = –1$

$d + 2c = 7, 2d + k = 5$

$\Rightarrow a = 1, b = –2, c = 3, d = 1$

177. (a) $(x-a)(x-b)(x-c)$

$= (x^2 - ax - bx + ab)(x - c)$

$= x^3 - ax^2 - bx^2 + abx - cx^2 + acx + bcx - abc$

$= x^3 - x^2(a + b + c) + x(ab + ac + bc) - abc$

178. (a) $f(x) = x^3 + 4x^2 + px + q$

Now, $(x-1)$ and $(x+2)$ divide

$f(x)$. so, $f(1) = f(-2) = 0$

$f(1) = 1^3 + 4 \times 1^2 + p\times 1 + q = 0 \Rightarrow \quad p + q = -5$

$f(2) = -8 + 16 - 2p + q = 0 \quad \Rightarrow q - 2p = -8$

on solving $p = 1$, $q = -6$

179. (b) $\dfrac{1}{a^{m-n}-1}+\dfrac{1}{a^{n-m}-1} = \dfrac{1}{\dfrac{a^m}{a^n}-1}+\dfrac{1}{\dfrac{a^n}{a^m}-1}$

$= \dfrac{a^n}{a^m-a^n}+\dfrac{a^m}{a^n-a^m} = \dfrac{a^n}{a^m-a^n}-\dfrac{a^m}{a^m-a^n}$

$= -\dfrac{a^m-a^n}{a^m-a^n} = -1$

180. (c) $f(x) = c(x-\alpha)(x-\beta) + ax + b$

Required Remainder is $ax + b$

$f(\alpha) = a\alpha + b$; $f(\beta) = a\beta + b$

$a = \dfrac{f(\alpha)-f(\beta)}{(\alpha-\beta)}$

$b = f(\alpha) - a\alpha = f(\alpha) - \dfrac{f(\alpha)-f(\beta)}{(\alpha-\beta)}\alpha$

Solving the question

$ax + b = \dfrac{(x-\beta)f(\alpha)-(x-\alpha)f(\beta)}{\alpha-\beta}$

181. (d) $x^2 - 1 = 0 \qquad \therefore \quad x = \pm 1$

take $x = -1$

$a - b + c - d + e = 0$

$\therefore \quad a + c + e = b + d$

182. (c) $x^8 + \dfrac{1}{x^8} = 47$

$\left(x^4 + \dfrac{1}{x^4}\right)^2 = x^8 + \dfrac{1}{x^8} + 2$

$\Rightarrow \quad x^4 + \dfrac{1}{x^4} = \sqrt{47+2} = 7$

$\left(x^2 + \dfrac{1}{x^2}\right)^2 = x^4 + \dfrac{1}{x^4} + 2$

$\Rightarrow \quad x^2 + \dfrac{1}{x^2} = \sqrt{7+2} = 3$

$\left(x^2 + \dfrac{1}{x^2}\right)^3 = x^6 + \dfrac{1}{x^6} + 3\left(x^2 + \dfrac{1}{x^2}\right)$

$\Rightarrow \quad x^6 + \dfrac{1}{x^6} = (3)^3 - 3(3) = 18$

CHAPTER 14

Linear Equations in One and Two Variables

1. **Assertion (A) :** The equations $2x - 3y = 5$ and $6y - 4x = 11$ cannot be solved graphically.
Reason (R) : The equations given above represent parallel lines. **[2007-I]**
(a) A and R are correct and R is correct explanation of A
(b) A and R are correct but R is not correct explanation of A
(c) A is correct but R is wrong
(d) A is wrong but R is correct

2. If a two-digit number is added to a number obtained by reversing the digits of the given number, then the sum is always divisible by which one of the following numbers? **[2007-II]**
(a) 7 (b) 9 (c) 10 (d) 11

3. A train started from a station with a certain number of passengers. At the first halt, $\frac{1}{3}$rd of its passengers got down and 120 passengers got in. At the second halt, half of the passengers got down and 100 persons got in. Then, the train left for its destination with 240 passengers. How many passengers were there in the train when it started ? **[2008-I]**
(a) 540 (b) 480 (c) 360 (d) 240

4. A person bought a certain number of books for ₹80. If he had bought 4 more books for the same sum, each book would have cost ₹ 1 less. What is the price of each book ? **[2008-I]**
(a) ₹ 10 (b) ₹ 8 (c) ₹ 5 (d) ₹ 4

5. What is/are the solutions of the set of homogeneous equations $(4x + 2y = 0)$ and $(6x + 3y = 0)$? **[2008-I]**
(a) Only $x = 0, y = 0$
(b) Only $x = 0, y = 0$ and $x = 1, y = 2$
(c) An infinite number of solutions
(d) No solution

6. The cost of 4 books and 3 pencils is same as that of 8 books and 1 pencil. This cost will be same as that of which one of the following ? **[2008-I]**
(a) 2 books and 6 pencils (b) 5 books and 5 pencils
(c) 6 books and 2 pencils (d) 12 books and 4 pencils

7. If one-third of a two-digit number exceeds its one-fourth by 8, then what is the sum of the digits of the number? **[2008-I]**
(a) 6 (b) 13 (c) 15 (d) 17

8. What is the sum of two numbers whose difference is 45 and the quotient of the greater number by the lesser number is 4 ? **[2008-I]**
(a) 100 (b) 90 (c) 80 (d) 75

9. The solution of the equations
$$\frac{3x - y + 1}{3} = \frac{2x + y + 2}{5} = \frac{3x + 2y + 1}{6}$$
is given by which one of the following? **[2008-II]**
(a) $x = 2, y = 1$ (b) $x = 1, y = 1$
(c) $x = -1, y = -1$ (d) $x = 1, y = 2$

10. A person bought 5 tickets from a station P to a station Q and 10 tickets from the station P to a station R. He paid ₹ 350. If the sum of a ticket from P to Q and a ticket from P to R is ₹ 42, then what is the fare from P to Q? **[2009-I]**
(a) ₹ 12 (b) ₹ 14 (c) ₹ 16 (d) ₹ 18

11. Pooja started her job with certain monthly salary and gets a fixed increment every year. If her salary was ₹ 4200 after 3 years and ₹ 6800 after 8 years of service, then what are her initial salary and the annual increment, respectively? **[2009-I]**
(a) ₹ 2640, ₹ 320 (b) ₹ 2460, ₹ 320
(c) ₹ 2460, ₹ 520 (d) ₹ 2640, ₹ 520

12. What is the solution of the equations $x - y = 0.9$ and $11(x + y)^{-1} = 2$? **[2009-I]**
(a) $x = 3.2$ and $y = 2.3$ (b) $x = 1$ and $y = 0.1$
(c) $x = 2$ and $y = 1.1$ (d) $x = 1.2$ and $y = 0.3$

13. What is the value of k for which the system of equations $x + 2y - 3 = 0$ and $5x + ky + 7 = 0$ has no solution? **[2009-I]**
(a) $-\frac{3}{14}$ (b) $-\frac{14}{3}$ (c) $\frac{1}{10}$ (d) 10

14. A number consists of two digits, whose sum is 10. If 18 is subtracted from the number, digits of the number are reversed. What is the product? **[2009-II]**
(a) 15 (b) 18 (c) 24 (d) 32

15. A railway ticket for a child costs half the full fare but the reservation charge is the same on half tickets as much as on full ticket. One reserved first class ticket for a journey between two stations is ₹ 362, one full and one half reserved first class tickets cost ₹ 554. What is the reservation charge? **[2009-II]**
(a) ₹ 18 (b) ₹ 22 (c) ₹ 38 (d) ₹ 46

16. Let there be three simultaneous linear equations in two unknowns, which are non-parallel and non-collinear them. What can be the number of solutions (if they do exist)? **[2010-I]**
(a) One or infinite (b) Only one
(c) Exactly two (d) Exactly three

17. Under what condition do the equations $kx - y = 2$ and $6x - 2y = 3$ have a unique solution? **[2010-II]**
(a) $k = 3$ (b) $k \neq 3$ (c) $k = 0$ (d) $k \neq 0$

18. If 1 is added to the denominator of a fraction, it becomes $\frac{1}{2}$ and if 1 is added to the numerator, the fraction becomes 1. What is the fraction? **[2010-II]**

(a) $\frac{5}{9}$ (b) $\frac{2}{3}$ (c) $\frac{4}{7}$ (d) $\frac{10}{11}$

19. If $\frac{2}{x}+\frac{3}{y}=\frac{9}{xy}$ and $\frac{4}{x}+\frac{9}{y}=\frac{21}{xy}$, where $x \neq 0$ and $y \neq 0$, then what is the value of x + y? **[2011-I]**

(a) 2 (b) 3 (c) 4 (d) 8

20. The sum of two numbers is 80. If the larger number exceeds four times the smaller by 5, what is the smaller number? **[2011-I]**

(a) 5 (b) 15 (c) 20 (d) 25

21. If a and b are positive integers, x and y are non-negative integers and a = bx + y, then which one of the following is correct? **[2011-I]**

(a) $0 \le y < a$ (b) $0 < y \le b$
(c) $0 < y < a$ (d) $0 \le y < b$

22. If $\frac{2x-3y+1}{2}=\frac{x+4y+8}{3}=\frac{4x-7y+2}{5}$, then what is (x + y) equal to? **[2011-I]**

(a) 3 (b) 2 (c) 0 (d) –2

23. If (x, y) = (4, 1) is the solution of the pair of linear equations mx + y = 2x + ny = 5, then what is m + n equal to? **[2011-II]**

(a) –2 (b) –1 (c) 2 (d) 1

24. The sum of two numbers is 10 and their product is 20. What is the sum of their reciprocals? **[2011-II]**

(a) $\frac{1}{10}$ (b) $\frac{1}{2}$ (c) 1 (d) 2

25. The sum of digits of a two-digit number is 8 and the difference between the number and that formed by reversing the digits is 18. What is the difference between the digits of the number? **[2011-II]**

(a) 1 (b) 2 (c) 3 (d) 4

26. The system of equations x + 2y = 3 and 3x + 6y = 9 has **[2011-II]**

(a) unique solution
(b) no solution
(c) infinitely many solutions
(d) finite number of solutions

27. If $\frac{5x-7y+10}{1}=\frac{3x+2y+1}{8}=\frac{11x+4y-10}{9}$, then what is x + y equal to? **[2011-II]**

(a) 1 (b) 2 (c) 3 (d) –3

28. If $\frac{3}{x+y}+\frac{2}{x-y}=2$ and $\frac{9}{x+y}-\frac{4}{x-y}=1$, then what is the value of $\frac{x}{y}$? **[2013-I]**

(a) $\frac{3}{2}$ (b) 5 (c) $\frac{2}{3}$ (d) $\frac{1}{5}$

29. The graphs of ax + by = c, dx + ey = f will be:

I. parallel, if the system has no solution.
II. co-incident, if the system has finite number of solutions.
III. intersecting, if the system has only one solution.

Which of the above statements are correct? **[2013-I]**

(a) Both I and II (b) Both II and III
(c) Both I and III (d) All of these

30. The sum of two numbers is 20 and their product is 75. What is the sum of their reciprocals? **[2013-I]**

(a) $\frac{1}{15}$ (b) $\frac{1}{5}$ (c) $\frac{4}{15}$ (d) $\frac{7}{15}$

31. If the sum of a number and its reciprocal is $\frac{10}{3}$, then the numbers are **[2013-I]**

(a) $3, \frac{1}{3}$ (b) $3, -\frac{1}{3}$ (c) $-3, \frac{1}{3}$ (d) $-3, -\frac{1}{3}$

32. A number consists of two digits. The sum of the digits is 10. On reversing the digits of the number, the number decreases by 36. What is the product of the two digits? **[2013-I]**

(a) 21 (b) 24 (c) 36 (d) 42

33. If $2x + 3y \le 6$, $x \ge 0$, $y \ge 0$, then one of the solutions is **[2013-I]**

(a) x = –2 and y = 3 (b) x = 1 and y = 2
(c) x = 1 and y = 1 (d) x = –1 and y = –1

34. There are some benches in a class room having the number of rows 4 more than the number of columns. If each bench is seated with 5 students, there are two seats vacant in a class of 158 students. The number of rows is **[2013-I]**

(a) 4 (b) 8 (c) 6 (d) 10

35. If $\frac{a}{b}-\frac{b}{a}=\frac{x}{y}$ and $\frac{a}{b}+\frac{b}{a}=x-y$, then what is the value of x? **[2013-I]**

(a) $\frac{a+b}{a}$ (b) $\frac{a+b}{b}$
(c) $\frac{a-b}{a}$ (d) None of these

36. A number consists of two digits whose sum is 8. If 18 is added to the number, the digits are reversed. The number is equal to **[2013-I]**

(a) 26 (b) 35 (c) 53 (d) 62

37. The sum of the squares of two numbers is 97 and the squares of their difference is 25. The product of the two numbers is **[2013-I]**

(a) 45 (b) 36 (c) 54 (d) 63

38. If $x + \frac{1}{x} = 2$, then what is value of $x - \frac{1}{x}$? **[2013-I]**

(a) 0 (b) 1 (c) 2 (d) –2

39. The system of equations 3x + y – 4 = 0 and 6x + 2y – 8 = 0 has **[2013-I]**

(a) a unique solution x = 1, y = 1
(b) a unique solution x = 0, y = 4

(c) no solution
(d) infinite solution

40. The sum of two numbers is 7 and the sum of their squares is 25. The product of the two numbers is **[2013-I]**
(a) 6 (b) 10 (c) 12 (d) 15

41. A number consists of two digits whose sum is 10. If the digits of the number are reversed, then the number decreased by 36. Which of the following is/are correct?
I. The number is divisible by a composite number.
II. The number is a multiple of a prime number.
Select the correct answer using the codes given below: **[2013-I]**
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

42. Ten chairs and six tables together cost ₹ 6200, three chairs and two tables together cost ₹ 1900. The cost of 4 chairs and 5 tables is **[2013-I]**
(a) ₹ 3000 (b) ₹ 3300
(c) ₹ 3500 (d) ₹ 3800

43. If $x + y - 7 = 0$ and $3x + y - 13 = 0$, then what is $4x^2 + y^2 + 4xy$ equal to? **[2013-II]**
(a) 75 (b) 85 (c) 91 (d) 100

44. If $\frac{x}{2}+\frac{y}{3} = 4$ and $\frac{2}{x}+\frac{3}{y} = 1$, then what is $x + y$ equal to? **[2013-II]**
(a) 11 (b) 10 (c) 9 (d) 8

45. The average age of male employees is 52 years and that of female employees is 42 years. The mean age of all employees is 50 years. The percentage of male and female employees are respectively **[2014-I]**
(a) 80% and 20% (b) 20% and 80%
(c) 50% and 50% (d) 52% and 48%

46. The present age of Ravi's father is 4 times of Ravi's present age. 5 years back, Ravi's father was seven times as old as Ravi was at that time. What is the present age of Ravi's father? **[2014-I]**
(a) 84 years (b) 70 years
(c) 40 years (d) 35 years

47. A positive number, when increased by 10 equals 200 times its reciprocal. What is number? **[2014-I]**
(a) 100 (b) 10 (c) 20 (d) 200

48. The sum of two positive numbers x and y is 2.5 times their difference. If the product of numbers is 84, then what is the sum of those two numbers? **[2014-I]**
(a) 26 (b) 24 (c) 22 (d) 20

49. Ravi's brother is 3 years elder to him. His father was 28 years of age when his sister was born while his mother was 26 years of age when he was born. If his sister was 4 years of age when his brother was born, the ages of Ravi's father and mother, respectively when his brother was born were **[2014-I]**
(a) 32 years and 23 years
(b) 32 years and 29 years
(c) 35 years and 29 years
(d) 35 years and 33 years

50. Two chairs and one table cost ₹700 and 1 chair and 2 tables cost ₹ 800. If the cost m tables and m chairs is ₹ 30,000, then what is m equal to? **[2014-I]**
(a) 60 (b) 55 (c) 50 (d) 45

51. A certain number of two digits is three times the sum of its digits. If 45 is added to the number, then the digits will be reversed. What is the sum of the squares of the two digits of the number ? **[2014-II]**
(a) 41 (b) 45 (c) 53 (d) 64

52. If $ax + by - 2$ and $axby = 1$, where $a \neq 0, b \neq 0$, then what is $(a^2x + b^2y)$ equal to ? **[2014-II]**
(a) $a + b$ (b) 2ab
(c) $a^3 + b^3$ (d) $a^4 + b^4$

53. If m and n $(m > n)$ are the roots of the equation $7(x + 2a)^2 + 3a^2 = 5a\,(7x + 23a)$, where $a > 0$, then what is $3m - n$ equal to ? **[2014-II]**
(a) $12a$ (b) $14a$ (c) $15a$ (d) $18a$

54. Consider the following statements **:** **[2015-I]**
1 The equation $1990x - 173y = 11$ has no solution in integers for x and y.
2. The equation $3x - 12y = 7$ has no solution in integers for x and y.
Which of the above statements is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

55. If $k = x - y + 2z$ where $-2 \leq x \leq 1, -1 \leq y \leq 2$ and $3 \leq z \leq 6$, then which one of the following is correct ? **[2015-II]**
(a) $0 \leq k \leq 9$ (b) $5 \leq k \leq 11$
(c) $2 \leq k \leq 14$ (d) $2 \leq k \leq 11$

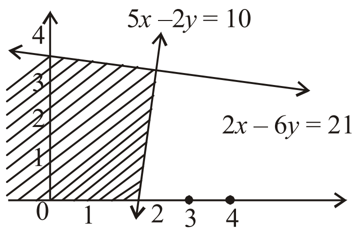
56. The number of pairs (x, y) where x, y are integers satisfying the equation $21x + 48y = 5$ is **[2015-II]**
(a) Zero (b) One (c) Two (d) Infinity

57. Let x and y be positive integers such that x is prime and y is composite. Which of the following statements are correct? **[2015-II]**
1. $(y - x)$ can be an even integer.
2. xy can be an even integer.
3. $0.5\,(x + y)$ can be an even integer
Select the correct answer using the code given below
(a) 1 and 2 only (b) 2 and 3 only
(c) land 3 only (d) 1, 2 and 3

58. If $\frac{p}{x}+\frac{q}{y} = m$ and $\frac{q}{x}+\frac{p}{y} = n$, then what is $\frac{x}{y}$ equal to? **[2016-I]**
(a) $\frac{np+mq}{mp+nq}$ (b) $\frac{np+mq}{mp-nq}$
(c) $\frac{np-mq}{mp-nq}$ (d) $\frac{np-mq}{mp+nq}$

59. The value of k, for which the system of equation $3x - ky - 20 = 0$ and $6x - 10y + 40 = 0$ has no solution, is **[2016-I]**
(a) 10 (b) 6 (c) 5 (d) 3

60. There are three brothers. The sums of ages of two of them at a time are 4 years, 6 years and 8 years. The age difference between the eldest and the youngest is **[2016-I]**
(a) 3 years (b) 4 years
(c) 5 years (d) 6 years

61. 

The linear inequations, for which the shaded area in the figure given above is the solution set, are **[2016-I]**

(a) $2x+6y\le 21, 5x-2y\le 10$
(b) $2x+6y\le 21, 5x-2y\ge 10$
(c) $2x+6y\ge 21, 5x-2y\le 10$
(d) $2x+6y\ge 21, 5x-2y\ge 10$

62. If a and b are negative real numbers and c is a positive real number, then which of the following is/are correct?

1. $a-b<a-c$ **[2016-I]**
2. If $a<b$ then $\frac{a}{c}<\frac{b}{c}$.
3. $\frac{1}{b}<\frac{1}{c}$

Select the Correct answer using the code given below.
(a) 1 (b) 2 only (c) 3 only (d) 2 and 3

63. The pair of linear equations kx + 3y + 1 = 0 and 2x + y + 3 = 0 intersect each other, if **[2017-I]**
(a) k = 6 (b) k ≠ 6 (c) k = 0 (d) k ≠ 0

64. The system of equations 2x + 4y = 6 and 4x + 8y = 8 is **[2017-I]**
(a) Consistent with a unique solution
(b) Consistent with infinitely many solutions
(c) Inconsistent
(d) None of the above

65. What is the value of u in the system of equations 3 (2u + v) = 7uv, 3 (u + 3v) = 11uv? **[2017-II]**
(a) 0 (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) 1

66. If a + b = 2c, then what is the value of $\frac{a}{a-c}+\frac{c}{b-c}$? **[2018-1]**
(a) –1 (b) 0 (c) 1 (d) 2

67. If 65x – 33y = 97 and 33x – 65y = 1, then what is xy equal to? **[2018-1]**
(a) 2 (b) 3 (c) –2 (d) –3

68. If x + y + z = 0, then what is $(y + z - x)^3 + (z + x - y)^3 + (x + y - z)^3$ equal to? **[2018-1]**
(a) $(x+y+z)^3$ (b) $3(x+y)(y+z)(z+x)$
(c) 24xyz (d) –24xyz

69. The solution of linear inequalities $x+y\ge 5$ and $x-y\le 3$ lies **[2018-II]**
(a) Only in the first quadrant
(b) In the first and second quadrants
(c) In the second and third quadrants
(d) In the third and fourth quadrants

70. It is given that the equations $x^2 - y^2 = 0$ and $(x - a)^2 + y^2 = 1$ have single positive solution. For this, the value of 'a' is **[2018-II]**
(a) $\sqrt{2}$ (b) 2 (c) $-\sqrt{2}$ (d) 1

71. Which of the following pair of numbers is the solution of the equation $3^{x+2} + 3^{-x} = 10$? **[2019-I]**
(a) 0, 2 (b) 0, –2 (c) 1, –1 (d) 1, 2

72. A man who recently died left a sum of ₹ 3,90,000 to be divided among his wife, five sons and four daughters. He directed that each son should receive 3 times as much as each daughter receives and that each daughter should receive twice as much as their mother receives. What was the wife's share? **[2019-I]**
(a) ₹ 14, 000 (b) ₹ 12, 000
(c) ₹ 10, 000 (d) ₹ 9, 000

73. If $x=\frac{1+\sqrt{3}}{2}$ and $y = x^3$, then y satisfies which one of the following equations ? **[2019-II]**
(a) $8y^2-20y-1=0$ (b) $8y^2+20y-1=0$
(c) $8y^2+20y+1=0$ (d) $8y^2-20y+1=0$

74. What is the point on the *xy*-plane satisfying $5x + 2y = 7xy$ and $10x + 3y = 8xy$? **[2020-I]**
(a) $\left(-1,\frac{1}{6}\right)$ (b) $\left(\frac{1}{6},-1\right)$
(c) $\left(1,\frac{1}{6}\right)$ (d) $\left(-\frac{1}{6},-1\right)$

75. Students of a class are made to sit in rows of equal number of chairs. If number of students is increased by 2 in each row, then the number of rows decreases by 3. If number of students is increased by 4 in each row, then the number of rows decreases by 5. What is the number of students in the class ? **[2020-I]**
(a) 100 (b) 105 (c) 110 (d) 120

76. The maximum marks in a Test are converted from 250 to 50 for the purpose of an Internal Assessment. The highest marks scored were 170 and lowest marks were 70. What is the difference between the maximum and minimum marks scored in the Internal Assessment ? **[2020-I]**
(a) 15 (b) 17 (c) 20 (d) 24

77. How many integral values of *x* and *y* satisfy the equation $5x + 9y = 7$, where $-500 < x < 500$ and $-500 < y < 500$? **[2020-I]**
(a) 110 (b) 111
(c) 112 (d) None of the above

HINTS & SOLUTIONS

1. (a) Given that,
$2x - 3y = 5$ ... (i)
and $-4x + 6y = 11$ ... (ii)
Also, $\frac{a_1}{a_2} = \frac{2}{-4} = \frac{-1}{2}$
$\frac{b_1}{b_2} = \frac{-3}{6} = \frac{-1}{2}$
$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{-1}{2} \neq \frac{c_1}{c_2}$
So, both equations are in parallel. So, it is not solved graphically.
Hence, A and R are individually true and R is correct explanation of A.

2. (d) Let a two-digit number be $(10x + y)$ and reversing number be $(10y + x)$
$\therefore$ Required sum $= 10x + y + 10y + x$
$= 11x + 11y = 11(x + y)$
Thus, it is divisible by 11.

3. (d) Suppose number of passengers be x in the starting.
Number of passengers after 1st halt
$= \left(x - \frac{x}{3}\right) + 120 = \frac{2x}{3} + 120$
Number of passengers after 2nd halt
$= \frac{1}{2}\left(\frac{2x}{3} + 120\right) + 100$
According to question,
Number of passengers after 2nd halt
$= \frac{1}{2}\left(\frac{2x}{3} + 120\right) + 100 = 240$
$\Rightarrow \frac{2x}{3} + 120 = (240 - 100) \times 2$
$\Rightarrow \frac{2x}{3} = 280 - 120$
$\frac{2x}{3} = 160$
$x = \frac{\overset{80}{\cancel{160}} \times 3}{\underset{1}{\cancel{2}}}$
$x = 240$

4. (c) Let the price of each book is ₹ x and the number of books is y.
$\therefore xy = 80$ ... (i)
and $(y + 4)(x - 1) = 80$
$\Rightarrow xy - y + 4x - 4 = 80$
$\Rightarrow 80 - y + 4x = 84$ using equation (i)]
$\Rightarrow 4x - y = 4$
$\Rightarrow y = 4(x - 1)$
On putting this value of y in equation (i), we get
$4(x - 1)x = 80$
$\Rightarrow (x - 1)x = 20$
$\Rightarrow x^2 - x - 20 = 0$
$\Rightarrow x^2 - 5x + 4x - 20 = 0$
$\Rightarrow (x - 5)(x + 4) = 0$
$\therefore x = 5$ $(\because x \neq -4)$
Thus, the price of each is ₹ 5.

5. (c) Given equations are:
$4x + 2y = 0$ ... (i)
and $6x + 3y = 0$ ... (ii)
Here, $a_1 = 4$, $b_1 = 2$, $c_1 = 0$
and $a_2 = 6$, $b_2 = 3$, $c_2 = 0$
Here, $\frac{a_1}{a_2} = \frac{b_1}{b_2}$
$\therefore \frac{4}{6} = \frac{2}{3} \Rightarrow \frac{2}{3} = \frac{2}{3}$
So, it has infinite solutions.

6. (c) Let the cost of one book = ₹ x
and the cost of one pencil = ₹ y
According to question,
$4x + 3y = 8x + y$
$\Rightarrow 2y = 4x \Rightarrow y = 2x$
$\therefore$ Cost of 4 books and 3 pencils
$= 4x + 3y = 4x + 6x = 10x$
Cost of 6 books and 2 pencils
$= 6x + 2y = 6x + 4x = 10x$

7. (c) Let the number be y.
$\therefore \frac{y}{3} = \frac{y}{4} + 8$
$\Rightarrow \frac{4y - 3y}{12} = 8$
$\Rightarrow y = 12 \times 8 = 96$
$\therefore$ Sum of digits $= 9 + 6 = 15$

8. (d) Let the greater number be x and smaller number be y.
$\therefore x - y = 45$ ... (i)
and $x = 4y$ ... (ii)
From equations (i) and (ii),
$4y - y = 45$
$\Rightarrow y = \frac{45}{3} = 15$
On putting the value of y in equation (i), we get
$x = 4 \times 15 = 60$
$\therefore$ Required sum $= x + y = 60 + 15 = 75$

9. (b) Given equations are:
$\frac{3x - y + 1}{3} = \frac{2x + y + 2}{5} = \frac{3x + 2y + 1}{6}$

On taking I^{st} and II^{nd} terms,
$5(3x - y + 1) = 3(2x + y + 2)$
$\Rightarrow 9x - 8y = 1$ ... (i)
On taking II^{nd} and III^{rd} terms,
$6(2x + y + 2) = 5(3x + 2y + 1)$
$\Rightarrow 3x + 4y = 7$ ... (ii)
On solving equations (i) and (ii), we get
$y = 1$ and $x = 1$

10. (b) Let,
fare from $P \rightarrow Q$ = ₹ x
fare from $P \rightarrow R$ = ₹ y
According to question,
$x + y$ = ₹ 42 ... (i)
$5x + 10y$ = ₹ 350 ... (ii)
By solving equations (i) and (ii), then we get
$x = 14$
$y = 28$
$\therefore$ fare from station $P \rightarrow R$ = ₹ 14

11. (d) Let Pooja's initial salary is ₹ x and fixed increment every year is ₹ y.
According to question,
$x + 3y = 4200$ ... (i)
and $x + 8y = 6800$ ... (ii)
On solving equations (i) and (ii), we get
x = ₹ 2640, y = ₹ 520

12. (a) Given,
$x - y = 0.9$ ... (i)
and $11(x + y)^{-1} = 2$
$\Rightarrow 2x + 2y = 11$ ... (ii)
On multiplying equation (i) by 2 and adding equations (i) and (ii), we get
$4x = 12.8$
$\Rightarrow x = 3.2$
From equation (i),
$y = 3.2 - 0.9 = 2.3$

13. (d) $x + 2y - 3 = 0$ and $5x + ky + 7 = 0$
Then, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
$\therefore \frac{1}{5} = \frac{2}{k} \neq \frac{-3}{7}$
$\Rightarrow k = 10$
So, it has no solution.

14. (c) Let the two-digit number be $10y + x$.
According to question,
$x + y = 10$ ... (i)
and $10y + x - 18 = 10x + y$
$\Rightarrow 9x - 9y = -18$
$\Rightarrow x - y = -2$ ... (ii)
On solving equations (i) and (ii), we get
$x = 4$ and $y = 6$
$\therefore$ Required product $= xy = 4 \times 6 = 24$.

15. (b) Let full fare = ₹ x and reservation charges = ₹ y
$\therefore x + y = 362$ ... (i)
and $1\frac{1}{2}x + 2y = 554$
$\Rightarrow 3x + 4y = 1108$ ... (ii)
On solving equations (i) and (ii), we get
$\therefore x = 340$ and $y = 22$
Reservation charge = ₹ 22

16. (d) If three lines are parallel, then there are infinite solution. If all three lines are collinear, then there is only one solution. If all three lines are non-parallel and non-collinear, then there is only three solutions. Hence, the given system of equation have here exactly three solutions.

17. (b) The equations $kx - y = 2$ and $6x - 2y = 3$ have a unique solution. Then,
$\therefore \frac{k}{6} \neq \frac{1}{2} \Rightarrow k \neq 3$

18. (b) Let the numerator = x and denominator = y
According to question,
$\frac{x}{y+1} = \frac{1}{2}$
$\Rightarrow 2x = y + 1$
$\Rightarrow 2x - y = 1$ ... (i)
and $\frac{x+1}{y} = 1 \Rightarrow x + 1 = y$
$\Rightarrow x - y = -1$ ... (ii)
$$\begin{array}{rl} & 2x - y = 1 \\ \Rightarrow & \underline{-x \mp y = \mp 1} \\ & x \quad = 2 \end{array}$$
Putting the value of x in equation (ii),
$2 - y = -1$
$\Rightarrow y = 3 \Rightarrow x = 2$ and $y = 3$
$\therefore$ Required fraction $= \frac{x}{y} = \frac{2}{3}$.

19. (c) Given,
$\frac{2}{x} + \frac{3}{y} = \frac{9}{xy}$
$\Rightarrow 2y + 3x = 9$ ... (i)
and $\frac{4}{x} + \frac{9}{y} = \frac{21}{xy}$
$\Rightarrow 4y + 9x = 21$ ... (ii)
On solving equations (i) and (ii), we get
$x = 1$ and $y = 3$
$\therefore x + y = 1 + 3 = 4$

20. (b) Let smaller number = x
Larger number = $80 - x$
According to question,
$80 - x = 4x + 5$
$\Rightarrow 5x = 75 \Rightarrow x = 15$

21. (c) Let $a, b \geq 0$ and $x, y > 0$
$\because a = bx + y$
$\Rightarrow x = \frac{a-y}{b} \Rightarrow \frac{a-y}{b} > 0$
$\Rightarrow a - y > 0 \Rightarrow a > y$
$\therefore 0 < y < a$

22. (d) $\frac{2x-3y+1}{2} = \frac{x+4y+8}{3} = \frac{4x-7y+2}{5}$
$= \frac{(2x-3y+1)+(x+4y+8)-(4x-7y+2)}{(2+3)-5}$

$= -x + 8y + 7 = x - 8y = 7$

$\Rightarrow x = 7 + 8y$ ... (i)

Now taking first two terms,

$3(2x - 3y + 1) = 2(x + 4y + 8)$

$4x - 17y = 13$

Putting the value of x from equation (i), then we get

$4(7 + 8y) - 17y = 13$

$28 + 32y - 17y = 13$

$15y = -15$

$y = -1$

Now putting the value of y in equation (i), then we get

$x = 7 - 8 = -1$

Then, $x + y = -1 - 1 = -2$

Hence, $x + y = -2$.

23. (a) Given,

$(x, y) = (4, 1)$ and $mx + y = 2x + ny = 5$

$\therefore$ $m(4) + 1 = 2 \times 4 + n = 5$

$\therefore$ $4m + 1 = 5$ and $8 + n = 5$

$\Rightarrow$ $m = 1$ and $n = -3$

$\therefore$ $m + n = 1 - 3 = -2$

24. (b) Let the two numbers be x and y.

$\therefore$ $x + y = 10$ and $xy = 20$

$\therefore$ $\frac{1}{x} + \frac{1}{y} = \frac{x+y}{xy} = \frac{10}{20} = \frac{1}{2}$.

25. (b) Let x be the first digit and y be the second digit of two digit number.

According to question,

$x + y = 8$ ... (i)

$(10x + y) - (10y + x) = 18$

$\Rightarrow$ $9x - 9y = 18$

$\Rightarrow$ $x - y = 2$ ... (ii)

Adding (i) and (ii),

$$\begin{array}{l} x + y = 8 \\ x - y = 2 \\ \hline 2x \quad = 10 \end{array}$$

$\Rightarrow$ $x = 5$

$\Rightarrow$ $y = 8 - 5 = 3$

$\Rightarrow$ $x = 5$ and $y = 3$

$\therefore$ Required difference of digits, $x - y = 5 - 3 = 2$.

26. (c) Given system of equations are:

$x + 2y = 3$ and $3x + 6y = 9$

$\Rightarrow$ $x + 2y = 3$

Here, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{3}$

So, it has infinitely many solutions.

27. (c) Given,

$$\frac{5x - 7y + 10}{1} = \frac{3x + 2y + 1}{8} = \frac{11x + 4y - 10}{9}$$

$$= \frac{(5x + 10 - 7y) + (3x + 2y + 1) - (11x + 4y - 10)}{(1 + 8) - 9}$$

$$= \frac{-3x - 9y + 21}{0}$$

$\Rightarrow$ $x + 3y = 7$ ... (i)

On taking first two terms,

$8(5x - 7y + 10) = 3x + 2y + 1$

$37x - 58y + 79 = 0$ ... (ii)

From equation (i), on putting the value of x in equation (ii), we get

$37(7 - 3y) - 58y + 79 = 0$

$\Rightarrow$ $259 - 111y - 58y + 79 = 0$

$\Rightarrow$ $169y = 338$

$\Rightarrow$ $y = 2$

From equation (i),

$x = 7 - 3(2) = 1$

$\therefore$ $x + y = 1 + 2 = 3$

28. (b) Given,

$$\frac{3}{x+y} + \frac{2}{x-y} = 2 \quad \text{... (i)}$$

and $$\frac{9}{x+y} - \frac{4}{x-y} = 1 \quad \text{... (ii)}$$

Let $\frac{1}{x+y} = a$ and $\frac{1}{x-y} = b$

$\therefore$ $3a + 2b = 2$ ... (iii)

$9a - 4b = 1$ ... (iv)

On multiplying equation (iii) by 2 and addition of equation (iv) and new one, then we get

$$\begin{array}{l} 6a + 4b = 4 \\ 9a - 4b = 1 \\ \hline 15a \quad = 5 \end{array}$$

$\Rightarrow$ $a = \frac{5}{15} = \frac{1}{3}$

$\therefore$ $\frac{1}{x+y} = \frac{1}{3}$

$\Rightarrow$ $x + y = 3$ ... (v)

On putting the value of (a) in equation (iii), we get

$3 \times \frac{1}{3} + 2b = 2$

$\Rightarrow$ $2b = 2 - 1 = 1$

$\Rightarrow$ $b = \frac{1}{2} \Rightarrow \frac{1}{x-y} = \frac{1}{2}$

$\Rightarrow$ $x - y = 2$ ... (vi)

$$\therefore \begin{array}{l} x + y = 3 \\ x - y = 2 \\ \hline 2x \quad = 5 \end{array}$$

$\Rightarrow$ $x = \frac{5}{2}$

From equation (v),

$y = 3 - \frac{5}{2} = \frac{1}{2}$

$\therefore$ $\frac{x}{y} = \frac{\frac{5}{2}}{\frac{1}{2}} = 5$

29. (c)

30. (c) Let the numbers be x and y.
According to the question,
$x + y = 20$
and
$xy = 75$
$\Rightarrow \frac{1}{x} + \frac{1}{y} = \frac{y + x}{xy}$
$\Rightarrow \frac{20}{75} = \frac{4}{15}$

31. (a) Let the numbers are x and $\frac{1}{x}$. Then,
$x + \frac{1}{x} = \frac{10}{3}$
$\frac{x^2 + 1}{x} = \frac{10}{3}$
$\Rightarrow 3x^2 - 10x + 3 = 0$
$\Rightarrow 3x^2 - 9x - x + 3 = 0$
$\Rightarrow 3x(x - 3) - 1(x - 3) = 0$
$\Rightarrow (3x - 1)(x - 3) = 0$
$\therefore x = \frac{1}{3}, x = 3$

32. (a) Let the unit digit of the number be x and tens digit be y.
$\therefore$ Number = 10y + x
According to the question,
$x + y = 10$ ... (i)
and $10x + y = (10y + x) - 36$
$\Rightarrow 10x + y - 10y - x = -36$
$\Rightarrow 9x - 9y = -36$
$\Rightarrow x - y = -4$ ... (ii)

$$\begin{array}{l} x + y = 10 \\ x - y = -4 \\ \hline 2x \quad = 6 \end{array}$$

$\Rightarrow x = 3$
From equation (i),
$3 + y = 10$
$\Rightarrow y = 7$
$\Rightarrow x = 3$ and $y = 7$
$\therefore$ Required product of two digits $= 3 \times 7 = 21$

33. (c) Given,
$2x + 3y \le 6, x \ge 0, y \ge 0$
Now,
$2x + 3y \le 6$
Put $x = 1, y = 1$
$\Rightarrow 2 \times 1 + 3 \times 1 \le 6$
$\Rightarrow 5 < 6$
x = 1 and y = 1 is only solution according to given condition.

34. (b) Let the number of column be x.
Number of rows = x + 4
According to the question,
$x(x + 4) \times 5 - 2 = 158$
$\Rightarrow 5x(x + 4) = 160$
$\Rightarrow x(x + 4) = 32$
$\Rightarrow x^2 + 4x - 32 = 0$
$\Rightarrow x^2 + 8x - 4x - 32 = 0$
$\Rightarrow x(x + 8) - 4(x + 8) = 0$
$\Rightarrow (x + 8)(x - 4) = 0$
So, x = 4 as x = –8 is not possible
$\therefore$ Number of rows = x + 4 = 4 + 4 = 8

35. (d) Given equations are:
$\frac{a}{b} - \frac{b}{a} = \frac{x}{y}$ ... (i)
and $\frac{a}{b} + \frac{b}{a} = x - y$ ... (ii)
From equations (i) and (ii), we get

$$\frac{a}{b} + \frac{b}{a} = x - \frac{x}{\left(\frac{a}{b} - \frac{b}{a}\right)} = \frac{\left(\frac{a}{b} - \frac{b}{a}\right)x - x}{\frac{a}{b} - \frac{b}{a}}$$

$$\Rightarrow \left(\frac{a}{b} + \frac{b}{a}\right)\left(\frac{a}{b} - \frac{b}{a}\right) = x\left(\frac{a}{b} - \frac{b}{a} - 1\right)$$

$$\Rightarrow \left(\frac{a^2}{b^2} - \frac{b^2}{a^2}\right) = x\left(\frac{a^2 - b^2 - ab}{ab}\right)$$

$$\Rightarrow x = \frac{ab}{\left(a^2 - b^2 - ab\right)} \times \left(\frac{a^4 - b^4}{a^2 b^2}\right)$$

$$\Rightarrow x = \frac{\left(a^4 - b^4\right)}{\left(a^2 - b^2 - ab\right)} \cdot \frac{1}{ab} = \frac{(a - b)(a + b)\left(a^2 + b^2\right)}{ab\left(a^2 - b^2 - ab\right)}.$$

36. (b) Let tens digit and unit digits are x and y, respectively.
Sum of digits = 8
$x + y = 8$ ... (i)
According to question,
$(10x + y) + 18 = (10y + x)$
$\Rightarrow 9y - 9x = 18$
$\Rightarrow y - x = 2$ ... (ii)
On solving equations (i) and (ii), we get
$\therefore$ y = 5 and x = 3
$\therefore$ Required number = 10x + y
= 10(3) + 5 = 30 + 5 = 35

37. (b) Let the two numbers are x and y.
According to question,
Sum of squares of two numbers = 97
$x^2 + y^2 = 97$ ... (i)
Square of their difference = 25
i.e. $(x - y)^2 = 25$ ... (ii)
$\Rightarrow (x^2 + y^2) - 2xy = 25$
$\Rightarrow 97 - 2xy = 25$ [from equation (i)]
$\Rightarrow 2xy = 72$
$\therefore xy = 36$ ... (iv)

38. (a) Given that $x + \frac{1}{x} = 2$ ... (i)
Squaring both sides, we get

$$\left(x+\frac{1}{x}\right)^2 = 4$$

$\Rightarrow$ $x^2 + \frac{1}{x^2} + 2 = 4$

$\Rightarrow$ $x^2 + \frac{1}{x^2} = 2$ ... (ii)

Now, $\left(x-\frac{1}{x}\right)^2 = \left(x^2+\frac{1}{x^2}\right) - 2$

$= 2 - 2 = 0$ [from equation (ii)]

$\therefore$ $x - \frac{1}{x} = 0$

39. (d) Given equations of system
$3x + y = 4$ ... (i)
$6x + 2y = 8$ ... (ii)
$\because$ $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2}$
Therefore, the system of equation has infinite solutions because it is a coincident line.

40. (c) Let the two numbers be x and y.
Given,
$x + y = 7$
$x^2 + y^2 = 25$
$xy = ?$
Now,
$(x + y)^2 = (x^2 + y^2) + 2xy$
$(7)^2 = 25 + 2xy$
$49 - 25 = 2xy$
$24 = 2xy$
$x = 12$

41. (d) Let the two-digit number be $10x + y$.
According to the question,
$x + y = 10$ ... (i)
and $10y + x + 36 = 10x + y$
$\Rightarrow$ $-9y + 9x = 36$
$\Rightarrow$ $x + y = 10$
$\Rightarrow$ $x - y = 4$ ... (ii)
on adding (i) and (ii), we get
$\Rightarrow$ $2x = 14 \Rightarrow x = 7$
On putting the value of x in equation (i), we get
$7 + y = 10$
$\Rightarrow$ $y = 3$
The required number is 73 which is neither divisible by a composite number nor a multiple of a prime number.

42. (a) Let the cost of one chair = ₹ x
Cost of one table = ₹ y
According to question,
$10x + 6y = 6200$ ... (i)
and $3x + 2y = 1900$ ... (ii)
$\Rightarrow$ $9x + 6y = 5700$ ... (iii)
On subtracting equation (iii) from equation (i), we get
$10x + 6y = 6200$
$\Rightarrow$ $-9x + 6y = 5700$
$\Rightarrow$ $x = 500$
$\therefore$ x = ₹ 500
From equation (i),
$5000 + 6y = 6200$
$\Rightarrow$ $6y = 1200$
$\Rightarrow$ y = ₹ 200
$\therefore$ Cost of 4 chairs and 5 tables = $4x + 5y$
$= 4 \times 500 + 5 \times 200 = 2000 + 1000 =$ ₹ 3000

43. (d) We have,
$x + y - 7 = 0$
$\Rightarrow$ $x + y = 7$ ... (i)
and $3x + y - 13 = 0$
$\Rightarrow$ $3x + y = 13$ ... (ii)
By subtracting equation (i) from equation (ii), we get

$$\begin{array}{r} 3x + y = 13 \\ \underset{-}{x} + \underset{-}{y} = \underset{-}{7} \\ \hline 2x = 6 \end{array}$$

$\therefore$ $x = 3$
On putting the value of x in equation (i), we get
$3 + y = 7$
$\Rightarrow$ $y = 4$
Now, $4x^2 + y^2 + 4xy$
$= 4 \times (3)^2 + (4)^2 + 4 \times 3 \times 4 = 4 \times 9 + 16 + 48$
$= 36 + 16 + 48 = 100$

44. (b) $\frac{x}{2} + \frac{y}{3} = 4$
$\Rightarrow$ $3x + 2y = 24$ ... (i)
and $\frac{2}{x} + \frac{3}{y} = 1$
$2y + 3x = xy$ ... (ii)
From equations (i) and (ii), we get
$xy = 24$ ... (iii)
Now, multiply by y on both the sides in equation (i),
$\Rightarrow$ $3xy + 2y^2 = 24y$
$\Rightarrow$ $y^2 - 12y + 36 = 0$
This can be written as $(y - 6)^2 = 0$
$y = 6$
Now, put y = 6 in equation (iii),
$x = 4$
Then, $x + y = 4 + 6 = 10$.

45. (a) Let the number of male employees be x and female employees be y.
Total average age of male employees = 52x
Total average age of female employees = 42y
Mean age of all employees = $\frac{52x + 42y}{x + y}$
According to the question,
$\frac{52x + 42y}{x + y} = 50$
$\Rightarrow$ $52x + 42y = 50x + 50y$
$\Rightarrow$ $2x = 8y$
$\Rightarrow$ $\frac{x}{y} = \frac{4}{1}$

i.e. x : y = 4 : 1

Percentage of male employees = $\frac{4}{5} \times 100 = 80\%$

Percentage of female employees = $\frac{1}{5} \times 100 = 20\%$

46. (c) Let present age of Ravi be x.
$\therefore$ Present age of Ravi's father = 4x
According to question,
5 years before,
Ravi's father age = 7 × Ravi's age
$4x - 5 = 7(x - 5)$
$\Rightarrow 4x - 5 = 7x - 35$
$\Rightarrow 3x = 30$
$\therefore x = 10$
$\therefore$ Ravi's present age = x = 10 years
Ravi's father's present age
= 4 × 10 = 40 years

47. (b) Let the positive number be x.
According to the question,

$$x + 10 = \frac{200}{x}$$

$\Rightarrow x^2 + 10x = 200$
$\Rightarrow x^2 + 10x - 200 = 0$
$\Rightarrow (x - 10)(x + 20) = 0$
$\therefore x = 10, -20$
But $x \neq -20$, since x is a positive number
The required number is 10.

48. (d) According to the question,
$(x + y) = 2.5(x - y)$
$\Rightarrow x + y = 2.5x - 2.5y$
$\Rightarrow 3.5y = 1.5x$

$$\Rightarrow \frac{x}{y} = \frac{7}{3} \quad \text{... (i)}$$

$$\Rightarrow x = \frac{7}{3}y$$

Now, xy = 84

$$\Rightarrow \frac{7}{3}y \times y = 84$$

$$\Rightarrow y^2 = \frac{84 \times 3}{7}$$

$\Rightarrow y^2 = 12 \times 3$

$$\therefore y = 6 \Rightarrow x = \frac{7}{3} \times 6 = 14$$

$\therefore$ Sum of numbers = x + y = 14 + 6 = 20

49. (a) Mother's age when Ravi was born
= 26 years ... (i)
Father's age when his sister was born
= 28 years ... (ii)
Sister's age when his brother was born
= 4 years ... (iii)
Ravi's brother is 3 years elder to him ... (iv)
From (i) and (iv),
Mother's age when brother was born
= 26 – 3 = 23 years
From (ii) and (iii),
Father's age when brother was born
= 28 + 4 = 32 years

50. (a) Let the cost of chair be x and the cost of table be y, then
$2x + y = 700$ ... (i)
and $x + 2y = 800$ ... (ii)
From equations (i) and (ii), we get

$$\begin{array}{r} 2x + y = 700 \\ 2x + 4y = 1600 \\ - \quad - \quad \; - \\ \hline -3y = -900 \end{array}$$

$\Rightarrow y = 300$
$\therefore x = 800 - 600 = 200$
Given the number of chairs and tables are m to be purchased for ₹ 30000
According to question,
$\therefore 200m + 300m = 30000$

$$\Rightarrow m = \frac{30000}{500} \Rightarrow m = 60$$

51. (c) Let the two digit number be $10x + y$.
According to the question, $10x + y = 3(x + y)$
$\Rightarrow 10x + y - 3x - 3y = 0$
$\Rightarrow 7x - 2y = 0$ ... (i)
According to question
$\Rightarrow (10x + y) + 45 = 10y + x$
$\Rightarrow 10x + y + 45 - 10y - x = 0$
$\Rightarrow 9x - 9y + 45 = 0$
$\Rightarrow x - y = -5$ ... (ii)
Now, solving equations (i) and (ii), we get
$x = 2$ and $y = 7$
digit = 10 × 2 + 7 = 27
$\therefore$ sum of the squares of digits = $(2)^2 + (7)^2$
= 4 + 49 = 53

52. (a) Given, $ax + by - 2 = 0$
$ax + by = 2$
By Squaring both sides.
$(ax + by)^2 = (2)^2$
$a^2x^2 + b^2y^2 + 2axby = 4$
Given $(ax\ by = 1)$
$\Rightarrow a^2x^2 + b^2y^2 + 2 = 4$
$\Rightarrow a^2x^2 + b^2y^2 = 2$
$\Rightarrow a^2x^2 = 1\ b^2y^2 = 1$
$\Rightarrow ax = 1$ and $by = 1$

$$\Rightarrow x = \frac{1}{a} \text{ and } y = \frac{1}{b}$$

$$\therefore a^2x + b^2y = a^2 \cdot \frac{1}{a} + b^2 \cdot \frac{1}{b} = a + b$$

53. (c) $7(x + 2a)^2 + 3a^2 = 5a(7x + 23a)$
$\Rightarrow 7(x^2 + 4a^2 + 4ax) + 3a^2 = 35ax + 115a^2$
$\Rightarrow x^2 - ax - 12a^2 = 0$
$\Rightarrow (x + 3a)(x - 4a) = 0$
$\Rightarrow x = -3a$ and $x = 4a$
m and n are the roots of the given equation.
Let $m = 4a$ and $n = -3a$
Now, $3m - n = 3(4a) - (-3a) = 12a + 3a = 15a$

54. (c) **Statement 1**
$1990x - 173y = 11$

$x = 0,\ y = \dfrac{-11}{173},\ x = 1,\ y = \dfrac{1990-11}{173} = 11.43$

$y = 0,\ x = \dfrac{11}{1990}\ y = 1,\ x = \dfrac{11+173}{1990} = 0.0924$

This equation has no integers solution for any value of x and y.
Statement 2
$3x - 12y = 7$

$x = 0,\ y = \dfrac{-7}{12} = -0.5833,\ x = 1,\ y = -0.333$

$y = 0,\ x = \dfrac{7}{3} = 2.33\ y = 1,\ x = 6.33.$

So statement 1 and statement 2 both are correct.

55. (c) $-2 \le x \le 1 \Rightarrow -2 \le x \le 1$ ----------(i)
$-1 \le y \le 2 \Rightarrow -2 \le -y \le 1$ ---------(ii)
$3 \le 2 \le 6 \Rightarrow 6 \le 2z \le 12$ ----------(iii)
Adding equ. (i), (ii), (iii)
$-2 - 2 + 6 \le x - y + 2z \le 1 + 1 + 12$
$2 \le x - y + 2z \le 14$
So, option (c) is correct.

56. (a) $21x + 48y = 5$ for any integer value of x, y is not a integer value and vice-versa.
So, option (a) is correct.

57. (d) Let $x = 2$ (even prime no.)
$y = 6$ (composite no.)
$\Rightarrow 4 - x = 6 - 2 = 4$, it is even integer
$\Rightarrow xy = 2 \times 6 = 12$, it is even integer
$\Rightarrow 0.5(x + y) = 0.5\,(2 + 6) = 4$, it is even integer.
So, option (d) is correct.

58. (c) Given $\dfrac{p}{x} + \dfrac{q}{y} = m$ and $\dfrac{q}{x} + \dfrac{p}{y} = n$

$\Rightarrow \dfrac{py + qx}{xy} = m$

$\Rightarrow \dfrac{qy + px}{xy} = n$

$\Rightarrow py + qx = mxy$ ....(1)
$\Rightarrow qy + px = nxy$ ...(2)
Dividing (1) by (2) we get

$$\frac{py + qx}{qy + px} = \frac{m}{n}$$

$$\Rightarrow \frac{y\left[p + q\dfrac{x}{y}\right]}{y\left[q + p\dfrac{x}{y}\right]} = \frac{m}{n}$$

$$\Rightarrow n\left[p + q\frac{x}{y}\right] = m\left[q + p\frac{x}{y}\right]$$

$$\Rightarrow np + nq\frac{x}{y} = mq + mp\frac{x}{y}$$

$$\Rightarrow np - mq = (mp - nq)\frac{x}{y}$$

$$\Rightarrow \frac{x}{y} = \frac{np - mq}{mp - nq}$$

$\therefore$ Option (c) is correct.

59. (c) Given that system of equations has no solution
$\Rightarrow |A| = 0$

$|A| = \begin{vmatrix} 3 & -k \\ 6 & -10 \end{vmatrix} \qquad B = \begin{vmatrix} 20 \\ -40 \end{vmatrix}$

$|A| = -30 + 6k$
But $|A| = 0$
$\Rightarrow 6k - 30 = 0$
$\Rightarrow k = 5$
$\therefore$ Option (c) is correct.

60. (b) Let the age of 1st brother = x years
The age of 2nd brother = y years
The age of 3rd brother = z years
According to question
$x + y = 4 \Rightarrow y = 4 - x$ ...(1)
$y + z = 6$ ...(2)
$z + x = 8 \Rightarrow z = 8 - x$ ...(3)
Substitute the values of y and z in (2) from (1) and (3) we get
$x = 3$ years
$y = 1$years
$z = 5$ years
Eldest Brother = 5 years
Youngest Brother = 1 years
Difference is their ages = 5 – 1 = 4 years
$\therefore$ Option (b) is correct.

61. (a) Given equations are
$5x - 2y = 10$
$2x + 6y = 21$
On solving, we get
$x = 3$ and $y = 5/2$
Their interaction point is (3, 5/2)
The shaded region shows the graph below the original region.
So solution set is
$5x - 2y \le 10$ and $2x + 6y \le 21$
$\therefore$ Option (a) is correct.

62. (b) Since a, b are negative numbers.
$a < 0$ and $b < 0$
C is a positive real number
$\Rightarrow c > 0$
(1) $a - b < a - c$
$\Rightarrow -b < -c$
$\Rightarrow b > c$
It is not true as $b < c$

(2) if $a < b \Rightarrow \dfrac{a}{c} < \dfrac{b}{c}$ this is true.

(3) $\dfrac{1}{b} < \dfrac{1}{c}$

Since $c > b \Rightarrow \frac{1}{c} < \frac{1}{b}$ this is not true.

$\therefore$ option (b) is correct.

63. (b) equation
$kx + 3y + 1 = 0$, and $2x + y + 3 = 0$ intersect i.e. have one solution

$$\frac{k}{2} \neq \frac{3}{1} \left(\frac{a_1}{a_2} \neq \frac{b_1}{b_2}\right)$$

$k \neq 6$

64. (c) $2x + 4y = 6$
$4x + 8y = 8$

$$\frac{2}{4} = \frac{4}{8} \neq \frac{6}{8}$$

$$\frac{1}{2} = \frac{1}{2} \neq \frac{3}{4}$$

So the form is $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\therefore$ the system of equation is inconsistant.

65. (d) Given that,

$3(2u + v) = 7uv$

$3(u + 3v) = 11uv$

$6u + 3v = 7uv$ ...(1)

$3u + 9v = 11uv$ ...(2)

Dividing equation (1) and equation (2) by uv, we get

$$\frac{6}{v} + \frac{3}{u} = 7$$

$$\frac{3}{v} + \frac{9}{u} = 11$$

Let $\frac{1}{u} = x$ and $\frac{1}{v} = y$

Multiplying equation (4) by 2, we get

$3x + 6y = 7$

$18x + 6y = 22$

Solving these equations by elimination method, we get

$-15x = -15$

$x = 1$

Therefore, equation (3) gives $y = \frac{4}{6} = \frac{2}{3}$

Thus, $x = 1 \Rightarrow \frac{1}{u} = 1 \Rightarrow u = 1$

66. (c) $a + b = 2c$

$\Rightarrow a + b = c + c$

$\Rightarrow a - c = c - b$ $\Rightarrow b - c = -(a - c)$

$$\frac{a}{a-c} + \frac{c}{b-c} = \frac{a}{a-c} + \frac{c}{-(a-c)}$$

$$= \frac{a}{a-c} - \frac{c}{a-c} = \frac{a-c}{a-c} = 1$$

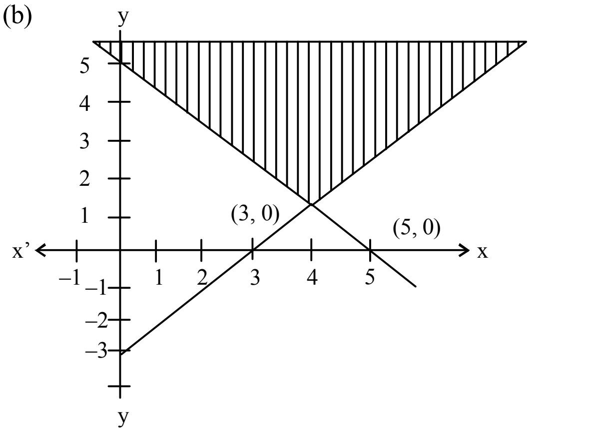
67. (a) $65x - 33y = 97$ .... (i)
(given)
$33x - 65y = 1$ ....(ii)
(given)
From eq (i) + eq (ii)
$98x - 98y = 98$
$98(x - y) = 98 \ \therefore x - y = 1$ ....(iii)
Now
From eq (i) $\times$ 1 – eq (iii) $\times$ 33

$$\begin{array}{r} 65x - 33y = 97 \\ -33x + 33y = -33 \\ \hline 32x = 64 \end{array} \qquad \therefore x = \frac{64}{32} = 2$$

From eq (iii)
$x - y = 1$
$2 - y = 1$ $\therefore y = 2 - 1 = 1$
Hence, $xy = 2 \times 1 = 2$

68. (d) $x + y + z = 0$
$(y + z - x)^3 + (z + x - y)^3 + (x + y - z)^3$
Here
$y + z - x + z + x - y + x + y - z = x + y + z = 0$
If $a + b + c = 0$ then value of $a^3 + b^3 + c^3 = 3abc$
$\therefore$ $(y + z - x)^3 + (z + x - y)^3 + (x + y - z)^3 = 3\,(y + z - x)$
$(z + x - y)\,(x + y - z)$
According to the question
$x + y = -z$
$y + z = -x$
$x + z = -y$
$\therefore 3\,(-x - x)\,(-y - y)\,(-z - z) = 3 \times (-2x) \times (-2y) \times (-2z)$
$= -24xyz$

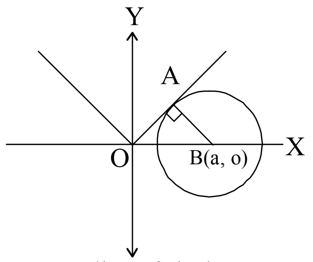
69. (b)



$x + y \geq 5$
let first draw graph of $x + y = 5$ ... (i)
put $y = 0$ in (i)
$x = 5$
put $x = 0$ in (i)
$y = 5$
Checking for (0, 0)
$x + y \geq 5$
$0 \geq 5$, which is false
Hence, origin does not lie in $x + y \geq 5$
Now,
$x - y \leq 3$
let first drawn graph of $x - y = 3$ ...(ii)
put $x = 0$ in (ii)
$y = -3$

put y = 0 in (ii)
x = 3'
Checking for (0, 0)
$x - y \le 3$
$0 \le 3$, which is true
Hence, origin lies in $x - y \le 3$
So, shaded region will be the solution of these linear inequalities lies in the first and second quadrant.

70. (a) $x^2 - y^2 = 0$
(Equation of $y = |x|$)
$(x - a)^2 + y^2 = 1$
[Equation of a circle with centre (a, 0) and radius 1]
These have single positive solution
$\Rightarrow$ $y = |x|$ to be tangent to the circle



AB = radius of circle
$\angle AOB = 45°$
$\angle OAB = 90°$
[tangent to the circle]
$\angle ABO = 45°$

Length of OB = $\sqrt{AB^2 + OA^2} = \sqrt{1^2 + 1^2}$

OB = $\sqrt{2}$

71. (b) $3^{x+2} + 3^{-x} = 10$
$3^2 + 3^0 = 10$
$x + 2 = 0$
$x = -2$ solution is consistent
Or $x + 2 = 2$
$x = 0$ solution is consistent
Thus $x = 0, -2$ are the solutions
Alternatively, we can put values from the options and check

72. (c) Let his wife get a share of ₹ x
Each of the 4 daughters get = ₹ $2x$
Each of the 5 sons get share = ₹ $6x$
So $x + 4 \times 2x + 5 \times 6x = 390000$
So $39x = 390000$
$x = 10000$
$\therefore$ wifes share = ₹10000

73. (a) $y = x^3 \Rightarrow x = \sqrt[3]{y}$

$\Rightarrow \sqrt[3]{y} = \dfrac{\sqrt{3}+1}{2}$

$\Rightarrow 2\sqrt[3]{y} = \sqrt{3}+1$

$\Rightarrow 8y = 3\sqrt{3} + 9 + 3\sqrt{3} + 1$ (cutting)
$\Rightarrow (4y - 5)^2 = 27$
$\Rightarrow 16y^2 - 40y + 25 = 27$
$\Rightarrow 16y^2 - 40y - 2 = 0$
$\Rightarrow 8y^2 - 20y - 1 = 0$

74. (b) $10x + 4y = 14xy$
$\underline{-10x - 3y = -8xy}$
On solving two equation
$y = 6xy \Rightarrow 6x = 1, x = 1/6$
$y = -5/6/5/6 = -1$

75. (d) Let number of student = R and number of Row = C
RC = (R + 2) (C – 3) = (R + 4) (C – 5)
Now, RC = (R + 2) (C – 3)
RC = RC + 2C – 3R – 6
2C – 3R = 6 ... (i)
Also, RC = (R + 4)(C – 5)
RC = RC + 4C – 5R – 20
4C – 5R = 20 ...(ii)
Now On applying (i) × 2 – (ii) solving equation
R = 8, C = 15
$\therefore$ Number of student = R × C = 15 × 8 = 120

76. (c) $\dfrac{170-70}{250/50} = \dfrac{100}{5} = 20$

77. (b) $5x + 9y = 7 \Rightarrow y = \dfrac{7-5x}{9}$

$\therefore$ 7 – 5x is divisible by 9
So x = 5, 14, ...491
or x = 5, –4, –11, ... –499
So total possible values of x

$= \dfrac{491+499}{9} + 1 = 111$

CHAPTER

15 Quadratic Equations

1. If $x^2 - kx - 21 = 0$ and $x^2 - 3kx + 35 = 0$ have one common root, then what is the value of k? **[2007-I]**
(a) +4 (b) – 4 (c) ± 4 (d) ± 1

2. What are the roots of the equation, $(a + b + x)^{-1} = a^{-1} + b^{-1} + x^{-1}$? **[2007-II]**
(a) a, b (b) –a, b (c) a, –b (d) –a, –b

3. What is one of the value of x in the equation $\sqrt{\frac{x}{1-x}} + \sqrt{\frac{1-x}{x}} = \frac{13}{6}$? **[2007-II]**
(a) $\frac{5}{13}$ (b) $\frac{7}{13}$ (c) $\frac{9}{13}$ (d) $\frac{11}{3}$

4. If the equations,
$2x^2 - 7x + 3 = 0$ and $4x^2 + ax - 3 = 0$
have a common root, then what is the value of a? **[2007-II]**
(a) –11 or 4 (b) –11 or –4
(c) 11 or –4 (d) 11 or 4

5. If one root of $px^2 + qx + r = 0$ is double of the other root, then which one of the following is correct? **[2007-II]**
(a) $2q^2 = 9pr$ (b) $2q^2 = 9p$
(c) $4q^2 = 9r$ (d) $9q^2 = 2pr$

6. If $(2x - 3y < 7)$ and $(x + 6y < 11)$, then which one of the following is correct? **[2008-I]**
(a) $x + y < 5$ (b) $x + y < 6$
(c) $x + y \le 5$ (d) $x + y \le 6$

7. What is one of the roots of the equation $\sqrt{\frac{2x}{3-x}} - \sqrt{\frac{3-x}{2x}} = \frac{3}{2}$ **[2008-I]**
(a) 1 (b) 2 (c) 3 (d) 4

8. If α and β are the roots of the equation $(x^2 - 3x + 2 = 0)$, then which equation has the roots $(\alpha + 1)$ and $(\beta + 1)$? **[2008-I]**
(a) $x^2 + 5x + 6 = 0$ (b) $x^2 - 5x - 6 = 0$
(c) $x^2 + 5x - 6 = 0$ (d) $x^2 - 5x + 6 = 0$

9. If α and β are the roots of the equation $(ax^2 + bx + c = 0)$, then what is the value of $\alpha^3 + \beta^3$? **[2008-I]**
(a) $\frac{b^3 + 3abc}{a^3}$ (b) $\frac{a^3 - b^3}{3ab}$
(c) $\frac{3abc - b^3}{a^3}$ (d) $\frac{b^3 - 3abc}{a^3}$

10. For what value of k, does the equation $[kx^2 + (2k + 6)x + 16 = 0]$ have equal roots? **[2008-I]**
(a) 1 and 9 (b) –9 and 1
(c) –1 and 9 (d) –1 and –9

11. If the product of the roots of $x^2 - 3kx + 2k^2 - 1 = 0$ is 7 for a fixed k, then what is the nature of roots? **[2008-I]**
(a) Integral and positive
(b) Integral and negative
(c) Irrational
(d) Rational but not integral

12. Which one of the following is the quadratic equation whose roots are reciprocal to the roots of the quadratic equation $2x^2 - 3x - 4 = 0$? **[2008-II]**
(a) $3x^2 - 2x - 4 = 0$ (b) $4x^2 + 3x - 2 = 0$
(c) $3x^2 - 4x - 2 = 0$ (d) $4x^2 - 2x - 3 = 0$

13. The value of y which will satisfy the equations $2x^2 + 6x + 5y + 1$ and $2x + y + 3 = 0$ may be found by solving which one of the following equations? **[2008-II]**
(a) $y^2 + 14y - 7 = 0$ (b) $y^2 + 8y + 1 = 0$
(c) $y^2 + 10y - 7 = 0$ (d) $y^2 - 8y + 7 = 0$

14. If a polynomial equation has rational co-efficients and has exactly three real roots, then what is the degree of the polynomial? **[2008-II]**
(a) Equal to 3
(b) Greater than or equal to 3
(c) Strictly greater than 3
(d) Less than 3

15. If α and β are the roots of $ax^2 + bx + c = 0$, then what is the value of $\left(\frac{1}{\alpha^2} - \frac{1}{\beta^2}\right)^2$? **[2008-II]**
(a) $\frac{b^2\left(b^2 - 4ac\right)}{c^4}$ (b) $\frac{b\left(b^2 - 4ac\right)}{c^2}$
(c) $\frac{\left(b^2 - 4ac\right)}{c^2}$ (d) $\frac{\left(b^2 - 4ac\right)}{c^4}$

16. Which one of the following is one of the two consecutive positive integers, the sum of whose squares is 761? **[2008-II]**
(a) 15 (b) 20 (c) 24 (d) 25

17. If $3^x + 27(3^{-x}) = 12$, then what is the value of x? **[2009-I]**
(a) Only 1 (b) Only 2
(c) 1 or 2 (d) 0 or 1

18. What is the magnitude of difference of the roots of $x^2 - ab + b = 0$? **[2009-I]**

(a) $\sqrt{a^2 - 4b}$ (b) $\sqrt{b^2 - 4a}$

(c) $2\sqrt{a^2 - 4b}$ (d) $\sqrt{b^2 - 4ab}$

19. If $a + b = 2m^2$, $b + c = 6m$, $a + c = 2$, where m is a real number and $a \le b \le c$, then which one of the following is correct? **[2009-II]**

(a) $0 \le m \le \frac{1}{2}$ (b) $-1 \le m \le 0$

(c) $\frac{1}{3} \le m \le 1$ (d) $1 < m \le 2$

20. Students of a class are made to stand in rows. If one student is extra in a row, there would be two rows less. If one student is less in a row, there would be three rows more. Then, what is the number of students in the class? **[2009-II]**

(a) 65 (b) 55 (c) 60 (d) 50

21. What is the ratio of sum of squares of roots to the product of the roots of the equation $7x^2 + 12x + 18 = 0$? **[2009-II]**

(a) 6 : 1 (b) 1 : 6 (c) – 6 : 1 (d) –6 : 7

22. If the roots of the equation $\frac{x(x-1)-(m+1)}{(x-1)(m-1)} = \frac{x}{m}$ are equal, then what is the value of m? **[2009-II]**

(a) 1 (b) $\frac{1}{2}$ (c) 0 (d) $-\frac{1}{2}$

23. What is the least integral value of k for which equation $x^2 - 2(k-1)x + (2k+1) = 0$ has both the roots positive? **[2010-I]**

(a) 1 (b) $-\frac{1}{2}$ (c) 4 (d) 0

24. If one of the roots of the equation $ax^2 + x - 3 = 0$ is –1.5, then what is the value of a? **[2010-I]**

(a) 4 (b) 3 (c) 2 (d) –2

25. r is a non-zero real number such that $r^{75} > r^{90}$. This is possible only when **[2010-I]**

(a) $-1 < r < 0$ (b) $0 < r < 1$

(c) $1 < r$ (d) $-1 < r < 1$

26. When the roots of the quadratic equation $ax^2 + bx + c = 0$ are negative of reciprocals of each other, then which one of the following is correct? **[2010-I]**

(a) b = 0 (b) c = 0 (c) a = c (d) a = –c

27. What are the roots of the equation $\log_{10}(x^2 - 6x + 45) = 2$? **[2010-I]**

(a) 9, –5 (b) –9, 5 (c) 11, –5 (d) –11, 5

28. The sum of the roots of the equation $\frac{1}{x+a} + \frac{1}{x+b} = \frac{1}{c}$ is zero. What is the product of the roots of the equation? **[2010-I]**

(a) $-\frac{(a+b)}{2}$ (b) $\frac{(a+b)}{2}$

(c) $-\frac{(a^2+b^2)}{2}$ (d) $\frac{(a^2+b^2)}{2}$

29. For what value of k, will the roots of the equation $kx^2 - 5x + 6 = 0$ be in the ratio of 2 : 3? **[2010-I]**

(a) 0 (b) 1 (c) –1 (d) 2

30. If α and β are the roots of the equation $x^2 + px + q = 0$, then $-\alpha^{-1}$, $-\beta^{-1}$ are the roots of which one of the following equations? **[2010-II]**

(a) $qx^2 - px + 1 = 0$ (b) $q^2 + px + 1 = 0$

(c) $x^2 + px - q = 0$ (d) $x^2 - px + q = 0$

31. If one root of the equation $ax^2 + x - 3 = 0$ is –1, then what is the other root? **[2010-II]**

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) 1

32. If the equation

$$(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$$

has equal roots, then which one of the following is correct? **[2010-II]**

(a) ab = cd (b) ad = bc

(c) $a^2 + c^2 = b^2 + d^2$ (d) ac = bd

33. What is the solution of the equation

$$\sqrt{\frac{x}{x+3}} - \sqrt{\frac{x+3}{x}} = -\frac{3}{2}?$$

[2010-II]

(a) 1 (b) 2

(c) 4 (d) None of these

34. What are the roots of the equation $4^x - 3.2^{x+2} + 32 = 0$? **[2010-II]**

(a) 1, 2 (b) 3, 4 (c) 2, 3 (d) 1, 3

35. If α and β are the roots of the equation $x^2 - x - 1 = 0$, then what is the value of $(\alpha^4 + \beta^4)$? **[2010-II]**

(a) 7 (b) 0

(c) 2 (d) None of these

36. If sum as well as product of roots of a quadratic equation is 9, then what is the equation? **[2010-II]**

(a) $x^2 + 9x - 18 = 0$ (b) $x^2 - 18x + 9 = 0$

(c) $x^2 + 9x + 9 = 0$ (d) $x^2 - 9x + 9 = 0$

37. If one root of the equation $2x^2 + 3x + c = 0$ is 0.5, then what is the value of c? **[2011-I]**

(a) –1 (b) –2 (c) –3 (d) –4

38. What is the condition that the quation $ax^2 + bx + c = 0$, where $a \neq 0$ has both the roots positive? **[2011-I]**

(a) a, b and c are of same sign.

(b) a and b are of same sign.

(c) b and c have the same sign opposite to that of a.

(d) a and c have the same sign opposite to that of b.

39. The equation $(1 + n^2)x^2 + 2ncx + (c^2 - a^2) = 0$ will have equal roots, if **[2011-I]**

(a) $c^2 = 1 + a^2$ (b) $c^2 = 1 - a^2$

(c) $c^2 = 1 + n^2 + a^2$ (d) $c^2 = (1 + n^2)a^2$

40. The equation whose roots are twice the roots of the equation $x^2 - 2x + 4 = 0$ is **[2011-I]**
(a) $x^2 - 2x + 4 = 0$ (b) $x^2 - 2x + 16 = 0$
(c) $x^2 - 4x + 8 = 0$ (d) $x^2 - 4x + 16 = 0$

41. If $\sin\theta$ and $\cos\theta$ are the roots of the equation $ax^2 - bx + c = 0$, then which one of the following is correct? **[2011-II]**
(a) $a^2 + b^2 + 2ac = 0$ (b) $a^2 - b^2 + 2ac = 0$
(c) $a^2 + c^2 + 2ab = 0$ (d) $a^2 - b^2 - 2ac = 0$

42. If α and β are the roots of the equation $x^2 - 6x + 6 = 0$, what is $\alpha^3 + \beta^3 + \alpha^2 + \beta^2 + \alpha + \beta$ equal to? **[2011-II]**
(a) 150 (b) 138 (c) 138 (d) 124

43. If $6 \le x \le 8$, then which one of the following is correct? **[2011-II]**
(a) $(x - 6)(x - 8) \ge 0$ (b) $(x - 6)(x - 8) > 0$
(c) $(x - 6)(x - 8) \le 0$ (d) $(x - 6)(x - 8) < 0$

44. What are the roots of the quadratic equation $a^2b^2x^2 - (a^2 + b^2)x + 1 = 0$? **[2011-II]**
(a) $\frac{1}{a^2}, \frac{1}{b^2}$ (b) $-\frac{1}{a^2}, -\frac{1}{b^2}$
(c) $\frac{1}{a^2}, -\frac{1}{b^2}$ (d) $-\frac{1}{a^2}, \frac{1}{b^2}$

45. If the roots of the equation
$x^2 - 2ax + a^2 + a - 3 = 0$
are real and less than 3, then which one of the following is correct? **[2012-I]**
(a) $a < 2$ (b) $2 < a < 3$ (c) $3 < a < 4$ (d) $a > 4$

46. Two students *A* and *B* solve an equation of the form $x^2 + px + q = 0$. *A* starts with a wrong value of p and obtains the roots as 2 and 6. *B* starts with a wrong value of q and gets the roots as 2 and –9. What are the correct roots of the equation? **[2012-I]**
(a) 3 and – 4 (b) – 3 and –4
(c) –3 and 4 (d) 3 and 4

47. If one of the roots of quadratic equation $7x^2 - 50x + k = 0$ is 7, then what is the value of k? **[2012-I]**
(a) 7 (b) 1 (c) $\frac{50}{7}$ (d) $\frac{7}{50}$

48. The quadratic equation whose roots are 3 and –1, is **[2012-II]**
(a) $x^2 - 4x + 3 = 0$ (b) $x^2 - 2x - 3 = 0$
(c) $x^2 + 2x - 3 = 0$ (d) $x^2 + 4x + 3 = 0$

49. If one root of the equation $\frac{x^2}{a} + \frac{x}{b} + \frac{1}{c} = 0$ is reciprocal of the other, then which one of the following is correct? **[2012-II]**
(a) a = b (b) b = c (c) ac = 1 (d) a = c

50. If f(x) is a polynomial with all coefficients are integers and constant term 10 having a factor (x – k), where k is an integer, then what is the possible value of k? **[2012-II]**
(a) –20 (b) 20 (c) 8 (d) 5

51. The difference in the roots of the equation $2x^2 - 11x + 5 = 0$ is **[2013-I]**
(a) 4.5 (b) 4 (c) 3.5 (d) 3

52. If one of the roots of the equation $x^2 - bx + c = 0$ is the square of the other, then which of the following option is correct? **[2013-I]**
(a) $b^3 = 3bc + c^2 + c$ (b) $c^3 = 3bc + b^2 + b$
(c) $3bc = c^3 + b^2 + b$ (d) $3bc = c^3 + b^3 + b^2$

53. Consider the following statements in respect of the quadratic equation $ax^2 + bx + b = 0$, where $a \neq 0$.
I. The product of the roots is equal to the sum of the roots.
II. The roots of the equation are always unequal and real.
Which of the statements given above is/are correct? **[2013-II]**
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

54. If $x^2 = 6 + \sqrt{6 + \sqrt{6 + \sqrt{6 + \ldots \infty}}}$, then what is one of the values of x equal to? **[2013-II]**
(a) 6 (b) 5 (c) 4 (d) 3

55. If α and β are the roots of the equation $x^2 - x - 1 = 0$, then what is $\frac{\alpha^2 + \beta^2}{(\alpha^2 - \beta^2)(\alpha - \beta)}$ equal to? **[2013-II]**
(a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) $\frac{4}{5}$ (d) 4

56. Which one is one of the factors of
$x^2 + \frac{1}{x^2} + 8\left(x + \frac{1}{x}\right) + 14$? **[2013-II]**
(a) $x + \frac{1}{x} + 1$ (b) $x + \frac{1}{x} + 3$
(c) $x + \frac{1}{x} + 6$ (d) $x + \frac{1}{x} + 7$

57. In solving a problem, one student makes a mistake in the coefficient of the first degree term and obtains –9 and –1 for the roots. Another student makes a mistake in the constant term of the equation and obtains 8 and 2 for the roots. The correct equation was **[2014-I]**
(a) $x^2 + 10x + 9 = 0$ (b) $x^2 - 10x + 16 = 0$
(c) $x^2 - 10x + 9 = 0$ (d) None of these

58. If m and n are the roots of the equation $ax^2 + bx + c = 0$, then the equation whose roots are $\frac{(m^2 + 1)}{m}$ and $\frac{(n^2 + 1)}{n}$ is **[2014-I]**
(a) $acx^2 + (ab + bc)x + b^2 + (a - c)^2 = 0$
(b) $acx^2 + (ab - bc)x + b^2 + (a - c)^2 = 0$
(c) $acx^2 - (ab - bc)x + b^2 - (a - c)^2 = 0$
(d) $acx^2 - (ab + bc)x + b^2 - (a - c)^2 = 0$

59. The value of $x^2 - 4x + 11$ can never be less than **[2014-I]**

(a) 7 (b) 8 (c) 11 (d) 22

60. The expression $2x^3 + x^2 - 2x - 1$ is divisible by **[2014-I]**

(a) $x + 2$ (b) $2x + 1$ (c) $x - 2$ (d) $2x - 1$

61. If $x + y = 5$, $y + z = 10$ and $z + x = 15$, then which one of the following is correct? **[2014-I]**

(a) $z > x > y$ (b) $z > y > x$
(c) $x > y > z$ (d) $x > z > y$

62. If the roots of the equation
$(a^2 - bc)x^2 + 2(b^2 - ac)x + (c^2 - ab) = 0$
are equal, where $b \neq 0$, then which one of the following is correct? **[2014-I]**

(a) $a + b + c = abc$ (b) $a^2 + b^2 + c^2 = 0$
(c) $a^3 + b^3 + c^3 = 0$ (d) $a^3 + b^3 + c^3 = 3abc$

63. If the roots of the equation $Ax^2 + Bx + C = 0$ are -1 and 1, then which one of the following is correct? **[2014-I]**

(a) A and C are both zero
(b) A and B are both positive
(c) A and C are both negative
(d) A and C are of opposite sign

64. If one the roots of the equation $px^2 + qx + r = 0$ is three times the other, then which one of the following relations is correct ? **[2014-II]**

(a) $3q^2 = 16\,pr$ (b) $q^2 = 24\,pr$
(c) $p = q + r$ (d) $p + q + r = 1$

65. If m% of $m + n$% of n = 2% of $(m \times n)$, then what percentage of m is n? **[2014-II]**

(a) 50% (b) 75%
(c) 100% (d) Cannot be determined

66. If m and n are the roots of the equation $x^2 + ax + b = 0$ and m^2, n^2 are the roots of the equation $x^2 - cx + d = 0$, then which of the following is / are correct ?

1. $2b - a^2 = c$ 2. $b^2 = d$

Select the correct answer using the codes given below:

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2 **[2014-II]**

67. If $a_n = 3 - 4n$, then what is $a_1 + a_2 + a_3 + ... + a_n$ equal to ? **[2014-II]**

(a) $-n(4n - 3)$ (b) $-n(2n - 1)$
(c) $-n^2$ (d) $-n(2n + 1)$

68. For which value of k does the pair of equations $x^2 - y^2 = 0$ and $(x - k)^2 + y^2 = 1$ yield a unique positive solution of x? **[2015-I]**

(a) 2 (b) 0
(c) $\sqrt{2}$ (d) $-\sqrt{2}$

69. If the roots of the quadratic equation $x^2 - 4x - \log_{10} N = 0$ are all real, then the minimum value of N is **[2015-II]**

(a) $\frac{1}{100}$ (b) $\frac{1}{1000}$
(c) $\frac{1}{10000}$ (d) 10000

70. The difference of maximum values of the expressions $(6 + 5x - x^2)$ and $(y - 6 - y^2)$ for any real values of x and y is **[2015-II]**

(a) 16 (b) 17 (c) 18 (d) 19

71. If the roots of the equation $lx^2 + mx + m = 0$ are in the ratio $p : q$, then **[2016-I]**

$$\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{m}{l}} \text{ is equal to}$$

(a) 0 (b) 1
(c) 2 (d) 3

72. If $\sqrt{3x^2 - 7x - 30} - \sqrt{2x^2 - 7x - 5} = x - 5$ **[2016-I]**
has α and β as its roots, then the value of $\alpha\beta$ is

(a) -15 (b) -5
(c) 0 (d) 5

73. If $a^2 - by - cz = 0$, $ax - b^2 + cz = 0$ and $ax + by - c^2 = 0$, then the value of $\frac{x}{a+x} + \frac{y}{b+y} + \frac{z}{c+z}$ will be **[2016-I]**

(a) $a + b + c$ (b) 3
(c) 1 (d) 0

74. Let p and q be non-zero integers. Consider the polynomial $A(x) = x^2 + px + q$
It is given that $(x - m)$ and $(x - km)$ are aimple factora of $A(x)$, where m is a non-zero integer and k is a positive integer, $k \geq 2$. Which one of the following is correct? **[2016-I]**

(a) $(k + 1)^2p^2 = kq$ (b) $(k + 1)^2q = kp^2$
(c) $k^2q = (k + 1)p^2$ (d) $k^2p^2 = (k + 1)^2q$

75. If the linear factors of $ax^2 - (a^2 + 1)x + a$ are p and q then $p + q$ is equal to **[2016-I]**

(a) $(x - 1)(a + 1)$ (b) $(x + 1)(a + 1)$
(c) $(x - 1)(a - 1)$ (d) $(x + 1)(a - 1)$

76. If the sum of the roots of $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocais, then which one of the following relations is correct? **[2016-I]**

(a) $ab^2 + bc^2 = 2a^2c$ (b) $ac^2 + bc^2 = 2b^2a$
(c) $ab^2 + bc^2 = a^2c$ (d) $a^2 + b^2 + c^2 = 1$

77. Under what condition on p and q, one of the roots of the equation $x^2 + px + q = 0$ is the square of the other? **[2016-I]**

(a) $1 + q + q^2 = 3pq$ (b) $1 + p + p^2 = 3pq$
(c) $p^3 + q + q^2 = 3pq$ (d) $q^3 + p + p^2 = 3pq$

78. The solution of the inequation **[2016-I]**

$$1 + \frac{1}{x} - \frac{1}{x^2} \geq 0$$

is (given that $x \neq 0$)

(a) $x > 0$
(b) $x < 0$
(c) $\frac{-1-\sqrt{5}}{2} \leq x \leq \frac{-1+\sqrt{5}}{2}$
(d) $x \leq \frac{-1-\sqrt{5}}{2}$ or $x \geq \frac{-1+\sqrt{5}}{2}$

79. If λ it is an integer and α, β are the roots of $4x^2 - 16x + \frac{\lambda}{4} = 0$ such that $1 < \alpha < 2$ and $2 < \beta < 3$, then how many values can λ take ? **[2016-II]**

(a) 3 (b) 9 (c) 14 (d) 15

80. If α and β are the two zeros of the polynomial $25x^2 - 15x + 2$, then what is a quadratic polynomial whose zeros are $(2\alpha)^{-1}$ and $(2\beta)^{-1}$? **[2016-II]**

(a) $x^2 + 30x + 2$ (b) $8x^2 - 30x + 25$

(c) $8x^2 - 30x$ (d) $x^2 + 30x$

81. If p and q are the roots of $x^2 + px + q = 0$, then which one of the following is correct ? **[2016-II]**

(a) $p = 0$ or 1 (b) $p = 1$ only

(c) $p = -2$ or 0 (d) $p = -2$ only

82. If α and β are the roots of the quadratic equation $2x^2 + 6x + k = 0$, Where $k < 0$, then what is the maximum value of $\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)$? **[2017-I]**

(a) 2 (b) – 2 (c) 9 (d) – 9

83. If one root of $(a^2 - 5a + 3)x^2 + (3a - 1)x + 2 = 0$ is twice the other, then what is the value of 'a'? **[2017-I]**

(a) $\frac{2}{3}$ (b) $-\frac{2}{3}$ (c) $\frac{1}{3}$ (d) $-\frac{1}{3}$

84. If $x = \frac{\sqrt{a+b} - \sqrt{a-b}}{\sqrt{a+b} + \sqrt{a-b}}$, then what is $bx^2 - 2ax + b$ equal to $(b \neq 0)$? **[2017-I]**

(a) 0 (b) 1 (c) ab (d) 2ab

85. If α and β are the roots of the equation $x^2 + px + q = 0$, then what is $\alpha^2 + \beta^2$ equal to ? **[2017-I]**

(a) $p^2 - 2q$ (b) $q^2 - 2p$

(c) $p^2 + 2q$ (d) $q^2 - q$

86. Aman and Alok attempted to solve a quadratic equation. Aman made a mistake in writing down the constant term and ended up in roots (4, 3). Alok made a mistake in writing down the coefficient of x to get roots (3, 2). The correct roots of the equation are **[2017-I]**

(a) – 4, – 3 (b) 6, 1 (c) 4, 3 (d) – 6, – 1

87. What is the positive value of m for which the roots of the equation $12x^2 + mx + 5 = 0$ are in the ratio 3 : 2? **[2017-II]**

(a) $5\sqrt{10}$ (b) $\frac{5\sqrt{10}}{12}$ (c) $\frac{5}{12}$ (d) $\frac{12}{5}$

88. If the roots of the equation $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$ are equal, then which one of the following is correct? **[2017-II]**

(a) $2b = a + c$ (b) $b^2 = ac$

(c) $\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$ (d) $\frac{1}{b} = \frac{1}{a} + \frac{1}{c}$

89. If k is an integer, then $x^2 + 7x - 14\left(k^2 - \frac{7}{8}\right) = 0$ has **[2017-II]**

(a) Both integral roots

(b) At least one integral root

(c) No integral root

(d) Both positive integral roots

90. If the roots of the equation $px^2 + x + r = 0$ are reciprocal to each other, then which one of the following is correct? **[2018-I]**

(a) $p = 2r$ (b) $p = r$

(c) $2p = r$ (d) $p = 4r$

91. If α and β are the roots of the equation $ax^2 + bx + c = 0$, then what is the value of the expression $(\alpha + 1)(\beta + 1)$? **[2018-I]**

(a) $\frac{a+b+c}{a}$ (b) $\frac{b+c-a}{a}$

(c) $\frac{a-b+c}{a}$ (d) $\frac{a+b-c}{a}$

92. A quadratic polynomial $ax^2 + bx + c$ is such that when it is divided by x, (x – 1) and (x + 1), the remainders are 3, 6 and 4 respectively. What is the value of (a + b)? **[2018-I]**

(a) 3 (b) 2 (c) 1 (d) –1

93. If $x^2 - 6x - 27 > 0$, then which one of the following is correct? **[2018-II]**

(a) $-3 < x < 9$ (b) $x < 9$ or $x > -3$

(c) $x > 9$ or $x < -3$ (d) $x < -3$ only

94. If α and β are the roots of the equation $ax^2 + bx + c = 0$, then the value of $\frac{1}{a\alpha + b} + \frac{1}{a\beta + b}$ is **[2018-II]**

(a) $\frac{a}{bc}$ (b) $\frac{b}{ac}$ (c) $\frac{c}{ab}$ (d) $\frac{1}{abc}$

95. The minimum value of the expression $2x^2 + 5x + 5$ is

(a) 5 (b) 15/8 (c) –15/8 (d) 0

96. If α, β and γ are the zeroes of the polynomial $f(x) = ax^3 + + bx^2 + cx + d$, then $\alpha^2 + \beta^2 + \gamma^2$ is equal to **[2018-II]**

(a) $\frac{b^2 - ac}{a^2}$ (b) $\frac{b^2 - 2ac}{a}$

(c) $\frac{b^2 + 2ac}{b^2}$ (d) $\frac{b^2 - 2ac}{a^2}$

97. If p and q are the roots of the equation $x^2 - 15x + r = 0$ and $p - q = 1$, then what is the value of r ? **[2019-I]**

(a) 55 (b) 56 (c) 60 (d) 64

98. For the inequation $x^2 - 7x + 12 > 0$, which one of the following is correct ? **[2019-I]**

(a) $3 < x < 4$ (b) $-\infty < x < 3$ only

(c) $4 < x < \infty$ only (d) $-\infty < x < 3$ or $4 < x < \infty$

99. The equation $x^2 + px + q = 0$ has roots equal to p and q where $q \neq 0$. What are the values of p and q respectively? **[2019-II]**

(a) 1, –2 (b) 1, 2 (c) –1, 2 (d) –1, –2

100. If $(b-6)$ is one of the quadratic equation $x^2 - 6x + b = 0$, where b is an integer, then what is the maximum value of b^2? **[2019-II]**
(a) 36 (b) 49 (c) 64 (d) 81

101. Two numbers p and q are such that the quadratic equation $px^2 + 3x + 2q = 0$ has -6 as the sum and the product of the roots. What is the value of $(p-q)$? **[2019-II]**
(a) -1 (b) 1 (c) 2 (d) 3

102. If the equations $x^2 + 5x + 6 = 0$ and $x^2 + kx + 1 = 0$ have a common root, then what is the value of k? **[2019-II]**
(a) $-\frac{5}{2}$ or $-\frac{10}{3}$ (b) $\frac{5}{2}$ or $\frac{10}{3}$
(c) $\frac{5}{2}$ or $-\frac{10}{3}$ (d) $-\frac{5}{2}$ or $\frac{10}{3}$

103. If α and β are the roots of the quadratic equation $x^2 + kx - 15 = 0$ such that $\alpha - \beta = 8$, then what is the positive value of k? **[2020-I]**
(a) 2 (b) 3 (c) 4 (d) 5

HINTS & SOLUTIONS

1. (c) Let the common root be α, then
$\alpha^2 - k\alpha - 21 = 0$ ... (i)
and $\alpha^2 - 3k\alpha + 35 = 0$ ... (ii)
Solving by the rules of cross multiplication

$$\frac{\alpha^2}{-35-63k} = \frac{\alpha}{-21-35} = \frac{1}{-3k+k}$$

$$\Rightarrow \frac{\alpha^2}{-98k} = \frac{\alpha}{-56} = \frac{1}{-2k}$$

$$\Rightarrow \frac{\alpha^2}{-98k} = \frac{-1}{2k}$$

$$\Rightarrow \alpha^2 = 49 \text{ and } \alpha = \frac{-56}{-2k} = \frac{28}{k}$$

Then, $\left(\frac{28}{k}\right)^2 = 49$

$$\Rightarrow \frac{28 \times 28}{49} = k^2$$

$\Rightarrow 16 = k^2$
$\therefore k = \pm 4$.

2. (d) Given, $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$

$$\therefore \frac{1}{a+b+x} - \frac{1}{x} = \frac{1}{a} + \frac{1}{b} \Rightarrow \frac{-(a+b)}{(a+b+x)x} = \frac{(a+b)}{ab}$$

$\Rightarrow x^2 + (a+b)x + ab = 0 \Rightarrow (x+a)(x+b) = 0$
Hence, $x = -a, -b$.

3. (c) Let $\sqrt{\frac{x}{1-x}} = y$

$$\therefore y + \frac{1}{y} = \frac{13}{6} \Rightarrow (y^2+1)\,6 = 13y$$

$\Rightarrow 6y^2 - 13y + 6 = 0 \Rightarrow 6y^2 - 9y - 4y + 6 = 0$
$\Rightarrow 3y(2y-3) - 2(2y-3) = 0$
$\Rightarrow (3y-2)(2y-3) = 0$

$$\therefore y = \frac{2}{3} \text{ and } \frac{3}{2}$$

When, we put $y = \frac{2}{3} \Rightarrow \frac{x}{1-x} = \frac{4}{9}$

$$\Rightarrow 9x = 4 - 4x \Rightarrow x = \frac{4}{13}$$

When we put $y = \frac{3}{2}$

$$\Rightarrow \frac{x}{1-x} = \frac{9}{4} \Rightarrow 4x = 9 - 9x$$

$$\therefore x = \frac{9}{13}$$

4. (a) Given equation, $2x^2 - 7x + 3 = 0$
$\therefore 2x^2 - 6x - x + 3 = 0$
$\Rightarrow 2x(x-3) - 1(x-3) = 0$
$\Rightarrow (2x-1)(x-3) = 0$
Both equation have a common root.

So, we put $x = \frac{1}{2} \Rightarrow 4\left(\frac{1}{2}\right)^2 + a\left(\frac{1}{2}\right) - 3 = 0$

$$\Rightarrow 1 + \frac{a}{2} - 3 = 0 \Rightarrow \frac{a}{2} = 2 \Rightarrow a = 4$$

Again, we put $x = 3$
$4(3)^2 + a(3) - 3 = 0$
$\Rightarrow 36 + 3a - 3 = 0 \Rightarrow a = -11$
$a = -11$ or 4.

5. (a) Given, $px^2 + qx + r = 0$
Let α and β are the roots of equation.
According to the question,
$\beta = 2\alpha$

Product of roots $(\alpha\beta) = \frac{r}{p} = 2\alpha^2$

$\Rightarrow \alpha^2 = \frac{r}{2p}$ ... (i)

Sum of roots $(\alpha + \beta) = -\frac{q}{p} = 3\alpha$

$\Rightarrow \alpha = \frac{-q}{3p}$ ... (ii)

We put the value of α in equation (i),

Now,

$\Rightarrow \left(\frac{q}{3p}\right)^2 = \frac{r}{2p}$

$2q^2 = 9pr$

6. (b) Given,

$2x - 3y < 7$ ...(i)

and $x + 6y < 11$ ...(ii)

On adding equations (i) and (ii),

$3x + 3y < 18$

$\Rightarrow x + y < 6$

7. (b) Given equation,

$$\sqrt{\frac{2x}{3-x}} - \sqrt{\frac{3-x}{2x}} = \frac{3}{2}$$

Let $\sqrt{\frac{2x}{3-x}} = a$

$\therefore a - \frac{1}{a} = \frac{3}{2}$

$\Rightarrow 2(a^2 - 1) = 3a$

$\Rightarrow 2a^2 - 3a - 2 = 0$

$\Rightarrow 2a^2 - 4a + a - 2 = 0$

$\Rightarrow 2a(a - 2) + 1(a - 2) = 0$

$\Rightarrow (2a + 1)(a - 2) = 0$

If $a - 2 = 0$

Now, put $a = 2$

$\Rightarrow \sqrt{\frac{2x}{3-x}} = 2$

Squaring both sides, then we get

$\Rightarrow 2x = 4(3 - x)$

$\Rightarrow 6x = 12 \Rightarrow x = 2$

If $2a + 1 = 0$,

$\Rightarrow a = -\frac{1}{2}, \ a \neq \frac{-1}{2}$

$x = 2$ is the root of equation.

8. (d) Since, α and β are the roots of the equation

$x^2 - 3x + 2 = 0$

$\therefore \alpha + \beta = 3$ and $\alpha\beta = 2$ ... (i)

Now, $\alpha + 1 + \beta + 1 = \alpha + \beta + 2$

$= 3 + 2 = 5$

and $(\alpha + 1)(\beta + 1) = \alpha\beta + \alpha + \beta + 1$

$= 2 + 3 + 1 = 6$

Required equation is

$x^2 - (\alpha + 1 + \beta + 1)x + (\alpha + 1)(\beta + 1) = 0$

$\Rightarrow x^2 - 5x + 6 = 0$

9. (c) α and β are the roots of the equation

$ax^2 + bx + c = 0$, then

$\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

$\therefore \alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

$= \left(-\frac{b}{a}\right)^3 - 3\left(\frac{c}{a}\right)\left(-\frac{b}{a}\right)$

$= -\frac{b^3}{a^3} + \frac{3bc}{a^2} = \frac{3abc - b^3}{a^3}$

10. (c) Given equation is

$kx^2 + (2k + 6)x + 16 = 0$

For equal roots then D must be zero.

$\Rightarrow (2k + 6)^2 - 4k \,.\, 16 = 0$

$\Rightarrow 4k^2 + 24k + 36 - 64k = 0$

$\Rightarrow 4k^2 - 40k + 36 = 0$

$\Rightarrow k^2 - 10k + 9 = 0$

$\Rightarrow k^2 - 9k - k + 9 = 0$

$\Rightarrow k(k - 9) - 1(k - 9) = 0$

$\Rightarrow (k - 1)(k - 9) = 0$

$\therefore k = 1$ and 9.

11. (c) Let the roots of equations are α and β.

$x^2 - 3kx + 2k^2 - 1 = 0$

$\therefore \alpha\beta = 2k^2 - 1$

But given, $-\alpha\beta = 7$

$\therefore 2k^2 - 1 = 7$

$\Rightarrow 2k^2 = 8 \Rightarrow k^2 = 4$

$k \neq 2$

On putting $k = \pm 2$ in the equation, then we get

$x^2 \mp 6x + 7 = 0$

$D = \sqrt{b^2 - 4ac} = \sqrt{(6)^2 - 4 \times 7} = \sqrt{36 - 28} = 2\sqrt{2}$

$D = 2\sqrt{2}$, so roots of equation are irrational.

12. (b) Given equation,

$2x^2 - 3x - 4 = 0$

For a reciprocal roots, we replace x by $\frac{1}{x}$, we get

$2\left(\frac{1}{x}\right)^2 - 3\left(\frac{1}{x}\right) - 4 = 0$

$\Rightarrow -4x^2 - 3x + 2 = 0$

$\Rightarrow 4x^2 + 3x - 2 = 0$

13. (c) Given that,

$2x^2 + 6x + 5y + 1 = 0$ ... (i)

and $2x + y + 3 = 0 \Rightarrow x = \dfrac{-y-3}{2}$ ... (ii)

Now, putting the value of x from (ii) in equation (i),

$$2\left(\frac{-3-y}{2}\right)^2 + 6\left(\frac{-3-y}{2}\right) + 5y + 1 = 0$$

$$\Rightarrow \frac{9+y^2+6y}{2} - \frac{(18y+6y)}{2} + 5y + 1 = 0$$

$\Rightarrow y^2 + 10y - 7 = 0$

14. (a) Since, a polynomial equation has rational coefficient and has exactly three real roots, then degree of the polynomial must be equal to 3.

15. (a) α and β are the roots of the equation $ax^2 + bx + c = 0$

$$\therefore \alpha + \beta = -\frac{b}{a} \text{ and } \alpha\beta = \frac{c}{a}$$

$$\therefore \left(\frac{1}{\alpha^2} - \frac{1}{\beta^2}\right)^2 = \left(\frac{\beta^2 - \alpha^2}{\alpha^2\beta^2}\right)^2$$

$$= \frac{(\alpha+\beta)^2 \{(\alpha+\beta)^2 - 4\alpha\beta\}}{(\alpha^2\beta^2)^2}$$

$$= \frac{\frac{b^2}{a^2}\left(\frac{b^2}{a^2} - \frac{4c}{a}\right)}{\left(\frac{c^2}{a^2}\right)^2}$$

$$= \frac{b^2}{c^4}(b^2 - 4ac)$$

16. (b) Let two positive integers be x and x + 1.

According to question,

$\Rightarrow x^2 + (x+1)^2 = 761$

$\Rightarrow 2x^2 + 2x - 760 = 0$

$\Rightarrow x^2 + x - 380 = 0$

$\Rightarrow (x+20)(x-19) = 0$

$\therefore x = 19$ $(\because x \neq -20)$

So, two consecutive integers are 19 and 20.

17. (c) Given, $3^x + 27(3^{-x}) = 12$

Let $3^x = y$

$$\therefore y + \frac{27}{y} = 12$$

$\Rightarrow y^2 - 12y + 27 = 0$

$\Rightarrow y^2 - 9y - 3y + 27 = 0$

$\Rightarrow (y-3)(y-9) = 0 \Rightarrow y = 3, 9$

when y = 3; when y = 9

$\Rightarrow 3^x = 3$ $3^x = 9$

$\therefore x = 1$ $x = 2$

x = 1, 2 are value of x.

18. (a) Given expression

$$x^2 - ax + b = 0$$

Here α and β are roots of the equation

$\therefore \alpha + \beta = a$ and $\alpha\beta = b$

So, $|\alpha - \beta| = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = \sqrt{a^2 - 4b}$

19. (c) Given, $a + b = 2m^2$ ... (i)

$b + c = 6m$ ... (ii)

and $a + c = 2$ ... (iii)

On adding equations (i), (ii) and (iii), we get

$2(a + b + c) = 2m^2 + 6m + 2$

$\Rightarrow a + b + c = m^2 + 3m + 1$ ... (iv)

On subtracting equation (ii) from equation (iv), we get

$a = m^2 + 3m - 1$

On subtracting equation (iii) from equation (iv), we get

$b = m^2 + 3m - 1$

On subtracting equation (i) from equation (iv), we get

$c = -m^2 + 3m + 1$

As $a \le b$ and $b \le c$

$\Rightarrow m^2 - 3m + 1 \le m^2 + 3m - 1$ and

$m^2 + 3m - 1 \le -m^2 + 3m + 1$

$\Rightarrow 6m \ge 2$ and $2m^2 \le 2$

$$\Rightarrow m \ge \frac{1}{3} \text{ and } -1 \le m \le 1$$

$$\therefore \frac{1}{3} \le m \le 1.$$

20. (c) Let number of student in each row = x

and number of rows = y

Hence total number of students = xy

According to the question,

$xy = (x + 1)(y - 2)$

$\Rightarrow xy = xy - 2x + y - 2$

$\Rightarrow 2x - y = -2$ ...(i)

and $xy = (x - 1)(y + 3)$

$\Rightarrow xy = xy + 3x - y - 3$

$\Rightarrow -3x + y = 3$

$3x - y = 3$

From equation (i) and (ii)

x = 5, y = 12

Hence number students = xy

$= 5 \times 12 = 60$

21. (d) Let α and β are roots of equation.

$7x^2 + 12x + 18 = 0$

$$\text{then, } \alpha + \beta = \frac{-12}{7} \text{ and } \alpha\beta = \frac{18}{7}$$

$$\Rightarrow \alpha^2 + \beta^2 + 2\alpha\beta = \frac{144}{49}$$

$\Rightarrow \quad \alpha^2 + \beta^2 = \frac{144}{49} - \frac{36}{7} = -\frac{108}{49}$

$\therefore \quad \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{-\frac{108}{49}}{\frac{18}{7}} = -\frac{6}{7}$

22. (d) Given, $\frac{x(x-1)-(m+1)}{(x-1)(m-1)} = \frac{x}{m}$

$\Rightarrow$ $m(x^2 - x - m - 1) = x(mx - x - m + 1)$

$\Rightarrow$ $mx^2 - mx - m(m + 1) = mx^2 - x^2 - mx + x$

$\Rightarrow$ $x^2 - x - m(m + 1) = 0$

Let roots be α and α.

$\therefore$ $\alpha + \alpha = 1, \alpha.\alpha = -m(m + 1)$

$\Rightarrow \quad \alpha = \frac{1}{2} \Rightarrow \left(\frac{1}{2}\right)^2 = -m(m+1)$

$\Rightarrow$ $4m^2 + 4m + 1 = 0 \Rightarrow (2m + 1)^2 = 0$

$\therefore \quad m = -\frac{1}{2}$

23. (a) The condition for both the roots of the equation $ax^2 + bx + c = 0$ are positive, if

$-\frac{b}{a} > 0$ and $\frac{c}{a} > 0$

Given, equation is $x^2 - 2(k - 1)x + (2k + 1) = 0$, whose roots are positive

$-\frac{b}{a} = \frac{2(k-1)}{1} > 0 \;\; \Rightarrow k > 1$

and $\frac{c}{a} = \frac{2k+1}{1} > 0 \;\; \Rightarrow k > -\frac{1}{2}$

$\therefore$ $k > 1$

Hence, the least value $k = 1$.

24. (c) -1.5 is a root of $ax^2 + x - 3 = 0$

then, $a(-1.5)^2 + (-1.5) - 3 = 0$

$\Rightarrow$ $2.25a - 4.5 = 0$

$\Rightarrow \quad a = \frac{4.5}{2.25}$

$\therefore$ $a = 2$

25. (b) $r^{75} > r^{90}$ is possible only when r lies between 0 and 1 $(0 < r < 1)$

26. (c) Let the roots of equation,

$ax^2 + bx + c = 0$ are $-\alpha$ and $-\frac{1}{\alpha}$.

$\therefore \quad (-\alpha)\left(-\frac{1}{\alpha}\right) = \frac{c}{a}$

$\Rightarrow \quad 1 = \frac{c}{a} \Rightarrow c = a$

27. (c) Given, $\log_{10}(x^2 - 6x + 45) = 2$

$\Rightarrow$ $(x^2 - 6x + 45) = 10^2 = 100$

$\Rightarrow$ $x^2 - 6x - 55 = 0$

$\Rightarrow$ $x^2 - 11x + 5x - 55 = 0$

$\Rightarrow$ $x(x - 11) + 5(x - 11) = 0$

$\Rightarrow$ $(x + 5)(x - 11) = 0$

$\therefore$ $x = 11, -5$

28. (c) Given, $\frac{1}{x+a} + \frac{1}{x+b} = \frac{1}{c}$

$\Rightarrow \quad \frac{(x+b)+(x+a)}{(x+a)(x+b)} = \frac{1}{c}$

$\Rightarrow$ $2cx + (a + b)c = x^2 + (a + b)x + ab$

$\Rightarrow$ $x^2 + (a + b - 2c)x + ab - ac - bc = 0$

Let the roots of above equation be α and β.

Given, $\alpha + \beta = 0$

$\Rightarrow$ $-a(a + b - 2c) = 0$

$\Rightarrow$ $a + b = 2c$ ... (i)

Now, $\alpha\beta = ab - ac - bc = ab - (a + b)c$

$= ab - (a + b)\frac{(a+b)}{2}$ [from equation (i)]

$= \frac{2ab-(a^2+b^2+2ab)}{2} = -\frac{(a^2+b^2)}{2}$

29. (b) Let the roots of the equation be α and β.

$\therefore \quad \alpha+\beta = \frac{5}{k}$ and $\alpha\beta = \frac{6}{k}$

Given, $\frac{\alpha}{\beta} = \frac{2}{3}$

$\Rightarrow \quad \alpha = \frac{2}{3}\beta$

$\therefore \quad \frac{2}{3}\beta + \beta = \frac{5}{k}$ and $\frac{2}{3}\beta^2 = \frac{6}{k}$

$\Rightarrow \quad \frac{5}{3}\beta = \frac{5}{k}$ and $\beta^2 = \frac{9}{k}$

$\beta = \frac{3}{k}$ and $\beta^2 = \frac{9}{k}$

$\Rightarrow \quad \frac{9}{k^2} = \frac{9}{k}$

$\Rightarrow$ $k = 1$ and $k \neq 0$. It is not satisfy the given condition.

30. (a) α and β be the roots of the equation : $x^2 + px + q = 0$

$\therefore$ $\alpha + \beta = -p$ and $\alpha\beta = q$

$-\alpha^{-1} - \beta^{-1} = -\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) = -\left(\frac{\alpha+\beta}{\alpha\beta}\right) = \frac{p}{q}$

and $\left(-\frac{1}{\alpha}\right)\left(-\frac{1}{\beta}\right)=\frac{1}{\alpha\beta}=\frac{1}{q}$

$x^2-(-\alpha^{-1}-\beta^{-1})x+(-\alpha^{-1})(-\beta^{-1})=0$

$\Rightarrow\ x^2-\frac{p}{q}x+\frac{1}{q}=0$

$\Rightarrow\ qx^2-px+1=0$

31. (c) Given, one root of the equation $ax^2+x-3=0$ is -1

$\therefore\ a(-1)^2+(-1)-3=0\Rightarrow a=4$

$\therefore\ 4x^2+x-3=0$

Let other root of the euqation is α

$\therefore\ -1\,.\,\alpha=-\frac{3}{4}\Rightarrow\alpha=\frac{3}{4}$

32. (b) The roots of equation

$(a^2+b^2)x^2-2(ac+bd)x+(c^2+d^2)=0$ are equal.

$B^2-4AC=0,\ B^2=4AC$

$\Rightarrow\ 4(ac+bd)^2=4(a^2+b^2)(c^2+d)^2$

$\Rightarrow\ a^2c^2+b^2d^2+2abcd=a^2c^2+a^2d^2+b^2c^2+b^2d^2$

$\Rightarrow\ (ad-bc)^2=0$

$\therefore\ ad=bc$

33. (a) Given, $\sqrt{\frac{x}{x+3}}-\sqrt{\frac{x+3}{x}}=-\frac{3}{2}$

Let $y=\sqrt{\frac{x}{x+3}}$, then

$y-\frac{1}{y}=-\frac{3}{2}$

$\Rightarrow\ 2y^2-2=-3y$

$\Rightarrow\ 2y^2+3y-2=0$

$\Rightarrow\ 2y^2+4y-y-2=0$

$\Rightarrow\ 2y(y+2)-1(y+2)=0\Rightarrow(2y-1)(y+2)=0$

Either, $(2y-1)=0$ or $y+2=0$

$\Rightarrow\ y=\frac{1}{2}$ $\qquad y=-2,\ y\neq-2$

$\Rightarrow\ y=\sqrt{\frac{x}{x+3}}$ $\qquad$ it can not be negative.

$\Rightarrow\ \sqrt{\frac{x}{x+3}}=\frac{1}{2}$

On squaring both sides,

$\frac{x}{x+3}=\frac{1}{4}$

$\Rightarrow\ 4x=x+3$

$\Rightarrow\ x=1$

So, the solution of $x=1$.

34. (c) Given, $4^x-3.2^{x+2}+32=0$

$\Rightarrow\ 2^{2x}-8.2^x-4.2^x+32=0$

$\Rightarrow\ (2^x-8)(2^x-4)=0$

On comparing both sides :

Either $2^x=8\Rightarrow x=3$

or $2^x=4\Rightarrow x=2$

35. (a) α and β be the roots of the equation

$x^2-x-1=0$

$\therefore\ \alpha+\beta=1$ and $\alpha\beta=-1$

We know that

$\alpha^4+\beta^4=(\alpha^2+\beta^2)^2-2(\alpha\beta)^2$

$=[(\alpha+\beta)^2-2\alpha\beta]^2-2(\alpha\beta)^2=(1+2)^2-2=9-2=7$

36. (d) Let roots of equations are α and β

$\alpha+\beta=9$ and $\alpha\beta=9$

Now, equation is

$x^2-(\alpha+\beta)x+(\alpha\beta)=0$

$\Rightarrow\ x^2-9x+9=0$

37. (b) Given, $2x^2+3x+c=0$

and 0.5 is the root of equation. So, we put $x=0.5$

$2(0.5)^2+3(0.5)+c=0$

$\Rightarrow\ 0.5+1.5+c=0$

$\therefore\ c=-2$

38. (d) a and c have the same sign opposite to that of b.

39. (d) The equation will have equal roots, if

$B^2-4AC=0$

$\therefore\ (2nc)^2-4(1+n^2)(c^2-a^2)=0$

$\Rightarrow\ 4n^2c^2-4(c^2-n^2c^2-a^2-n^2a^2)=0$

$\Rightarrow\ -4c^2+4a^2+4n^2a^2=0$

$\Rightarrow\ c^2=a^2(1+n^2)$

40. (d) Let the roots of equation a and b

$\alpha+\beta=2$ and $\alpha\beta=4$

On taking $\alpha\to2\alpha$ and $\beta\to2\beta$

$2\alpha+2\beta=4$ and $2\alpha\,.\,2\beta=+4\times4=+16$

Thus, the new equation $x^2-4x+16=0$.

41. (b) Since, $\sin\theta$ and $\cos\theta$ are the roots of the equation $ax^2-bx+c=0$.

$\therefore\ \sin\theta+\cos\theta=\frac{b}{a}$ ... (i)

and $\sin\theta\cos\theta=\frac{c}{a}$ ... (ii)

Now, squaring on both sides of equation (i)

$\Rightarrow\ \sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta=\frac{b^2}{a^2}$

Put the value of $\sin\theta\cos\theta$

$\Rightarrow\ 1+2\left(\frac{c}{a}\right)=\frac{b^2}{a^2}$

$\Rightarrow\ 2\left(\frac{c}{a}\right)=\frac{b^2-a^2}{a^2}$

$\Rightarrow\ 2ac=b^2-a^2$

$\therefore\ a^2-b^2+2ac=0$

42. (b) $\alpha+\beta=6$ and $\alpha\beta=6$

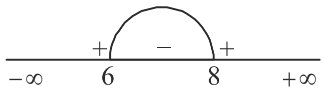
$\therefore\ (\alpha+\beta)^2=6^2$

$\Rightarrow\ \alpha^2+\beta^2+2\alpha\beta=36$

$\Rightarrow\ \alpha^2+\beta^2=36-2(6)=24$

$\therefore$ $(\alpha^3 + \beta^3) + (\alpha^2 + \beta^2) + (\alpha + \beta)$
$= (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta) + (\alpha^2 + \beta^2) + (\alpha + \beta)$
$= 6(24 - 6) + (24) + (6)$
$= 6(18) + 30 = 108 + 30 = 138$

43. (c) $6 \le x \le 8$
i.e.,



or $x \in [6, 8]$
$\Rightarrow$ $(x - 6)(x - 8) \le 0$

44. (a) Let roots of equation are α and β.

$\therefore \quad \alpha + \beta = \dfrac{a^2 + b^2}{a^2b^2}$ and $\alpha\beta = \dfrac{1}{a^2b^2}$

We know that

$$\alpha - \beta = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta} = \sqrt{\left(\frac{a^2 + b^2}{a^2b^2}\right)^2 - \frac{4}{a^2b^2}}$$

$$\Rightarrow \quad \alpha - \beta = \sqrt{\frac{(a^2 - b^2)^2}{(a^2b^2)^2}} = \frac{a^2 - b^2}{a^2b^2}$$

On solving, we get $\alpha = \dfrac{1}{b^2}$ and $\beta = \dfrac{1}{a^2}$.

45. (a) If the roots of the equation
$x^2 - 2ax + a^2 + a - 3 = 0$ are real and less than 3.
Then, $D \ge 0$ and $f(3) > 0$
$\Rightarrow$ $4a^2 - 4(a^2 + a - 3) \ge 0$
and $(3)^2 - 2a(3) + a^2 + a - 3 > 0$
$\Rightarrow$ $-a + 3 \ge 0$
and $9 - 6a + a^2 + a - 3 > 0$
$\Rightarrow$ $-a + 3 \ge 0$ and $a^2 - 5a + 6 > 0$
$\Rightarrow$ $a - 3 \le 0$ and $(a - 3)(a - 2) > 0$
$\Rightarrow$ $a \le 3$ ... (i)
and $a < 2$ or $a > 3$ ... (ii)
From equations (i) and (ii)
$a < 2$.

46. (b) Let, the roots of the quadratic equation
$x^2 + px + q = 0$ is (α, β).
Given that, *A* starts with a wrong value of p and obtains the roots as 2 and 6. But this time q is correct.
i.e., Product of roots
$q = \alpha \, . \, \beta = 6 \times 2 = 12$ ...(i)
and *B* starts with a wrong value of q and gets the roots as 2 and – 9. But this time p is correct.
i.e., Sum of roots $p = \alpha + \beta = -9 + 2 = -7$ ... (ii)
$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$
$= (-7)^2 - 4.12 = 49 - 48 = 1$
[From equations (i) and (ii)]
$\Rightarrow$ $\alpha - \beta = 1$ ... (iii)
From equations (ii) and (iii),
$\alpha = -3$ and $\beta = -4$
which is correct roots.

47. (a) Quadratic equation $7x^2 - 50x + k = 0$
Here, $a = 7$, $b = -50$ and $c = k$

Since, $\alpha + \beta = \dfrac{-b}{a}$

$\therefore$ $\alpha + \beta = \dfrac{50}{7}$

$\Rightarrow$ $\beta = \dfrac{1}{7}$ $(\because \alpha = 7$, given)

and $\alpha\beta = \dfrac{c}{a}$ or $7 \times \dfrac{k}{7} = \dfrac{k}{7} \Rightarrow k = 7$

48. (b) Here, given roots of equation are 3 and –1.
So, expression can be written as
$(x - 3)(x + 1) = 0$
$= x^2 - 3x + x - 3 = 0$
$= x^2 - 2x - 3 = 0$

49. (d) Given equation,

$\dfrac{x^2}{a} + \dfrac{x}{b} + \dfrac{1}{c} = 0$

$bcx^2 + acx + ab = 0$

Let roots are α and $\dfrac{1}{\alpha}$

Product of roots $= \alpha \cdot \dfrac{1}{\alpha} = \dfrac{ab}{bc} \Rightarrow 1 = \dfrac{a}{c}$

$\Rightarrow$ $\boxed{\therefore c = a}$

50. (d) Given that, f(x) is a polynomial with constant term 10 and all coefficients are integer. Let k, $k_1, k_2 \ldots k_{n-1}$ be roots of nth degree polynomial.
Now $(x - k)$ is a factor of f(x), where k is an integer.

Product of roots $= \dfrac{\text{Constant term}}{\text{Coefficient of } x^n}$

i.e., $k.k_1.k_2 \ldots k_{n-1} = \dfrac{10}{1}$ Coefficient of x^n is 1)

$\Rightarrow$ $k.k_1.k_2 \ldots k_{n-1} = 10 = 5.2.1 \ldots 1$
Therefore, the possible value of k is 5.

51. (a) Let α and β be the roots of this quadratic equation $2x^2 - 11x + 5 = 0$

$\therefore$ $\alpha + \beta = -\dfrac{(-11)}{2} = \dfrac{11}{2}$ ... (i)

and $\alpha \cdot \beta = \dfrac{5}{2}$

We know that
$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$$= \left(\frac{11}{2}\right)^2 - 4\left(\frac{5}{2}\right) = \frac{121}{4} - \frac{40}{4} = \frac{81}{4} = \left(\frac{9}{2}\right)^2$$

$\therefore$ Difference of roots $= (\alpha - \beta) = \dfrac{9}{2} = 4.5$.

52. (a) According to question
Let one roots of equation is α then others roots of equation is α^2.

$\therefore$ Sum of roots $= \alpha + \alpha^2 = -\frac{(-b)}{1}$

$\Rightarrow \alpha(\alpha + 1) = b$ ... (i)

Product of roots $= \alpha \cdot \alpha^2 = \frac{c}{1}$

$\Rightarrow \alpha^3 = c \Rightarrow \alpha = c^{1/3}$ ... (ii)

From equations (i) and (ii),

$$c^{\frac{1}{3}}\left(c^{\frac{1}{3}}+1\right) = b \quad \text{... (iii)}$$

On cubing both sides, we get

$$c\left(c^{\frac{1}{3}}+1\right)^3 = b^3$$

$$\Rightarrow c\left\{c+1+3c^{\frac{1}{3}}\left(c^{\frac{1}{3}}+1\right)\right\} = b^3$$

$\Rightarrow c(c + 1 + 3b) = b^3$ [from equation (iii)]

$\Rightarrow b^3 = 3bc + c^2 + c$

53. (d) $ax^2 + bx + b = 0$

$$\Rightarrow x^2 + \frac{b}{a}x + \frac{b}{a} = 0$$

$\therefore$ Sum of roots, $\alpha + \beta = \frac{-b}{a}$

and Products of roots, $\alpha\beta = \frac{b}{a}$

Hence, product of roots is not equal to the sum of roots, so Statement I not correct.
Now, for roots to be real and uequal.

$\therefore$ Determinant, $D > 0$

$\Rightarrow b^2 - 4ac > 0$

$\Rightarrow b^2 - 4a\,(b) > 0$

$\Rightarrow b^2 - 4ab > 0$

$\Rightarrow b^2 > 4ab$

$\therefore b > 4a$

So, if $b > 4a$, then roots are unequal and real, so Statement II is not always true it will depend on values of a and b.

54. (d) $x^2 = 6 + \sqrt{6 + \sqrt{6 + \sqrt{6 + \ldots \infty}}}$

$x^2 = 6 + \sqrt{x^2}$

$\Rightarrow x^2 = 6 + x$

$\Rightarrow x^2 - x - 6 = 0$

$\Rightarrow x^2 + 2x - 3x - 6 = 0$

$\Rightarrow x(x + 2) - 3(x + 2) = 0$

$\Rightarrow (x - 3)(x + 2) = 0$

$\therefore x = 3$

Alternate Method :
Given,

$$x^2 = 6 + \sqrt{6 + \sqrt{6 + \sqrt{6 + \ldots \infty}}}$$

$$x = \sqrt{6 + \sqrt{6 + \sqrt{6 + \sqrt{6 + \ldots \infty}}}}$$

Here, factor of 6 = 2 and 3
Sign in expression is positive.
So that x = 3.

55. (b) $\alpha + \beta = \frac{-(-1)}{1} = 1$

$$\alpha.\beta = \frac{-1}{1} = -1$$

Now, $(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$

$\Rightarrow \alpha^2 + \beta^2 = (1)^2 - 2 \times (-1) = 3$

and $\alpha^2 - \beta^2 = \sqrt{(\alpha^2+\beta^2)^2 - 4\alpha^2\beta^2}$

$= \sqrt{9 - 4(-1)^2} = \sqrt{5}$

$\alpha - \beta = \sqrt{(\alpha-\beta)^2 - 4\alpha\beta}$

$= \sqrt{1 - 4(-1)} = \sqrt{5}$

Now, $\frac{\alpha^2+\beta^2}{(\alpha^2-\beta^2)(\alpha-\beta)} = \frac{3}{\sqrt{5}.\sqrt{5}} = \frac{3}{5}$

56. (c) We know that

$$\left(x+\frac{1}{x}\right)^2 - 2 = x^2 + \frac{1}{x^2}$$

Now, put this value in expression

$$\left(x+\frac{1}{x}\right)^2 - 2 + 8\left(x+\frac{1}{x}\right) + 14$$

$$\Rightarrow \left(x+\frac{1}{x}\right)^2 + 8\left(x+\frac{1}{x}\right) + 12$$

$\Downarrow$ $\Downarrow$
6 + 2 6 × 2

So factor of this expression $= \left(x + \frac{1}{x} + 6\right)$

57. (c) When mistake is done in first degree term the roots of the equation are – 9 and – 1.

$\therefore$ Equation is $(x + 1)(x + 9) = x^2 + 10x + 9$ ...(i)

When mistake is done in constant term, the roots of equation are 8 and 2.

$\therefore$ Equation is $(x - 2)(x - 8) = x^2 - 10x + 16$ ... (ii)

$\therefore$ Required equation from equations (i) and (ii), we get
$x^2 - 10x + 9$

58. (a) For the given equation $ax^2 + bx + c = 0$, m and n are the roots

$\therefore$ Sum of roots = $m + n = -\frac{b}{a}$

and Product of roots $mn = \frac{c}{a}$

Sum of roots & of required equation

$$= \frac{m^2+1}{m} \text{ and } \frac{n^2+1}{n}$$

$$= \frac{m^2+1}{m} + \frac{n^2+1}{n} = \frac{m^2n + n + mn^2 + m}{mn}$$

$$= \frac{mn(m+n)+(m+n)}{mn} = \frac{(m+n)(mn+1)}{mn}$$

$$= \frac{-\frac{b}{a}\left(\frac{c}{a}+1\right)}{\frac{c}{a}} = \frac{-b(a+c)}{ac}$$

Put $m+n = -\frac{b}{a}$ and $mn = \frac{c}{a}$

$\therefore$ Product of

$$\text{roots} = \frac{m^2+1}{m} \times \frac{n^2+1}{n} = \frac{(m^2+1)(n^2+1)}{mn}$$

$$= \frac{m^2n^2 + n^2 + m^2 + 1}{mn}$$

$$= \frac{(mn)^2 + (m+n)^2 - 2mn + 1}{mn}$$

$$= \frac{\left(\frac{c}{a}\right)^2 + \left(\frac{-b}{a}\right)^2 - 2\left(\frac{c}{a}\right) + 1}{\frac{c}{a}}$$

$$= \frac{c^2 + b^2 - 2ac + a^2}{ac} = \frac{b^2 + (a-c)^2}{ac}$$

We know that, quadratic equation is of the form
x^2 – (Sum of roots) x + Product of roots = 0

$$\Rightarrow \quad x^2 - \left(\frac{-b(a+c)}{ac}\right)x + \left(\frac{b^2+(a-c)^2}{ac}\right) = 0$$

$\Rightarrow$ $acx^2 + b(a + c)x + b^2 + (a - c)^2 = 0$
$\Rightarrow$ $acx^2 + (ab + bc)x + b^2 + (a - c)^2 = 0$

59. (a) $x^2 - 4x + 11$
This can be written as
$= (x - 2)^2 + 7$
Here, $(x - 2)^2 \geq 0$
So, given function can be not be less than 7.

60. (b) $f(x) = 2x^3 + x^2 - 2x - 1$
$= x^2(2x + 1) - 1(2x + 1) = (2x + 1)(x^2 - 1)$
$= (2x + 1)(x + 1)(x - 1)$
So given expression is divisible by $2x + 1$.

61. (a) Given,
$x + y = 5$
$y + z = 10$
$z + x = 15$
By solving equations (i), (ii) and (iii), then
$x = 5$, $y = 0$ and $z = 10$
$\therefore$ $z > x > y$

62. (d) Given equation
$(a^2 - bc)x^2 + 2(b^2 - ac)x + (c^2 - ab) = 0$
The given roots are equal, then D must be zero.
$\therefore$ $D = 0$
i.e., $[2(b^2 - ac)]^2 - 4(a^2 - bc)(c^2 - ab) = 0$
$\Rightarrow$ $4(b^4 + a^2c^2 - 2ab^2c) - 4(a^2c^2 - bc^3 - a^3b + ab^2c) = 0$
$\Rightarrow$ $4b^4 + 4a^2c^2 - 8ab^2c - 4a^2c^2 + 4bc^3 + 4a^3b - 4ab^2c = 0$
$\Rightarrow$ $4b^4 - 12ab^2c + 4bc^3 + 4a^3b = 0$
$\Rightarrow$ $b^3 + c^3 + a^3 - 3abc = 0$
$\therefore$ $a^3 + b^3 + c^3 = 3abc$

63. (c) Given equation,
$Ax^2 + Bx + C = 0$
Since, the given roots are –1 and 1.
$\therefore$ Sum of roots = –1 + 1 = 0
and Product of roots = 1 × (–1) = –1
Standard equation is
x^2 – (Sum of roots)x + Product of roots = 0
On comparing with above equation from equation (i),

$$x^2 + \frac{B}{A}x + \frac{C}{A} = 0$$

$\therefore$ $\frac{C}{A}$ = Product of roots $\Rightarrow C = -A$

So, A and C are of opposite signs.

64. (a) Given equation $px^2 + qx + r = 0$.
Let one root of the equation be α.
So, other root = 3α

$\therefore$ Sum of roots = $\alpha + 3\alpha = -\frac{q}{p}$

$\Rightarrow$ $4\alpha = \frac{-q}{p} \Rightarrow \alpha = \frac{-q}{4p}$ ... (i)

Product of roots = $(\alpha) . (3\alpha) = \frac{r}{p}$

Put the value of α

$\Rightarrow\ 3\alpha^2 = \frac{r}{p} \Rightarrow 3\left(\frac{-q}{4p}\right)^2 = \frac{r}{p}$

$\Rightarrow\ \frac{3q^2}{16p^2} = \frac{r}{p} \Rightarrow 3q^2p = 16p^2r$

$\Rightarrow\ 3q^2 = 16pr$

65. (c) $\because$ m% of m + n% of n = 2% of $(m \times n)$

$\Rightarrow\ \frac{m}{100}\times m + \frac{n}{100}\times n = \frac{2}{100}\times(mn)$

$\Rightarrow\ \frac{m^2}{100}+\frac{n^2}{100}=\frac{2mn}{100}$

$\Rightarrow\ m^2 + n^2 = 2mn$

$\Rightarrow\ m^2 + n^2 - 2mn = 0$

$\Rightarrow\ (m-n)^2 = 0$

$\Rightarrow\ m - n = 0$

$\Rightarrow\ m = n$

$\therefore$ Though, both are equal, so 100% of m is n.

66. (b) Here m and n are the roots of the equation $x^2 + ax + b = 0$.

$m + n = -a$ ...(i)

$mn = b$ ...(ii)

Also, m^2 and n^2 are the roots of the equation of $x^2 - cx + d = 0$.

$m^2 + n^2 = c$ ...(iii)

$m^2n^2 = d$ ...(iv)

by squaring Eq. (i) both sides, we get

$m^2 + n^2 + 2mn = a^2$ [from Eqs. (i) and (ii)]

$\Rightarrow c + 2b = a^2 \quad \Rightarrow c = a^2 - 2b$

$\Rightarrow 2b - a^2 = -c$

Therefore, Statement 1 is incorrect.

From Eq. (ii)

$m^2n^2 = b^2 \Rightarrow b^2 = d$

Therefore , Statements 2 is correct.

67. (b) Given, $a_n = 3 - 4n$

$\therefore\ \sum a_n = \sum(3-4n) = \sum 3 - 4\sum n$

$= 3n - 4\frac{[n\times(n+1)]}{2}$

$= 3n - 2n^2 - 2n$

$= n - 2n^2 = -n(2n-1)$

68. (a) $x^2 - y^2 = 0 \Rightarrow x^2 = y^2$

$(x-k)^2 + y^2 = 1$

$x^2 + k^2 - 2kx + y^2 - 1 = 0$

Here we put $y^2 = x^2$

$x^2 + k^2 - 2kx + x^2 - 1 = 0$

$2x^2 - 2kx + k^2 - 1 = 0$

$$x = \frac{2k \pm \sqrt{(-2k)^2 - 4(2)(2k^2-1)}}{2\times 2}$$

$$= \frac{2k \pm \sqrt{4k^2 - 8k^2 + 8}}{4}$$

$$= \frac{2k \pm \sqrt{8-4k^2}}{4}$$

$$= \frac{2k \pm 2\sqrt{2-k^2}}{4}$$

$$= \frac{k \pm \sqrt{2-k^2}}{4}$$

For unique positive solution we put k = 2

$x = \frac{2}{2} = 1$.

69. (c) $x^2 - 4x - \log_{10} N = 0$

if roots are real, $b^2 - 4ac \ge 0$

$\Rightarrow (-4)^2 + 4.\log_{10}N \ge 0$

$\Rightarrow 16 + 4\log_{10}N \ge 0$

$\Rightarrow 16\log_{10}10 + 4\log_{10}N \ge 0$

$\Rightarrow \log_{10}(10)^{16} + \log_{10}N^4 \ge 0$

$\Rightarrow \log_{10}[(10)^{16}.N^4] \ge \log_{10}1$

$\Rightarrow (10)^{16}N^4 \ge 1$

$\Rightarrow N^4 \ge \left(\frac{1}{10}\right)^{16\div 4}$

$\Rightarrow N \ge \left(\frac{1}{10}\right)^4$

$\Rightarrow N \ge \frac{1}{10000}$

So, minimum value of N is $\frac{1}{10000}$.

Option (c) is correct.

70. (c) $6 + 5x - x^2 - \left(x - \frac{5}{2}\right)^2 + \frac{25}{4} + 6$

$= \frac{49}{4} - \left(x - \frac{5}{2}\right)^2$

So, maximum value of $6 + 5x - x^2$ is $\frac{49}{4}$.

$y - 6 - y^2 = -\left(y - \frac{1}{2}\right)^2 + \frac{1}{4} - 6$

$= -\frac{23}{4} - \left(4 - \frac{1}{2}\right)^2$

So maximum value of $y-6-4^2$ is $-\frac{23}{4}$.

Difference between the maximum values

$= \frac{49}{4} - \left(-\frac{23}{4}\right)$

$= \frac{49+23}{4} = \frac{72}{4} = 18$

So, option (c) is correct

71. (a) Let α, β be the roots of the equation $lx^2 + mx + m = 0$

Given $\frac{\alpha}{\beta} = \frac{p}{q}$ ...(1)

Now $\alpha + \beta$ (sum of roots) $= \frac{-m}{l}$

and $\alpha\beta$ (product of roots) $= \frac{m}{l}$

Consider $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{m}{l}}$

Using (1)

$= \sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}} + \sqrt{\frac{m}{l}}$

$= \frac{\alpha+\beta}{\sqrt{\alpha\beta}} + \sqrt{\frac{m}{l}}$

$= \frac{-\frac{m}{l}}{\sqrt{\frac{m}{l}}} + \sqrt{\frac{m}{l}} = -\sqrt{\frac{m}{l}} + \sqrt{\frac{m}{l}} = 0$

$\therefore$ option (a) is correct.

72. (a) $\sqrt{3x^2 - 7x - 30} - \sqrt{2x^2 - 7x - 5} = x - 5$

$\Rightarrow \sqrt{3x^2 - 7x - 30} - (x-5) = \sqrt{2x^2 - 7x - 5}$

Squaring on both sides, we get

$\Rightarrow (3x^2 - 7x - 30) + (x-5)^2 - 2(x-5)\sqrt{3x^2 - 7x - 30} = 2x^2 - 7x - 5$

$\Rightarrow 3x^2 - 7x - 30 + x^2 + 25 - 10x - 2(x-5)\sqrt{3x^2 - 7x - 30} = 2x^2 - 7x - 5$

$\Rightarrow 3x^2 - 30 + x^2 + 25 - 10x - 2x^2 + 5 = 2(x-5)\sqrt{3x^2 - 1x - 30}$

$\Rightarrow 2x^2 - 10x = 2(x-5)\sqrt{3x^2 - 7x - 30}$

$\Rightarrow 2x(x-5) = 2(x-5)\sqrt{3x^2 - 7x - 30}$

$\Rightarrow x = \sqrt{3x^2 - 7x - 30}$

Again squaring on both sides, we get

$\Rightarrow x^2 = 3x^2 - 7x - 30$

$\Rightarrow 2x^2 - 7x - 30 = 0$

Let α and β be roots of this equation

$\Rightarrow \alpha\beta = -\frac{30}{2}$

$= -15$

$\therefore$ Option (a) is correct.

73. (c) Given $a^2 - by - cz = 0$ ...(1)

$ax - b^2 + cz = 0$ ...(2)

$ax + by - c^2 = 0$ ...(3)

Adding (1) and (3), we get

$a^2 + ax - cz - c^2 = 0$

$a(a+x) = c(c+z)$

$\Rightarrow a + x = \frac{c}{a}(c+z)$ ...(4)

Substracting (2) from (3), we get

$by + b^2 - c^2 - cz = 0$

$b(b+y) = c(c+z) \Rightarrow b + y = \frac{c}{b}(c+z)$ ...(5)

Consider $\frac{x}{a+x} + \frac{y}{b+y} + \frac{z}{c+z}$

Using (4) and (5), we get

$\frac{ax}{c(c+z)} + \frac{by}{c(c+z)} + \frac{z}{c+z}$

$= \frac{ax+by}{c(c+z)} + \frac{z}{c+z}$

[Using (3) $ax + by = c^2$]

$= \frac{c^2}{c(c+z)} + \frac{z}{c+z}$

$= \frac{c}{c+z} + \frac{z}{c+z} = \frac{c+z}{c+z}$

$= 1$

$\therefore$ Option (c) is correct.

74. (b)

75. (a) Consider $ax^2 - (a^2+1)x + a$ ...(1)

$\Rightarrow ax^2 - a^2x - x + a$

$\Rightarrow a^x(x-a) - 1(x-a)$

$= (x-a)(ax-1)$

Given p and q are two linear factors of (1)

$\therefore$ $p = x + a$ and $q = ax - 1$

$\Rightarrow p + q = x - a + ax - 1$

$= x(a+1) - 1(a+1)$

$= (x-1)(a+1)$

$\therefore$ Option (a) is correct

76. (a) Let α and β be two roots of the given equation $ax^2 + bx + c = 0$

Then $\alpha + \beta = -\frac{b}{a}$ & $\alpha\beta = \frac{c}{a}$

According to question,

$$\alpha+\beta=\frac{1}{\alpha^2}+\frac{1}{\beta^2}$$

$$=\frac{\alpha^2+\beta^2+2\alpha\beta-2\alpha\beta}{\alpha^2\beta^2}=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha^2\beta^2}$$

Substitute the value of $\alpha+\beta$ and $\alpha\beta$, we get

$$-\frac{b}{a}=\frac{\left(-\frac{b}{a}\right)^2-\frac{2c}{a}}{\frac{c^2}{a^2}}$$

$$\Rightarrow -\frac{b}{a}=\frac{b^2-2ac}{a^2}\times\frac{a^2}{c^2}=\frac{b^2-2ac}{c^2}$$

$\Rightarrow -bc^2 = a(b^2-2ac)$

$\Rightarrow ab^2+bc^2-2a^2c=0$

$\therefore$ Option (a) is correct.

77. (c) Let α, β be two roots of the equation $x^2+px+q=0$

According to question

$\beta=\alpha^2$

Sum of roots $=\alpha+\alpha^2=-p$ ... (1)

Product of roots $=\alpha^3=q$ ...(2)

Dividing (1) by (2) we get

$$\frac{\alpha(\alpha+1)}{\alpha^3}=\frac{-p}{q}$$

$$\frac{\alpha+1}{\alpha^2}=-\frac{p}{q}$$

Cubing on both sides, we get

$q^3(\alpha+1)^3=(-p)^3(\alpha^2)^3$

$$q^3(\alpha^3+1+3\alpha^2+3\alpha)=-p^3\alpha^6 \quad \begin{bmatrix}\alpha^3=q\\ \alpha^6=q^2\end{bmatrix}$$

$q^3[q+1+3(\alpha^2+\alpha)]=-p^3q^2$

$q[q+1+3(-p)]=-p^3$

$q^2+q-3pq=-p^3$

$\Rightarrow p^3+q^2+q=3pq$

$\therefore$ Option (c) is correct

78. (d) $1+\frac{1}{x}-\frac{1}{x^2}\geq 0$

$x^2+x-1\geq 0$

$$\Rightarrow\left(x-\frac{(-1-\sqrt{5})}{2}\right)\left(x-\frac{(-1+\sqrt{5})}{2}\right)=0$$

$$\Rightarrow x\leq\frac{-1-\sqrt{5}}{2}$$

and $x\geq\dfrac{-1+\sqrt{5}}{2}$

79. (d) $4x^2-16x+\frac{\lambda}{4}=0$

$$x^2-4x+\frac{\lambda}{16}=0$$

$$\alpha\beta=\frac{\lambda}{16},\alpha+\beta=4$$

$$\begin{bmatrix}1<\alpha<2\\2<\beta<3\end{bmatrix},3<\alpha\beta<4$$

$$3<\frac{\lambda}{16}<4$$

$48<\lambda<64$, $\lambda=49, 50......63$ (No of λ's values $=15$)

80. (b) $25x^2-15x+2=0$

$25x^2-10x-5x+2=0$

$5x(5x-2)-1(5x-2)=0$

$(5x-1)(5x-2)=0$

$$x=\frac{1}{5},\ x=\frac{2}{5}$$

$$\alpha=\frac{1}{5},\ \beta=\frac{2}{5}$$

New roots $=(2\alpha)^{-1}=\left(\frac{5}{2}\right)$

$$(2\beta)^{-1}=\left(\frac{5}{2}\right)$$

equation

$$x^2-\left[\frac{5}{2}+\frac{5}{4}\right]x+\frac{5}{2}\times\frac{5}{4}=0$$

$$\Rightarrow x^2-\frac{15}{4}x+\frac{25}{8}=0$$

$\Rightarrow 8x^2-30x+25=0$

81. (b) Given p and q are roots of eq

$x^2+px+q=0$

i.e. $p+q=-P$ $\quad p\times q=q$

i.e. p is 1 only

$\because\ 1\times q=q$

which is divisible by 11.

82. (a) $x^2+3x+\frac{k}{2}=0$

$\alpha+\beta=-3$

$$\alpha+\beta=\frac{k}{2}$$

$$\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}=\frac{\alpha^2+\beta^2}{\alpha\beta}=\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$$

$$\Rightarrow\frac{9-2\times\frac{k}{2}}{\frac{k}{2}}=\frac{18}{k}-2,\qquad\Leftrightarrow\qquad k<O$$

83. (a) Let the roots of equation be α and 2α

Sum of root

$$-(2\alpha+\alpha)=\frac{3a-1}{a^2-5a+3}$$

$$-3\alpha=\frac{3a-1}{a^2-5a+3}....$$

Squaring Both the side

$$9\alpha^2=\frac{(3a-1)^2}{(a^2-5a+3)^2} \quad ...(i)$$

Product of root

$$2\alpha\times\alpha=\frac{2}{a^2-5a+3}$$

$$\alpha^2=\frac{1}{a^2-5a+3} \quad ...(2)$$

Dividing equation (i) from (ii)

$$\frac{9\alpha^2}{\alpha^2}=\frac{(3a-1)^2}{(a^2-5a+3)^2}\times\frac{a^2-5a+3}{1}$$

$$9=\frac{(3a-1)^2}{(a^2-5a+3)}$$

$9a^2-45a+27=9a^2+1-6a$

$26=39a$

$$a=\frac{2}{3}$$

84. (a) $x=\dfrac{\sqrt{a+b}-\sqrt{a-b}}{\sqrt{a+b}+\sqrt{a-b}}$

By using componendo and dividendo

$$\frac{x+1}{x-1}=\frac{2\sqrt{a+b}}{-2\sqrt{a-b}}=\frac{\sqrt{a+b}}{-\sqrt{a-b}}$$

Squaring both side and using componendo and dividendo

$$\frac{(x+1)^2+(x-1)^2}{(x+1)^2-(x-1)^2}=\frac{a+b+a-b}{a+b-a+b}$$

$$\frac{2[x^2+1^2]}{4x}=\frac{2a}{2b}$$

$$\frac{x^2+1}{2x}=\frac{a}{b}$$

$bx^2+b-2ax=0$

85. (a) Sum of root $\alpha+\beta=\dfrac{-b}{a}$ product of root $\alpha\beta=\dfrac{c}{a}$

$\alpha+\beta=-p \qquad \alpha\beta=q$

$(\alpha+\beta)^2=p^2$

$\alpha^2+\beta^2+2\alpha\beta=p^2$

$\alpha^2+\beta^2=p^2-2q$

86. (b) Roots of Aman (4, 3)

equation $=x^2-(4+3)x+(4)(3)$

eq $=x^2-(\alpha+\beta)x+\alpha\beta$

$x^2-7x+\boxed{12}$

here cosntant 12 is wrong

Roots of Alok (3, 2)

equation $=x^2-(3+2)x+(3)(2)$

$=x^2-5x+6$

Here -5 is wrong

So the correct equation is

x^2-7x+6

Solution

$x^2-x-6x-16$

$x(x-1)-6(x-1)$

$(x-6)(x-1)$

$x=6$ and $x=1$

87. (a) Let the two roots be $3x$ and $2x$. Let $\alpha=3x$ and $\beta=2x$

Sum of the roots,

$a+b=$

$$3x+2x=\frac{-m}{12}\Rightarrow 5x=\frac{-m}{12}\Rightarrow m=-60x \quad ...(1)$$

Product of the roots,

$$\alpha\beta=3x.2x=\frac{5}{12}\Rightarrow 6x^2=\frac{5}{12}\Rightarrow x^2=\frac{5}{72}$$

$$\Rightarrow x=\pm\frac{\sqrt{5}}{\sqrt{72}}=\pm\frac{\sqrt{5}}{\sqrt{2\times2\times2\times3\times3}}=\pm\frac{\sqrt{5}}{6\sqrt{2}}$$

Putting this value of x in (1), we get

$$m=-60\times\pm\frac{\sqrt{5}}{6\sqrt{2}}$$

Since we need positive value of m, therefore,

$$m=60\times\frac{\sqrt{5}}{6\sqrt{2}}=\frac{10\sqrt{5}}{\sqrt{2}}=5\sqrt{10}$$

88. (c) Since the roots of the given equation are equal, therefore the discriminant of the given equation is zero. Thus,

$$b^2-4ac=0\Rightarrow[b^2(c-a)^2-4.a(b-c).c(a-b)]=0$$

$$\Rightarrow[b^2(c^2+a^2-2ac)-4ac(ab-b^2-ac+bc)]=0$$

$$\Rightarrow[(ab)^2+(bc)^2+(-2ac)^2+2ab^2c-4a^2bc-4abc^2]=0$$

$$\Rightarrow[(ab)^2+(bc)^2+(-2ac)^2+2.ab.bc+2.ab.(-2ac)+2.bc.(-2ac)]=0$$

$$\Rightarrow [b^2c^2 + b^2a^2 - 2ab^2c - 4a^2bc + 4ab^2c + 4a^2c^2 - 4abc^2] = 0$$

$$\Rightarrow [b^2a^2 + b^2c^2 + 4a^2c^2 - 2ab^2c - 4a^2bc + 4ab^2c - 4abc^2] = 0$$

$$\Rightarrow (ab + bc - 2ac)^2 = 0$$

$$\Rightarrow ab + bc - 2ac = 0$$

$$\Rightarrow ab + bc = 2ac$$

$$\Rightarrow b(a + c) = 2ac$$

$$\Rightarrow \frac{2}{b} = \frac{a+c}{ac}$$

$$\Rightarrow \frac{2}{b} = \frac{1}{c} + \frac{1}{a}$$

89. (c) We are given that $x^2 + 7x - 14\left(k^2 - \frac{7}{8}\right) = 0$. Let us check for the nature of roots.

We have,

$$D = b^2 - 4ac = 7^2 - 4.1.14\left(k^2 - \frac{7}{8}\right)$$

$$= 49 + 56k^2 - 49 = 56$$

Now,

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-7 \pm \sqrt{56k^2}}{2} = \frac{-7 \pm 2k\sqrt{14}}{2}$$

Since $\sqrt{14}$ is an irrational number, therefore any value of k will give an irrational number only.

Therefore, the given equation has no integral roots.

90. (b) 91. (c) 92. (a)

93. (c) $x^2 - 6x - 27 > 0$; $x^2 - 9x + 3x - 27 > 0$

$(x - 9)(x + 3) > 0 \;\therefore\; x = (-\infty, 3) \cup (9, “)$

94. (b) Equation, $ax^2 + bx + c = 0$ have root α and β then $a\alpha^2 + b\alpha + c = 0 \Rightarrow a\alpha^2 + b\alpha = -c$

$$\Rightarrow (a\alpha + b) = \frac{-c}{\alpha}$$

$$a\beta^2 + b\beta + c = 0 \Rightarrow a\beta^2 + b\beta = -c \Rightarrow (a\beta + b) = \frac{-c}{\beta}$$

$$\text{Now, } \frac{1}{(a\alpha + b)} + \frac{1}{(a\beta + b)} = -\frac{\alpha}{c} - \frac{\beta}{c}$$

$$= -\frac{(\alpha+\beta)}{c} = -\frac{\left(-\frac{b}{a}\right)}{c} = \frac{b}{ac}$$

95. (b) Let $y = 2x^2 + 5x + 5$

$$= 2\left\{x^2 + \frac{5}{2}x + \frac{25}{16}\right\} + 5 - \frac{25}{8} \quad = \quad 2\left\{x + \frac{5}{4}\right\}^2 + \frac{15}{8}$$

For minimum value, $\left(x + \frac{5}{4}\right) = 0$

$$\therefore \quad y_{min} = \frac{15}{8}$$

96. (d) $f(x) = ax^3 + bx^2 + cx + d$ ...(i)

α, β, γ are the zeroes of f(x)

Then

$$\alpha + \beta + \gamma = \frac{-b}{a} \quad \text{...(ii)}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} \quad \text{...(iii)}$$

Squaring (ii), we get

$$\alpha^2 + \beta^2 + \gamma^2 + 2(\alpha\beta + \beta\gamma + \gamma\alpha) = \frac{b^2}{a^2}$$

$$\alpha^2 + \beta^2 + \gamma^2 + \frac{2c}{a} = \frac{b^2}{a^2}$$

[Form (iii)]

$$\alpha^2 + \beta^2 + \gamma^2 = \frac{b^2}{a^2} - \frac{2c}{a}$$

$$= \frac{b^2 - 2ac}{a^2}$$

97. (b) Given equation

$x^2 - 15x + r = 0$

Sum of roots = 15

$p + q = 15$ ...(i)

and $p - q = 1$ ...(ii)

From equation (i) and (ii) we have

$p = 8, q = 7$

Now, $p^2 - 15p + r = 0$

$(8)^2 - 15(8) + r = 0$

$\therefore r = 56$

98. (d) Root of the equation

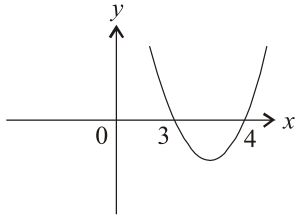
$x^2 - 7x + 12 = 0$

$(x - 3)(x - 4) = 0$

$\therefore x = 3$ and 4

For, $x^2 - 7x + 12 > 0$

$x = (-\infty, 3) \text{ U } (4, \infty)$.



As we can see from the graph of the quadratic equation, that the value of the equation is greater than zero for the values of $x < 3$ and $x > 4$

99. (a) p and q are roots of

$x^2 + px + q = 0$

$\Rightarrow \; p + q = -p$ and $pq = q$

$\Rightarrow \; pq - q = 0$

$\Rightarrow \; q(p - 1) = 0 \Rightarrow \; (p - 1) = 0 \Rightarrow p = 1$

Adding p in equation $p + q = -p$

$\Rightarrow \ 1 + q = -1$

$\Rightarrow \ q = -2$

100. (d) $(b - 6)$ is a root of

$x^2 - 6x + b = 0$

$\Rightarrow \ (b - 6)^2 - 6(b - 6) + b = 0$

$b^2 + 36 - 12b - 6b + 36 + b = 0$

$b^2 - 17b + 72 = 0$

$b^2 - 9b - 8b + 72 = 0$

$b(b - 9) - 8(b - 9) = 0$

$(b - 8)(b - 9) = 0$

$\Rightarrow \ b = 9$ or $b = 8$

Maximum value of $b^2 = 9^2 = 81$

101. (c) $Px^2 + 3x + 2q = 0$

Let two root are α and β

then, $\alpha + \beta = -6 = \dfrac{-3}{P}$

$\therefore \ P = \dfrac{1}{2}$

$\alpha.\beta = -6 = \dfrac{2q}{P}$

$\dfrac{q}{p} = -3$

$q = -3p = -3 \times \dfrac{1}{2} = -1.5$

$(p - q) = \left(\dfrac{1}{2} + 1.5\right) = 2$

102. (b) We know that two equation

$a_1x^2 + b_1x + c_1 = 0$

$a_2x^2 + b_2 + c_2 = 0$

have common root when

$(c_1a_2 - a_1c_2)^2 = (b_1c_2 - c_1b_2)(a_1b_2 - b_1a_2)$

So, for $x^2 + 5x + 6 = 0$ and $x^2 + kx + 1 = 0$

we have $(5)^2 = (5 - 6x)(x - 5)$

$\Rightarrow \ 25 = -6x^2 + 35x - 25$

$\Rightarrow \ 6x^2 - 35x + 50 = 0$

$\Rightarrow \ x = \dfrac{5}{2}$ or $\dfrac{10}{3}$

103. (a) As α and β are roots of the equation $x^2 + kx - 15 = 0$

Then, sum of roots $(\alpha + \beta) = -k$,

product of roots $\alpha\beta = -15$ and $(\alpha - \beta) = 8$ {given}

$(\alpha - \beta)^2 + 4\alpha\beta = (\alpha + \beta)^2$

$64 + (4 \times -15) = k^2$

$\Rightarrow \ k^2 = 4 \ \Rightarrow \ k = 2$

CHAPTER 16

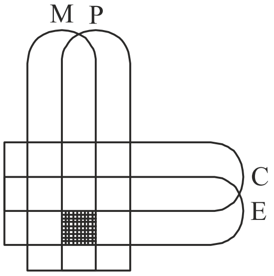
Set Theory

1. What is $\left\{\left[(A \cup B)' \cap A\right]\right\} - (A - B)$ equal to? **[2007-I]**
   (a) ϕ (b) A (c) B (d) B′

2. If A = $\{(2^{2n} - 3n - 1) \mid n \in N\}$
   and B = $\{9(n - 1) \mid n \in N\}$,
   then which one of the following is correct? **[2007-I]**
   (a) $A \subset B$
   (b) $B \subset A$
   (c) A = B
   (d) Neither A is a subset of B nor B is a subset of A

3. Which one of the following is correct? **[2007-I]**
   (a) $\{\phi\} \subset \{\{\phi\}, \{\{\phi\}\}\}$ (b) $\{\phi\} \in \{\{\phi\}, \{\{\phi\}\}\}$
   (c) $\phi \in \{\{\phi\}, \{\{\phi\}\}\}$ (d) $\phi = \{\{\phi\}, \{\{\phi\}\}\}$

4. Let :
   P = Set of all integral multiples of 3
   Q = Set of all integral multiples of 4
   R = Set of all integral multiples of 6
   Consider the following relations:
   I. $P \cup Q = R$
   II. $P \subset R$
   III. $R \subset (P \cup Q)$
   Which of the relations given above is/are correct? **[2007-II]**
   (a) Only I (b) Only II
   (c) Only III (d) II and III

5. 

   The Venn diagram given above represents four sets of students who have opted for Mathematics, Physics, Chemistry and Electronics. What does the shaded region represent? **[2007-II]**
   (a) Students who opted for Physics, Chemistry and Electronics
   (b) Students who opted for Mathematics, Physics and Chemistry
   (c) Students who opted for Mathematics, Physics and Electronics only
   (d) Students who opted for Mathematics, Chemistry and Electronics only

6. If two sets A and B have 2n and 4n elements, respectively. When n is a natural number. What can be the minimum number of elements in $A \cup B$? **[2007-II]**
   (a) 2n (b) 3n (c) 4n (d) 6n

7. If A = $\{x : x^2 - 6x + 8 = 0\}$ and
   B = $\{x : 2x^2 + 3x - 2 = 0\}$
   Then, which one of the following is correct? **[2007-II]**
   (a) $A \subseteq B$
   (b) $B \subseteq A$
   (c) Neither $A \subseteq B$ nor $B \subseteq A$
   (d) A = B

8. If A = {1, 2, 3, 4}, then what is the number of subsets of A with atleast three elements? **[2008-I]**
   (a) 3 (b) 4 (c) 5 (d) 10

9. The set {2, 4, 16, 256, ...} can be represented as which one of the following? **[2008-II]**
   (a) $\left\{x \in N \mid x = 2^{2^n}, n \in N\right\}$
   (b) $\left\{x \in N \mid x = 2^{2^n}, n = 0, 1, 2, ...\right\}$
   (c) $\left\{x \in N \mid x = 2^{4n}, n = 0, 1, 2, ...\right\}$
   (d) $\left\{x \in N \mid x = 2^{2n}, n = 0, 1, 2, ...\right\}$

10. Which one of the following is a correct statement? **[2008-II]**
    (a) $\phi \in \phi$ (b) $\phi \notin P(\phi)$
    (c) $\phi = P(\phi)$ (d) $\phi \in P(\phi)$

11. The set of integers is closed with respect to which one of the following? **[2008-II]**
    (a) Addition only
    (b) Multiplication only
    (c) Both addition and Multiplication
    (d) Division

12. Which one of the following is not correct in respect of the sets A and B? **[2009-II]**
    (a) If $A \subseteq B$, then $B \cup A = B$
    (b) If $A \subseteq B$, then $A \cap (A - B) = \phi$
    (c) If $A \subseteq B$, then $B \cap A = A$
    (d) If $A \cap B = \phi$, then either $A = \phi$ or $B = \phi$

13. Which one of the following is a correct statement? **[2010-I]**
    (a) $\{a\} \in \{\{a\}, \{b\}, c\}$
    (b) $\{a\} \subseteq \{\{a\}, b, c\}$
    (c) $\{a, b\} \subseteq \{\{a\}, b, c\}$
    (d) $a \subseteq \{\{a\}, b, c\}$

14. Which one of the following is an infinite set? **[2010-I]**
(a) $\{x : x$ is a whole number less than or equal to $1000\}$
(b) $\{x : x$ is a natural number less than $1000\}$
(c) $\{x : x$ is a positive integer less than or equal to $1000\}$
(d) $\{x : x$ is an integer and less than $1000\}$

15. In an examination, 52% candidates failed in English and 42% failed in Mathematics. If 17% candidates failed in both English and Mathematics, what percentage of candidates passed in both the subjects? **[2011-I]**
(a) 18% (b) 21% (c) 23% (d) 25%

16. The set $S = \{x \in N : x + 3 = 3\}$ is a **[2011-I]**
(a) null set (b) singleton set
(c) infinite set (d) None of the above

17. Consider the following statements:
I. Set of points of a given line is a finite set.
II. Intelligent students in a class is a set.
III. Good books in a school library is a set.
Which of the above statements is/are not correct? **[2011-II]**
(a) Only I (b) Both II and III
(c) Both I and II (d) I, II and III

18. If $A = \{x : x$ is an even natural number$\}$,
$B = \{x : x$ is a natural number and multiple of $5\}$ and
$C = \{x : x$ is a natural number and multiple of $10\}$, then what is the value of $A \cap (B \cup C)$? **[2012-I]**
(a) $\{10, 20, 30, ...\}$ (b) $\{5, 10, 15, 20, ...\}$
(c) $\{2, 4, 6, ...\}$ (d) $\{20, 40, 60, ...\}$

19. If a set A contains 60 elements and another set B contains 70 elements and there are 50 elements in common, then how many elements does $A \cup B$ contain? **[2012-I]**
(a) 130 (b) 100 (c) 80 (d) 70

20. Let $x \in \{2, 3, 4\}$ and $y \in \{4, 6, 9, 10\}$. If A be the set of all order pairs (x, y) such that x is a factor of y. Then, how many elements does the set A contain? **[2012-II]**
(a) 12 (b) 10 (c) 7 (d) 6

21. Which one of the following is a null set? **[2012-II]**
(a) $A = \{x$ is a real number $: x > 1$ and $x < 1\}$
(b) $B = \{x : x + 3 = 3\}$
(c) $C = \{\phi\}$
(d) $D = \{x$ is a real number $: x \geq 1$ and $x \leq 1\}$

22. In a school there are 30 teachers who teach Mathematics or Physics. Of these teachers, 20 teach Mathematics and 15 teach Physics, 5 teach both Mathematics and Physics. The number of teachers teaching only Mathematics is **[2013-I]**
(a) 5 (b) 10 (c) 15 (d) 20

23. If $A = \{x : x$ is an odd integer$\}$ and $B = \{x : x^2 - 8x + 15 = 0\}$. Then, which one of the following is correct? **[2013-I]**
(a) $A = B$ (b) $A \subseteq B$
(c) $B \subseteq A$ (d) $A \subseteq B^C$

24. Consider the following in respect of the sets A and B.
I. $(A \cap B) \subseteq A$
II. $(A \cap B) \subseteq B$
III. $A \subseteq (A \cup B)$
Which of the above are correct? **[2013-I]**
(a) I and II (b) II and III
(c) I and III (d) I, II and III

25. The set of natural numbers is closed under
I. addition II. subtraction
III. multipliction IV. division **[2013-I]**
Which of the above is/are correct?
(a) Only I (b) Both I and III
(c) I, II and III (d) Both III and IV

26. In a class of 110 students, x students take both Mathematics and Statistics, 2x + 20 students take Mathematics and 2x + 30 students take Statistics. There are no students who take neither Mathematics nor Statistics. What is x equal to? **[2013-II]**
(a) 15 (b) 20 (c) 25 (d) 30

27. Out of 105 students taking an examination English and Mathematics, 80 students pass in English, 75 students pass in Mathematics 10 students fail in both the subjects. How many students fail in only one subject? **[2014-I]**
(a) 26 (b) 30 (c) 35 (d) 45

28. If A and B are any two non-empty subsets of a set E, then what is $A \cup (A \cap B)$ equal to? **[2014-I]**
(a) $A \cap B$ (b) $A \cup B$
(c) A (d) B

29. If A is a non-empty subset of a set E, then what is $E \cup (A \cap \phi) - (A - \phi)$ equal to? **[2014-I]**
(a) A (b) Complement of A
(c) f (d) E

30. Let A and B be finite non-empty sets with the number of elements in $A = m$ and number of elements in $B = n$. Let $m > n$. If for some integer $k \geq 1$, the number of non-empty subsets of $A = 2^k$ + the number of non-empty subsets of B, then which one of the following is correct? **[2015-I]**
(a) $m = n + 2$
(b) $m = n + 1$
(c) $m = n + p$ for some odd prime number p
(d) $m = n + t$ for some composite number t

31. Let A denote the set of quadrilaterals having two diagonals equal and bisecting each other. Let B denote the set of quadrilaterals having diagonals bisecting each other at 90°. Then $A \cap B$ denotes **[2015-II]**
(a) the set of parallelograms
(b) the set of rhombuses
(c) the set of squares
(d) the set of rectangles

32. In a gathering of 100 people, 70 of them can speak Hindi, 60 can speak English and 30 can speak French Further, 30 of them can speak both Hindi and English. 20 can speak both Hindi and French. If x is the number of people who can speak both English and French, then which one of the following is correct? (Assume that everyone can speak at least one of the three languages) **[2016-I]**

(a) $9 < x \le 30$ (b) $0 \le x < 8$
(c) $x = 9$ (d) $x = 8$

33. A is a set of positive integers such that when divided by 2, 3, 4, 5 and 6 leaves the remainder 1, 2, 3, 4 and 5 respectively. How many integers between 0 and 100 belong to the set A ? **[2016-II]**

(a) No integer (b) One
(c) Two (d) Three

34. In the quadratic equation $x^2 + ax + b = 0$, a and b can take any value from the set {1, 2, 3, 4}. How many pairs of values of a and b are possible in order that the quadratic equation has real roots? **[2016-II]**

(a) 6 (b) 7 (c) 8 (d) 16

35. Let A = {7, 8, 9, 10, 11, 12} and B = {7, 10, 14, 15}. What is the number of elements in (A–B) and (B–A) respectively ? **[2016-II]**

(a) 2 and 4 (b) 4 and 2 (c) 2 and 2 (d) 4 and 4

36. Let S be a set of first ten natural numbers. What is the possible number of pairs (a, b) where a, b E S and $a \neq b$ such that the product ab (>12) leaves remainder 4 when divided by 12 ? **[2016-II]**

(a) 4 (b) 6 (c) 8 (d) 10

37. If A = (x : x is a multiple of 7),
B = (x : x is a multiple of 5) and
C = (X : x is a multiple of 35),
then which one of the following is a null set ? **[2017-I]**

(a) $(A - B) \cup C$ (b) $(A - B) - C$
(c) $(A \cap B) \cap C$ (d) $(A \cap B) - C$

38. Which one of the following is a correct statement? **[2017-II]**

(a) $\{x : x + 5 = 5\} = \phi$ (b) $\{x : x + 5 = 5\} = \{0\}$
(c) $\{x : x + 5 = 5\} = 0$ (d) $\{x : x + 5 = 5\} = \{\phi\}$

39. In an examination, 35% students failed in Hindi, 45% students failed in English and 20% students failed in both the subjects. What is the percentage of students passing in both the subjects? **[2017-II]**

(a) 0 (b) 20 (c) 30 (d) 40

40. In an examination, 50% of the candidates failed in English, 40% failed in Hindi and 15% failed in both the subjects. The percentage of candidates who passed in both English and Hindi is **[2018-II]**

(a) 20% (b) 25% (c) 60% (d) 75%

41. In a class of 60 boys, there are 45 boys who play chess and 30 boys who plays carrom. If every boy of the class plays at least one of the two games, then how many boys play carrom only? **[2018-II]**

(a) 30 (b) 20 (c) 15 (d) 10

Consider the following for the next 04 (four) items that follow :

In an examination of Class XII, 55% students passed in Biology, 62% passed in Physics, 60% passed in Chemistry, 25% passed in Physics and Biology, 30% passed in Physics and Chemistry, 28% passed in Biology and Chemistry. Only 2% failed in all the subjects. **[2018-II]**

42. What percentage of students passed in all the three subjects? **[2018-II]**

(a) 6 (b) 5 (c) 4 (d) 3

43. What percentage of students passed in exactly one subject? **[2018-II]**

(a) 21 (b) 23 (c) 25 (d) 27

44. If the number of students is 360, then how many passed in at least two subjects? **[2018-II]**

(a) 270 (b) 263 (c) 265 (d) 260

45. What is the ratio of number of students who passed in both Physics and Chemistry to number of students who passed in both Biology and Physics but not Chemistry? **[2018-II]**

(a) 7 : 10 (b) 10 : 7 (c) 9 : 7 (d) 7 : 9

Consider the following for the next three (03) items :

In a certain town of population size 1,00,000 three types of newspapers (I, II and III) are available. The percentages of the people in the town who read these papers are as follows : **[2019-I]**

Newspaper	*Proportion of readers*
I	10%
II	30%
III	5%
Both I and II	8%
Both II and III	4%
Both I and III	2%
All the three (I, II and III)	1%

46. What is the number of people who read only one newspaper? **[2019-I]**

(a) 20,000 (b) 25,000 (c) 30,000 (d) 35,000

47. What is the number of people who read at least two newspapers? **[2019-I]**

(a) 12,000 (b) 13,000 (c) 14,000 (d) 15,000

48. What is the number of people who do ***not*** read any of these three newspapers? **[2019-I]**

(a) 62,000 (b) 64,000 (c) 66,000 (d) 68,000

49. In an examination, 52% candidates failed in English and 42% failed in Mathematics. If 17% failed in both the subjects, then what percent passed in both the subjects ? **[2019-I]**

(a) 77 (b) 58 (c) 48 (d) 23

50. If $X = \{a, \{b\}, c\}$, $Y = \{\{a\}, b, c\}$ and $Z = \{a, b, \{c\}\}$, then $(X \cap Y) \cap Z$ equals to **[2019-II]**

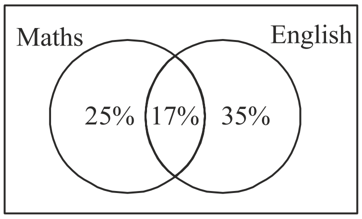
(a) $\{a, b, c\}$ (b) $\{\{a\}, \{b\}, \{c\}\}$
(c) $\{\Phi\}$ (d) Φ

51. In a competitive examination, 250 students have registered. Out of these, 50 students have registered for Physics, 75 students for Mathematics and 35 students for both Mathematics and Physics. What is the number of students who have registered neither for Physics nor for Mathematics? **[2020-I]**

(a) 90 (b) 100 (c) 150 (d) 160

HINTS & SOLUTIONS

1. (a) $\left\{(A \cup B)' \cap A\right\} - (A - B)$
$= \{(U - (A \cup B)) \cap A\} - (A - B)$
$= \{(U \cap A) - \{(A \cup B) \cap A\}\} - (A - B)$
$= \{A - A\} - (A - B)$
$= \phi - (A - B) = \phi$
2. (a) Given:
$A = \{(2^{2n} - 3n - 1) \mid n \in N\}$
$= \{0, 9, 54, 243, ...\}$
and $B = \{9(n - 1) \mid n \in N\}$
$= \{0, 9, 18, 27, ...\}$
From the above, it is clear that $A \subset B$.
3. (b) Here, $\{\phi\}$ is an element of $\{\phi\} \in \{\{\phi\}, \{\{\phi\}\}\}$.
4. (c) Here, $P = \{..., -6, -3, 0, 3, 6, ...\}$
$Q = \{..., -8, -4, 0, 4, 8, ...\}$
and $R = \{..., -36, -6, 0, 6, 36, ...\}$
I. $P \cup Q$
$= \{..., -8, -6, -4, -3, 0\ 3, 4, 6, 8, ...\} \neq R$
II. Here, $P \not\subset R$
III. Here, $R \subset (P \cup Q)$ is true.
5. (c) It is clear from the given Venn diagram that shaded portion represent the students who opted for Mathematics, Physics and Electronics only.
6. (c) Here, $n(A \cap B) = 2n$
$\therefore n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$= 2n + 4n - 2n = 4n$
Hence, minimum number of elements of $A \cup B$ is 4n.
7. (c) Given, $A \equiv x^2 - 6x + 8 = 0$
$\Rightarrow (x - 4)(x - 2) = 0$
$\Rightarrow x = 4, 2$
and $B \equiv 2x^2 + 3x - 2 = 0$
$\Rightarrow (2x - 1)(x + 2) = 0$
$\Rightarrow x = \frac{1}{2}, -2$
Hence, neither $A \subseteq B$ nor $B \subseteq A$.
8. (c) Given, $A = \{1, 2, 3, 4\}$
So, the required subsets are $\{1, 2, 3\}$, $\{1, 2, 4\}$, $\{1, 3, 4\}$, $\{2, 3, 4\}$ and $\{1, 2, 3, 4\}$
9. (b) Let $A = \{2, 4, 16, 256, ...\}$
This set can be re-written as :
$$\left\{x \in N \mid x = 2^{2^n},\ n = 0, 1, 2, ...\right\}$$
10. (d) In the given options, the correct statement is $\phi \in P(\phi)$.
11. (c) The set of integers is closed with respect to addition and multiplication.
e.g., Let $z = \{..., -3, -2, -1, 0, 1, 2, 3, ...\}$
$1 + 2 = 3$ and $-2 - 1 = -3$
(for addition)
$1 \times 2 = 2$ and $2 \times 1 = 2$
(for multiplication)
Hence, the number of subsets is 5.
12. (d) If $A \cap B = \phi$, then it is not necessary that either A = f or B = f
13. (a) Here, $\{a\}$ is an element of $\{\{a\}, \{b\}, \{c\}\}$.
14. (d) In a given option only,
$\{x : x$ is an integer and less than $1000\}$
i.e., $x \in (-\infty, 1000)$ is an infinite set.
15. (c) Total number of candidates = 100%

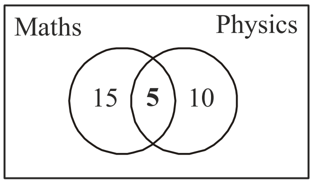


Percentage of candidates passed in both the subjects $= \{100 - (25 + 17 + 35)\}\% = 23\%$
16. (a) Given:
$S = x \in N : \{x + 3 = 3\}$
$S = \{\ \}$
Thus, S is a null set.
17. (d) I. The set of points of a given line is not a finite set.
II. Here, we cannot decide, which students are intelligent.
III. Here, we cannot decide, which books are good a school library.
18. (a) We know that
$A \cap (B \cup C) = \{A \cap B) \cup (A \cap C)$
Example:
A = Set of an even natural number
$A = \{2, 4, 6, 8, 10, 12, ...\}$
B = Set of natural number and multiples of 5.
$B = \{5, 10, 15, 20, 25, ...\}$
C = Set of natural number and multiple of 10.
$C = \{10, 20, 30, 40, 50, ...\}$
$A \cap B = \{2, 4, 6, 8, 10, 12, ...\} \cap \{5, 10, 15, 20, 25, ...\}$
$= \{10, 20, 30, ...\}$
$A \cap C = \{2, 4, 6, 8, 10, 12, ...\} \cap \{10, 20, 30, 40, 50, ...\}$
$= \{10, 20, 30, 40, ...\}$
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
$= \{10, 20, 30, ...\} \cup \{10, 20, 30, 40, ...\}$
$= \{10, 20, 30, 40, ...\}$
19. (c) Here, $n(A) = 60$, $n(B) = 70$, $n(A \cap B) = 50$ and $n(A \cup B) = ?$
We know that:
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$= 60 + 70 - 50 = 130 - 50 = 80$

20. (d) Given that
$x \in \{2, 3, 4\}$
and
$y \in \{4, 6, 9, 10\}$
$A = x \times y$
But, A is set of pairs in which 1^{st} number is factor of second number.
$A = \{2, 3, 4\} \times \{4, 6, 9, 10\}$
$= \{(2, 4); (2, 6); (2, 10); (3, 6); (3, 9); (4, 4)\}$
Total number of elements = 6

21. (a) From option (a),
$A = \{x \text{ is a real number} : x > 1 \text{ and } x < 1\}$.
So, there is no element which is greater or less than 1.
So, A is a null set.
From option (b),
$B = \{x : x + 3 = 3\} = \{0\}$
= Singleton set
From option (c):
$C = \{\phi\}$ = Singleton set
From option (d),
$D = \{x \text{ is a real number} : x \geq 1 \text{ and } x \leq 1\}$
$= \{1\}$ = Singleton set

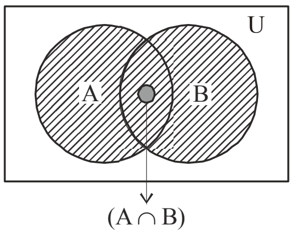
22. (c) Total number of teachers = 30.



Number of teachers who teaches only Math
$= 20 - 5 = 15$.

23. (c) Given that,
$A = \{x : x \text{ is an odd integer}\}$
and $B = \{x : x^2 - 8x + 15 = 0\}$
$= (x : x^2 - 5x - 3x + 15 = 0)$
$= \{x : x(x - 5) - 3(x - 5) = 0\}$
$= \{x : (x - 5)(x - 3) = 0\} = \{3, 5\}$
Since, B has the odd elements,
$\therefore B \subseteq A$

24. (d) From figure,



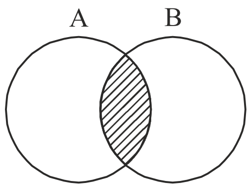
$(A \cap B) \subseteq A$ (true)
$(A \cap B) \subseteq B$ (true)
and $A \subseteq (A \cup B)$ also (true)
Thus, all three statements are correct.
Shaded region = $(A \cup B)$.

25. (b) Set of natural numbers,
$N = \{1, 2, 3, 4, 5, 6, ...$
(i) Addition: $2 + 3 = 5$ is also an element of N.
(ii) Subtraction: $2 - 3 = -1 \notin N$
(iii) Multiplication: $2 \times 3 = 6 \in N$
(iv) Division: $\frac{3}{2} = 1.5 \notin N$ (since, N contains only positive integers)
Therefore, the set of natural numbers is closed under addition and multiplication.

26. (b) $n(M) = 2x + 20$
$n(S) = 2x + 30$
$n(M \cap S) = x$
$n(M \cup S) = 110$
We know that,
$n(M \cup S) = n(M) + n(S) - n(M \cap S)$
$\Rightarrow 110 = 2x + 20 + 2x + 30 - x$
$\Rightarrow 110 = 3x + 50$
$\Rightarrow 3x = 60$
$\therefore x = 20$

27. (d) Number of students failing in Mathematics
$= 105 - 75 = 30$
Number of students failing in English
$= 105 - 80 = 25$
$\therefore$ Number of students failing in 1 subject
$= (25 + 30) - 10 = 45$

28. (c) A and B are non-empty subsets of E.



$\therefore A \cup (A \cap B) = A \cup$ (Shaded portion) $= A$

29. (b) $E \cup (A \cap \phi) - (A - \phi)$
$= E \cup \phi - A = E - A = A'$

30. (b) A and B are non-empty set
$A = m$ elements
$B = n$ elements
$m > n, A > B$
$m = n + 1$

31. (b) Externality is a result of an economic activity which is realised by third one. It may be of two types - negative and positive. Pollution caused by a factory is negative one and increase in land price of a plot due to construction of a road.

32. (c) Let n(A) be no. of people who speak Hindi
$\Rightarrow n(A) = 70$
Let n(B) be no. of people who speak English

$\Rightarrow n(B) = 60$

Let n(C) be no. of people who speak French

$\Rightarrow n(C) = 30$

Given $n(A \cup B \cup C) = 100$

$n(A \cap B) = 30$, $n(A \cap C) = 20$

$n(B \cap C) = x$, $n(A \cap B \cap C) = 1$

We know that

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B)$

$-n(B \cap C) - n(C \cap A) - n(A \cap B \cap C)$

$100 = 70 + 60 + 30 - 30 - x - 20 - 1$

$100 = 109 - x$

$\Rightarrow x = 109 - 100$

$\Rightarrow x = 9$

$\therefore$ Option (c) is correct.

33. (b) LCM of 2, 3, 4, 5, 6 = 60
Number when divided by 2, 3, 4, 5, 6 gives remainder 1, 2, 3, 4, 5 respectively here 2 – 1 = 1
3 – 2 = 1, 4 – 3, = 1, 5 – 4 = 1, 6 – 5 = 1
So required no. = 60 – 1 = 59
other no. 59 × 2 = 118
i.e. there is one no. below 100.

34. (b) $x^2 + ax + b = 0$ value of (a, b)
$a^2 - ab \geq 0$ [1, 2, 3, 4]
for real roots
$1^2 - 4 \times 1 \geq 0$ (1, 1)
not possible
$(2, 1) \rightarrow 2^2 - 4 \times 1 \geq 0$ possible
$(3, 1) \rightarrow$ possible
(3, 2), (4, 1), (4, 2) (4, 3) (4, 4) $\rightarrow$ possible
So $\rightarrow$ possible values can be possible

35. (b) A = {7, 8, 9, 10, 11, 12}; B = {7, 10, 14, 15}
(A – B) = {8, 9, 11, 12} (B – A) = {14, 15}
n (A – B) = 4; n (B – A) = 2

36. (c) Since numbers which and leave 4 as Remainder when devided by 12 are
16, 28, 40, 52, 64, 76, 88, 100 and 124
$16 = 2 \times 8, 8 \times 2$
$28 = 4 \times 7, 7 \times 4$
$40 = 4 \times 10, 10 \times 4, 5 \times 5, 8 \times 5$
All remaining numbers doesn't meet the requirement
Aswer is 8.

37. (d) A = {7, 14, 21............}
B = {5, 10, 15.............}
C = {35, 70, 105.............}
$(A \cap B)$ = {35, 70, 105.......} i.e. LCM of 7 and 5 i.e. multiple of 35
$A \cap B - C = \phi$

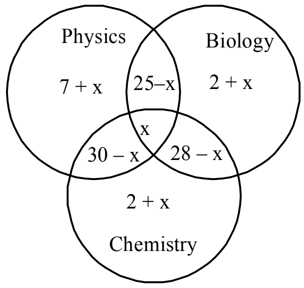
38. (b) Solving the given equation for x, we have
$x + 5 = 5 \Rightarrow x = 5 - 5 = 0$
Therefore, in the given set we have only one element i.e. 0. Element φ is a null set i.e. no elements in the set.
Hence, option (b) is the correct answer.

39. (d) Let the total number of students be 100%
Number of students failed in Hindi = 35%
Number of students failed in English = 45%
Number of students failed in both the subjects = 20%
Total number of students failed = (35 + 45 – 20)% = 60%
Number of students passing in both the subjects = (100 – 60)% = 40%

40. (b) Percent of candidates, who failed in either Hindi or English = (50 + 40 – 15)% = 75%
$\therefore$ Percent of candidates who passed in both subject = 100 – 75 = 25%

41. (c) Total number of boys in the class
$n(A \cup B) = 60$
Number of boy play chess n(A) = 45
Number of boy play carrom n(B) = 30
$\therefore$ Number of boy play both chess and Carrom
$n(A \cap B) = n(A) + n(B) - n(A \cup B) = 45 + 30 - 60 = 15$
Number of boys plays only carrom $n(B) - n(A \cap B) = 30 - 15 = 15$

Sol. (42-45):



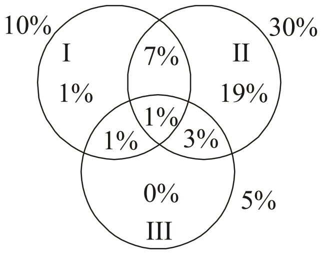
42. (c) Total passed student = 98%
$7 + x + 2 + x + 2 + x + 30 - x + 25 - x + 28 - x + x = 98$
$94 + x = 98$; $x = 4\%$

43. (b) Percentage of students passed in exactly one subject
$= 7 + x + 2 + x + 2 + x = 11 + 3x = 11 + 12 = 23\%$

44. (a) Passed in at least two subject
$$= \frac{25 - x + 30 - x + 28 - x + x}{100} \times 360$$
$$= \frac{83 - 2x}{100} \times 360 = 270$$

45. (b) Ratio $= \frac{30}{25 - x} = \frac{30}{25 - 4} = \frac{30}{21} = \frac{10}{7}$

46. (a)



The number of people who read only 1, only II and only II are

$1\% + 19\% + 0\% = 20\%$ of total population $= 20/100 \times 100000 = 20000$

47. (a) As we can see from the above venn diagram the number of people who read two or more newspapers are $1\% + 1\% + 3\% + 7\% = 12\% = 12/100 \times 100000$

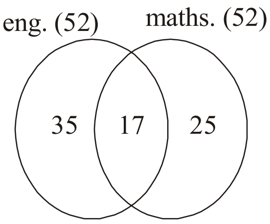
$= 12000$

48. (d) Number of people who do not read any of these newspaper = total population – number of people who read atleast one of these newspapers.

number of people who read atleast one of these newspapers $= 1\% + 1\% + 3\% + 1\% + 7\% + 19\% = 32\%$ of total population $= 32000$

required number of people $= 100000 - 32000 = 68000$

49. (d)



venn diagram of no. of failed students

No. of students failed in English only $= 52 - 17 = 35$

No. of students failed in maths only $= 42 - 17 = 25$

Total no. of failed students in either of the subjects $= 35 + 17 + 25 = 77$

No. of passed student in both subjects $= 100 - 77 = 23$

50. (d) $(X \cap Y) = \{a, \{b\}, c\} \cap \{\{a\}, b, c\} = c$

Now, $(X \cap Y) \cap Z$

$= c \cap \{a, b, \{c\}\} = \phi$

51. (d) Number of students registered for

Physics $n(P) = 50$

Mathematics $n(M) = 75$

Number of students registered for both subjects $n(P \cap M) = 35$.

Number of students, registered for either physics or mathematics $n(P \cup M) = n(P) + n(M) - n(P \cap M)$.

$= 50 + 75 - 35 = 90$

$\therefore$ Number of students registered neither for physics nor for mathematics

$n(\overline{P \cup M}) = 250 - 90 = 160$.

CHAPTER

17 Trigonometric Ratios & Identities

1. Which one of the following is true for some value of θ, where $0° \le \theta \le 90°$? **[2007-I]**
(a) $\sin\theta = \sqrt{2}$ (b) $\sin\theta + \cos\theta = 2$
(c) $\sin\theta + \cos\theta = 0$ (d) $\sin\theta - \cos\theta = 1$

2. If $\sin\theta + \text{cosec}\,\theta = 2$, then what is the value of $\sin^4\theta + \cos^4\theta$? **[2007-I]**
(a) 2 (b) 2^2 (c) 2^3 (d) 1

3. If $\theta \in R$ be such that $\sec\theta > 0$ and $2\sec^2\theta + \sec\theta - 6 = 0$. Then, what is the value of cosec θ? **[2007-I]**
(a) $\sqrt{5}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{3}{\sqrt{5}}$ (d) $\frac{2}{\sqrt{3}}$

4. Under which one of the following conditions is the trigonometrical identify $\frac{\sin x}{(1+\cos x)} = \frac{(1-\cos x)}{\sin x}$ true? **[2007-I]**
(a) x is not a multiple of 360º
(b) x is not an odd multiple of 180º
(c) x is not a multiple of 180º
(d) None of the above

5. If $3\sin\theta + 4\cos\theta = 5$, then what is $3\cos\theta - 4\sin\theta$ equal to? **[2007-I]**
(a) 0 (b) 3 (c) 4 (d) 5

6. If $\sec\theta = \frac{13}{5}$, then what is the value of $\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta}$? **[2007-I]**
(a) 1 (b) 2 (c) 3 (d) 4

7. If $p = \sin^{10} x$, then which one of the following is correct for any value x? **[2007-II]**
(a) $p \ge 1$ (b) $0 \le p \le 1$
(c) $1 \le p \le 2$ (d) None of these

8. What is the value of the expression :
$$\cos^2\frac{\pi}{8} + 4\cos^2\frac{\pi}{4} - \sec\frac{\pi}{3} + 5\tan^2\frac{\pi}{3} + \sin^2\frac{\pi}{8}?$$
[2007-II]
(a) 8 (b) 10 (c) 16 (d) 18

9. Which of the following expression for $0° < \theta < 90°$ is/are independent of θ? **[2007-II]**
I. $\cos\theta(1 - \sin\theta)^{-1} + \cos\theta(1 + \sin\theta)^{-1}$
II. $\cos\theta(1 + \text{cosec}\,\theta)^{-1} + \cos\theta(\text{cosec}\,\theta - 1)^{-1}$
Select the correct answer unsing the codes given below :
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

10. If $\text{cosec}\,\theta = \frac{p}{q}$ and θ is acute, then what is the value of $\left(\sqrt{p^2 - q^2}\right)\tan\theta$? **[2007-II]**
(a) p (b) q (c) pq (d) $\sqrt{p^2 + q^2}$

11. If $a\cos\theta - b\sin\theta = c$, then what is the value of $a\sin\theta + b\cos\theta$? **[2007-II]**
(a) $\pm\sqrt{a^2 + b^2 + c^2}$ (b) $\pm\sqrt{a^2 - b^2 + c^2}$
(c) $\pm\sqrt{a^2 + b^2 - c^2}$ (d) $\pm\sqrt{a^2 - b^2 - c^2}$

12. If $2x^2\cos 60° - 4\cot^2 45° - 2\tan 60° = 0$, then what is the value of x? **[2007-II]**
(a) 2 (b) 3 (c) $\sqrt{3} - 1$ (d) $\sqrt{3} + 1$

13. Which one of the following statements is true in respect of the expression sin 31º + sin 32º ? **[2007-II]**
(a) Its value is 0
(b) Its value is 1
(c) Its value is less than 1
(d) Its value is greater than 1.

14. **Assertion (A) :** tan 50º > 1.
Reason (R) : $\tan\theta > 1$ for $0° < \theta < 90°$. **[2007-II]**
(a) A and R are correct and R is correct explanation of A.
(b) A and R are correct but R is not correct explanation of A.
(c) A is true but R is false.
(d) A is true but R is false.

15. Which one of the following is correct? **[2007-II]**
(a) sin 35º > cos 55º (b) $\cos 61° > \frac{1}{2}$
(c) $\sin 32° > \frac{1}{2}$ (d) tan 44º > 1

16. What is the value of x in the equation
$$x\frac{\text{cosec}^2 30°\ \sec^2 45°}{8\cos^2 45°\ \sin^2 60°} = \tan^2 60° - \tan^2 30°\ ?$$
[2007-II]
(a) x = 1 (b) x = 2 (c) $x = \frac{1}{2}$ (d) $x = \frac{3}{2}$

17. The smallest side of a right angled triangle has length 2 cm. The tangent of one acute angle is $\frac{3}{4}$. What is the hypotenuse of the triangle? **[2008-I]**

(a) 5 cm (b) 2.5 cm (c) 1.25 cm (d) $\frac{10}{3}$ cm

18. If $\sin x - \cos x = 0$, then what is the value of $\sin^4 x + \cos^4 x$? **[2008-I]**

(a) 1 (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

19. What is the expression $\frac{\tan x}{1+\sec x} - \frac{\tan x}{1-\sec x}$ equal to? **[2008-I]**

(a) cosec x (b) 2 cosec x
(c) 2 sin x (d) 2 cos x

20. If $\tan x = \frac{3}{4}$, where $0^\circ < x < 90^\circ$, then what is the value of sin x cos x? **[2008-I]**

(a) $\frac{3}{5}$ (b) $\frac{4}{5}$ (c) $\frac{12}{25}$ (d) $\frac{13}{25}$

21. Which one of the following is correct? **[2008-I]**

(a) $\tan x > 1$, $45^\circ < x < 90^\circ$

(b) $\sin x > \frac{1}{2}$, $0^\circ < x < 30^\circ$

(c) $\cos x > \frac{1}{2}$, $60^\circ < x < 90^\circ$

(d) sin x = cos x for some value of x, $30^\circ < x < 45^\circ$

22. What is the expression :

$(\sin^4 x - \cos^4 x + 1) \operatorname{cosec}^2 x$ equal to? **[2008-I]**

(a) 1 (b) 2 (c) 0 (d) –1

23. If $x + y = 90^\circ$, then what is $\sqrt{\cos x \operatorname{cosec} y - \cos x \sin y}$ equal to? **[2008-I]**

(a) cos x (b) sin x
(c) $\sqrt{\cos x}$ (d) $\sqrt{\sin x}$

DIRECTIONS (Qs. 24-25): *The following two questions consists of two statements, one labelled as the 'Assertion (A)' and the other as 'Reason (R)'. You are to examine these two statements carefully and select the answers to these items using the codes given below* :

Codes :

(a) Both A and R are individually true and R is the correct explanation of A.
(b) Both A and R are individually true and R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

24. **Assertion (A) :** $\sec^2 23^\circ - \tan^2 23^\circ = 1$.
Reason (R) : $\sec^2 \theta - \tan^2 \theta = 1$ for all real values of θ. **[2008-I]**

25. **Assertion (A) :** $\sin 1^\circ < \cos 1^\circ$.
Reason (R) : $\sin \theta < \cos \theta$ when $0^\circ < \theta < 90^\circ$. **[2008-I]**

26. What is the value of $\cot^2 \theta - \left(\frac{1}{\sin^2 \theta}\right) = ?$ **[2008-II]**

(a) $\frac{1}{2}$ (b) –1 (c) $-\frac{1}{2}$ (d) $\frac{3}{2}$

27. If $\sin^2 60^\circ + \cos^2 (3x - 9^\circ) = 1$, then what is the value of x? **[2008-II]**

(a) 24° (b) 23° (c) 22° (d) 21°

28. If $\cos A = \frac{5}{13}$, then what is the value of $\frac{\sin A - \cot A}{2 \tan A}$? **[2008-II]**

(a) $\frac{395}{3644}$ (b) $\frac{395}{3844}$ (c) $\frac{395}{3744}$ (d) $\frac{385}{3744}$

29. If sin x = cos y, ∠x and ∠y are acute angle then what is the relation between ∠x and ∠y = ? **[2008-II]**

(a) $x - y = \frac{\pi}{2}$ (b) $x + y = \frac{3\pi}{2}$

(c) $x + y = \frac{\pi}{2}$ (d) $x + y = \frac{\pi}{4}$

30. If $\sin \theta = \frac{m^2 - n^2}{m^2 + n^2}$, then what is the value of tan θ? **[2008-II]**

(a) $\frac{m^2 + n^2}{m^2 - n^2}$ (b) $\frac{2mn}{m^2 + n^2}$

(c) $\frac{m^2 - n^2}{2mn}$ (d) $\frac{m^2 + n^2}{2mn}$

31. If $\sin(x - y) = \frac{1}{2}$ and $\cos(x + y) = \frac{1}{2}$, then what is the value of x? **[2008-II]**

(a) 15° (b) 30° (c) 45° (d) 60°

32. If $1 + \tan \theta = \sqrt{2}$, then what is the value of $\cot \theta - 1$? **[2008-II]**

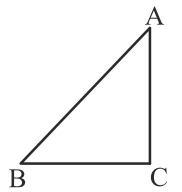
(a) $\frac{1}{\sqrt{2}}$ (b) $\sqrt{2}$ (c) 2 (d) $\frac{1}{2}$

33. If $\sin(x + 54^\circ) = \cos x$, where $0 < x, x + 54^\circ < 90^\circ$, then what is the value of x? **[2008-II]**

(a) 54° (b) 36° (c) 27° (d) 18°

34. If clock started at noon, then what is the angle turned by hour hand at 3 : 45 pm? **[2008-II]**
(a) 67.5º (b) 97.5º (c) 112.5º (d) 142.5º

35. If the given figure, BC = 15 cm and sin B = $\frac{4}{5}$. What is the value of AB? **[2008-II]**



(a) 25 cm (b) 20 cm (c) 5 cm (d) 4 cm

36. If x cos 60º + y cos 0º = 3 and 4x sin 30º – y cot 45º = 2, then what is the value of x? **[2009-I]**
(a) –1 (b) 0 (c) 1 (d) 2

37. What is the angle (in radian) included between the hands of a clock, when the time is 10 min past 5? **[2009-I]**
(a) $\frac{17\pi}{36}$ (b) $\frac{19\pi}{36}$ (c) $\frac{5\pi}{9}$ (d) $\frac{7\pi}{12}$

38. What is log(tan 1º) + log(tan 2º) + log(tan 3º) + ... + log(tan 89º) equal to? **[2009-I]**
(a) 0 (b) 1 (c) 2 (d) –1

39. Consider the following equations:
I. $\text{cosec}^2 x + \sec^2 x = \text{cosec}^2 x \sec^2 x$
II. $\sec^2 x + \tan^2 x = \sec^2 x \tan^2 x$
III. $\text{cosec}^2 x + \tan^2 x = \cot^2 x + \sec^2 x$
Which of the above statements are correct? **[2009-I]**
(a) I and II (b) II and III
(c) I and III (d) All of these

40. If $\cos x + \cos^2 x = 1$, then what is the value of $\sin^2 x + \sin^4 x = 1 =$? **[2009-I]**
(a) 0 (b) 1 (c) 2 (d) 4

41. If $\sin x \cos x = \frac{1}{2}$, then what is the value of sin (x – cos x)? **[2009-I]**
(a) 2 (b) 1 (c) 0 (d) –1

42. If $\tan^2 y \,\text{cosec}^2 x - 1 = \tan^2 y$, then which one of the following is correct? **[2009-I]**
(a) x – y = 0 (b) x = 2y
(c) y = 2x (d) x – y = 1º

43. If $\frac{\cos x}{1+\text{cosec } x} + \frac{\cos x}{\text{cosec } x - 1} = 2$, then which one of the following is one of the values of x? **[2009-I]**
(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

44. If x + y = 90º and sin x : sin y $= \sqrt{3} : 1$, then what is x : y equal to? **[2009-I]**
(a) 1 : 1 (b) 1 : 2 (c) 2 : 1 (d) 3 : 2

45. If $\frac{\cos x}{\cos y} = n$ and $\frac{\sin x}{\sin y} = m$, then $(m^2 - n^2) \sin^2 y$ is equal to **[2009-I]**
(a) $1 - n^2$ (b) $1 + n^2$ (c) m^2 (d) n^2

46. If $0 \le x \le \frac{\pi}{2}$, then which one of the following is always correct? **[2009-I]**
(a) $\sin^2 x < \frac{1}{2}$ and $\cos^2 x > \frac{1}{2}$
(b) $\sin^2 x > \frac{1}{2}$ and $\cos^2 x < \frac{1}{2}$
(c) $\sin^2 x < \frac{1}{2}$ and $\cos^2 x < \frac{1}{2}$
(d) Atleast one of $\sin^2 x$, $\cos^2 x$ is less than 1

47. $p = \tan^2 x + \cot^2 x$, then which one of the following is correct? **[2009-I]**
(a) $p \le 2$ (b) $p \ge 2$ (c) $p < 2$ (d) $p > 2$

48. What is the value of
$$\frac{5 \sin 75^\circ \sin 77^\circ + 2 \cos 13^\circ \cos 15^\circ}{\cos 15^\circ \sin 77^\circ} - \frac{7 \sin 81^\circ}{\cos 9^\circ} ?$$
[2009-I]
(a) –1 (b) 0 (c) 1 (d) 2

49. If sin x + sin y = a and cos x + cos y = b, what is sin x · sin y + cos x · cos y equal to? **[2009-I]**
(a) a + b – ab (b) a + b + ab
(c) $a^2 + b^2 - 2$ (d) $\left(\frac{a^2+b^2-2}{2}\right)$

50. If α is the angle of first quadrant such that $\text{cosec}^4 \alpha = 17 + \cot^4 \alpha$, then what is the value of sin α? **[2009-I]**
(a) $\frac{1}{3}$ (b) $\frac{1}{4}$ (c) $\frac{1}{9}$ (d) $\frac{1}{16}$

51. If $x + \left(\frac{1}{x}\right) = 2 \cos \alpha$, then what is the value of $x^2 + \left(\frac{1}{x^2}\right)$? **[2009-I]**
(a) $4 \cos^2 \alpha$ (b) $4 \cos^2 \alpha - 1$
(c) $2 \cos^2 \alpha - 2 \sin^2 \alpha$ (d) $\cos^2 \alpha - \sin^2 \alpha$

52. If $\cot \theta = \frac{8}{15}$, then what is the value of $\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$ where, θ is a positive acute angle? **[2009-II]**
(a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

53. Consider the following :

I. $\dfrac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta} = \cos^2\theta\,(1 + \tan\theta)\,(1 - \tan\theta)$

II. $\dfrac{1+\sin\theta}{1-\sin\theta} = (\tan\theta + \sec\theta)^2$

[2009-II]

Which of the statements given above is/are correct?

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

54. $\dfrac{\cos\theta}{1-\sin\theta} - \dfrac{\cos\theta}{1+\sin\theta} = 2$ is satisfied by which one of the following values of θ? **[2009-II]**

(a) $\dfrac{\pi}{2}$ (b) $\dfrac{\pi}{3}$ (c) $\dfrac{\pi}{4}$ (d) $\dfrac{\pi}{6}$

55. The difference of the two angles in degree measure is 1 and their sum in circular measure is also 1. What are the angles in circular measure? **[2009-II]**

(a) $\left(\dfrac{1}{2}-\dfrac{\pi}{360}\right),\left(\dfrac{1}{2}+\dfrac{\pi}{360}\right)$

(b) $\left(\dfrac{1}{2}-\dfrac{90}{\pi}\right),\left(\dfrac{1}{2}+\dfrac{90}{\pi}\right)$

(c) $\left(\dfrac{1}{2}-\dfrac{\pi}{180}\right),\left(\dfrac{1}{2}+\dfrac{\pi}{180}\right)$

(d) None of these

56. If $0 < x < 45^\circ$ and $45^\circ < y < 90^\circ$, then which one of the following is correct? **[2009-II]**

(a) sin x = sin y (b) sin x < sin y
(c) sin x > sin y (d) sin x ≤ sin y

57. What is the value of
$\sin^3 60^\circ \cot 30^\circ - 2\sec^2 45^\circ + 3\cos 60^\circ \tan^2 45^\circ - \tan^2 60^\circ$?

[2009-II]

(a) $\dfrac{35}{0}$ (b) $\dfrac{-35}{8}$ (c) $\dfrac{-11}{8}$ (d) $\dfrac{11}{8}$

58. If $\tan\theta = \dfrac{p}{q}$, then what is $\dfrac{p\sec\theta - q\,\mathrm{cosec}\,\theta}{p\sec\theta + q\,\mathrm{cosec}\,\theta}$ equal to? **[2009-II]**

(a) $\dfrac{p-q}{p+q}$ (b) $\dfrac{q^2-p^2}{q^2+p^2}$

(c) $\dfrac{p^2-q^2}{q^2+p^2}$ (d) 1

59. The value of $\mathrm{cosec}^2\theta - 2 + \sin^2\theta$ is always **[2009-II]**

(a) less than zero (b) non-negative
(c) zero (d) 1

60. If $\cot\theta = \dfrac{2xy}{x^2 - y^2}$, then what is cos θ equal to?

[2009-II]

(a) $\dfrac{x^2-y^2}{x^2+y^2}$ (b) $\dfrac{x^2+y^2}{x^2-y^2}$

(c) $\dfrac{2xy}{x^2+y^2}$ (d) $\dfrac{2xy}{\sqrt{x^2+y^2}}$

61. For what value of θ is (sin θ + cosec θ) = 2.5, where $0 < \theta < 90^\circ$? **[2009-II]**

(a) 30° (b) 45° (c) 60° (d) 90°

62. If $0 < \theta < \phi < 90^\circ$, then which one of the following is correct? **[2009-II]**

(a) $(\sin\theta + \cos\theta)^2 > 2$ (b) $(\sin^2\theta + \cos^2\phi) \le 2$
(c) $(\sin^2\theta + \cos^2\phi)^2 < 2$ (d) $(\sin^2\theta + \cos^2\phi)^2 > 2$

63. What is the angle subtended at the centre of a circle of radius 8 m after traversing 4π m along its circumference? **[2009-II]**

(a) $\dfrac{\pi}{3}$ (b) $\dfrac{\pi}{2}$ (c) $\dfrac{2\pi}{3}$ (d) $\dfrac{3\pi}{4}$

64. If cos 1° = p and cos 89° = q, then which one of the following is correct? **[2010-I]**

(a) p is close to 0 and q is close to 1
(b) p < q
(c) p = q
(d) p is close to 1 and q is close to 0

65. Which one of the following is correct? **[2010-I]**

(a) There is only one θ with $0^\circ < \theta < 90^\circ$ such that sin θ = a, where a is a real number.
(b) There is more than one θ with $0^\circ < \theta < 90^\circ$ such that sin θ = a, where a is a real number.
(c) There is no θ with $0^\circ < \theta < 90^\circ$ such that sin θ = a, where a is a real number.
(d) There are exactly θ's with $0^\circ < \theta < 90^\circ$ such that sin θ = a, where a is a real number.

66. If $7\cos^2\theta + 3\sin^2\theta = 4$ and $0 < \theta < \dfrac{\pi}{2}$, then what is the value of tan θ? **[2010-I]**

(a) $\sqrt{7}$ (b) $\dfrac{7}{3}$ (c) 3 (d) $\sqrt{3}$

67. What is the value of $[(1 - \sin^2\theta)\sec^2\theta + \tan^2\theta](\cos^2\theta + 1)$ when $0^\circ < \theta < 90^\circ$? **[2010-I]**

(a) 2 (b) > 2 (c) ≥ 2 (d) < 2

68. If $0 \le \theta < \dfrac{\pi}{2}$ and $p = \sec^2\theta$, then which one of the following is correct? **[2010-I]**

(a) p < 1 (b) p = 1 (c) p > 1 (d) p ≥ 1

69. In a ΔABC, ∠ABC = 90°, ∠ACB = 30°, AB = 5 cm. What is the length of AC? **[2010-I]**

(a) 10 cm (b) 5 cm (c) $5\sqrt{2}$ cm (d) $5\sqrt{3}$ cm

70. If $0 \le \theta \le \frac{\pi}{2}$ and $\cos\theta + \sqrt{3}\sin\theta = 2$, then what is the value of θ? **[2010-I]**

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

71. If ABC is a right angled triangle at C and having u units, v units and w units as the lengths of its sides opposite to be vertices A, B and C respectively, then what is tan A + tan B equal to? **[2010-I]**

(a) $\frac{u^2}{uw}$ (b) 1 (c) u + v (d) $\frac{w^2}{uv}$

72. ABC is a right triangle with right angle at A. If the value of $\tan B = \frac{1}{\sqrt{3}}$, then for any real k the length of the hypotenuse is of the form **[2010-I]**

(a) 3 k (b) 2 k (c) 5 k (d) 9 k

73. What is the value of

$\sin^{12} 15^\circ + \sin^2 20^\circ + \sin^2 25^\circ + ... + \sin^2 75^\circ$? **[2010-I]**

(a) $\tan^2 15^\circ + \tan^2 20^\circ + \tan^2 25^\circ + ... + \tan^2 75^\circ$
(b) $\cos^2 15^\circ + \cos^2 20^\circ + \cos^2 25^\circ + ... + \cos^2 75^\circ$
(c) $\cot^2 15^\circ - \cot^2 20^\circ + \cot^2 25^\circ - ... + \cot^2 75^\circ$
(d) $\sec^2 15^\circ + \sec^2 20^\circ + \sec^2 25^\circ + ... + \sec^2 75^\circ$

74. If α is an acute angle and $\sin\alpha = \sqrt{\frac{x-1}{2x}}$, then what is $\tan\alpha$ equal to? **[2010-I]**

(a) $\sqrt{\frac{x-1}{x+1}}$ (b) $\sqrt{\frac{x+1}{x-1}}$

(c) $\sqrt{x^2-1}$ (d) $\sqrt{x^2+1}$

75. If $\cos \ge \frac{1}{2}$ in the first quadrant, then which one of the following is correct? **[2010-II]**

(a) $\theta \le \frac{\pi}{3}$ (b) $\theta \ge \frac{\pi}{3}$ (c) $\theta \le \frac{\pi}{6}$ (d) $\theta \ge \frac{\pi}{6}$

76. What is the value of $\cos 1^\circ \cos 2^\circ \cos 3^\circ ..... \cos 90^\circ$? **[2010-II]**

(a) $\frac{1}{2}$ (b) 0 (c) 1 (d) 2

77. If $\sin\theta + \cos\theta = 1$, then what is the value of $\sin\theta\cos\theta$? **[2010-II]**

(a) 2 (b) 0 (c) 1 (d) $\frac{1}{2}$

78. What is $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$ equal to? **[2010-II]**

(a) $\sec\theta - \tan\theta$ (b) $\sec\theta + \tan\theta$
(c) $\operatorname{cosec}\theta + \cot\theta$ (d) $\operatorname{cosec}\theta - \cot\theta$

79. Two sides of an acute angle triangle are 6 cm and 2 cm, respectively. Which one of the following represents the correct range of the third side in cm? **[2010-II]**

(a) $(4, 8)$ (b) $(4, 2\sqrt{10})$

(c) $(4\sqrt{2}, 8)$ (d) $(4\sqrt{2}, 2\sqrt{10})$

80. What is $\cot 15^\circ \cot 20^\circ \cot 70^\circ \cot 75^\circ$ equal to? **[2011-I]**

(a) –1 (b) 0 (c) 1 (d) 2

81. If $\sin 3\theta = \cos(\theta - 2^\circ)$, where 3θ and $(\theta - 2^\circ)$ are acute angles, what is the value of θ? **[2011-I]**

(a) 22° (b) 23° (c) 24° (d) 25°

82. What is $\frac{\sin^6\theta - \cos^6\theta}{\sin^2\theta - \cos^2\theta}$ equal to? **[2011-I]**

(a) $\sin^4\theta - \cos^4\theta$ (b) $1 - \sin^2\theta\cos^2\theta$
(c) $1 + \sin^2\theta\cos^2\theta$ (d) $1 - 3\sin^2\theta\cos^2\theta$

83. Consider the following :

I. $\tan^2\theta - \sin^2\theta = \tan^2\theta\sin^2\theta$
II. $(\operatorname{cosec}\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta) = 1$

Which of the identities above is/are correct? **[2011-I]**

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

84. If $\tan A = \frac{1-\cos B}{\sin B}$, then what is $\frac{2\tan A}{1-\tan^2 A}$ equal to? **[2011-I]**

(a) $\frac{\tan B}{2}$ (b) 2 tan B (c) tan B (d) 4 tan B

85. Assume the Earth to be a sphere of radius R. What is the radius of the circle of latitude 40° S? **[2011-I]**

(a) R cos 40° (b) R sin 80°
(c) R sin 40° (d) R tan 40°

86. If α and β are complimentary angles, then what is

$$\sqrt{\operatorname{cosec}\alpha \cdot \operatorname{cosec}\beta}\left(\frac{\sin\alpha}{\sin\beta} + \frac{\cos\alpha}{\cos\beta}\right)^{-\frac{1}{2}}$$ equal to?

[2011-II]

(a) 0 (b) 1
(c) 2 (d) None of these

87. If A, B, C and D are the successive angles of a cyclic quadrilateral, then what is cos A + cos B + cos C + cos D are equal to? **[2011-II]**

(a) 4 (b) 2 (c) 1 (d) 0

88. A unit radian is approximately equal to **[2011-II]**

(a) 57° 17′ 43″ (b) 57° 16′ 22″
(c) 57° 17′ 47″ (d) 57° 17′ 49″

89. How many degrees are there in an angle which equals two-third of its complement? **[2011-II]**

(a) 36° (b) 45° (c) 48° (d) 60°

90. If $\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}=2$ with $0 < \theta < 90°$, then what is θ equal to? **[2011-II]**
(a) 30° (b) 45° (c) 60° (d) 75°

91. If $A=\frac{\pi}{6}$ and $B=\frac{\pi}{3}$, then which of the following is/are correct?
I. sin A + sin B = cos A + cos B
II. tan A + tan B = cot A + cot B
Select the correct answer using the codes given below. **[2011-II]**
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

92. The Earth takes 24 h to rotate about its own axis. Through what angle will it turn in 4 h and 12 min? **[2011-II]**
(a) 63° (b) 64° (c) 65° (d) 70°

93. Consider the following :
I. $\sin^2 1° + \cos^2 1° = 1$
II. $\sec^2 33° - \cot^2 57° = \text{cosec}^2 37° - \tan^2 53°$
Which of the above statements is/are correct? **[2012-I]**
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

94. If p = a sin x + b cos x and q = a cos x – b sin x, then what is the value of $p^2 + q^2$? **[2012-I]**
(a) a + b (b) ab
(c) $a^2 + b^2$ (d) $a^2 - b^2$

95. The expression $\sin^2 x + \cos^2 x - 1 = 0$ is satisfied by how many values of x? **[2012-I]**
(a) Only one value of x (b) Two values of x
(c) Infinite values of x (d) No value of x

96. Consider the following statements :
I. The angular measure in radian of a circular arc of fixed length subtending at its centre decreases, if the radius of the arc increases.
II. 1800° is equal to 5π radian.
Which of the above statements is/are correct? **[2012-I]**
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

97. Consider the following statements :
I. There is only one value of x in the first quadrant that satisfies six + cos x = 2.
II. There is only one value of x in the first quadrant that satisfies sin x – cos x = 0.
Which of the statements above is/are correct? **[2012-I]**
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

98. If x lies in the first quadrant and $\cos x = \frac{5}{13}$, what is the value of tan x – cot x? **[2012-I]**
(a) $\frac{-139}{60}$ (b) $\frac{139}{60}$
(c) $\frac{119}{60}$ (d) None of these

99. Consider the following :
I. $\frac{\cot 30° + 1}{\cot 30° - 1} = 2(\cos 30° + 1)$
II. 2 sin 45° cos 45° – tan 45° cot 45° = 0
Which of the above identities is/are correct? **[2012-I]**
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

100. If 3 sin x + 5 cos x = 5, then what is the value of (3 cos x – 5 sin x)? **[2012-I]**
(a) 0 (b) 2 (c) 3 (d) 5

101. If $\tan\theta = \frac{3}{4}$ and θ is acute, then what is the value of sin θ? **[2012-II]**
(a) $-\frac{3}{5}$ (b) $\frac{3}{5}$
(c) $\frac{4}{5}$ (d) $-\frac{4}{5}$

102. What is the value of sec (90 – θ)° · sin θ sec 45°? **[2012-II]**
(a) 1 (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{2}$ (d) $\sqrt{3}$

103. If an angle measures p degrees and q radians, then which one of the following is correct? **[2012-II]**
(a) πp = 90q (b) πp = 360q
(c) πp = 180q (d) πp = 180p

104. If the angle θ is in the first quadrant and tan θ = 3, them what is the value of (sinθ + cosθ)? **[2012-II]**
(a) $\frac{1}{\sqrt{10}}$ (b) $\frac{2}{\sqrt{10}}$
(c) $\frac{3}{\sqrt{10}}$ (d) $\frac{4}{\sqrt{10}}$

105. If 0° < θ < 90°, then all the trigonometric ratios can be obtained when **[2012-II]**
(a) only sin θ is given
(b) only cos θ is given
(c) only tan θ is given
(d) any one of the six ratios is given

106. What is the value of sin A cos A tan A + cos A sin A cot A? **[2012-II]**
(a) $\sin^2 A + \cos A$
(b) $\sin^2 A + \tan^2 A$
(c) $\sin^2 A + \cot^2 A$
(d) $\text{cosec}^2 A - \cot^2 A$

107. What is the value of $\frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}$? **[2012-II]**

(a) 2 cosec θ (b) 2 sec θ
(c) sec θ (d) cosec θ

108. If sin θ cos θ = $\frac{\sqrt{3}}{4}$, then $\sin^4\theta + \cos^4\theta$ is equal to **[2012-II]**

(a) $\frac{7}{8}$ (b) $\frac{5}{8}$ (c) $\frac{3}{8}$ (d) $\frac{1}{8}$

109. A wheel makes 12 revolutions per min. The angle in radian described by a spoke of the wheel in 1 *s* is **[2012-II]**

(a) 5π/2 (b) 2π/5 (c) 3π/5 (d) 4π/5

110. If $\cos A + \cos^2 A = 1$, then what is the value of $2(\sin^2 A + \sin^4 A)$? **[2013-I]**

(a) 4 (b) 2 (c) 1 (d) $\frac{1}{2}$

111. $(1 - \tan A)^2 + (1 + \tan A)^2 + (1 - \cot A)^2 + (1 + \cot A)^2$ is equal to **[2013-I]**

(a) $\sin^2 A \cos^2 A$ (b) $\sec^2 A \operatorname{cosec}^2 A$
(c) $2\sec^2 A \operatorname{cosec}^2 A$ (d) None of these

DIRECTIONS (Qs. 112-115) : *Read the following information carefully to answer the questions that follow.*

The angles A, B, C and D of a quadrilateral ABCD are in the ratio 1 : 2 : 4 : 5.

A = 30º, B = 60º, C = 120º, D = 150º

112. What is the value of cos(A + B)? **[2013-I]**

(a) 0 (b) $\frac{1}{2}$
(c) 1 (d) None of these

113. What is the value of cosec(C – D + B)? **[2013-I]**

(a) 1 (b) 2 (c) 3 (d) 4

114. Consider the following statements :
I. ABCD is a cyclic quadrilateral.
II. sin(B – A) = cos(D – C)
Which of the above statements is/are correct? **[2013-I]**

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

115. What is the value of $\sec^2 D - \tan^2 D$? **[2013-I]**

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$
(c) 1 (d) None of these

116. What is the value of $\frac{\tan A - \sin A}{\sin^3 A}$? **[2013-I]**

(a) $\frac{\sec A}{1-\cos A}$ (b) $\frac{\sec A}{1+\cos^2 A}$
(c) $\frac{\sec A}{1+\cos A}$ (d) None of these

117. Consider the following statements for 0 ≤ θ ≤ 90º.
I. The value of sin θ + cos θ is always greater than 1.
II. The value of tan θ + cot θ is always greater than 1.
Which of the above statements is/are correct? **[2013-I]**

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

118. If sin A = $\frac{3}{5}$ and A is an acute angle, then tan A + sec A is equal to **[2013-I]**

(a) 0 (b) 1 (c) 2 (d) –1

119. If $\sin\theta = \frac{x^2 - y^2}{x^2 + y^2}$, then which one of the following is correct? **[2013-I]**

(a) $\cos\theta = \frac{2xy}{x^2 - y^2}$ (b) $\cos\theta = \frac{2xy}{x^2 + y^2}$
(c) $\cos\theta = \frac{x - y}{x^2 + y^2}$ (d) $\cos\theta = \frac{xy(x-y)}{x^2 + y^2}$

120. If $a^2 = \frac{1+2\sin\theta\cos\theta}{1-2\sin\theta\cos\theta}$, then what is the value of $\frac{a+1}{a-1}$? **[2013-I]**

(a) sec θ (b) 1 (c) 0 (d) tan θ

121. If 5 sin θ + 12 cos θ = 13, then what is 5 cos θ – 12 sin θ equal to? **[2013-II]**

(a) –2 (b) –1 (c) 0 (d) 1

122. If 4 tan θ = 3, then what is $\frac{4\sin\theta - \cos\theta}{4\sin\theta + 9\cos\theta}$ equal to? **[2013-II]**

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{6}$

123. If sin θ – cos θ = 0, then what is $\sin^4\theta + \cos^4\theta$ equal to? **[2013-II]**

(a) 1 (b) $\frac{3}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

124. Consider the following statements :
I. tan θ increases faster than sin θ as θ increases.
II. The value of sin θ + cos θ is always greater than 1.
Which of the statements given above is/are correct? **[2013-II]**

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

125. What is $\frac{(\sin\theta+\cos\theta)(\tan\theta+\cot\theta)}{\sec\theta+\operatorname{cosec}\theta}$ equal to? **[2013-II]**

(a) 1 (b) 2 (c) sin θ (d) cos θ

126. What is $\dfrac{\cos^2(45^\circ+\theta)+\cos^2(45^\circ-\theta)}{\tan(60^\circ+\theta)\tan(30^\circ-\theta)}$ equal to? **[2013-II]**

(a) -1 (b) 0 (c) 1 (d) 2

127. What is $\sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta$ equal to? **[2013-II]**

(a) 0 (b) 1 (c) 2 (d) 4

128. What is $\dfrac{(1+\sec\theta-\tan\theta)\cos\theta}{(1+\sec\theta+\tan\theta)(1-\sin\theta)}$ equal to? **[2013-II]**

(a) 1 (b) 2
(c) $\tan\theta$ (d) $\cot\theta$

129. If $\sin\theta + \cos\theta = \sqrt{3}$, then what is $\tan\theta + \cot\theta$ equal to? **[2013-II]**

(a) 1 (b) $\sqrt{2}$ (c) 2 (d) $\sqrt{3}$

130. If $\tan\theta + \sec\theta = m$, then what is $\sec\theta$ equal to? **[2013-II]**

(a) $\dfrac{m^2-1}{2m}$ (b) $\dfrac{m^2+1}{2m}$ (c) $\dfrac{m+1}{m}$ (d) $\dfrac{m^2+1}{m}$

131. What is $\text{cosec}(75^\circ+\theta) - \sec(15^\circ-\theta)$ equal to? **[2013-II]**

(a) 0 (b) 1
(c) $2\sin\theta$ (d) $2\cos\theta$

132. If ΔABC is right angled at C, then what is $\cos(A+B)+\sin(A+B)$ equal to? **[2013-II]**

(a) 0 (b) $\dfrac{1}{2}$ (c) 1 (d) 2

133. If α, β and γ are acute angles such that $\sin\alpha = \dfrac{\sqrt{3}}{2}$, $\cos\beta = \dfrac{\sqrt{3}}{2}$ and $\tan\gamma = 1$, then what is $\alpha+\beta+\gamma$ equal to? **[2013-II]**

(a) 105º (b) 120º (c) 135º (d) 150º

134. The value of $\cos 25^\circ - \sin 25^\circ$ is **[2014-I]**

(a) positive but less than 1
(b) positive but greater than 1
(c) negative
(d) 0

135. In a right angled ΔABC, right angle at B, if $\cos A = \dfrac{4}{5}$, then what is the value of sice? **[2014-I]**

(a) $\dfrac{3}{5}$ (b) $\dfrac{4}{5}$ (c) $\dfrac{3}{4}$ (d) $\dfrac{2}{5}$

136. If α and β are complementary angles, then what is $\sqrt{\cos\alpha\,\text{cosec}\,\beta - \cos\alpha\sin\beta}$ equal to? **[2014-I]**

(a) $\sec\beta$ (b) $\cos\alpha$
(c) $\sin\alpha$ (d) $-\tan\beta$

137. If $2\cot\theta = 3$, then what is $\dfrac{2\cos\theta-\sin\theta}{2\cos\theta+\sin\theta}$ equal to? **[2014-I]**

(a) $\dfrac{2}{3}$ (b) $\dfrac{1}{3}$ (c) $\dfrac{1}{2}$ (d) $\dfrac{3}{4}$

138. If $\sin\theta\cos\theta = \dfrac{1}{2}$, then what is $\sin^6\theta + \cos^6\theta$ equal to? **[2014-I]**

(a) 1 (b) 2 (c) 3 (d) $\dfrac{1}{4}$

139. If $\sec\theta + \tan\theta = 2$, then what is the value of $\sec\theta$? **[2014-I]**

(a) $\dfrac{3}{2}$ (b) $\sqrt{2}$ (c) $\dfrac{5}{2}$ (d) $\dfrac{5}{4}$

140. What is $\text{cosec}(75^\circ+\theta) - \sec(15^\circ-\theta) - \tan(55^\circ+\theta) + \cot(35^\circ-\theta)$ equal to? **[2014-I]**

(a) -1 (b) 0 (c) 1 (d) $\dfrac{3}{2}$

141. If $\sin\theta + 2\cos\theta = 1$, where $0 < \theta < \dfrac{\pi}{2}$, what is $2\sin\theta - \cos\theta$ equal to? **[2014-I]**

(a) -1 (b) $\dfrac{1}{2}$ (c) 2 (d) 1

142. If $\cos x + \sec x = 2$, then what $\cos^n x + \sec^n x$ equal to, where n is a positive integer? **[2014-I]**

(a) 2 (b) 2^{n-2} (c) 2^{n-1} (d) 2^n

143. What is $\sin 25^\circ \sin 35^\circ \sec 65^\circ \sec 55^\circ$ equal to? **[2014-I]**

(a) -1 (b) 0 (c) $\dfrac{1}{2}$ (d) 1

144. If $\tan 8\theta = \cot 2\theta$, where $0 < 8\theta < \dfrac{\pi}{2}$, then what is the value of $\tan 5\theta$? **[2014-I]**

(a) $\dfrac{1}{\sqrt{3}}$ (b) 1 (c) $\sqrt{3}$ (d) 0

145. If $\sin(A+B) = 1$, where $0 < B < 45^\circ$, then what is $\cos(A-B)$ equal to? **[2014-I]**

(a) $\sin 2B$ (b) $\sin B$ (c) $\cos 2B$ (d) $\cos B$

146. At what point of time after 3 o'clock, hour hand and the minute hand of a clock occur at right angles for the first time? **[2014-I]**

(a) 9 o'clock (b) $4\text{ h }37\dfrac{1}{6}$ min

(c) $3\text{ h }30\dfrac{8}{11}$ min (d) $3\text{ h }32\dfrac{8}{11}$ min

147. If from the top of a post a string twice the length of the post is stretched tight to a point on the ground, then what angle will the string make with the post? **[2014-II]**

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{5\pi}{12}$ (d) $\frac{\pi}{3}$

148. If $0 < \theta < \frac{\pi}{4}$, then what is $\sqrt{1-2\sin\theta\cos\theta}$ equal to? **[2014-II]**

(a) $\cos\theta - \sin\theta$ (b) $\sin\theta - \cos\theta$
(c) $\pm(\cos\theta - \sin\theta)$ (d) $\cos\theta\sin\theta$

149. If $\tan\theta + \cot\theta = 2$, then what is $\sin\theta + \cos\theta$ equal to? **[2014-II]**

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\sqrt{2}$ (d) 1

150. What is $\frac{\sec x}{\cot x + \tan x}$ equal to ? **[2014-II]**

(a) $\sin x$ (b) $\cos x$ (c) $\tan x$ (d) $\cot x$

151. What is $\frac{\sin x - \cos x + 1}{\sin x + \cos x - 1}$ equal to ? **[2014-II]**

(a) $\frac{\sin x - 1}{\cos x}$ (b) $\frac{\sin x + 1}{\cos x}$

(c) $\frac{\sin x - 1}{\cos x + 1}$ (d) $\frac{\sin x + 1}{\cos x + 1}$

152. What is $(\sin^2 x - \cos^2 x)(1 - \sin^2 x\cos^2 x)$ equal to? **[2014-II]**

(a) $\sin^4 x - \cos^4 x$ (b) $\sin^6 x - \cos^6 x$
(c) $\cos^8 x - \sin^8 x$ (d) $\sin^8 x - \cos^8 x$

153. What is $(\sin x\cos y + \cos x\sin y)(\sin x\cos y - \cos x\sin y)$ equal to? **[2014-II]**

(a) $\cos^2 x - \cos^2 y$ (b) $\cos^2 x - \sin^2 y$
(c) $\sin^2 x - \cos^2 y$ (d) $\sin^2 x - \sin^2 y$

154. What is $(1 + \cot x - \operatorname{cosec} x)(1 + \tan x + \sec x)$ equal to ? **[2014-II]**

(a) 1 (b) 2 (c) $\sin x$ (d) $\cos x$

155. What is $(\operatorname{cosec} x - \sin x)(\sec x - \cos x)(\tan x + \cot x)$ equal to ? **[2014-II]**

(a) $\sin x + \cos x$ (b) $\sin x - \cos x$
(c) 2 (d) 1

156. Consider the following statements **[2014-II]**

1. $\sin 1° > \sin 1$ 2. $\cos 1° < \cos 1$

Which of the above statements is / are correct ?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

157. If $\sin x + \operatorname{cosec} x = 2$, then what is $\sin^9 x + \operatorname{cosec}^9 x$ equal to ? **[2014-II]**

(a) 2 (b) 18 (c) 512 (d) 1024

158. If $\sin x + \cos x = p$ and $\sin^3 x + \cos^3 x = q$, then what is $p^3 - 3p$ equal to ? **[2014-II]**

(a) 0 (b) $-2q$ (c) $2q$ (d) $4q$

159. If $\tan(A + B) = \sqrt{3}$ and $\tan A = 1$, then $\tan(A - B)$ is equal to **[2015-I]**

(a) 0 (b) 1 (c) $\frac{1}{\sqrt{3}}$ (d) $\sqrt{2}$

160. If cos A = tan B, cos B = tan C and cos C = tan A then sin A is equal to **[2015-I]**

(a) $\frac{\sqrt{5}-1}{4}$ (b) $\frac{\sqrt{5}-1}{2}$ (c) $\frac{\sqrt{3}-1}{4}$ (d) $\frac{\sqrt{3}-1}{2}$

161. If $\frac{3-\tan^2 A}{1-3\tan^2 A} = K$

where K is a real number, then cosec A(3 sinA – 4 $\sin^3$A) is equal to **[2015-I]**

(a) $\frac{2K}{K-1}$

(b) $\frac{2K}{K-1}$, where $\frac{1}{3} \le K \le 3$

(c) $\frac{2K}{K-1}$, where $K < \frac{1}{3}$ or K > 3

(d) $\frac{2K}{K+1}$

162. If tan A + cot A = 4

then $\tan^4$ A + $\cot^4$ A is equal to **[2015-I]**

(a) 110 (b) 191 (c) 80 (d) 194

163. If $p = \sqrt{\frac{1-\sin x}{1+\sin x}}, q = \frac{1-\sin x}{\cos x}, r = \frac{\cos x}{1+\sin x}$

then which of the following is/are correct ? **[2015-I]**

1. $p = q = r$
2. $p^2 = qr$

Select the correct answer using the code given below.

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

164. Consider the following : **[2015-I]**

1. $\frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A} = \cos A + \sin A$
2. $(1-\sin A-\cos A)^2 = 2(1-\sin A)(1+\cos A)$

Which of the above is/are identity/identities?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

165. ABC is a triangle right angled at B and $AB : BC = 3 : 4$. What is $\sin A + \sin B + \sin C$ equal to? **[2015-I]**

(a) 2 (b) $\frac{11}{5}$ (c) $\frac{12}{5}$ (d) 3

166. The value of $\operatorname{cosec}^2 67° + \sec^2 57° - \cot^2 33° - \tan^2 23°$ is **[2015-I]**

(a) $2\sqrt{2}$ (b) 2 (c) $\sqrt{2}$ (d) 0

167. Consider the following statements : **[2015-I]**

1. There exists at least one value of x between 0 and $\frac{\pi}{2}$ which satisfies the equation $\sin^4 x - 2\sin^2 x - 1 = 0$.
2. sin 1.5 is greater than cos 1.5. Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

168. If $\sin x + \cos x = c$ then $\sin^6 x + \cos^6 x$ is equal to **[2015-I]**

(a) $\frac{1+6c^2-3c^4}{16}$ (b) $\frac{1+6c^2-3c^4}{4}$
(c) $\frac{1+6c^2+3c^4}{16}$ (d) $\frac{1+6c^2+3c^4}{4}$

169. Consider the following statements : **[2015-I]**

1. There exists no value of x such that
$$\frac{1}{1-\sin x} = 4+2\sqrt{3}, \quad 0 < x < \frac{\pi}{2}$$
2. $\sin x = 3^{\sin^2 x}$ does not hold good for any real x.

Which of the above statements is /are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

170. The complement angle of 80° is **[2015-I]**

(a) (b) $\frac{5\pi}{9}$ radian
(c) $\frac{\pi}{18}$ radian (d) $\frac{9}{5\pi}$ radian

171. Consider the following statements : **[2015-II]**

1. $\frac{1+\tan^2\theta}{1+\cot^2\theta} = \left(\frac{1-\tan\theta}{1-\cot\theta}\right)^2$ is true for all $0 < \theta < \frac{\pi}{2}, \theta \neq \frac{\pi}{4}$.
2. $\cot\theta - \frac{1}{\tan\theta}$ is true for $\theta = 45°$ only.

Which of the above statements is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

172. If $x = a\cos\theta$ and $y = b\cot\theta$, then $(ax^{-1} - by^{-1})(ax^{-1} + by^{-1})$ is equal to **[2015-II]**

(a) 0 (b) 1 (c) $\tan^2\theta\theta$ (d) $\sin^2\theta$

173. $\frac{\cos\theta}{1-\sin\theta}$ is equal to $\left(\text{where } \theta \neq \frac{\pi}{2}\right)$ **[2015-II]**

(a) $\frac{\tan\theta-1}{\tan\theta+1}$ (b) $\frac{1+\sin\theta}{\cos\theta}$
(c) $\frac{\tan\theta+1}{\tan\theta-1}$ (d) $\frac{1+\cos\theta}{\sin\theta}$

174. If $\tan(x+40)°\tan(x+20)°\tan(3x)°\tan 70-x)°\tan(50-x)° = 1$, then the value of x is equal to **[2015-II]**

(a) 30 (b) 20 (c) 15 (d) 10

175. If θ is an acute angle and $\sin\theta\cos\theta = 2\cos^3\theta - 1.5\cos\theta$, then what is sin θ equal to? **[2015-II]**

(a) $\frac{\sqrt{5}-1}{4}$ (b) $\frac{1-\sqrt{5}}{4}$ (c) $\frac{\sqrt{5}+1}{4}$ (d) $-\frac{\sqrt{5}+1}{4}$

176. Consider the following statements : **[2015-II]**

1. sin66° is less than cos66°
2. sin26° is less than cos26°

Which of the above statements is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

177. If a and b are positive, then the relation $\sin\theta = \frac{2a+3b}{3b}$ is **[2015-II]**

(a) not possible (b) possible only if a = b
(c) possible if a > b (d) possible if a < b

178. If $\tan\theta + \sec\theta = 2$, then tanθ is equal to **[2015-II]**

(a) $\frac{3}{4}$ (b) $\frac{5}{4}$ (c) $\frac{3}{2}$ (d) $\frac{5}{2}$

179. An equilateral triangle BOC is drawn inside a square ABCD. If angle AOD= 2θ, what is tanθ equal to ? **[2015-II]**

(a) $2-\sqrt{3}$ (b) $1+\sqrt{2}$ (c) $4-\sqrt{3}$ (d) $2+\sqrt{3}$

180. The minimum value of $\cos^2 x + \cos^2 y - \cos^2 z$ is **[2015-II]**

(a) –1 (b) 0
(c) 1 (d) 2

181. The value of **[2015-II]**

$32\cot^2\left(\frac{\pi}{4}\right) - 8\sec^2\left(\frac{\pi}{3}\right) + 8\cos^3\left(\frac{\pi}{6}\right)$ is equal to

(a) $\sqrt{3}$ (b) $2\sqrt{3}$ (c) 3 (d) $3\sqrt{3}$

182. $\left(\frac{\sin 35°}{\cos 55°}\right)^2 - \left(\frac{\cos 55°}{\sin 35°}\right)^2 + 2\sin 30°$ is equal to **[2016-I]**

(a) –1 (b) 0 (c) 1 (d) 2

183. If $\frac{x}{a} - \frac{y}{b}\tan\theta = 1$ and $\frac{x}{a}\tan\theta + \frac{y}{b} = 1$, then the value of $\frac{x^2}{a^2} + \frac{y^2}{b^2}$ is **[2016-I]**

(a) $2\sec^2\theta$ (b) $\sec^2\theta$
(c) $2\cos^2\theta$ (d) $2\cos^2\theta$

184. Consider the following : **[2016-I]**

1. $\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \operatorname{cosec}\theta - \cot\theta$
2. $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \operatorname{cosec}\theta + \cot\theta$

Which of the above is/are identity identities?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

185. If $p = \cot\theta + \tan\theta$ and $q = \sec\theta - \cos\theta$, then $(p^2 p)^{\frac{2}{3}}\ (q^2 p)^{\frac{2}{3}}$ is equal to **[2016-I]**
(a) 0 (b) 1 (c) 2 (d) 3

186. If $\dfrac{\cos^2\theta - 3\cos\theta + 2}{\sin^2\theta} = 1$ **[2016-I]**

where $0<\theta<\frac{\pi}{2}$, then which of the following statements is/are correct?
1. There are two values of θ satisfying the above equation.
2. θ = 60° is satisfied by the above equation.

Select the correct answer using the code given below.
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

187. Which of the following is correct in respect of the equation $3-\tan^2\theta = \alpha(1-3\tan^2\theta)$? (Given that α is a real number.) **[2016-I]**
(a) $\alpha \in \left[\frac{1}{3}, 3\right]$ (b) $\alpha \in \left[-\infty, \frac{1}{3}\right][3, \infty]$
(c) $\alpha \in \left[-\infty, \frac{1}{3}\right][3, \infty]$ (d) None of the above

188. If $\tan\theta + \cot\theta = \dfrac{4}{\sqrt{3}}$, where $0<\theta<\frac{\pi}{2}$, then $\sin\theta + \cos\theta$ is equal to **[2016-I]**
(a) 1 (b) $\dfrac{\sqrt{3}-1}{2}$
(c) $\dfrac{\sqrt{3}+1}{2}$ (d) $\sqrt{2}$

189. If $\sin\theta + \cos\theta = \dfrac{\sqrt{7}}{2}$, then what is $\sin\theta - \cos\theta$ equal to? **[2016-II]**
(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) $\sqrt{2}$

190. If $\sin x + \sin^2 x = 1$, then what is the value of $\cos^8 x + 2\cos^6 x + \cos^4 x$? **[2016-II]**
(a) 0 (b) 1 (c) 2 (d) 4

191. What is the value of $\operatorname{cosec}^2 68° + \sec^2 56° - \cot^2 34° - \tan^2 22°$? **[2016-II]**
(a) 0 (b) $\frac{1}{2}$ (c) 1 (d) 2

192. If $2y\cos\theta = x\sin\theta$ and $2x\sec\theta - y\operatorname{cosec}\theta = 3$, then what is $x^2 + 4y^2$ equal to ? **[2016-II]**
(a) 1 (b) 2 (c) 4 (d) 8

193. If $\sin\theta + \cos\theta = \dfrac{1+\sqrt{3}}{2}$ where $0<\theta<\frac{\pi}{2}$, then what is $\tan\theta + \cot\theta$ equal to ? **[2016-II]**
(a) $\dfrac{\sqrt{3}}{4}$ (b) $\dfrac{1}{\sqrt{3}}$ (c) $\sqrt{3}$ (d) $\dfrac{4}{\sqrt{3}}$

194. If $A = \sin^2\theta + \cos^4\theta$ where $0 \le \theta < \frac{\pi}{2}$, then which one of the following is correct? **[2016-II]**
(a) $1 \le A \le 2$ (b) $\frac{3}{4} \le A \le 1$
(c) $\frac{13}{16} \le A \le 2$ (d) $\frac{3}{4} \le A \le \frac{13}{16}$

195. What is $\dfrac{\cot A + \operatorname{cosec} A - 1}{\cot A - \operatorname{cosec} A + 1}$ equal to ? **[2016-II]**
(a) $\dfrac{1+\cos A}{\sin A}$ (b) $\dfrac{1-\cos A}{\sin A}$
(c) $\dfrac{1+\sin A}{\cos A}$ (d) $\dfrac{1-\sin A}{\cos A}$

196. Consider the following : **[2016-II]**
1. $\sin 1° > \sin 1^c$
2. $\cos 1° < \cos 1^c$
3. $\tan 1° > \tan 1^c$.

Which of the above are not correct? **[2016-II]**
(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

197. If $\tan^2 x + \dfrac{1}{\tan^2 x} = 2$ and $0° < x < 90°$, then what is the value of x ? **[2016-II]**
(a) 15° (b) 30° (c) 45° (d) 60°

198. Consider the following : **[2016-II]**
1. $\dfrac{\cos 75°}{\sin 15°} + \dfrac{\sin 12°}{\cos 78°} - \dfrac{\cos 18°}{\sin 72°} = 1$
2. $\dfrac{\cos 35°}{\sin 55°} - \dfrac{\sin 11°}{\cos 79°} + \cos 28° \operatorname{cosec} 62° = 1$
3. $\dfrac{\sin 80°}{\cos 10°} - \sin 59° \sec 31° = 0$

Which of the above are correct?
(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

199. What is the value of $\tan 1° \tan 2° \tan 3° \tan 4° \ldots \tan 89°$? **[2016-II]**
(a) 0 (b) 1 (c) 2 (d) $\sqrt{3}$

200. What is the minimum value of $9\tan^2\theta + 4\cot^2\theta$? **[2017-I]**
(a) 6 (b) 9 (c) 12 (d) 13

201. If $x \sin\theta = y\cos\theta = \frac{2z\tan\theta}{1-\tan^2\theta}$, then what is $4z^2(x^2+y^2)$ equal to ?
(a) $(x^2+y^2)^3$ (b) $(x^2-y^2)^2$
(c) $(x^2-y^2)^2$ (d) $(x^2+y^2)^2$

202. If $\cos\theta_1 + \cos\theta_2 + \cos\theta_3 = 3$, then what is $\sin\theta_1 + \sin\theta_2 + \sin\theta_3$ equal to ? **[2017-I]**
(a) 0 (b) 1 (c) 2 (d) 3

203. What is the value of θ which satisfies the equation $\cos\theta + \tan\theta = 1$? **[2017-I]**
(a) 0° (b) 30° (c) 45° (d) 60°

204. What is the value of **[2017-I]**
$\sin x\sqrt{\frac{1}{1+\cos x}+\frac{1}{1-\cos x}}$?
(a) $\sqrt{2}$ (b) $2\sqrt{2}$ (c) $\sqrt{2}\tan x$ (d) 0

205. What is $\frac{\cos^4 A-\sin^4 A}{\cos^2 A-\sin^2 A}$ equal to ? **[2017-I]**
(a) $\cos^2 A - \sin^2 A$ (b) $\cos A - \sin A$
(c) 1 (d) 2

206. If $7\sin^2 x + 3\cos^2 x = 4$, $0 < x < 90°$, then what is the value of $\tan x$? **[2017-I]**
(a) $\sqrt{2}$ (b) 1 (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1}{\sqrt{3}}$

207. If a triangle has sides 5, 13 and 12 units and θ is the acute angle of the triangle, then what is the value of $(\sin\theta + \cos\theta)$? **[2017-II]**
(a) $\frac{5}{13}$ (b) $\frac{7}{13}$ (c) $\frac{12}{13}$ (d) $\frac{17}{13}$

208. If $0 < x < \frac{\pi}{2}$, then $(\sin x + \operatorname{cosec} x)$ is **[2017-II]**
(a) > 2 (b) < 2 (c) ≥ 2 (d) ≤ 2

209. If $\sin\theta = \frac{m^2-n^2}{m^2+n^2}$ and $0 < \theta < \frac{\pi}{2}$, then what is the value of $\cos\theta$? **[2017-II]**
(a) $\frac{2mn}{m^2+n^2}$ (b) $\frac{2mn}{m^2-n^2}$
(c) $\frac{m^2+n^2}{2mn}$ (d) $\frac{m^2-n^2}{2mn}$

210. If $A = \frac{\sin 45° - \sin 30°}{\cos 45° + \cos 60°}$ and $B = \frac{\sec 45° - \tan 45°}{\operatorname{cosec} 45° + \cot 45°}$, then which one of the following is correct? **[2017-II]**
(a) $A = B$ (b) $A > B > 0$
(c) $A < B$ (d) $B < A < 0$

211. Consider the following statements : **[2017-II]**
1. If $45° < \theta < 60°$, then $\sec^2\theta + \operatorname{cosec}^2\theta = \alpha^2$ for some real number $\alpha > 1$.
2. If $0° < \theta < 45°$, then $\frac{1+\cos\theta}{1-\cos\theta} = x^2$ for some real number $x > 2$.
3. If $0° < \theta < 45°$, then $\frac{\cos\theta}{1-\tan\theta}+\frac{\sin\theta}{1-\cot\theta} \geq 2$.

What is the number of true statements?
(a) Zero (b) One (c) Two (d) Three

212. The value of $\frac{\sin 1°}{\sin 1^c}$ where 1^c represents 1 radian is **[2017-II]**
(a) Equal to 1
(b) Less than 1
(c) Greater than 1 but less than 2
(d) Greater than 2

213. What is $\sin^4\theta - \cos^4\theta$ equal to for any real number θ? **[2018-I]**
(a) 1 (b) $1 - 2\sin^2\theta$
(c) $2\cos^2\theta + 1$ (d) $1 - 2\cos^2\theta$

214. What is cot 1° cot 23° cot 45° cot 67° cot 89° equal to? **[2018-I]**
(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{3}$

215. Consider the following statements : **[2018-I]**
1. $(\sec^2\theta - 1)(1 - \operatorname{cosec}^2\theta) = 1$
2. $\sin\theta(1+\cos\theta)^{-1} + (1+\cos\theta)(\sin\theta)^{-1} = 2\operatorname{cosec}\theta$

Which of the above is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

216. If $\sec x \operatorname{cosec} x = 2$, then what is $\tan^n x + \cot^n x$ equal to? **[2018-I]**
(a) 2 (b) 2^{n+1} (c) $2n$ (d) 2^{n-1}

217. If $\cos x + \cos^2 x = 1$, then what is $\sin^2 x + \sin^4 x$ equal to? **[2018-I]**
(a) 1 (b) 1.5 (c) 2 (d) 3

218. If $\sin A + \cos A = p$ and $\sin^3 A + \cos^3 A = q$, then which one of the following is correct? **[2018-I]**
(a) $p^3 - 3p + q = 0$ (b) $q^3 - 3q + 2p = 0$
(c) $p^3 - 3p + 2q = 0$ (d) $p^3 + 3p + 2q = 0$

219. If $x = \frac{\sec^2\theta - \tan\theta}{\sec^2\theta + \tan\theta}$, then which one of the following is correct? **[2018-I]**
(a) $\frac{1}{3} < x < 3$ (b) $x \notin \left[\frac{1}{3}, 3\right]$
(c) $-3 < x < -\frac{1}{3}$ (d) $\frac{1}{3} \leq x \leq 3$

220. ABC is a right angled triangle with base BC and height AB. The hypotenuse AC is four times the length of the perpendicular drawn to it from the opposite vertex. What is tan C equal to? **[2018-I]**
(a) $2-\sqrt{3}$ (b) $\sqrt{3}-1$
(c) $2+\sqrt{3}$ (d) $\sqrt{3}+1$

221. If $\cos\theta = \frac{1}{\sqrt{5}}$, where $0 < \theta < \frac{\pi}{2}$, then $\frac{2\tan\theta}{1-\tan^2\theta}$ is equal to **[2018-II]**

(a) 4@3 (b) –4/3 (c) 1@3 (d) –2/3

222. If $0 < \theta < 90°$, $0 < \phi < 90°$ and $\cos\theta < \cos\phi$, then which one of the following is correct? **[2018-II]**

(a) $\theta < \phi$

(b) $\theta > \phi$

(c) $\theta + \phi = 90°$

(d) No conclusion can be drawn

223. Let $\sin(A+B) = \frac{\sqrt{3}}{2}$ and $\cos B = \frac{\sqrt{3}}{2}$, where A, B are acute angles. What is tan (2A – B) equal to? **[2018-II]**

(a) 1@2 (b) $\sqrt{3}$ (c) $\frac{1}{\sqrt{3}}$ (d) 1

224. Consider the following statements : **[2018-II]**

1. If $\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 4$, where $0<\theta<90°$, then $\theta=60°$.
2. If $3\tan\theta + \cot\theta = 5\operatorname{cosec}\theta$, where $0<\theta<90°$, then $\theta=60°$.

Which of the statements given above is/are correct?

(a) 1 only

(b) 2 only

(c) Both 1 and 2

(d) Neither 1 nor 2

225. Consider the following statements : **[2018-II]**

1. $\cos^2\theta = 1 - \frac{p^2+q^2}{2pq}$, where p, q are non-zero real numbers, is possible only when p = q.
2. $\tan^2\theta = \frac{4pq}{(p+q)^2} - 1$, where p, q are non-zero real numbers, is possible only when p = q.

Which of the statements given above is/are correct?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

226. Consider the following statements : **[2018-II]**

1. $\cos\theta + \sec\theta$ can never be equal to 1.5.
2. $\sec^2\theta + \operatorname{cosec}^2\theta$ can never be less than 4.

Which of the statements given above is/are correct?

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

227. If $\sin^2 x + \sin x = 1$, then what is the value of $\cos^{12} x + 3\cos^{10} x + 3\cos^8 x + \cos^6 x$? **[2018-II]**

(a) &1 (b) 0 (c) 1 (d) 8

228. If $3\sin\theta + 5\cos\theta = 4$, then what is the value of $(3\cos\theta - 5\sin\theta)^2$? **[2018-II]**

(a) 9 (b) 12 (c) 16 (d) 18

229. If $\cot\theta(1+\sin\theta) = 4m$ and $\cot\theta(1-\sin\theta) = 4n$, then which of the following is correct? **[2018-II]**

(a) $(m^2+n^2)^2 = mn$

(b) $(m^2-n^2)^2 = mn$

(c) $(m^2-n^2)^2 = m^2n^2$

(d) $(m^2+n^2)^2 = m^2n^2$

230. If $\tan x = 1$, $0 < x < 90°$, then what is the value of $2\sin x\cos x$? **[2019-I]**

(a) $\frac{1}{2}$ (b) 1

(c) $\frac{\sqrt{3}}{2}$ (d) $\sqrt{3}$

231. What is the value of sin 46° cos 44° + cos 46° sin 44° ? **[2019-I]**

(a) sin 2° (b) 0 (c) 1 (d) 2

232. Suppose $0 < \theta < 90°$, then for every θ, $4\sin^2\theta + 1$ is greater than or equal to **[2019-I]**

(a) 2 (b) 4 sin θ (c) 4 cos θ (d) 4 tan θ

233. What is the value of tan 1° tan 2° tan 3°.......... tan 89° ? **[2019-I]**

(a) 0 (b) 1 (c) 2 (d) ¥

234. If $3\tan\theta = \cot\theta$ where $0 \le \theta \le \frac{\pi}{2}$, then what is the value of θ? **[2019-I]**

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

235. What is the value of $\sin^2 25° + \sin^2 65°$? **[2019-I]**

(a) 0 (b) 1 (c) 2 (d) 4

236. What is the value of $\sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta - 1$? **[2019-I]**

(a) 0 (b) 1 (c) 2 (d) 4

237. Consider the following for real numbers α, β, γ and δ : **[2019-I]**

1. $\sec\alpha = 1/4$
2. $\tan\beta = 20$
3. $\operatorname{cosec}\gamma = 1/2$
4. $\cos\delta = 2$

How many of the above statements are ***not*** possible ?

(a) One (b) Two (c) Three (d) Four

238. If $\cos^2 x + \cos x = 1$, then what is the value of $\sin^{12}x + 3\sin^{10}x + 3\sin^8 x + \sin^6 x$ **[2019-II]**

(a) 1 (b) 2

(c) 4 (d) 8

239. If $0 < \theta < 90°$, $\sin\theta = \frac{3}{5}$ and $x = \cot\theta$, then what is the value of $1 + 3x + 9x^2 + 27x^3 + 81x^4 + 243x^5$? **[2019-II]**

(a) 941 (b) 1000

(c) 1220 (d) 1365

240. What is the value of $\frac{\sin 19°}{\cos 71°}+\frac{\cos 73°}{\sin 17°}$? **[2019-II]**

(a) 0 (b) 1
(c) 2 (d) 4

241. A wheel makes 360 revolutions in one minute. What is the number of radians it turns in one second? **[2020-I]**

(a) 4π (b) 6π
(c) 12π (d) 16π

242. What is the least value of $(25\ \text{cosec}^2 x + \sec^2 x)$? **[2020-I]**

(a) 40 (b) 36 (c) 26 (d) 24

243. Let $0 < \theta < 90°$ and $100\theta = 90°$. If $\alpha = \Pi_{n=1}^{99} \cot n\theta$, then which one of the following is correct ? **[2020-I]**

(a) $\alpha = 1$ (b) $\alpha = 0$ (c) $\alpha > 1$ (d) $0 < \alpha < 1$

244. If $\tan 6\theta = \cot 2\theta$, where $0 < 6\theta < \frac{\pi}{2}$, then what is the value of $\sec 4\theta$? **[2020-I]**

(a) $\sqrt{2}$ (b) 2
(c) $\frac{2}{\sqrt{3}}$ (d) $\frac{4}{3}$

245. What is the value of $\sin^2 6° + \sin^2 12° + \sin^2 18° + ...+ \sin^2 84° + \sin^2 90°$? **[2020-I]**

(a) 1 (b) 2 (c) 4 (d) 8

246. What is $\frac{\cos\theta}{1+\sin\theta}+\frac{1}{\cot\theta}$ equal to? **[2020-I]**

(a) $\text{cosec}\theta$
(b) $\sec\theta$
(c) $\sec\theta + \text{cosec}\theta$
(d) $\text{cosec}\theta - \cot\theta$

247. What is $\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}-\frac{\sin\theta+1}{\cos\theta}$ equal to? **[2020-I]**

(a) 0 (b) 1 (c) $2\sin\theta$ (d) $2\cos\theta$

248. What is $(\tan x + \tan y)(1 - \cot x \cot y) + (\cot x + \cot y)(1 - \tan x \tan y)$ equal to ? **[2020-I]**

(a) 0 (b) 1 (c) 2 (d) 4

249. What is $\sqrt{\frac{\sec x-\tan x}{\sec x+\tan x}}$ equal to ? **[2020-I]**

(a) $\frac{1}{\sin x+\cos x}$

(b) $\frac{1}{\tan x+\cot x}$

(c) $\frac{1}{\sec x+\tan x}$

(d) $\frac{1}{\text{cosec}\ x+\cot x}$

250. If θ lies in the first quadrant and $\cot\theta = \frac{63}{16}$, then what is the value of $(\sin\theta + \cos\theta)$? **[2020-I]**

(a) 1 (d) $\frac{69}{65}$ (c) $\frac{79}{65}$ (d) 2

251. What is the value of $\frac{1-2\sin^2\theta\cos^2\theta}{\sin^4\theta+\cos^4\theta}+4$ equal to? **[2020-I]**

(a) 0 (b) 1 (c) 2 (d) 5

252. A rectangle is 48 cm long and 14 cm wide. If the diagonal makes an angle θ with the longer side, then what is $(\sec\theta + \text{cosec}\ \theta)$ equal to? **[2020-I]**

(a) $\frac{775}{168}$ (b) $\frac{725}{168}$ (c) $\frac{375}{84}$ (d) $\frac{325}{84}$

HINTS & SOLUTIONS

1. (d) (a) $\sin\theta = \sqrt{2}$ is not possible, since $\sin\theta \le 1$.

(b) $\sin\theta + \cos\theta = 2$ is not possible, since

$-\sqrt{2} \le \sin\theta + \cos\theta \le \sqrt{2}$.

(c) $\sin\theta + \cos\theta = 0$

$\Rightarrow \sin\theta = -\cos\theta \Rightarrow \tan\theta = -1$

$\Rightarrow \tan\theta = \tan\frac{3\pi}{4} \Rightarrow \theta = \frac{3\pi}{4}$

So, θ does not lie in $0° \le \theta \le 90°$.

Thus, option (c) is not correct.

(d) $\sin\theta - \cos\theta = 1$

Squaring both sides,

$\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta = 1$

$\therefore 1 - \sin 2\theta = 1 \Rightarrow \sin 2\theta = 0 = \sin 0°$

$\Rightarrow \theta = \frac{n\pi}{2}, n \in N$

$\theta = 0, \frac{\pi}{2}$

Thus, option (d) is correct.

2. (d) $\sin\theta + \text{cosec}\,\theta = 2$

$\Rightarrow \sin\theta + \frac{1}{\sin\theta} = 2$

$\Rightarrow \sin^2\theta - 2\sin\theta + 1 = 0$

$\Rightarrow (\sin\theta - 1)^2 = 0$

$\Rightarrow \sin\theta = 1$

$\Rightarrow \sin\theta = \sin 90°$

$\Rightarrow \theta = 90°$

$\therefore \sin^4\theta + \cos^4\theta = \sin^4 90° + \cos^4 90°$

$= 1 + 0 = 1$

Aliternate Method

Given that

$\sin\theta + \text{cosec}\theta = 2$

Now, put – $\theta = 90°$

$1 + 1 = 2$

Similary put $\theta = -90°$

$\sin^4\theta + \cos^4\theta$

$= \sin^4 90 + \cos^4 90°$

$= 1 + 0 = 1$

3. (c) Given, $2\sec^2\theta + \sec\theta - 6 = 0$

$\Rightarrow 2\sec^2\theta + 4\sec\theta - 3\sec\theta - 6 = 0$

$\Rightarrow 2\sec\theta(\sec\theta + 2) - 3(\sec\theta + 2) = 0$

$\Rightarrow (2\sec\theta - 3)(\sec\theta + 2) = 0$

$\Rightarrow \sec\theta = \frac{3}{2}$ $\quad (\because \sec\theta \ne -2, \sec\theta > 0)$

$\Rightarrow \cos\theta = \frac{2}{3}$

$\Rightarrow \sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{4}{9}} = \frac{\sqrt{5}}{3}$

$\therefore \text{cosec}\,\theta = \frac{1}{\sin\theta}$

$= \frac{1}{\frac{\sqrt{5}}{3}} = \frac{3}{\sqrt{5}}$

4. (c) $\frac{\sin x}{1+\cos x} = \frac{1-\cos x}{\sin x}$

$\Rightarrow \sin^2 x = (1 - \cos x)(1 + \cos x)$

$\Rightarrow (1 - \cos^2 x) = (1 - \cos^2 x)$

which is possible for all values of x except multiples of 180°.

For x = 180°, sin x = 0 and 1 + cos x = 0.

5. (a) $(3\sin\theta + 4\cos\theta) = 5$

Now, squaring both sides,

$9\sin^2\theta + 16\cos^2\theta + 24\sin\theta\cos\theta = 25$

$\Rightarrow 9(1 - \cos^2\theta) + 16(1 - \sin^2\theta) + 24\sin\theta\cos\theta = 25$

$\Rightarrow 9\cos^2\theta + 16\sin^2\theta - 24\sin\theta\cos\theta = 0$

$\Rightarrow (3\cos\theta - 4\sin\theta)^2 = 0$

$= 3\cos\theta - 4\sin\theta = 0$

6. (c) $\sec\theta = \frac{13}{5}$

$\Rightarrow \sec^2\theta = \frac{169}{25}$

$\Rightarrow 1 + \tan^2\theta = \frac{169}{25}$

$\Rightarrow \tan^2\theta = \frac{169}{25} - 1$

$\Rightarrow \tan^2\theta = \frac{144}{25}$

$\Rightarrow \tan\theta = \frac{12}{5}$ ... (i)

$\therefore \frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} = \frac{2\frac{\sin\theta}{\cos\theta} - 3}{4\frac{\sin\theta}{\cos\theta} - 9}$

Puting the value of $\tan\theta$ from equation (i)

$= \frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} = \frac{2\tan\theta - 3}{4\tan\theta - 9} = \frac{2\left(\frac{12}{5}\right) - 3}{4\left(\frac{12}{5}\right) - 9}$

$= \frac{24-15}{48-45} = \frac{9}{3} = 3$

7. (b) We know that,
$0 \le \sin^2 x \le 1$
$\Rightarrow 0 \le \sin^{10} x \le 1$
$\Rightarrow 0 \le p \le 1 \qquad (\because p = \sin^{10} x)$

8. (c) $\left(\cos^2 \frac{\pi}{8} + \sin^2 \frac{\pi}{8}\right) + 4\cos^2 \frac{\pi}{8} - \sec \frac{\pi}{3} + 5\tan^2 \frac{\pi}{3}$

$= 1 + 4 \times \left(\frac{1}{\sqrt{2}}\right)^2 - 2 + 5\left(\sqrt{3}\right)^2$
$(\because \sin^2\theta + \cos^2\theta = 1)$
$= 1 + 2 - 2 + 15 = 16$

9. (d) I. $\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta}$

$= \frac{\cos\theta\,[1+\sin\theta+1-\sin\theta]}{(1-\sin\theta)(1+\sin\theta)}$

$= \frac{2\cos\theta}{1-\sin^2\theta} = \frac{2\cos\theta}{\cos^2\theta} = \frac{2}{\cos\theta}$

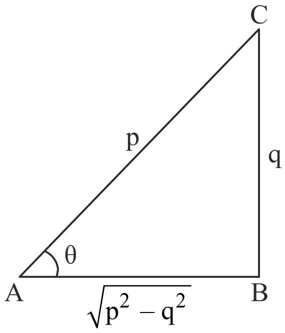
II. $\frac{\cos\theta}{1+\operatorname{cosec}\theta} + \frac{\cos\theta}{\operatorname{cosec}\theta-1}$

$= \frac{\cos\theta[\operatorname{cosec}\theta-1+1+\operatorname{cosec}\theta]}{\operatorname{cosec}^2\theta-1}$

$= \frac{2\cos\operatorname{cosec}\theta}{\cot^2\theta} = \frac{2\cot\theta}{\cot^2\theta} = \frac{2}{\cot\theta}$

Thus, neither I nor II independent of θ.

10. (b) Given, $\operatorname{cosec}\theta = \frac{p}{q}$



$b = \sqrt{p^2 - q^2}$

In ΔABC,

$\tan\theta = \frac{q}{\sqrt{p^2-q^2}}$

$\Rightarrow \sqrt{p^2-q^2} \cdot \tan\theta = q$

11. (c) Given, $a\cos\theta - b\sin\theta = c$
On squaring both sides, we get
$a^2\cos^2\theta + b^2\sin^2\theta - 2ab\cos\theta\sin\theta = c^2$
$\Rightarrow a^2(1-\sin^2\theta) + b^2(1-\cos^2\theta) - 2ab\sin\theta\cos\theta = c^2$
$\Rightarrow a^2 + b^2 - c^2 = a^2\sin^2\theta + b^2\cos^2\theta + 2ab\sin\theta\cos\theta$
$\Rightarrow (a\sin\theta + b\cos\theta)^2 = a^2 + b^2 - c^2$
$\Rightarrow a\sin\theta + b\cos\theta = \pm\sqrt{a^2+b^2-c^2}$

12. (d) Given, $2x^2\cos 60^\circ - 4\cot^2 45^\circ - 2\tan 60^\circ = \theta$

$\Rightarrow 2x^2 \times \frac{1}{2} - 4(1)^2 - 2 \times \sqrt{3} = 0$

$\Rightarrow x^2 - 4 - 2\sqrt{3} = 0$

$\Rightarrow x^2 = 4 + 2\sqrt{3}$
$\Rightarrow x^2 = 3 + 1 + 2\sqrt{3}$
$\Rightarrow x^2 = \left(\sqrt{3}\right)^2 + (1)^2 + 2\sqrt{3}\cdot 1$
$\Rightarrow x^2 = \left(\sqrt{3}+1\right)^2 \qquad \Rightarrow x = \sqrt{3} + 1$

13. (d) We know:

$\sin 30^\circ = \frac{1}{2}$

Value of sin increases 0° to 90°
$\therefore$ sin 31° > sin 30° and sin 32° > sin 30°

$\Rightarrow \sin 31^\circ > \frac{1}{2}$ and $\sin 32^\circ > \frac{1}{2}$

On adding both sides, we get
sin 31° + sin 32°

$\Rightarrow \frac{1}{2} + \frac{1}{2} \Rightarrow \sin 31^\circ + \sin 32^\circ > 1$

14. (c) We know that, tan θ is increasing in 0° to 90° and tan 45° = 1.
$\therefore$ tan 50° > 1.
So, A is true but R is false.

15. (c) We know that, sin θ is increasing in 0° to 90°.

$\because \sin 30^\circ = \frac{1}{2}$

$\therefore \sin 32^\circ > \frac{1}{2}$

16. (a) Given, $x\frac{\operatorname{cosec}^2 30^\circ \sec^2 45^\circ}{8\cos^2 45^\circ \sin^2 60^\circ} = \tan^2 60^\circ - \tan^2 30^\circ$

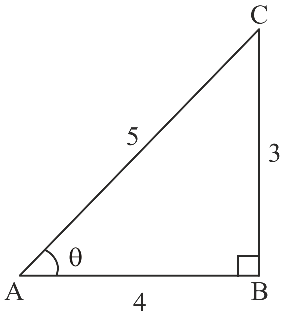
$\Rightarrow \frac{x \times (2)^2 \times \left(\sqrt{2}\right)^2}{8 \times \left(\frac{1}{\sqrt{2}}\right)^2 \times \left(\frac{\sqrt{3}}{2}\right)^2} = \left(\sqrt{3}\right)^2 - \left(\frac{1}{\sqrt{3}}\right)^2$

$\Rightarrow \frac{x \times 4 \times 2 \times 4}{8 \times \frac{1}{2} \times 3} = 3 - \frac{1}{3}$

$\Rightarrow \frac{8x}{3} = \frac{8}{3} \Rightarrow x = 1$

17. (d) Since, $\tan\theta = \frac{3}{4} = \frac{P}{B}$

$\therefore H = \sqrt{P^2 + B^2} = \sqrt{9+16} = \sqrt{25} = 5$
Let the length of hypotenuse = x cm



$\therefore \quad \sin\theta = \frac{2}{x} = \frac{3}{5} \Rightarrow x = \frac{2\times5}{3} = \frac{10}{3}$ cm

18. (c) $\sin x - \cos x = 0$

$\Rightarrow \sin x = \cos x$

$\Rightarrow x = 45^\circ = \frac{\pi}{4}$

Now, $x = 45^\circ = \frac{\pi}{4}$

$\therefore \sin^4 x + \cos^4 x = \sin^4\frac{\pi}{4} + \cos^4\frac{\pi}{4}$

$= \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

19. (b) Given that, $\frac{\tan x}{1+\sec x} - \frac{\tan x}{1-\sec x}$

$= \frac{\tan x(1-\sec x - 1 - \sec x)}{1-\sec^2 x}$

$= \frac{-2\tan x\sec x}{-\tan^2 x} = \frac{\frac{2}{\cos x}}{\frac{\sin x}{\cos x}} = 2\,\text{cosec}\,x$

20. (c) Given, $\tan x = \frac{3}{4}$, $0^\circ < x < 90^\circ$

$\therefore \sin x = \frac{\tan x}{\sqrt{1+\tan^2 x}}$, $0^\circ < x < 90^\circ$

$= \frac{\frac{3}{4}}{\sqrt{1+\frac{9}{16}}} = \frac{\frac{3}{4}}{\sqrt{\frac{25}{16}}} = \frac{3}{5}$

and $\cos x$

$= \frac{1}{\sqrt{1+\tan^2 x}} = \frac{1}{\sqrt{1+\left(\frac{3}{4}\right)^2}} = \frac{1}{\sqrt{\frac{16+9}{16}}} = \frac{4}{5}$

$\therefore \sin x\cos x = \frac{3}{5}\times\frac{4}{5} = \frac{12}{25}$

21. (a) Since, $\sin x < \frac{1}{2}$, $0^\circ < x < 30^\circ$

and $\cos x < \frac{1}{2}$, $60^\circ < x < 90^\circ$

then, $\sin x = \cos x$ only for $x = 45^\circ$ in first quadrant.

Hence, option (a) is correct.

22. (b) $(\sin^4 x - \cos^4 x + 1)\,\text{cosec}^2 x$

$= \{(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x) + 1\}\,\text{cosec}^2 x$

$[(\because a^2 - b^2 = (a+b)(a-b)]$

$= (\sin^2 x - \cos^2 x + 1)\,\text{cosec}^2 x$

$= (\sin^2 x + \sin^2 x)\,\text{cosec}^2 x$

$(\because 1 - \cos^2 x = \sin^2 x)$

$= 2\sin^2 x \cdot \frac{1}{\sin^2 x} = 2$

23. (b) $\sqrt{\cos x\,\text{cosec}\,y - \cos x\sin y}$

$(\because x + y = 90^\circ$, given)

$= \sqrt{\cos x \cdot \text{cosec}\,(90^\circ - x) - \cos x \cdot \sin(90^\circ - x)}$

$= \sqrt{\cos x \cdot \sec x - \cos^2 x}$

$= \sqrt{1-\cos^2 x} = \sqrt{\sin^2 x} = \sin x$

24. (a) Both A and R are individually true and R is correct explanation of A.

25. (c) A. It is true.

R. We know that, $\cos\theta > \sin\theta$, $0^\circ < \theta < 45^\circ$ and $\sin\theta > \cos\theta$,

$45^\circ < \theta < 90^\circ$.

Therefore, A is true but R is false.

26. (b) $\cot^2\theta - \frac{1}{\sin^2\theta} = \frac{\cos^2\theta}{\sin^2\theta} - \frac{1}{\sin^2\theta} = -\frac{\sin^2\theta}{\sin^2\theta} = -1$

27. (b) Given equation can be rewritten as

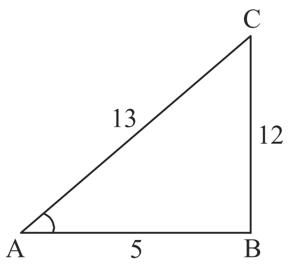
$\cos^2(3x - 9^\circ) = 1 - \sin^2 60^\circ$

$\Rightarrow \cos^2(3x - 9^\circ) = \cos^2 60^\circ$

$\Rightarrow 3x - 9^\circ = 60^\circ$

$\Rightarrow 3x = 69^\circ \Rightarrow x = 23^\circ$

28. (c) Given, $\cos A = \frac{5}{13}$



$\therefore \quad \dfrac{\sin A - \cot A}{2 \tan A} = \dfrac{\dfrac{12}{13} - \dfrac{5}{12}}{2 \times \dfrac{12}{5}}$

$= \dfrac{144 - 65}{13 \times 12 \times 2 \times \dfrac{12}{5}} = \dfrac{395}{3744}$

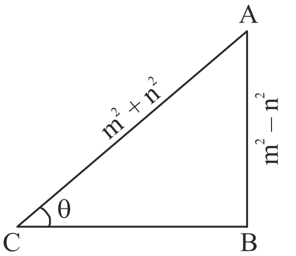
29. (c) Since, $\sin x = \cos y$
As $\angle x$ and $\angle y$ are acute angles, then

$x = y = \dfrac{\pi}{4}$

$\therefore \quad x + y = \dfrac{\pi}{2}$

$\left(\because\ 0 < x < \dfrac{\pi}{2} \text{ and } 0 < y < \dfrac{\pi}{2} \Rightarrow 0 < x + y < \dfrac{\pi}{12}\right)$

30. (c) $\sin \theta = \dfrac{m^2 - n^2}{m^2 + n^2}$



In ΔABC, $BC = \sqrt{(AC)^2 - (AB)^2}$

$= \sqrt{m^4 + n^4 + 2m^2 n^2 - \left(m^4 + n^4 - 2m^2 n^2\right)}$

$= \sqrt{4 m^2 n^2} = 2mn$

$\therefore \quad \tan \theta = \dfrac{m^2 - n^2}{2mn}$

31. (c) Given, $\sin (x - y) = \dfrac{1}{2}$ and $\cos (x + y) = \dfrac{1}{2}$

$\Rightarrow \sin (x - y) = \sin 30^\circ$
and $\cos (x + y) = \cos 60^\circ$
$\Rightarrow x - y = 30^\circ$ and $x + y = 60^\circ$
$\therefore \quad x = 45^\circ$ and $y = 15^\circ$

32. (b) $1 + \tan \theta = \sqrt{2}$

$\Rightarrow \tan \theta = \sqrt{2} - 1$

$\therefore \quad \cot \theta - 1 = \dfrac{1}{\sqrt{2} - 1} - 1 = \dfrac{\sqrt{2} + 1}{2 - 1} - 1 = \sqrt{2}$

33. (d) Given, $\sin(x + 54^\circ) = \cos x$
$\Rightarrow \sin(x + 54^\circ) = \sin(90^\circ - x) \qquad (\because\ 0^\circ < x < 90^\circ)$
$\Rightarrow x + 54^\circ = 90^\circ - x$
$\Rightarrow 2x = 36^\circ \Rightarrow x = 18^\circ$

34. (c) Given time =

$3 : 45 \text{ pm} = \left(3 + \dfrac{45}{60}\right)h$

$= \left(3 + \dfrac{3}{4}\right)h = \dfrac{15}{4}h$

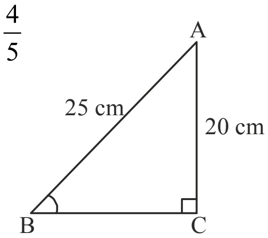
We know that
The hour hand revolve 360° in 12 hour.
So, 12h hour hand made 360°

$\therefore$ 1h hour hand made $\dfrac{360^\circ}{12}$

$\therefore \dfrac{15}{4}$h hour hand made $= \dfrac{360}{12} \times \dfrac{15}{4}$

$= \dfrac{450^\circ}{4} = 112.5^\circ$

35. (a) BC = 15 cm and $\sin B = \dfrac{4}{5}$



$\sin B = \dfrac{AC}{AB} = \dfrac{4}{5}$
then BC = 3m
But, BC = 15 (given)
then AC = 4 × 5 = 20
AB = 5 × 5 = 25.
Hence, the value of AB is 25 cm.

36. (d) Given that,
$x \cos 60^\circ + y \cos 0^\circ = 3$

$\Rightarrow \dfrac{x}{2} + y = 3$

$\Rightarrow x + 2y = 6 \qquad \ldots \text{(i)}$
and $4x \sin 30^\circ - y \cot 45^\circ = 2$

$\Rightarrow 4x \times \dfrac{1}{2} - y \cdot 1 = 2$

$\Rightarrow 2x - y = 2 \qquad \ldots \text{(ii)}$
On solving equations (i) and (ii), we get x = y = 2.

37. (b) In 60 min h hand gains = 5 min

In 1 min h hand gains $= \dfrac{5}{60}$ min

In 10 min h hand gains $= \dfrac{5}{60} \times 10 = \dfrac{5}{6}$ min

There is 15 min gap between hours and minutes hands but in 10 min h hand gains $\dfrac{5}{6}$ min more.

So, the actual gap $= 15 + \dfrac{5}{6} = \dfrac{95}{6}$ min

In 1 min, there are 6º,

In $\frac{95}{6}$ min, there are

$\frac{95}{6} \times 6° = \frac{95}{6} \times 6 \times \frac{\pi}{180} = \frac{19\pi}{36}$ radian

38. (a) $\log(\tan 1^o) + \log(\tan 2^o) + ... + \log(\tan 89^o)$
$= \log(\tan 1^o \tan 2^o ... \tan 45^o ... \tan 88^o \tan 89^o)$
$[\because \tan 89^o = \tan(90^o - 1^o) = \cot 1^o]$
$= \log[(\tan 1^o \cot 1^o)(\tan 2^o \cot 2^o) ... \tan 45^o]$
$= \log(1^o \cdot 1^o ... 1^o) = 0$

39. (c) I. L.H.S. $= \text{cosec}^2 x + \sec^2 x = \frac{\cos^2 x + \sin^2 x}{\sin^2 x \cos^2 x}$
$= \text{cosec}^2 x \sec^2 x =$ R.H.S.

II. L.H.S. $= \sec^2 x + \tan^2 x = \frac{1+\sin^2 x}{\cos^2 x} \neq$ R.H.S.

III. L.H.S. $= \text{cosec}^2 x + \tan^2 x$
$= \cot^2 x + 1 + \tan^2 x = \cot^2 x + \sec^2 x =$ R.H.S.

40. (b) Given, $\cos x + \cos^2 x = 1$
$\Rightarrow \cos = \sin^2 x$
On squaring both sides, we get
$\cos^2 x = \sin^4 x$
$\sin^2 x + \sin^4 x = 1$

41. (c): Now, $(\sin x - \cos x)^2 = (\sin^2 x + \cos^2 x) - 2 \sin x \cos x$

$= 1 - 2\left(\frac{1}{2}\right) \qquad \left(\because \sin x \cos x = \frac{1}{2}, \text{ given}\right)$

$= 0$

42. (a) Given, $\tan^2 y \,\text{cosec}^2 x - 1 = \tan^2 y$
$\Rightarrow \tan^2 y \,\text{cosec}^2 x - \tan^2 y = 1$
$\Rightarrow \tan^2 y (\text{cosec}^2 x - 1) = 1$
$\Rightarrow \tan^2 y \cdot \cot^2 x = 1$
$\Rightarrow \cot^2 x = \cot^2 y$
$\Rightarrow x = y$
$\therefore = x - y = 0$

43. (c) Given, $\frac{\cos x}{1+\text{cosec } x} + \frac{\cos x}{\text{cosec } x - 1} = 2$

$\Rightarrow \frac{2 \cos x \,\text{cosec } x}{\text{cosec}^2 x - 1} = 2$

$\Rightarrow \frac{\cos x \,\text{cosec } x}{\cot^2 x} = 1$

$\Rightarrow \tan x = 1$

$\Rightarrow x = \frac{\pi}{4}$

44. (c) $\sin x : \sin y = \sqrt{3} : 1 = \frac{\sqrt{3}}{2} : \frac{1}{2}$
$= \sin 60^o : \sin 30^o$
$\therefore x : y = 60 : 30$
$\Rightarrow x : y = 2 : 1.$

45. (a) Given, $\frac{\cos x}{\cos y} = n, \frac{\sin x}{\sin y} = m$ ... (i)

Now, $(m^2 - n^2) \sin^2 y = \left(\frac{\sin^2 x}{\sin^2 y} - \frac{\cos^2 x}{\cos^2 y}\right)\sin^2 y$

$= \frac{(1-\cos^2 x)\cos^2 y - \cos^2 x(1-\cos^2 y)}{\cos^2 y}$

$= \frac{\cos^2 y - \cos^2 x}{\cos^2 y} = 1 - n^2$ [from equation (i)]

46. (d) For $0 \le x \le \frac{\pi}{2}$, $\cos^2 x$ and $\sin^2 x$ lies between 0 to 1.
Hence, atleast one of $\sin^2 x$, $\cos^2 x$ is less than 1.

47. (b) Given, $p = \tan^2 x + \cot^2 x$
$= (\tan x + \cot x)^2 - 2$

$= \left(\frac{\sin^2 x + \cos^2 x}{\sin x \cos x}\right)^2 - 2 = \left(\frac{2}{\sin 2x}\right)^2 - 2$

$= \frac{4}{\sin^2 2x} - 2$

Since, the maximum value of sin 2x is 1.

$\therefore p_{min} = \frac{4}{1} - 2 = 2$

$\therefore p \ge 2$
Hence, $p \ge 2$.

Alternate Method

$p = \tan^2 x + \cot^2 x = \tan^2 x + \frac{1}{\tan^2 x}$

$\therefore$ A.M. $\ge$ G.M.

$\because \tan^2 x + \frac{1}{\tan^2 x} \ge 2\left(\tan^2 x \cdot \frac{1}{\tan^2 x}\right)^{\frac{1}{2}}$

$\Rightarrow \tan^2 x + \frac{1}{\tan^2 x} \ge 2 \Rightarrow P \ge 2$

48. (b) $\frac{5 \sin 75^o \sin 77^o + 2 \cos 13^o \cos 15^o}{\cos 15^o \sin 77^o} - \frac{7 \sin 81^o}{\cos 9^o}$

$= \frac{5 \cos 15^o \sin 77^o + 2 \sin 77^o \cos 15^o}{\cos 15^o \sin 77^o} - \frac{7 \cos 9^o}{\cos 9^o}$

$= \frac{7 \cos 15 \cdot \sin 77}{\cos 15 \cdot \sin 77} - \frac{7 \cos 9}{\cos 9} = 7 - 7 = 0$

49. (d) $\sin x + \sin y = a$ and $\cos x + \cos y = b$
$\Rightarrow \sin^2 x + \sin^2 y + 2 \sin x \sin y = a^2$ ... (i)
and $\cos^2 x + \cos^2 y + 2 \cos x \cos y = b^2$ ... (ii)
Adding equations (i) and (ii),
$\Rightarrow (\sin^2 x + \cos^2 x) + (\sin^2 y + \cos^2 y)$

$$+\,2(\sin x \sin y + \cos x \cos y)$$

$= a^2 + b^2$

$\Rightarrow$ $(\sin x \sin y + \cos x \cos y) = \dfrac{a^2+b^2-2}{2}$

50. (a) $\operatorname{cosec}^4 \alpha - \cot^4 \alpha = 17$ (Given)

$\Rightarrow$ $(\operatorname{cosec}^2 \alpha - \cot^2 \alpha)(\operatorname{cosec}^2 \alpha + \cot^2 \alpha) = 17$

$\Rightarrow$ $1 \cdot \left(\dfrac{1+\cos^2 \alpha}{\sin^2 \alpha}\right) = 17$

$\Rightarrow$ $2 - \sin^2 \alpha = 17 \sin^2 \alpha$

$\Rightarrow$ $18 \sin^2 \alpha = 2 \Rightarrow \sin^2 \alpha = \dfrac{1}{9}$

$\therefore$ $\sin \alpha = \dfrac{1}{3}$ (since, α lie in first quadrant)

51. (c) $x + \dfrac{1}{x} = 2 \cos \alpha$

Squaring both sides, then we get

$x^2 + \dfrac{1}{x^2} + 2 = 4 \cos^2 \alpha$

$\Rightarrow$ $x^2 + \dfrac{1}{x^2} = 2(2 \cos^2 \alpha - 1)$

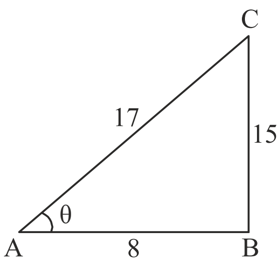
$= 2(2 \cos^2 \alpha - \sin^2 \alpha - \cos^2 \alpha)$

$= 2 \cos^2 \alpha - 2 \sin^2 \alpha$

52. (c) $\cot \theta = \dfrac{8}{15}$

$AC = \sqrt{8^2 + 15^2}$

$= \sqrt{64 + 225} = 17$



$\Rightarrow$ $\cos \theta = \dfrac{8}{17}$ $(\angle B = 90^\circ)$

$\therefore$ $\sqrt{\dfrac{1-\cos \theta}{1+\cos \theta}} = \sqrt{\dfrac{1-\dfrac{8}{17}}{1+\dfrac{8}{17}}} = \sqrt{\dfrac{9}{25}} = \dfrac{3}{5}$

53. (c) I. R.H.S. $= \cos^2 \theta\,(1 + \tan \theta)(1 - \tan \theta)$

$= \cos^2 \theta\,(1 - \tan^2 \theta)$

$= \cos^2 \theta \left(\dfrac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta}\right)$

$= \dfrac{\cos^2 \theta - \sin^2 \theta}{\cos^2 \theta + \sin^2 \theta} = $ L.H.S.

II. L.H.S. $= \dfrac{1+\sin \theta}{1-\sin \theta} = \dfrac{(1+\sin \theta)^2}{1-\sin^2 \theta}$

$= \left(\dfrac{1+\sin \theta}{\cos \theta}\right)^2 = (\sec \theta + \tan \theta)^2$

Hence, both statements are correct.

54. (c) Given, $\dfrac{\cos \theta}{1-\sin \theta} - \dfrac{\cos \theta}{1+\sin \theta} = 2$

$\Rightarrow$ $\dfrac{\cos \theta + \sin \theta \cos \theta - \cos \theta + \cos \theta \sin \theta}{1-\sin^2 \theta} = 2$

$\Rightarrow$ $2 \sin \theta \cos \theta = 2 \cos^2 \theta$

$\Rightarrow$ $\cos^2 \theta\,(\tan \theta - 1) = 0$

$\Rightarrow$ $\tan \theta = 1$ and $\cos \theta \neq 0$ $\Rightarrow$ $\theta = \dfrac{\pi}{4}$

55. (b) Let angles in circular meausres are A and B, then degree measures will be $\dfrac{\pi A}{180^\circ}$ and $\dfrac{\pi B}{180^\circ}$.

According to question,

$A + B = 1$ ... (i)

and $\dfrac{\pi A}{180^\circ} - \dfrac{\pi B}{180^\circ} = 1$ ... (ii)

On solving equations (i) and (ii), we get

$A = \dfrac{90}{\pi}\left(\dfrac{\pi}{180} + 1\right) \Rightarrow A = \left(\dfrac{1}{2} + \dfrac{90}{\pi}\right)$

From equation (i),

$\dfrac{1}{2} + \dfrac{90}{\pi} + B = 1 \Rightarrow B = \left(\dfrac{1}{2} - \dfrac{90}{\pi}\right)$

56. (b) As we know, sin x is increasing from 0 to 90º.

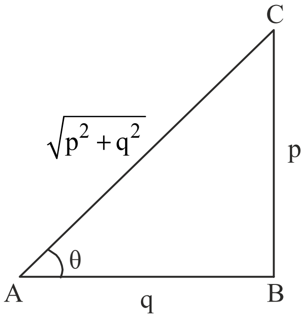
$\therefore$ $\sin y > \sin x$.

57. (b) $\sin^3 60^\circ \cot 30^\circ - 2 \sec^2 45^\circ + 3 \cos 60^\circ$ $\tan^2 45^\circ - \tan^2 60^\circ$

$= \left(\dfrac{\sqrt{3}}{2}\right)^3 (\sqrt{3}) - 2(\sqrt{2})^2 + 3\left(\dfrac{1}{2}\right)(1) - (\sqrt{3})^2$

$= \dfrac{9}{8} - 4 + \dfrac{3}{2} - 3 = \dfrac{-35}{8}$

58. (c) Given, $\tan\theta = \frac{p}{q}$



$\Rightarrow \sec\theta = \frac{\sqrt{p^2+q^2}}{q}$ and $\operatorname{cosec}\theta = \frac{\sqrt{p^2+q^2}}{p}$

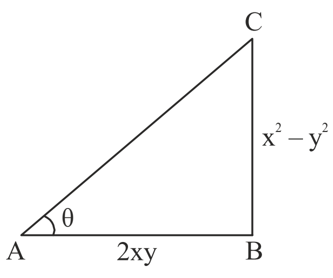
$\therefore \frac{p\sec\theta - q\operatorname{cosec}\theta}{p\sec\theta + q\operatorname{cosec}\theta}$

$$= \frac{p\left(\frac{\sqrt{p^2+q^2}}{q}\right) - q\left(\frac{\sqrt{p^2+q^2}}{p}\right)}{p\left(\frac{\sqrt{p^2+q^2}}{q}\right) + q\left(\frac{\sqrt{p^2+q^2}}{p}\right)}$$

$$= \frac{\frac{p}{q} - \frac{q}{p}}{\frac{p}{q} + \frac{q}{p}} = \frac{p^2-q^2}{p^2+q^2}$$

59. (b) $\operatorname{cosec}^2\theta - 2 + \sin^2\theta = (\sin\theta - \operatorname{cosec}\theta)^2$
Hence, it is always non-negative.

60. (c) Given, $\cot\theta = \frac{2xy}{x^2-y^2}$



In ΔABC,
$AC^2 = (x^2-y^2)^2 + (2xy)^2$
$\Rightarrow AC^2 = (x^2+y^2)^2 \Rightarrow AC = x^2+y^2$

$\therefore \cos\theta = \frac{AB}{AC} = \frac{2xy}{x^2+y^2}$

61. (a) Here, $(\sin\theta + \operatorname{cosec}\theta) = 2.5$

$\Rightarrow \left(\sin\theta + \frac{1}{\sin\theta}\right) = \frac{5}{2}$

$\Rightarrow 2\sin^2\theta - 5\sin\theta + 2 = 0$
$\Rightarrow 2\sin^2\theta - 4\sin\theta - \sin\theta + 2 = 0$
$\Rightarrow 2\sin\theta(\sin\theta - 2) - 1(\sin\theta - 2) = 0$
$\Rightarrow (2\sin\theta - 1)(\sin\theta - 2) = 0$

$\Rightarrow \sin\theta = \frac{1}{2}$ $(\because \sin\theta \neq 2)$

$\therefore \theta = 30^\circ$

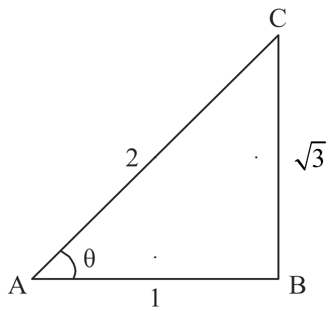
62. (c) For $0^\circ < \theta < \phi < 90^\circ$, $0 < \sin^2\theta, \cos^2\phi < 1$
$\therefore \sin^2\theta + \cos^2\phi < 2$.

63. (b) Angle subtended $= \frac{\text{Arc}}{\text{Radius}} = \frac{4\pi}{8} = \frac{\pi}{2}$

64. (d) We know that, the value of $\cos\theta$ is decreasing from 0 to 90°.
$\therefore \cos 1^\circ > \cos 89^\circ$
$\Rightarrow p > q$
Also, $\cos 1^\circ$ is close to 1 and $\cos 89^\circ$ is close to 0. Hence, option (d) is correct.

65. (a) It is true, for $0^\circ < \theta < 90^\circ$, there exist only one θ such that $\sin\theta = a$.

66. (d) Given, $7\cos^2\theta + 3\sin^2\theta = 4$ and $0 < \theta < \frac{\pi}{2}$



$\Rightarrow 7(1 - \sin^2\theta) + 3(\sin^2\theta) = 4$
$\Rightarrow 7 - 4\sin^2\theta = 4$
$\Rightarrow 4\sin^2\theta = 3$

$\Rightarrow \sin\theta = \pm\frac{\sqrt{3}}{2}$

For $0 < \theta < \frac{\pi}{2}$,

$\sin\theta = \frac{\sqrt{3}}{2}$

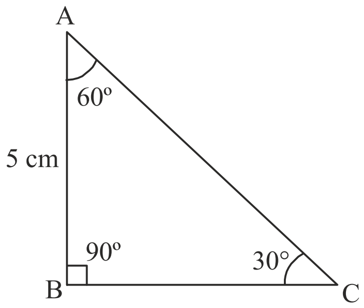
$\therefore \tan\theta = \frac{\sqrt{3}}{1} = \sqrt{3}$

67. (b) $[(1 - \sin^2\theta)\sec^2\theta + \tan^2\theta](\cos^2\theta + 1)$
$(\because \sin^2\theta + \cos^2\theta = 1)$
$= [\cos^2\theta \cdot \sec^2\theta + \tan^2\theta](\cos^2\theta + 1)$
$= (1 + \tan^2\theta)(\cos^2\theta + 1)$ $(\because \sec^2\theta - \tan^2\theta = 1)$
$= \sec^2\theta(\cos^2\theta + 1)$

$= \sec^2\theta \cdot \cos^2\theta + \sec^2\theta$
$= 1 + \sec^2\theta > 1 + 1 > 2$
$(\because \sec^2\theta > 1 \text{ for } \theta < \theta < 90^\circ)$

68. (d) We know the interval $\theta \in \left[0, \frac{\pi}{2}\right]$, $\sec^2\theta$ is increasing from 1 to ∞.
$\therefore$ $p \geq 1$.

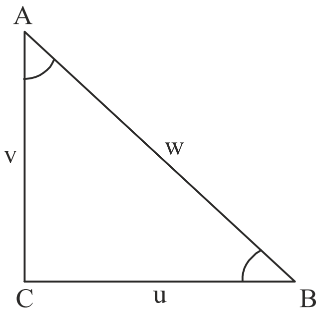
69. (a) In ΔBAC,



$\cos 60^\circ = \frac{AB}{AC} \Rightarrow \frac{1}{2} = \frac{5}{AC}$
$\therefore$ AC = 10 cm

70. (a) Given that, $\cos\theta + \sqrt{3}\sin\theta = 2$
$\Rightarrow \frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta = 1$
$\Rightarrow \sin 30^\circ \cos\theta + \cos 30^\circ \sin\theta = 1$
$\Rightarrow \sin(30^\circ + \theta) = \sin 90^\circ$
$30^\circ + \theta = 90^\circ$
$\therefore \theta = 60^\circ$

71. (d) In ΔABC,



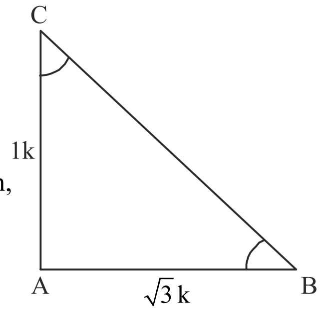
$\tan A = \frac{BC}{AC} = \frac{u}{v}$

and $\tan B = \frac{v}{u}$
Also, $u^2 + v^2 = w^2$ (by Pythagoras theorem) ... (i)

$\therefore \tan A + \tan B = \frac{u}{v} + \frac{v}{u} = \frac{u^2 + v^2}{uv} = \frac{w^2}{uv}$
[from equation (i)]

72. (b) Given,

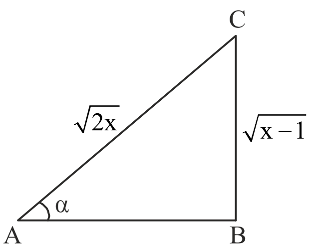
In ΔABC,



$\tan B = \frac{k}{\sqrt{3}k}$
By Pythogaros theorem,
$AB^2 + AC^2 = BC^2$
$\Rightarrow \left(\sqrt{3}\,k\right)^2 + (1k)^2 = BC^2$
$\Rightarrow BC^2 = 4k^2$
$\Rightarrow BC = 2k$

73. (b) $\sin^2 15^\circ + \sin^2 20^\circ + \sin^2 25^\circ + \ldots + \sin^2 75^\circ$
$= \sin^2(90^\circ - 75^\circ) + \sin^2(90^\circ - 70^\circ) + \ldots + \sin^2(90^\circ - 15^\circ)$
$= \cos^2 75^\circ + \cos^2 70^\circ + \ldots + \cos^2 15^\circ$

74. (a) Given that,
$\sin\alpha = \sqrt{\frac{x-1}{2x}}$
In ΔABC, using Pythagoras theorem,
$AC^2 = AB^2 + BC^2$



$\Rightarrow 2x = AB^2 + (x - 1)$
$\Rightarrow AB^2 = x + 1$
$\Rightarrow AB = \sqrt{x+1}$
$\therefore \tan\alpha = \frac{BC}{AB} = \frac{\sqrt{x-1}}{\sqrt{x+1}}$

75. (a) We know that, if value of $\cos\theta$ increases, then the value of θ decreases.
$\because \cos\theta \geq \frac{1}{2}$
$\therefore \cos\theta \geq \cos\frac{\pi}{3} \Rightarrow \theta \leq \frac{\pi}{3}$

76. (b) We know, $\cos 90^\circ = 0$
$\therefore \cos 1^\circ \cos 2^\circ \cos 3^\circ \ldots\ldots \cos 90^\circ = 0$.

77. (b) Given, $\sin\theta + \cos\theta = 1$
Squaring both sides,
$(\sin^2\theta + \cos^2\theta) + 2\sin\theta\cos\theta = 1$
$\Rightarrow 1 + 2\sin\theta\cos\theta = 1 \Rightarrow \sin\theta\cos\theta = 0$.

78. (b) $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \sqrt{\frac{(1+\sin\theta)(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}}$

$= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}} = \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}} = \frac{1+\sin\theta}{\cos\theta}$

$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = \sec\theta + \tan\theta$

79. (b) $\because \cos\theta = \frac{a^2 + b^2 - c^2}{2ab}$ By cosine rule

$$= \frac{6^2 + 2^2 - c^2}{2 \times 6 \times 2} = \frac{40 - c^2}{24}$$

For acute angle,

$$\cos \theta > 0 \Rightarrow \frac{40 - c^2}{24} > 0 \Rightarrow c^2 < 40$$

$\Rightarrow$ $0 < c < 2\sqrt{10}$ (since, c cannot be negative)... (i)

Also, b + c > a

$c > 6 - 2 \Rightarrow c > 4$

From equations (i) and (ii),

$c \in \left(4, 2\sqrt{10}\right)$

80. (c) $\cot 15^\circ \cot 20^\circ \cot 70^\circ \cot 75^\circ$

$= \tan (90^\circ - 15^\circ) \tan (90^\circ - 20^\circ) \cot 70^\circ \cot 75^\circ$

$$= \tan 75^\circ \tan 70^\circ \frac{1}{\tan 70^\circ} \cdot \frac{1}{\tan 75^\circ} = 1$$

81. (b) $\sin 3\theta = \cos (\theta - 2^\circ)$

$\Rightarrow$ $\sin 3\theta = \sin [90^\circ - (\theta - 2^\circ)]$

$\Rightarrow$ $3\theta = 90^\circ - \theta + 2^\circ$

$$\Rightarrow 4\theta = 92^\circ \Rightarrow \theta = \frac{92^\circ}{4} = 23^\circ$$

82. (b) $$\frac{\sin^6 \theta - \cos^6 \theta}{\sin^2 \theta - \cos^2 \theta} = \frac{(\sin^2 \theta)^3 - (\cos^2 \theta)^3}{\sin^2 \theta - \cos^2 \theta}$$

$$= \frac{(\sin^2 \theta - \cos^2 \theta)(\sin^4 \theta + \cos^4 \theta + \sin^2 \theta \cos^2 \theta)}{\sin^2 \theta - \cos^2 \theta}$$

$= \sin^4 \theta + \cos^4 \theta + 2 \sin^2 \theta \cos^2 \theta - \sin^2 \theta \cos^2 \theta$

$= (\sin^2 \theta + \cos^2 \theta)^2 - \sin^2 \theta \cos^2 \theta = 1 - \sin^2 \theta \cos^2 \theta$

83. (d) I. $$\tan^2 \theta - \sin^2 \theta = \frac{\sin^2 \theta}{\cos^2 \theta} - \sin^2 \theta, \theta \neq (2n+1)\frac{\pi}{2}$$

$$= \frac{\sin^2 \theta (1 - \cos^2 \theta)}{\cos^2 \theta}, \quad \theta \neq (2n+1)\frac{\pi}{2}$$

$$= \frac{\sin^2 \theta}{\cos^2 \theta} \sin^2 \theta, \quad \theta \neq (2n+1)\frac{\pi}{2}$$

$$= \tan^2 \theta \sin^2 \theta, \theta \neq (2n+1)\frac{\pi}{2}$$

II. $(\operatorname{cosec} \theta - \sin \theta)(\sec \theta - \cos \theta)(\tan \theta + \cot \theta)$

$$= \left(\frac{1}{\sin \theta} - \sin \theta\right)\left(\frac{1}{\cos \theta} - \cos \theta\right)\left(\tan \theta + \frac{1}{\tan \theta}\right)$$

$$\left[\theta \neq n\pi (2n+1)\frac{\pi}{2}\right]$$

$$= \frac{\cos^2 \theta}{\sin \theta} \frac{\sin^2 \theta}{\cos \theta} \frac{\sec^2 \theta}{\tan \theta}, \quad \theta \neq n\pi, \ (2n+1)\frac{\pi}{2}$$

$$= \sin \theta \cos \theta \frac{1}{\cos^2 \theta} \cdot \frac{\cos \theta}{\sin \theta}, \theta \neq n\pi, (2n+1)\frac{\pi}{2}$$

$= 1$

Since, to become an identity, both statements must be satisfied for every value of θ.

Therefore, neither I nor II are the identities.

84. (c) $$\tan A = \frac{1 - \cos B}{\sin B}$$

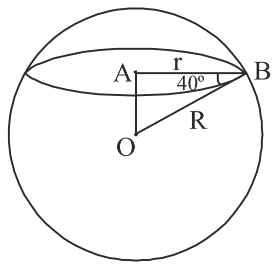
$$\therefore \quad \frac{2 \tan A}{1 - \tan^2 A} = \frac{2 \cdot \frac{1 - \cos B}{\sin B}}{1 - \left(\frac{1 - \cos B}{\sin B}\right)^2}$$

$$= \frac{2(1 - \cos B)\sin B}{\left(\sin^2 B - 1\right) - \cos^2 B + 2\cos B}$$

$$= \frac{2(1 - \cos B)\sin B}{-2\cos^2 B + 2\cos B}$$

$$= \frac{2 \sin B (1 - \cos B)}{2 \cos B (1 - \cos B)} = \frac{\sin B}{\cos B} = \tan B$$

85. (a) In ΔOAB,



$$\cos 40^\circ = \frac{AB}{OB} \Rightarrow \cos 40^\circ = \frac{r}{R}$$

$\Rightarrow$ $r = R \cos 40^\circ$

So, the radius of the circle of latitude 40º S is R cos 40º.

86. (b) Given, $\alpha + \beta = 90^\circ$ ... (i)

$$\therefore \quad \sqrt{\operatorname{cosec} \alpha \cdot \operatorname{cosec} \beta}\left(\frac{\sin \alpha}{\sin \beta} + \frac{\cos \alpha}{\cos \beta}\right)^{-\frac{1}{2}}$$

$$= \frac{1}{(\sin \alpha \sin \beta)^{\frac{1}{2}}}\left(\frac{\sin \alpha \cos \beta + \cos \alpha \sin \beta}{\sin \beta \cos \beta}\right)^{-\frac{1}{2}}$$

$$= \frac{1}{(\sin\alpha\sin\beta)^{\frac{1}{2}}}\left\{\frac{\sin(\alpha+\beta)}{\sin\beta\cos\beta}\right\}^{-\frac{1}{2}}$$

$$= \frac{1}{(\sin\alpha\sin\beta)^{\frac{1}{2}}}\left\{\frac{\sin 90^\circ}{\cos(90^\circ-\alpha)\sin\beta}\right\}^{-\frac{1}{2}}$$

[from equation (i)]

$$= \frac{1}{(\sin\alpha\sin\beta)^{\frac{1}{2}}}\times(\sin\alpha\sin\beta)^{\frac{1}{2}} = 1$$

87. (d) We know that in a cyclic quadrilateral sum of opposite angle is 180°.

$\therefore$ $A + C = 180^\circ$ ... (i)

and $B + D = 180^\circ$ ... (ii)

$\therefore$ $\cos A + \cos B + \cos C + \cos D$

$= \cos A + \cos B + \cos(180^\circ - A) + \cos(180^\circ - B)$

From equations (i) and (ii),

$= \cos A + \cos B - \cos A - \cos B = 0$

88. (b) We know that,

π radian = 180°

$\Rightarrow$ 1 radian $= \frac{180^\circ}{\pi} = \frac{180^\circ}{22}\times 7^\circ$

$= \frac{630^\circ}{11} = 57\frac{3^\circ}{11} = 57^\circ + \frac{3\times 60}{11}$ min

$= 57^\circ + 16' + \frac{4}{11}$ min

$= 57^\circ + 16' + \frac{4}{11}\times 60$ s $= 57^\circ + 16' + 21.8''$

$= 57^\circ\ 16'\ 21.8'' = 57^\circ\ 16'\ 22''$

89. (a) Given that, $\alpha + \beta = 90^\circ$ ... (i)

According to question,

$\beta = \frac{2}{3}\alpha$

$\therefore$ $\beta = \frac{2}{3}\alpha = \frac{2}{3}(90^\circ - \beta)$ [from equation (i)]

$\Rightarrow$ $\beta = 60^\circ - \frac{2}{3}\beta \Rightarrow \beta = 36^\circ$

90. (b) Given that, $\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = 2$

$\therefore$ $\sin^2\theta + \cos^2\theta = 2\sin\theta\cos\theta$

$\Rightarrow$ $\sin 2\theta = 1 = \sin 90^\circ$

$\Rightarrow$ $2\theta = 90^\circ \Rightarrow \theta = 45^\circ$

91. (c) Given, $A = \frac{\pi}{6}$ and $B = \frac{\pi}{3}$

I. L.H.S. = $\sin A + \sin B = \sin\frac{\pi}{6} + \sin\frac{\pi}{3}$

$= \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2}$

R.H.S. = $\cos A + \cos B = \cos\frac{\pi}{6} + \frac{\cos\pi}{3}$

$= \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{\sqrt{3}+1}{2}$

$\Rightarrow$ $\sin A + \sin B = \cos A + \cos B$

II. L.H.S. = $\tan A + \tan B = \tan\frac{\pi}{6} + \tan\frac{\pi}{3}$

$= \frac{1}{\sqrt{3}} + \sqrt{3} = \frac{4}{\sqrt{3}}$

R.H.S. = $\cot A + \cot B = \cot\frac{\pi}{6} + \cot\frac{\pi}{3}$

$= \sqrt{3} + \frac{1}{\sqrt{3}} = \frac{4}{\sqrt{3}}$

$\Rightarrow$ $\tan A + \tan B = \cot A + \cot B$

Both statements are true.

Alternate Method:

$$A + B = \frac{\pi}{6} + \frac{\pi}{3} = \frac{\pi}{2}$$

I. $\sin A + \sin B = \sin\left(\frac{\pi}{2} - B\right) + \sin\left(\frac{\pi}{2} - A\right)$

$= \cos B + \cos A = \cos A + \cos B$

II. $\tan A + \tan B = \tan\left(\frac{\pi}{2} - B\right) + \tan\left(\frac{\pi}{2} - A\right)$

$= \cot B + \cot A = \cot A + \cot B$

Hence, both statements are true.

92. (a) $\because$ In 24 h, Earth rotate about its own axis = 360°

In 1 h Earth rotate about its own axis $= \frac{360^\circ}{24} = 15^\circ$

In 4 h Earth rotate about its own axis = $15^\circ \times 4 = 60^\circ$

Since, in 60 min Earth rotate about its own axis = 15°

In 12 min Earth rotate about its own axis

$= \frac{15^\circ \times 12}{60} = 3^\circ$

$\therefore$ In 4 h 12 min Earth rotate about its own axis $= 60^\circ + 3^\circ = 63^\circ$

93. (a) We know that, $\sin^2\theta + \cos^2\theta = 1$

I. $\sin^2 1^\circ + \cos^2 1^\circ = 1$. It is true.

II. $\sec^2 33^\circ - \cot^2 57^\circ = \text{cosec}^2 37^\circ - \tan^2 53^\circ$

Now, $\sec^2(90^\circ - 57^\circ) = \text{cosec}^2 57^\circ$

and $\cot^2 57^\circ = \cot^2(90^\circ - 33^\circ) = \tan^2 33^\circ$

$\therefore$ $\sec^2 33^\circ - \cot^2 57^\circ = \text{cosec}^2 57^\circ - \tan^2 33^\circ$

Thus, Statement II is incorrect.

94. (c) Here, $p = a \sin x + b \cos x$ and $q = a \cos x - b \sin x$

On squaring both sides,

$\Rightarrow p^2 = a^2 \sin^2 x + b^2 \cos^2 x + 2ab \sin x \cos x$ ... (i)

$q = a \cos x - b \sin x$

On squaring both sides,

and $q^2 = a^2 \cos^2 x + b^2 \sin^2 x - 2ab \sin x \cos x$... (ii)

Now, add equation (i) and equation (ii), we get

$\therefore p^2 + q^2 = a^2 (\sin^2 x + \cos^2 x) + b^2 (\cos^2 x + \sin^2 x)$

$= a^2 + b^2$

95. (c) Given that, $\sin^2 x + \cos^2 x - 1 = 0$

$\Rightarrow \sin^2 x + \cos^2 x = 1$

which is an identity of trigonometric ratio and always true for every real value of x.

Therefore, the equation have an infinite solution.

96. (a) I. We know that,

$$\text{Radius} = \frac{\text{Arc}}{\text{Angle}} \text{ (given, arc length is constant)}$$

$$\text{Radius} \propto \frac{1}{\text{Angle}}$$

So, angular measure in radian decreases, if the radius on the arc increases.

II. $1800^\circ \times \dfrac{\pi}{180^\circ} = 10\pi$

Hence, only Statement I is correct.

97. (b) I. Given that, $\sin x + \cos x = 2$

$\Rightarrow (\sin x + \cos x)^2 = 4$

$\Rightarrow (\sin^2 x + \cos^2 x) + 2 \sin x \cos x = 4$

$\Rightarrow 1 + \sin 2x = 4$

$\Rightarrow \sin 2x = 3$

$\Rightarrow \sin 2x \neq 3$

Hence, there is no value of x in the first quadrant that satisfies $\sin x + \cos x = 2$

II. $\sin x - \cos x = 0$

$\Rightarrow \tan x = 1 = \tan \dfrac{\pi}{4} \Rightarrow x = \dfrac{\pi}{4}$

Also, there is only one value of x in the first quadrant that satisfies $\sin x - \cos x = 0$.

98. (c) Given that, $\cos x = \dfrac{5}{13} = \dfrac{\text{Base}}{\text{Hypotenuse}}$

$P = \sqrt{h^2 - b^2} = \sqrt{13^2 - 5^2}$

$= \sqrt{169 - 25} = \sqrt{144} = 12$

$\therefore \tan x - \cot x = \dfrac{p}{b} - \dfrac{b}{p}$

$= \dfrac{12}{5} - \dfrac{5}{12} = \dfrac{144 - 25}{60} = \dfrac{119}{60}$

99. (c) I. $\dfrac{\cot 30^\circ + 1}{\cot 30^\circ - 1} = 2(\cos 30^\circ + 1)$

$\Rightarrow \dfrac{\sqrt{3}+1}{\sqrt{3}-1} = 2\left(\dfrac{\sqrt{3}}{2} + 1\right)$

$\Rightarrow \dfrac{\sqrt{3}+1}{\sqrt{3}-1} \times \dfrac{\sqrt{3}+1}{\sqrt{3}+1} = 2\left(\dfrac{\sqrt{3}+2}{2}\right)$

$\Rightarrow \dfrac{3+1+2\sqrt{3}}{3-1} = \sqrt{3}+2$

$\Rightarrow \dfrac{4+2\sqrt{3}}{2} = \sqrt{3}+2$

$\Rightarrow \dfrac{2(2+\sqrt{3})}{2} = \sqrt{3}+2$

$\Rightarrow \sqrt{3}+2 = \sqrt{3}+2$

Hence, it is true.

II. $2 \sin 45^\circ \cos 45^\circ - \tan 45^\circ \cot 45^\circ = 0$

$\Rightarrow 2 \times \left(\dfrac{1}{\sqrt{2}} \times \dfrac{1}{\sqrt{2}}\right) - 1 \times 1 = 0$ or $2 \times \dfrac{1}{2} - 1 \times 1 = 0$

$\Rightarrow 1 - 1 = 0$

Hence, both Statements I and II are true.

100. (c) Given that, $3 \sin x + 5 \cos x = 5$

Now, squaring both sides,

$9 \sin^2 x + 25 \cos^2 x + 30 \sin x \cos x = 25$

$\Rightarrow 9(1 - \cos^2 x) + 25(1 - \sin^2 x) + 30 \sin x \cos x = 25$

$\Rightarrow 9 + 25 - \{9 \cos^2 x + 25 \sin^2 x - 30 \sin x \cos x\} = 25$

$\Rightarrow 9 = (3 \cos x - 5 \sin x)^2$

$\Rightarrow 3 \cos x - 5 \sin x = 3$

101. (b) Given that, $\tan \theta = \dfrac{3}{4}$ and $(0^\circ < \theta < 90^\circ)$ (acute)

$\because 1 + \tan^2 \theta = \sec^2 \theta$

$\Rightarrow \sec^2 \theta = 1 + \left(\dfrac{3}{4}\right)^2 = 1 + \dfrac{9}{16} = \dfrac{25}{16}$

$\Rightarrow \sec \theta = \dfrac{5}{4}$ (since, θ is acute)

$\therefore \cos \theta = \dfrac{4}{5}$

$\because \sin^2 \theta = 1 - \cos^2 \theta = 1 - \left(\dfrac{4}{5}\right)^2 = 1 - \dfrac{16}{25} = \dfrac{9}{25}$

$\therefore \sin \theta = \dfrac{3}{5}$ (since θ is acute)

102. (c) Given, $\sec(90-\theta)^\circ \sin\theta \sec 45^\circ$
$= \operatorname{cosec}\theta^\circ \sin\theta^\circ \sec 45^\circ$
$$= \frac{1}{\sin\theta}\cdot\sin\theta\cdot\left(\sqrt{2}\right) = \sqrt{2}$$

103. (c) Given that, $p^\circ = q^c$
$$\Rightarrow \left(p\cdot\frac{\pi}{180}\right)^c = q^c \qquad (\because 180^\circ = \pi^c)$$
$\therefore (p\pi)^c = (q180)^c$
$\therefore \pi p = 180q$

104. (d) θ lies is in first quadrant and $\tan\theta = 3$
On squaring both sides,
$\therefore \tan^2\theta = 9$
Add 1 both sides,
$\Rightarrow 1 + \tan^2\theta = 10$
$\Rightarrow \sec^2\theta = 10 \Rightarrow \sec\theta = \sqrt{10}$
$$\Rightarrow \cos\theta = \frac{1}{\sqrt{10}} \qquad \text{... (i)}$$
$$\because \sin^2\theta = 1-\cos^2\theta = 1-\frac{1}{10} = \frac{9}{10}$$
$$\Rightarrow \sin\theta = \frac{3}{\sqrt{10}} \qquad \text{... (ii)}$$
$$\text{Now, } \sin\theta + \cos\theta = \frac{3}{\sqrt{10}} + \frac{1}{\sqrt{10}} = \frac{4}{\sqrt{10}}$$
(θ lies in first quadrant)

105. (d) If $0^\circ < \theta < 90^\circ$, then all the trigonometric ratios can be obtained when any one of the six ratios is given. Since, we use any of the following identity to get any trigonometric ratios.
$\sin^2\theta + \cos^2\theta = 1$, $1 + \tan^2\theta = \sec^2\theta$
and $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

106. (d) $\sin A\cdot\cos A\cdot\tan A + \cos A\cdot\sin A\cdot\cot A$
$$= \sin A\cdot\cos A\cdot\frac{\sin A}{\cos A} + \cos A\cdot\sin A\cdot\frac{\cos A}{\sin A}$$
$= \sin^2 A + \cos^2 A = 1 \quad (\because \sin^2\theta + \cos^2\theta = 1)$
$= \operatorname{cosec}^2 A - \cot^2 A \quad (\because 1 + \cot^2\theta = \operatorname{cosec}^2\theta)$

107. (a) Let $f(\theta) = \dfrac{\sin\theta}{1+\cos\theta} + \dfrac{1+\cos\theta}{\sin\theta}$
$$= \frac{2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}}{1+2\cos^2\frac{\theta}{2}-1} + \frac{1+2\cos^2\frac{\theta}{2}-1}{2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}}$$
$$= \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} + \frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}$$
$$= \frac{2}{2}\cdot\frac{\left(\sin^2\frac{\theta}{2}+\cos^2\frac{\theta}{2}\right)}{\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}} = \frac{2}{\sin\theta} = 2\operatorname{cosec}\theta$$

108. (b) $\sin\theta\cdot\cos\theta = \dfrac{\sqrt{3}}{4}$ ... (i)
$\therefore \sin^4\theta + \cos^4\theta = (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cdot\cos^2\theta$
$= (1)^2 - 2(\sin\theta\cdot\cos\theta)^2$
$$= 1 - 2\left(\frac{\sqrt{3}}{4}\right)^2 = 1 - 2\cdot\frac{3}{16} = 1-\frac{3}{8} = \frac{5}{8}$$

109. (b) In 1 min = 60 s, distance travelled by the wheel
$= 12 \times$ Its circumference
$= 12 \times 2\pi r$
$\therefore$ In 1 s distance travelled by the wheel
$$= \frac{12\times 2\pi r}{60} = \frac{2}{5}\pi r$$
$$\because \text{Angle} = \frac{Ram}{\text{Radius}} = \frac{\frac{2}{5}\pi r}{r} = \frac{2}{5}\pi$$
Which is the required angle.

110. (b) Given that, $\cos A + \cos^2 A = 1$
$\Rightarrow \cos A = 1 - \cos^2 A = \sin^2 A$
Now, $2(\sin^2 A + \sin^4 A) = 2(\sin^2 A + \cos^2 A)$
$= 2\cdot(1) \qquad (\because \sin^2\theta + \cos^2\theta = 1)$
$= 2$

111. (c) $(1-\tan A)^2 + (1+\tan A)^2 + (1-\cot A)^2 + (1+\cot A)^2$
$= 1 + \tan^2 A - 2\tan A + 1 + \tan^2 A + 2\tan A + 1 + \cot^2 A - 2\cot A + 1 + \cot^2 A + 2\cot A$
$= 4 + 2(\tan^2 A + \cot^2 A)$
$= (2 + 2\tan^2 A) + (2 + 2\cot^2 A) = 2\sec^2 A + 2\operatorname{cosec}^2 A$
$$= 2\left(\frac{1}{\cos^2 A} + \frac{1}{\sin^2 A}\right) = 2\left(\frac{\sin^2 A + \cos^2 A}{\sin^2 A\cdot\cos^2 A}\right)$$
$$= \frac{2\cdot(1)}{\sin^2 A\cdot\cos^2 A} = 2\sec^2 A\cdot\operatorname{cosec}^2 A$$

112. (a) $\cos(A+B) = \cos(30^\circ + 60^\circ) = \cos 90^\circ = 0$

113. (b) $\operatorname{cosec}(C - D + B) = \operatorname{cosec}(120^\circ - 150^\circ + 60^\circ)$
$= \operatorname{cosec}(180^\circ - 150^\circ) = \operatorname{cosec} 30^\circ = 2$

114. (d) If ABCD is a cyclic quadrilateral, then sum of opposite angles is 180º.
$30^\circ + 120^\circ = 150^\circ \neq 180^\circ$ and $60^\circ + 150^\circ = 210^\circ \neq 180^\circ$
So, Statement I is not correct.
Statement II :
$\sin(B - A) = \cos(D - C)$
$\Rightarrow \sin(60^\circ - 30^\circ) = \cos(150^\circ - 120^\circ)$

$\Rightarrow$ sin 30º = cos 30º $\Rightarrow \frac{1}{2} \neq \frac{\sqrt{3}}{2}$

So, Statement II is also not correct.

115. (c) Required angles of a quadrilateral ABCD are 30º, 60º, 120º and 150º, respectively.

$\sec^2 D - \tan^2 D = \sec^2 (150°) - \tan^2 (150°)$

$= \sec^2 (90° + 60°) - \tan^2 (90° + 60°)$

$= \text{cosec}^2\, 60° - \cot^2 60°$

$$= \left(\frac{2}{\sqrt{3}}\right)^2 - \left(\frac{1}{\sqrt{3}}\right)^2 = \frac{4}{3} - \frac{1}{3} = \frac{3}{3} = 1$$

After we know that $\sec^2\theta - \tan^2\theta = 1$

Similary, $\sec^2 D - \tan^2 D$ is always equal to 1.

116. (c) $\frac{\tan A - \sin A}{\sin^3 A} = \frac{\frac{\sin A}{\cos A} - \sin A}{\sin^3 A}$

Multiply in Numerator and Denominator by (1 + cos A)

$$= \frac{(1 - \cos A)}{\cos A \cdot \sin^2 A} \times \frac{(1 + \cos A)}{(1 + \cos A)}$$

$$= \frac{(1 - \cos^2 A)}{\cos A \cdot \sin^2 A\,(1 + \cos A)}$$

$$= \frac{\sin^2 A}{\cos A \cdot \sin^2 A\,(1 + \cos A)}$$

$$= \frac{1}{\cos A} \cdot \frac{1}{1 + \cos A} = \frac{\sec A}{1 + \cos A}$$

117. (b) Let $f(\theta) = \sin\theta + \cos\theta$

Maximum and minimum value of $a\cos\theta + b\sin\theta$ is

$$-\sqrt{a^2 + b^2} \le a\cos\theta + b\sin\theta \le \sqrt{a^2 + b^2}$$

$\therefore\ -\sqrt{1+1} \le \cos\theta + \sin\theta \le \sqrt{1+1}$

$\Rightarrow\ -\sqrt{2} \le \cos\theta + \sin\theta \le \sqrt{2}$

$\Rightarrow\ -1.414 \le \cos\theta + \sin\theta \le 1.414$

$\therefore\ f(\theta) = (\sin\theta + \cos\theta) \in [-1.414, 1.414]$

and let $g(\theta) = \tan\theta + \cot\theta = \tan\theta + \frac{1}{\tan\theta}$

($\because$ AM $\ge$ GM)

$$\Rightarrow \frac{\tan\theta + \frac{1}{\tan\theta}}{2} \ge \left(\tan\theta \cdot \frac{1}{\tan\theta}\right)^{\frac{1}{2}}$$

$\Rightarrow$ $(\tan\theta + \cot\theta) \ge 2$

So, $(\tan\theta + \cot\theta)$ is always greater than 1.

Hence, Statement 1 is false and Statement II is true.

118. (c) Given that :

$\sin A = \frac{3}{5}$ (A is acute, i.e. $0 \le A < 90°$)

Then, $\cos A = \sqrt{1 - \sin^2 A}$

$$= \sqrt{1 - \left(\frac{3}{5}\right)^2} = \sqrt{1 - \frac{9}{25}}$$

$$= \sqrt{\frac{16}{25}} = \frac{4}{5}$$

$\therefore$ $\tan A + \sec A = \frac{\sin A}{\cos A} + \frac{1}{\cos A} = \frac{1 + \sin A}{\cos A}$

$$= \frac{1 + \frac{3}{5}}{\frac{4}{5}} = \frac{\frac{8}{5}}{\frac{4}{5}} = \frac{8}{4} = 2$$

119. (b) Here, given that: $\sin\theta = \frac{x^2 - y^2}{x^2 + y^2}$

$\because$ $\cos^2\theta = 1 - \sin^2\theta$

$$= 1 - \left(\frac{x^2 - y^2}{x^2 + y^2}\right)^2$$

$$= \frac{(x^2 + y^2)^2 - (x^2 - y^2)^2}{(x^2 + y^2)^2}$$

$$= \frac{2x^2 \cdot 2y^2}{(x^2 + y^2)^2}$$

$$= \frac{4x^2 y^2}{(x^2 + y^2)^2} = \left(\frac{2xy}{x^2 + y^2}\right)^2$$

$\therefore$ $\cos\theta = \frac{2xy}{x^2 + y^2}$

120. (d) Given that:

$$a^2 = \frac{1 + 2\sin\theta\cos\theta}{1 - 2\sin\theta\cos\theta}$$

$$\Rightarrow a^2 = \frac{(\sin^2\theta + \cos^2\theta) + 2\sin\theta \cdot \cos\theta}{(\sin^2\theta + \cos^2\theta) - 2\sin\theta \cdot \cos\theta}$$

$$\Rightarrow a^2 = \frac{(\sin\theta + \cos\theta)^2}{(\sin\theta - \cos\theta)^2} \Rightarrow \frac{a}{1} = \frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}$$

(applying componendo dividendo formula)

$\Rightarrow \dfrac{a+1}{a-1} = \dfrac{(\sin\theta+\cos\theta)+(\sin\theta-\cos\theta)}{(\sin\theta+\cos\theta)-(\sin\theta-\cos\theta)}$

$\Rightarrow \dfrac{a+1}{a-1} = \dfrac{2\sin\theta}{2\cos\theta} = \tan\theta$

121. (c) $\because$ $5\sin\theta + 12\cos\theta = 13$
Now, squaring both sides, we get
$25\sin^2\theta + 144\cos^2\theta + 120\sin\theta\cos\theta = 169$
$\Rightarrow 25(1-\cos^2\theta) + 144(1-\sin^2\theta) + 120\sin\theta\cos\theta = 169$
$\Rightarrow 25 - 25\cos^2\theta + 144 - 144\sin^2\theta + 120\sin\theta\cos\theta = 169$
$\Rightarrow 25\cos^2\theta + 144\sin^2\theta - 120\sin\theta\cos\theta = 169 - 169$
$\Rightarrow (5\cos\theta - 12\sin\theta)^2 = 0$
$\therefore$ $5\cos\theta - 12\sin\theta = 0$

122. (c) $\dfrac{4\sin\theta-\cos\theta}{4\sin\theta+9\cos\theta}$

On dividing both numerator and denominator by $\cos\theta$, we get

$= \dfrac{\dfrac{4\sin\theta}{\cos\theta}-\dfrac{\cos\theta}{\cos\theta}}{\dfrac{4\sin\theta}{\cos\theta}+\dfrac{9\cos\theta}{\cos\theta}}$

Now, put the value of 4 tan θ

$\dfrac{4\tan\theta-1}{4\tan\theta+9} = \dfrac{3-1}{3+9} = \dfrac{2}{12} = \dfrac{1}{6}$

123. (c) Given, $\sin\theta - \cos\theta = 0$
$\sin\theta = \cos\theta$, then
$\theta = 45^\circ$
$\therefore$ $\sin^4\theta + \cos^4\theta = (\sin 45^\circ)^4 + (\cos 45^\circ)^4$

$= \left(\dfrac{1}{\sqrt{2}}\right)^4 + \left(\dfrac{1}{\sqrt{2}}\right)^4$

$= \dfrac{1}{4}+\dfrac{1}{4} = \dfrac{1+1}{4} = \dfrac{2}{4} = \dfrac{1}{2}$

124. (a) Only Statement I is correct as tan θ increases faster than sin θ as θ increases while Statement II is wrong as the value of sin θ + cos θ is not always greater than 1. It may also be equal to 1.

125. (a) $\dfrac{(\sin\theta+\cos\theta)(\tan\theta+\cot\theta)}{\sec\theta+\operatorname{cosec}\theta}$

$= \dfrac{(\sin\theta+\cos\theta)\left(\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\sin\theta}\right)}{\dfrac{1}{\cos\theta}+\dfrac{1}{\sin\theta}}$

$= \dfrac{(\sin\theta+\cos\theta)\left(\dfrac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}\right)}{\dfrac{\sin\theta+\cos\theta}{\sin\theta\cos\theta}}$

$[\because \sin^2\theta+\cos^2\theta = 1]$

$= \dfrac{(\sin\theta+\cos\theta)\left(\dfrac{1}{\sin\theta\cos\theta}\right)}{\dfrac{\sin\theta+\cos\theta}{\sin\theta\cos\theta}}$

$= \dfrac{\dfrac{\sin\theta+\cos\theta}{\sin\theta\cos\theta}}{\dfrac{\sin\theta+\cos\theta}{\sin\theta\cos\theta}} = 1$

126. (c) $\dfrac{\cos^2(45^\circ+\theta)+\cos^2(45^\circ-\theta)}{\tan(60^\circ+\theta)\tan(30^\circ-\theta)}$

$= \dfrac{\dfrac{\cos(90^\circ+2\theta)+1}{2}+\dfrac{\cos(90^\circ-2\theta)+1}{2}}{\tan(60^\circ+\theta)\cdot\tan\left[90^\circ-(60^\circ+\theta)\right]}$

$(\because \cos 2\theta = 2\cos^2\theta - 1)$

$= \dfrac{\dfrac{\cos(90^\circ+2\theta)+\cos(90^\circ-2\theta)}{2}+1}{\tan(60^\circ+\theta)\cot(60^\circ+\theta)}$

$= \dfrac{\dfrac{-\sin 2\theta+\sin 2\theta}{2}+1}{1} = 1$

127. (b) $\sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta$
$= (\sin^2\theta)^3 + (\cos^2\theta)^3 + 3\sin^2\theta\cos^2\theta(\sin^2\theta + \cos^2\theta)$
$[\because (a+b)^3 = a^3 + b^3 + 3ab(a+b)]$
$= (\sin^2\theta + \cos^2\theta)^3$ $(\because \sin^2\theta + \cos^2\theta = 1)$
$= (1)^3 = 1$

128. (a) $\dfrac{(1+\sec\theta-\tan\theta)\cos\theta}{(1+\sec\theta+\tan\theta)(1-\sin\theta)}$

$= \dfrac{\left(1+\dfrac{1}{\cos\theta}-\dfrac{\sin\theta}{\cos\theta}\right)\cos\theta}{\left(1+\dfrac{1}{\cos\theta}+\dfrac{\sin\theta}{\cos\theta}\right)(1-\sin\theta)}$

$= \dfrac{(\cos\theta+1-\sin\theta)}{\dfrac{(\cos\theta+1+\sin\theta)(1-\sin\theta)}{\cos\theta}}$

$= \dfrac{\cos\theta+1-\sin\theta}{\dfrac{\cos\theta+1+\sin\theta-\sin\theta\cos\theta-\sin\theta-\sin^2\theta}{\cos\theta}}$

$$= \frac{\cos\theta + 1 - \sin\theta}{\frac{\cos\theta + 1 - \sin^2\theta - \sin\theta\cos\theta}{\cos\theta}}$$

$(\because\ 1 - \sin^2\theta = \cos^2\theta)$

$$= \frac{\cos\theta + 1 - \sin\theta}{\frac{\cos\theta + \cos^2\theta - \sin\theta\cos\theta}{\cos\theta}}$$

$$= \frac{\cos\theta + 1 - \sin\theta}{\frac{\cos(\cos\theta + 1 - \sin\theta)}{\cos\theta}} = 1$$

129. (a) $\sin\theta + \cos\theta = \sqrt{3}$

Squaring both sides, we get

$(\sin\theta + \cos\theta)^2 = (\sqrt{3})^2$

$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$

$\Rightarrow 1 + 2\sin\theta\cos\theta = 3$

$\Rightarrow \sin\theta\cos\theta = \frac{3-1}{2} = \frac{2}{2} = 1$ ... (i)

Now, $\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta}$

From equation (i),

$\tan\theta + \cot\theta = \frac{1}{1} = 1$

130. (b) $\tan\theta + \sec\theta = m$ ... (i)

We know that :

$\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow (\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$\Rightarrow \sec\theta - \tan\theta = \frac{1}{m}$ ... (ii)

Now, adding equations (i) and (ii), we get

$2\sec\theta = m + \frac{1}{m} = \frac{m^2+1}{m}$

$\Rightarrow \sec\theta = \frac{m^2+1}{2m}$

131. (a) $\text{cosec}(75° + \theta) - \sec(15° - \theta)$

$= \text{cosec}(75° + \theta) - \sec[90° - (75° + \theta)]$

$= \text{cosec}(75° + \theta) - \text{cosec}(75° + \theta)$

$= 0$

132. (c) In ΔABC, if $\angle C$ is 90°, then

$\angle A + \angle B = 180° - 90° = 90°$

then, $\cos(A + B) + \sin(A + B)$

$= \cos 90° + \sin 90°$

$= 0 + 1 = 1$

133. (c) Given $\sin\alpha = \frac{\sqrt{3}}{2}$

$\Rightarrow \alpha = 60°$ $\left(\because\ \sin 60° = \frac{\sqrt{3}}{2}\right)$

Now, $\cos\beta = \frac{\sqrt{3}}{2}$

$\Rightarrow \beta = 30°$ $\left(\because\ \cos 30° = \frac{\sqrt{3}}{2}\right)$

and $\tan\gamma = 1$

$\Rightarrow \gamma = 45°$ $(\because\ \tan 45° = 1)$

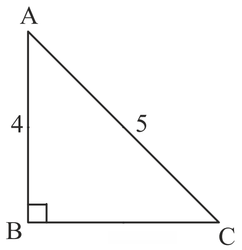
$\therefore\ \alpha + \beta + \gamma = 60° + 30° + 45° = 135°$

134. (a) Since, value of $\cos\theta$ decreases, from 0° to 90° and at 45°, it is equal to the value of $\sin\theta$.
Similarly, value of $\sin\theta$ increases from 0° to 90° and at 45°, it is equal to the value of $\cos\theta$.
For $0° < \theta < 45°$, $\cos\theta > \sin\theta$
So, value of cos 25° – sin 25° is always positive but less than 1.

135. (b) In ΔABC,

$\cos A = \frac{4}{5}$, i.e, AB = 4 and AC = 5

$\sin C = \frac{AB}{AC} = \frac{4}{5}$



136. (c) Given, α and β are complementary angles.

$\therefore\ \alpha = 90 - \beta$

$= \sqrt{\cos\alpha\,\text{cosec}\,\beta - \cos\alpha\sin\beta}$

$= \sqrt{\frac{\cos\alpha}{\sin\beta} - \cos\alpha\sin\beta}$

$= \sqrt{\frac{\cos\alpha}{\cos(90-\beta)} - \cos\alpha\cos(90-\beta)}$

$= \sqrt{\frac{\cos\alpha}{\cos\alpha} - \cos\alpha\cdot\cos\alpha}$

$= \sqrt{1 - \cos^2\alpha} = \sqrt{\sin^2\alpha} = \sin\alpha$

137. (c) $\because\ 2\cot\theta = 3$

$\Rightarrow \cot\theta = \frac{3}{2}$

$\therefore \dfrac{2\cos\theta - \sin\theta}{2\cos\theta + \sin\theta} = \dfrac{2\cot\theta - 1}{2\cot\theta + 1}$

$= \dfrac{2 \times \frac{3}{2} - 1}{2 \times \frac{3}{2} + 1} = \dfrac{3-1}{3+1} = \dfrac{2}{4} = \dfrac{1}{2}$

138. (d) $\sin^6\theta + \cos^6\theta = (\sin^2\theta)^3 + (\cos^2\theta)^3$
$= (\sin^2\theta + \cos^2\theta)(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta)$
$= (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta)$
$= (1 - 3\sin^2\theta\cos^2\theta) = 1 - 3 \times \dfrac{1}{4} = 1 - \dfrac{3}{4} = \dfrac{1}{4}$

139. (d) Given that
$\sec\theta + \tan\theta = 2$ ... (i)
We know that
$\sec^2\theta - \tan^2\theta = 1$
$(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$
$\Rightarrow \sec\theta - \tan\theta = \dfrac{1}{2}$ ... (ii)
Now, adding equations (i) and (ii), we get
$2\sec\theta = \dfrac{1}{2} + 2$
$\therefore \sec\theta = \dfrac{5}{4}$

140. (b) $\text{cosec}(75^\circ + \theta) - \sec(15^\circ - \theta)$
$- \tan(55^\circ + \theta) + \cot(35^\circ - \theta)$
$\Rightarrow \text{cosec}(75^\circ + \theta) - \text{cosec}[90^\circ - (75^\circ + \theta)]$
$- \tan(55^\circ + \theta) + \tan(90^\circ - (35^\circ - \theta)]$
$\Rightarrow \text{cosec}(75^\circ + \theta) - \text{cosec}(75^\circ + \theta)$
$- \tan(55^\circ + \theta) + \tan(55^\circ + \theta)] = 0$

141. (c) $\sin\theta + 2\cos\theta = 1$
On squaring both sides, we get
$(\sin\theta + 2\cos\theta)^2 = 1$
$\Rightarrow \sin^2\theta + 4\cos^2\theta + 4\sin\theta\cos\theta = 1$
$\Rightarrow (1 - \cos^2\theta) + 4(1 - \sin^2\theta) + 4\sin\theta\cos\theta = 1$
$\Rightarrow -(\cos^2\theta + 4\sin^2\theta) + 4\sin\theta\cos\theta = 1 - 5$
$\Rightarrow \cos^2\theta + 4\sin^2\theta - 4\sin\theta\cos\theta = 4$
$\Rightarrow (2\sin\theta - \cos\theta)^2 = 4 \Rightarrow 2\sin\theta - \cos\theta = 2$

142. (a) $\cos x + \sec x = 2$ ... (i)
On squaring both sides, we get
$\cos^2 x + \sec^2 x + 2 = 4$
$\Rightarrow \cos^2 x + \sec^2 x = 2$ ... (ii)
On cubing equation (i), we get
$\cos^3 x + \sec^3 x + 3(\cos x + \sec x) = 8$
$\Rightarrow \cos^3 x + \sec^3 x + (3 \times 2) = 8$
$\Rightarrow \cos^3 x + \sec^3 x = 2$ ... (iii)
Similarly, when we multiply n in power both sides,
$\cos^n x + \sec^n x = 2$

Alternate Method
of we put x = 90°, :
then $\cos 90^\circ + \sec 90^\circ = 1 + 1 = 2$
Similary $\text{cosec}^h x + \sec^h x$
$= \text{cosec}^h 90^\circ + \sec^h 90^\circ$
$= 1 + 1 = 2$

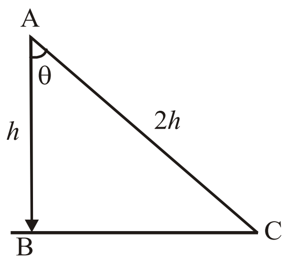
143. (d) $\sin 25^\circ \sin 35^\circ \sec 65^\circ \sec 55^\circ$
$= \sin 25^\circ \cdot \sin 35^\circ \cdot \dfrac{1}{\cos 65^\circ} \cdot \dfrac{1}{\cos 55^\circ}$
$= \sin 25^\circ \cdot \sin 35^\circ \cdot \dfrac{1}{\cos(90 - 25^\circ)} \cdot \dfrac{1}{\cos(90 - 35^\circ)}$
$= \sin 25^\circ \cdot \sin 35^\circ \cdot \dfrac{1}{\sin 25^\circ} \cdot \dfrac{1}{\sin 35^\circ} = 1$

144. (b) $\tan 8\theta = \cot 2\theta \Rightarrow \tan 8\theta = \tan(90 - 2\theta)$
$\Rightarrow 8\theta = 90 - 2\theta \Rightarrow \theta = 9^\circ$
$\therefore \tan 5\theta \Rightarrow \tan 45^\circ = 1$

145. (a) Given, $\sin(A + B) = 1$
$\Rightarrow A + B = \sin^{-1} 1 \Rightarrow (A + B) = 90^\circ$
$\therefore B = 90^\circ - A \Rightarrow A = 90^\circ - B$
$\cos(A - B) = \cos A \cos B + \sin A \sin B$
$= \cos(90 - B)\cos B + \sin(90 - B)\sin B$
$= \sin B \cos B + \cos B \sin B$
$= 2\sin B \cos B = \sin 2B$

146. (d) Clock will make right angle at $(5n + 15) \times \dfrac{12}{11}$ min past n.
Here n = 3,
$\therefore (5 \times 3 + 15) \times \dfrac{12}{11}$ min past 3
$= 30 \times \dfrac{12}{11}$ min past 3
$= 32\dfrac{8}{11}$ min past 3, i.e. 3 h $32\dfrac{8}{11}$ min.

147. (d) Let AB be the height, AC be the string and the angle made by string with the post be θ.



From figure $\cos\theta = \dfrac{AB}{AC} = \dfrac{h}{2h} = \dfrac{1}{2} = \cos\dfrac{\pi}{3}$

$\theta = \dfrac{\pi}{3}$

148. (a) Given, $0 < \theta < \dfrac{\pi}{4}$, then $\sqrt{1-2\sin\theta\cos\theta}$

$= \sqrt{\sin^2\theta+\cos^2\theta-2\sin\theta+\cos\theta}$
[$\because \sin^2\theta \cos^2\theta = 1$]

$= \sqrt{(\cos\theta-\sin\theta)^2}$ $(\cos\theta - \sin\theta)^2$ is always + ve

[$\because$ $0 < \theta < \dfrac{\pi}{4}$, $\cos\theta > \sin\theta$, so we take $(\cos\theta - \sin\theta)^2$]

$= \cos\theta - \sin\theta$

149. (c) Take $\theta = 45°$
$\tan\theta + \cot\theta = \tan 45° + \cot 45° = 1 + 1 = 2$
Now, $\sin\theta + \cos\theta$
$= \sin 45° + \cos 45°$

$$= \frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}=\frac{2}{\sqrt{2}}=\sqrt{2}$$

150. (a) $\dfrac{\sec x}{\cot x+\tan x}$

$$\Rightarrow \frac{\dfrac{1}{\cos x}}{\dfrac{\cos x}{\sin x}+\dfrac{\sin x}{\cos x}}$$

$$\Rightarrow \frac{\dfrac{1}{\cos x}}{\dfrac{\cos^2 x+\sin^2 x}{\sin x\cos x}}$$

$$\Rightarrow \frac{1}{(\cos x)}\times\cos x\times\sin x$$

$\Rightarrow \sin x$

151. (b) $\dfrac{\sin x-\cos x+1}{\sin x+\cos x-1}$

$$= \frac{(\sin x-\cos x)+1}{(\sin x+\cos x)-1}\times\frac{(\sin x+\cos x)+1}{(\sin x+\cos x)+1}$$

$$= \frac{(\sin x-\cos x+1)(\sin x+\cos x+1)}{(\sin x+\cos x)^2-1}$$

$$\frac{\sin^2 x+\sin x\cos x+\sin x-\cos x\sin x-\cos^2 x-\cos x+\sin x+\cos x+1}{\sin^2 x+\cos^2 x+2\sin x\cos x-1}$$

$$= \frac{\sin^2 x+2\sin x-\cos^2 x+1}{1+2\sin x\cos x-1}$$

$$= \frac{\sin^2 x+2\sin x-(1-\sin^2 x)x+1}{2\sin x\cos x}$$

$$= \frac{\sin^2 x+2\sin x-1+\sin^2 x+1}{2\sin x\cos x}$$

$$= \frac{2\sin^2 x+2\sin x}{2\sin\cos x}=\frac{2\sin x(\sin x+1)}{2\sin\cos x}$$

$$= \frac{\sin x+1}{\cos x}$$

152. (b) $(\sin^2x - \cos^2x)\,(1 - \sin^2x\cos^2x)$
$= (\sin^2x - \cos^2x)\,[(\sin^2x + \cos^2x)^2 - \sin^2x\cos^2x]$
[$\because \sin^2x + \cos^2x = 1$]
$(\sin^2 x - \cos^2x)\,[(\sin^4 x + \cos^4x + 2\sin^2x\cos^2x) - \sin^2x\cos^2x]$
$= (\sin^2x - \cos^2x)\,(\sin^4 x + \cos^4 x + \sin^2x\cos^2x)$
$= \sin^6x + \sin^2x\cos^4x + \sin^4 x\cos^2 x - \cos^2 x\sin^4x - \cos^6x - \sin^2x\cos^4x$
$= \sin^6 x - \cos^6x$

153. (d) $(\sin x.\cos y + \cos x.\sin y)\,\sin x.\cos y - \cos x.\sin y)$
$= \sin(x + y).\sin(x - y)$
$= \sin^2x - \sin^2y$

$$\left[\begin{array}{l}\because \sin^2 A-\sin^2 B=\\ \sin(A+B)\sin(A-B)\end{array}\right]$$

154. (b) $(1 + \cot x - \operatorname{cosec} x)\,(1 + \tan x + \sec x)$

$$= (1+\cot x-\operatorname{cosec} x)\left(1+\frac{1}{\cot x}+\sec x\right)$$

$$= \frac{(1+\cot x-\operatorname{cosec} x)(1+\cot x+\operatorname{cosec} x)}{\cot x}$$

$$= \frac{(1+\cot x)^2-(\operatorname{cosec} x)^2}{\cot x}$$

$$= \frac{1^2+\cot^2 x+2\cot x-\operatorname{cosec}^2 x}{\cot x}$$

$$= \frac{1+2\cot x-(\operatorname{cosec}^2 x-\cot^2 x)}{\cot x}$$

$$= \frac{1+2\cot x-1}{\cot x}=2$$

155. (d) $(\operatorname{cosec} x - \sin x)\,(\sec x - \cos x)\,(\tan x + \cot x)$

$$= \left(\frac{1}{\sin x}-\sin x\right)\left(\frac{1}{\cos x}-\cos x\right)\left(\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right)$$

$$= \frac{(1-\sin^2 x)(1-\cos^2 x)(\sin^2 x+\cos^2 x)}{\sin x\cos x.\sin x\cos x}$$

$$= \frac{\cos^2 x\sin^2 x\times 1}{\sin^2 x\cos^2 x}=1$$

156. (d) We know that, $\sin 1° > \sin 1$ and $\cos 1 < \cos 1°$
Hence, neither Statement 1 nor 2 is correct.

157. (a) Here put $\theta = 90°$

$\sin 90° + \text{cosec} 90°$

$= 1 + 1 = 2$

Now, $\sin^9 x + \text{cosec}^9 x = (\sin 90)^9 + (\text{cosec } 90)^9$

$= 1 + 1 = 2$

158. (b) Let $\sin x + \cos x = p$ ....(i)

$\sin^3 x + \cos^3 x = q$ ....(ii)

On cubing Eq. (i) both sides

$\sin^3 x + \cos^3 x + 3 \sin x \cos x (\sin x + \cos x) = p^3$

Put $\sin^3 x + \cos^3 x = q$ from equation (ii)

$\Rightarrow q + 3 \sin x \cos x(p) = p^3$....(iii)

On squaring Eq. (i) both sides, we get

$\sin^2 x + \cos^2 x + 2 \sin x \cos x = p^2$

$\Rightarrow \quad \sin x \cos x = \dfrac{p^2 - 1}{2} \quad [\because \sin^2 x + \cos^2 x = 1]$

From Eq. (iii),

$$q + \frac{3(p^2 - 1)}{2} = p^3$$

$\Rightarrow 2q + 3p^3 - 3p = 2p^3 \quad \Rightarrow p^3 - 3p = -2q$

159. (c) $\tan(A + B) = \sqrt{3}$

$\tan(A + B) = \tan 60°$

$A + B = 60°$ ....(i)

Now, $\tan A = 1$

$\tan A = \tan 45°$

$A = 45°$

Now putting the value of A in eqn (i)

$B = 60° - 45° = 15°$

$\tan(A - B) = \tan(45° - 15°) = \tan 30° = \dfrac{1}{\sqrt{3}}$

160. (*) $\cos A = \tan B$

Squaring on both sides

$\cos^2 A = \tan^2 B$

$$\Rightarrow \tan^2 B = \frac{\sin^2 B}{\cos^2 B} = \frac{1 - \cos^2 B}{\cos^2 B}$$

$$\therefore \cos^2 A = \frac{1 - \cos^2 B}{\cos^2 B}$$

$$\cos^2 A = \frac{1 - \tan^2 C}{\tan^2 C}$$

$(\because \cos B = \tan C)$

$$\Rightarrow \cos^2 A \tan^2 C = 1 - \tan^2 C = 1 - \frac{\sin^2 C}{\cos^2 C}$$

$$\Rightarrow \cos^2 A \frac{\sin^2 C}{\cos^2 C} = \frac{\cos^2 C - \sin^2 C}{\cos^2 C}$$

$\Rightarrow \cos^2 A (1 - \cos^2 C) = 2 \cos^2 C - 1$

$\Rightarrow \cos^2 A (1 - \tan^2 A) = 2\tan^2 A - 1$

$$\cos^2 A - \sin^2 A = \frac{2\sin^2 A}{\cos^2 A} - 1$$

$$\Rightarrow 1 - 2\sin^2 A = \frac{2\sin^2 A - \cos^2 A}{\cos^2 A}$$

$\Rightarrow \cos^2 A (1 - 2\sin^2 A) = 2\sin^2 A - \cos^2 A$

$\Rightarrow \cos^2 A (1 - 2\sin^2 A) = 2\sin^2 A - 1 + \sin^2 A$

$\Rightarrow (1 - \sin^2 A)(1 - 2\sin^2 A) = 3\sin^2 A - 1$

$\Rightarrow 1 - 2\sin^2 A - \sin^2 A + 2\sin^4 A = 3\sin^2 A - 1$

$\Rightarrow 1 - 3\sin^2 A + 2\sin^4 A = 3\sin^2 A - 1$

$\Rightarrow 2\sin^4 A - 6\sin^2 A + 2 = 0$

$\Rightarrow \sin^4 A - 3\sin^2 A + 1 = 0$

This is quadratic equation in $\sin^2 A$

$(\sin^2 A)^2 - 3(\sin^2 A) + 1 = 0$

$$\sin^2 A = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(1)}}{2}$$

$$= \frac{3 \pm \sqrt{5}}{2}$$

$\sin^2 A = \dfrac{3 + \sqrt{5}}{2}$ not possible because $\sin^2 A \not> 1$

So $\sin^2 A = \dfrac{3 - \sqrt{5}}{2}$.

So none of the options are correct.

161. (c) $\dfrac{3 - \tan^2 A}{1 - 3\tan^2 A} = K$

$3 - \tan^2 A = K - 3K\tan^2 A$

$3K\tan^2 A - \tan^2 A = K - 3$

$\tan^2 A(3K - 1) = K - 3$

$\tan^2 A = \dfrac{K - 3}{3K - 1}$ ...(i)

Subject to the condition $K > 3$ or $K < \dfrac{1}{3}$.

$\text{cosec}A(3\sin A - 4\sin^3 A) = 3 - 4\sin^2 A$

$\cot^2 A = \dfrac{3K - 1}{K - 3}$

$\text{cosec}^2 A = \dfrac{K - 3 + 3K - 1}{K - 3} = \dfrac{4K - 4}{K - 3}$

$\sin^2 A = \dfrac{K - 3}{4(K - 1)}$

$3 - 4\sin^2 A = 3 - \dfrac{4(K - 3)}{4(K - 1)}$

$$= \frac{3K-3-K+3}{K-1} = \frac{2K}{K-1}$$

where K > 3 or K < $\frac{1}{3}$.

162. (d) tan A + cot A = 4

$\Rightarrow$ Squaring both sides

$\tan^2 A + \cot^2 A + 2 = 16$

$\tan^2 A + \cot^2 A = 14$

Again, squaring both sides

$\tan^4 A + \cot^4 A + 2 = 196$

$\tan^4 A + \cot^4 A = 194$

163. (c) **Statement 1**

$$1 = \sqrt{\frac{1-\sin n}{1+\sin x}} = \sqrt{\frac{(1-\sin x)(1-\sin x)}{(1-\sin x)(1+\sin x)}}$$

$$= \frac{1-\sin x}{\sqrt{1-\sin^2 x}} = \frac{1-\sin x}{\sqrt{\cos^2 x}} = \frac{1-\sin x}{\cos x}$$

P = q

$$r = \frac{\cos x}{1+\sin x} = \frac{\cos x(1-\sin x)}{(1+\sin x)(1-\sin x)} = \frac{\cos x(1-\sin x)}{1-\sin^2 x}$$

$$= \frac{\cos x(1-\sin x)}{\cos^2 x} = \frac{1-\sin x}{\cos x}$$

P = q = r

Now, Statement 2 $P^2 = qr$

$$= \frac{1-\sin x}{\cos x}.\frac{\cos x}{1+\sin x} = \frac{1-\sin x}{1+\sin x} = P^2$$

So, Both are correct.

164. (c) **Statement 1**

$$\frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A}$$

$$\Rightarrow \frac{\cos A.\cos A}{\cos A-\sin A} + \frac{\sin A.\sin A}{\sin A-\cos A}$$

$$= \frac{\cos^2 A-\sin^2 A}{(\cos A-\sin A)} = \cos A + \sin A.$$

Statement 2

$(1 - \sin A - \cos A)^2 = 2(1 - \sin A)(1 + \cos A)$

LHS $= (1 - \sin A - \cos A)^2$

$= 1 + \sin^2 A + \cos^2 A - 2\sin A + 2\sin A\cos A - 2\cos A$

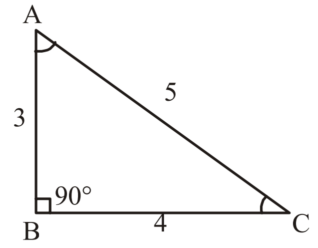
$= 2 - 2\sin A\ 2 + \cos A + 2\sin A\cos A$

$\Rightarrow 2\{(1 - \sin A) + \cos A(1 - \sin A)\}$

$= 2(1 - \sin A)(1 + \cos A)$

So both (1) and (2) are correct.

165. (c)



Given $\frac{AB}{BC} = \frac{3}{4}$

sin A + sin B + sin C

$$= \frac{4}{5} + 1 + \frac{3}{5} = \frac{4+3+5}{5} = \frac{12}{5}$$

166. (b) $\text{cosec}^2 67° + \sec^2 57° - \cot^2 33° - \tan^2 23°$

$= (\text{cosec}^2 67° - \tan^2 23°) + (\sec^2 57° - \cot^2 33°)$

$= (\sec^2 23° - \tan^2 23°) + (\text{cosec}^2 33° - \cot^2 33°)$

$(\because \text{cosec}(90 - \theta) = \sec\theta)$

= 1 + 1 = 2.

167. (b) **Statement (1)**

$\sin^4 x - 2\sin^2 x - 1 = 0$

Let $\sin^2 x = t$

$\Rightarrow t^2 - 2t - 1 = 0$

$$\Rightarrow t = \frac{2 \pm 2\sqrt{2}}{2} \Rightarrow t = 1 \pm \sqrt{2}$$

$\Rightarrow \sin^2 x = 1 \pm \sqrt{2}$ i.e., $1 - \sqrt{2}$ or $1 + \sqrt{2}$

$\sin^2 x$ cannot be –ve.

and sin x lies between –1 and 1

So, for $0 \le x \le \pi/2$ there is no value that satisfies the equation.

$\therefore$ (1) is not true

Statement (2) sin (1.5) > cos (1.5)

1.5 radian is $1.5 \times \frac{180°}{\pi} > 90°$ in 2nd quadrant.

sin (1.5) is +ve but cos(1.5) falls in second where it is –ve.

So, it is always < sin (1.5).

So, only (2) is correct.

168. (b) $\sin x + \cos x = c$ ....(i)

Squaring both sides.

$\Rightarrow \sin^2 x + \cos^2 x + 2\sin x\cos x = c^2$

$$\Rightarrow \sin x \cos = \frac{c^2 - 1}{2} \quad ...(ii)$$

Now, cubing eqn (i) both sides

$\Rightarrow \sin^3 x + \cos^3 x + 3\sin x\cos x(\sin x + \cos x) = c^3$

$\Rightarrow \sin^3 x + \cos^3 x + 3.\frac{(c^2-1)}{2} \times c = c^3$

$\Rightarrow \sin^3 x + \cos^3 x = c^3 - \frac{3}{2}(c^2-1)\ c$

$\Rightarrow \sin^3 x + \cos^3 x = c^3 - \frac{3c^3+3c}{2}$

$\sin^3 x + \cos^3 x = \frac{3c - c^3 2}{2}$ ...(iii)

On squaring both sides.

$\Rightarrow \sin^6 x + \cos^6 x + 2\sin^3 x\cos^3 x = \frac{(3c-c^3)^2}{4}$

$\Rightarrow \sin^6 x + \cos^6 x + 2\left\{\frac{(c^2-1)}{2}\right\}^3 = \frac{9c^2+c^6-6c^4}{4}$

$\Rightarrow \sin^6 x + \cos^6 x$

$= \frac{9c^2+c^6-6c^4-c^6+1+3c^2(c^2-1}{4}$

$\sin^6 x + \cos^6 x. = \frac{1+6c^2-3c^4}{4}$

169. (b) **Statement 1**

$\frac{1}{1-\sin x} = 4+2\sqrt{3}$

$\Rightarrow 1 = 4+2\sqrt{3}-4\sin x - 2\sqrt{3}\sin x$

$\Rightarrow \sin x = \frac{3+2\sqrt{3}}{4+2\sqrt{3}}$

$\Rightarrow \sin x = .866 < 1$

$0 < \sin x < 1$, Therefore, value of x exists between

0 to $\frac{\pi}{2}$

Statement 2

$\sin x = 3^{\sin^2 x}$

For example $x = 45$,

$\sin 45° = 3\sin^2 45$

$\Rightarrow \frac{1}{\sqrt{2}} = 3\left(\frac{1}{\sqrt{2}}\right)^2 ; \frac{1}{\sqrt{2}} = 3\left(\frac{1}{2}\right)$

So this does not hold good for any values.

So only statement 2 is true.

170. (d) Complementary angle of 80° = 90° – 80° = 10°

10° can be written as = $10 \times \frac{\pi}{180}$ Rad = $\frac{\pi}{18}$ rad.

171. (a) (LHS) $\frac{1+\tan^2\theta}{1+\cot^2\theta} = \frac{\sec^2\theta}{\cos ec^2\theta}$

$= \left(\frac{\sec\theta}{\cos ec\theta}\right)^2 = \left(\frac{\sin\theta}{\cos\theta}\right)^2 = \tan^2\theta$

R.H.S. $\left(\frac{1-\tan\theta}{1-\cot\theta}\right)^2 = \left(\frac{1-\tan\theta}{1-\frac{1}{\tan\theta}}\right)^2$

$\left[\tan\theta\left(\frac{1-\tan\theta}{\tan\theta-1}\right)\right]^2 = (-\tan\theta)^2$

$\Rightarrow \tan^2\theta$

LHS = RHS.

$\therefore$ Statement 1 is true when $\theta \neq \frac{\pi}{9}$.

Statement 2

$(\cot\theta)(\tan\theta) = 1$ for all values of θ. Except when $\theta =$ 0, 90°, 180°..... .

$\therefore$ Statement 2 is not true.

Option (d) is correct.

172. (b) $x = a\cos\theta,\ y = b\cot\theta$

$\Rightarrow (ax^{-1} - by^{-1})(ax^{-1} + by^{-1})$

$\Rightarrow \left(\frac{a}{x} - \frac{b}{y}\right)\left(\frac{a}{x} + \frac{b}{y}\right)$

$\Rightarrow \left(\frac{a}{a\cos\theta} - \frac{b}{b\cot\theta}\right)\left(\frac{a}{a\cos\theta} + \frac{b}{b\cot\theta}\right)$

$\Rightarrow (\sec\theta - \tan\theta)(\sec\theta + \tan\theta)$

$\Rightarrow \sec^2\theta - \tan^2\theta$

$\Rightarrow 1$

So, option (b) is correct.

173. (b) $\frac{\cos\theta}{(1-\sin\theta)} \times \frac{(1+\sin\theta)}{(1+\sin\theta)}$

$\Rightarrow \frac{\cos\theta(1+\sin\theta)}{(1-\sin^2\theta)}$

$\Rightarrow \frac{\cos\theta(1+\sin\theta)}{\cos^2\theta}$

$\Rightarrow \frac{(1+\sin\theta)}{\cos\theta}$

So, option (b) is correct.

174. (c) $\tan(x+40)°\tan(x+20)°\tan 3x°\tan(70-x)°\tan(50-x)° = 1$

$\Rightarrow \tan(x+40)\tan(x+20)\tan 3x\cot[90-(70-x)]\cot[90-(50-x] = 1$

$\Rightarrow \tan(x+40)\tan(x+20)\tan 3x\cot(x+20)\cot(x+40) = 1$

$\Rightarrow \tan 3x = \tan 45°$

$\Rightarrow 3x = 45°$

$\Rightarrow x = 15°$

So, option (c) is correct.

175. (a) $\sin\theta\cos\theta = 2\cos^3\theta - 1.5\cos\theta$

$\sin\theta\cos\theta = [2\cos^2\theta - 1.5]\cos\theta$

$\sin\theta = 2(1 - \sin 2\theta) - 1.5$

$2\sin^2\theta + \sin\theta - 0.5 = 0$

$$\sin\theta = \frac{-1 \pm \sqrt{(1)^2 + 4 \times 2 \times 0.5}}{4}$$

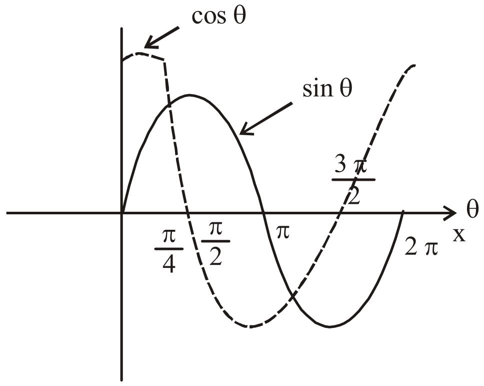
$$\frac{-1 \pm \sqrt{5}}{4}$$

as $-1 \le \sin\theta \le 1$

$$\sin\theta = \frac{-1 \pm \sqrt{5}}{4}$$

So, option (a) is correct.

176. (b)



From the graph it is clear that

$\sin\theta > \cos\theta$, when $\frac{\pi}{4}\theta \le \frac{\pi}{2}$

and $\sin\theta < \cos\theta$, when $0 \le 0 \le \frac{\pi}{4}$

So, option (b) is correct

177. (a) $\sin\theta = \frac{2a + 3b}{3b}$

$\Rightarrow \sin\theta = \frac{2a}{3b} + 1$

as a and b is positive so $\left(1 + \frac{2a}{3b}\right)$ will be always greater than 1 that is not possible for $\sin\theta$.

option (a) is correct.

178. (a) $\tan\theta + \sec\theta = 2$ ----------(i)

As we know

$\Rightarrow \sec^2\theta - \tan^2\theta = 1$

$\Rightarrow (\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$

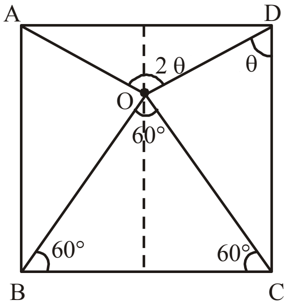
$\Rightarrow \sec\theta - \tan\theta = \frac{1}{2}$ ----------(ii)

equation (i) - eq (ii)-

$2\tan\theta = 2 - \frac{1}{2}$ $\qquad$ $2\tan\theta = \frac{3}{2} \Rightarrow \tan\theta = \frac{3}{4}$

So, option (a) is correct.

179. (d)



In ΔOCD

$CD = CO$

$\angle ODC = \angle DOC = \theta$

$\theta + \theta + 30° = 180°$

$\theta = 75°$

$$\tan 75° = \tan(45° + 30°) = \frac{\tan 45° + \tan 30°}{1 - \tan 45° \tan 30°}$$

$$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1.\frac{1}{\sqrt{3}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$$

$$= \frac{(\sqrt{3}+1)^2}{2} = \frac{4 + 2\sqrt{3}}{2}$$

$= 2 + \sqrt{3}$

So, option (d) is correct.

180. (a) $0 \le \cos^2 x \le 1$ ------------(i)

$0 \le \cos^2 y \le 1$ ------------(ii)

$0 \le \cos^2 z \le 1$

$0 \ge -\cos^2 z \ge -1$

$-1 \le -\cos^2 z \le 0$ ------------(iii)

Adding eq (i), (ii) and (iii)-

$0 + 0 - 1 \le \cos^2 x + \cos^2 y - \cos^2 z \le 1 + 1 + 0$

$-1 \le \cos^2 x + \cos^2 y - \cos^2 z \le 2$

so minimum value is -1.

So, option (a) is correct.

181. (d) $32\cot^2\left(\frac{\pi}{4}\right) - 8\sec^2\left(\frac{\pi}{3}\right) + 8\cos^3\left(\frac{\pi}{6}\right)$

$\Rightarrow 32^2(1)^2 - 8(2)^2 + 8\left(\frac{\sqrt{3}}{2}\right)^2$

$\Rightarrow 32 - 32 + 8 \times \frac{3\sqrt{3}}{8}$

$\Rightarrow 3\sqrt{3}$

So, option (d) is correct.

182. (c) $\left(\frac{\sin 35°}{\cos 55°}\right)^2 - \left(\frac{\cos 55°}{\sin 35°}\right)^2 + 2\sin 30°$ ...(1)

We know that

$\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta$

$\Rightarrow \sin(90-55°) = \cos 55°$

$\Rightarrow \sin 35° = \cos 55°$

So from (1) we get

$\left(\frac{\sin 35°}{\sin 35°}\right)^2 - \left(\frac{\cos 55°}{\cos 55°}\right)^2 + 2\times\frac{1}{2}$

$= (1)^2 - (1)^2 + 1$

$\therefore$ Option (c) is correct.

183. (d) $\frac{x}{a} - \frac{y}{b}\tan\theta = 1$ ...(1)

$\frac{x}{a}\tan\theta + \frac{y}{b} = 1$ ...(2)

Multiplying (2) by $\tan\theta$ and add in (1) we get

$\frac{x}{a} - \frac{y}{b}\tan\theta = 1$

$\frac{x}{a}\tan^2\theta + \frac{y}{b}\tan\theta = \tan\theta$

$\frac{x}{a}\left(1+\tan^2\theta\right) = 1+\tan\theta$

$\Rightarrow \frac{x}{a} = \frac{1+\tan\theta}{1+\tan^2\theta}$

$\Rightarrow \frac{y}{b} = 1 - \frac{x}{a}\tan\theta$

$= 1 - \frac{(1+\tan\theta)}{1+\tan^2\theta}\tan\theta$

$\frac{y}{b} = \frac{1-\tan\theta}{1+\tan^2\theta}$

$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{(1+\tan\theta)^2}{\left(1+\tan^2\theta\right)^2} + \frac{(1-\tan\theta)^2}{\left(1+\tan^2\theta\right)^2}$

$= \frac{1+\tan^2\theta + 2\tan\theta + 1 + \tan^2\theta - 2\tan\theta}{\left(1+\tan^2\theta\right)^2}$

$= \frac{2\left(1+\tan^2\theta\right)}{\left(1+\tan^2\theta\right)^2}$

$= \frac{2}{1+\tan\theta} = \frac{2}{\sec^2\theta} = 2\cos^2\theta$

$\therefore$ Option (d) is correct.

184. (c) Statement (1) $\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \operatorname{cosec}\theta - \cot\theta$

$\Rightarrow \sqrt{\frac{2\sin^2\theta/2}{2\cos^2\theta/2}} = \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}$

$\Rightarrow \sqrt{\tan^2\theta/2} = \frac{1-\cos\theta}{\sin\theta} = \frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$

$\Rightarrow \tan\frac{\theta}{2} = \tan\frac{\theta}{2}$

Hence, (1) is identity

(2) Statement $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \cos ec\theta + \cot\theta$

$\sqrt{\frac{2\cos^2\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}}} = \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta} = \frac{1+\cos\theta}{\sin\theta}$

$= \frac{2\cos^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$

$\sqrt{\cot^2\frac{\theta}{2}} = \cot^2\frac{\theta}{2}$

$\Rightarrow \cot\frac{\theta}{2} = \cot\frac{\theta}{2}$

Hence, (2) is also an identity

$\therefore$ Option (c) is correct.

185. (b) p = cotθ + tanθ q = secθ – cosθ

$= \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} = \frac{1}{\cos\theta} - \cos\theta$

$\Rightarrow p = \frac{1}{\sin\theta\cos\theta} \quad q = \frac{\sin^2\theta}{\cos\theta}$

$\therefore \quad \left(p^2q\right)^{\frac{2}{3}} - \left(q^2p\right)^{\frac{2}{3}}$

$$= \left(\frac{1}{\sin^2\theta.\cos^2\theta}\times\frac{\sin^2\theta}{\cos\theta}\right)^{\frac{2}{3}} - \left(\frac{\sin^4\theta}{\cos^2\theta}\times\frac{1}{\sin\theta\cos\theta}\right)^{\frac{2}{3}}$$

$$= \left(\frac{1}{\cos^3\theta}\right)^{\frac{2}{3}} - \left(\frac{\sin^3\theta}{\cos^3\theta}\right)^{\frac{2}{3}}$$

$$= \frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}$$

$$= \frac{1-\sin^2\theta}{\cos^2\theta} = \frac{\cos^2\theta}{\cos^2\theta} = 1$$

$\therefore$ Option (b) is correct.

186. (b) Given $\dfrac{\cos^2\theta - 3\cos\theta + 2}{\sin^2\theta} = 1$ ...(1)

$\Rightarrow \quad \dfrac{(2-\cos\theta)(1-\cos\theta)}{1-\cos^2\theta} = 1$

$\Rightarrow \quad \dfrac{2-\cos\theta}{1+\cos\theta} = 1$

$\Rightarrow \quad 2 - \cos\theta = 1 + \cos\theta$

$\Rightarrow \quad \cos\theta = \dfrac{1}{2} = \cos 60°$

$\Rightarrow \quad \theta = 60°$ As $0 < \theta < \dfrac{\pi}{2}$

There is only one value of θ satisfying the above equation.

Statement (1) is not correct.

Again Put $\theta = 60°$ in L.H.S. of (1)

$$\frac{\cos^2 60° - 3\cos 60° + 2}{\sin^2 60°}$$

$$= \frac{\frac{1}{4} - \frac{3}{2} + 2}{\frac{3}{4}} = \frac{1-6+8}{3}$$

$$= \frac{3}{3} = 1 = \text{RHS}$$

Statement (2) is correct.

$\therefore$ Option (b) is correct.

187. (d) $3 - \tan^2\theta = \alpha(1 - 3\tan^2\theta)$

$\Rightarrow \quad \alpha = \dfrac{3-\tan^2\theta}{1-3\tan^2\theta}$

$\Rightarrow \quad \alpha = \dfrac{4-4\tan^2\theta}{2-2\tan^2\theta}$ [By componendo and dividendo]

$= 2\left(\dfrac{1-\tan^2\theta}{1+\tan^2\theta}\right)$

$\Rightarrow \quad \alpha = 2\cos 2\theta$

Now $-1 \le \cos 2\theta \le 1 \Rightarrow -2 \le 2\cos 2\theta \le 2$

188. (c) $\tan\theta + \cot\theta = \dfrac{4}{\sqrt{3}}$

$\Rightarrow \quad \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta} = \dfrac{4}{\sqrt{3}}$

$\Rightarrow \quad \dfrac{1}{\sin\theta\cos\theta} = \dfrac{4}{\sqrt{3}}$

$\Rightarrow \quad \sin\theta\cos\theta = \dfrac{\sqrt{3}}{4}$

$\Rightarrow \quad 2\sin\theta\cos\theta = \dfrac{\sqrt{3}}{2}$

$\Rightarrow \quad \sin 2\theta = \sin 60° \Rightarrow 2\theta = 60° \Rightarrow \theta = 30°$

$\Rightarrow \quad \sin\theta + \cos\theta$

$= \sin 30° + \cos 30°$

$= \dfrac{1}{2} + \dfrac{\sqrt{3}}{2}$

$= \dfrac{\sqrt{3}+1}{2}$

$\therefore$ Option (c) is correct.

189. (b) $\sin\theta + \cos\theta = \dfrac{\sqrt{7}}{2}$

$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = \dfrac{7}{4}$

$2\sin\theta\cos\theta = \dfrac{7-4}{4} = \dfrac{3}{4}$

$(\sin\theta + \cos\theta)^2 - 4\sin\theta\cos\theta = \dfrac{7}{4} - 2\left(\dfrac{3}{4}\right)$

$(\sin\theta - \cos\theta)^2 = \dfrac{1}{4}$

$\sin\theta - \cos\theta = \dfrac{1}{2}$

190. (b) $\sin x + \sin^2 x = 1$
$\sin x = \cos^2 x$ $\quad \because 1 - \sin^2 x = \cos^2 x$
now
$\cos^8 x + 2\cos^6 x + \cos^4 x$
$\Rightarrow \sin^4 x + 2\sin^3 x + \sin^2 x$
$\Rightarrow (\sin^2 x + \sin x)^2 = 1$

191. (d) $\text{cosec}^2 68° + \sec^2 56° - \cot^2 34° - \tan^2 22°$
$\sec^2 22° - \tan^2 22° + \text{cosec}^2 34° - \cot^2 34°$
$1 + 1 = 2$

192. (c) $2y\cos\theta = x\sin\theta$
$2y\,\text{cosec}\,\theta = x\sec\theta$
$2y\,\text{cosec}\,\theta - x\sec\theta = 0$ ... (i)
$y\,\text{cosec}\,\theta + 2x\sec\theta = 3$ ... (ii)
Multiplying eq (ii) with 2 and adding both eq.
$3x\sec\theta = 6$
$x\sec\theta = 2$
$x = 2\cos\theta$
using
$2y\cos\theta = x\sin\theta$
$2y\cos\theta = 2\cos\theta\sin\theta$
$y = \sin\theta$
Putting value of x and y in $x^2 + 4y^2$
$(2\cos\theta)^2 + 4\sin^2\theta$
$4\cos^2\theta + 4\sin^2\theta = 4$

193. (d) $\sin\theta + \cos\theta = \frac{1+\sqrt{3}}{2} = \frac{1}{2} + \frac{\sqrt{3}}{2}$ i.e. $\theta = 30°$

than $\tan 30 + \cot 30 = \frac{1}{\sqrt{3}} + \sqrt{3} = \frac{4}{\sqrt{3}}$

194. (b) $A = \sin^2\theta + \cos^4\theta$
$= 1 - \cos^2\theta + \cos^4\theta$

$\cos^4\theta - 2\left(\frac{1}{2}\right)\cos^2\theta + 1 - \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2$

$\left(\cos^2\theta - \frac{1}{2}\right)^2 + \frac{3}{4}$

When $\left(\cos^2\theta - \frac{1}{2}\right)^2 = 0$

value is minimum that is $\frac{3}{4}$

Now from equation b and d are with minimum value

$\frac{3}{4}$, Maximum value is either 1 or $\frac{13}{16}$ since $1 > \frac{13}{16}$

When $\theta = 0°$ or $90°$

$\therefore \frac{3}{4} \le A \le 1$

195. (a) $\frac{\cot A + \text{cosec}\,A - 1}{\cot A - \text{cosec}\,A + 1}$

$\Rightarrow \frac{\cot A + \text{cosec}\,A - (\text{cosec}^2 A - \cot^2 A)}{\cot A - \text{cosec}\,A + 1}$

$\Rightarrow \frac{\cot A + \text{cosec}A\left[1 - \text{cosec}\,A + \cot A\right]}{\cot A - \text{cosec}\,A + 1}$

$\Rightarrow \cot A + \text{cosec}\,A$

$\Rightarrow \frac{\cos A}{\sin A} + \frac{1}{\sin A} \Rightarrow \frac{1+\cos A}{\sin A}$

196. (c) 1 and 3 only
$\sin 1^c = \sin 57°$ (approx)
$\cos 1^c = \cos 57°$

$\tan 1^c = \tan 57°$ $\quad \boxed{\therefore 180° = \frac{22^c}{7}}$

$\cos 0° = 1, \cos 1° = 0.99$
$\cos 57° = 0.54$

197. (c) $\tan^2 x + \frac{1}{\tan^2 x} = 2$
$\left(\tan^2 x + \frac{1}{\tan^2 x} - 2\right) = 0$
$(\tan x - \cot x)^2 = 0$
$\tan x = \cot x$
when $x = 45°$

198. (d) 1. $\frac{\cos 75°}{\sin 15°} + \frac{\sin 12°}{\cos 78°} - \frac{\cos 18°}{\sin 72°} = 1$

$\frac{\cos 75°}{\cos 75°} + \frac{\sin 12°}{\sin 12°} - \frac{\cos 18°}{\cos 18°} = 1$
$2 - 1 = 1$
Statement is correct

2. $\frac{\cos 35°}{\sin 55°} - \frac{\sin 11°}{\cos 79°} + \cos 28°\,\text{cosec}\,62° = 1$

$\frac{\cos 35°}{\cos 35°} - \frac{\sin 11°}{\sin 11°} + \cos 28° \sec 28° = 1$
$1 - 1 + 1 = 1$
2nd statement is also true.

3. $\frac{\sin 80°}{\cos 10°} - \sin 59° \sec 31° = 0$

$\frac{\sin 80°}{\sin 80°} - \sin 59°\,\text{cosec}\,59° = 0$
$1 - 1 = 0$
3rd statement is correct.

199. (b) $\tan 1° \tan 2° \tan 3° \tan 4° \ldots\ldots. \tan 89°$
$\tan 1° \tan 2° \ldots.. \tan 45° \ldots\ldots \tan 89°$
$= 1$

200. (c) $9\tan^2\theta + 4\cot^2\theta$
$\Rightarrow (3\tan\theta)^2 + (2\cot\theta)^2 - 2(3\tan\theta)(2)\cot\theta + 2(3\tan\theta)^2 \cot\theta$
$\Rightarrow (3\tan\theta - 2\cot\theta)^2 + 12$

since $(3 \tan \theta - 2 \cot \theta)^2 \geq 0$
Minimum value is 12

201. (b) $x \sin \theta = y \cos \theta = \dfrac{2z + \tan \theta}{1 - \tan^2 \theta}$

Let $\theta = 30°$

$$\frac{x}{2} = \frac{y\sqrt{3}}{2} = \frac{z}{\sqrt{3}} = K \because \frac{2 \tan \theta}{1 - \tan^2 \theta} = \tan^2 \theta = \tan 60°$$

$x = 2K, y = \dfrac{2K}{\sqrt{3}}, z = \sqrt{3}k$

putting value of x, y and z in $4z^2 (x^2 + y^2)$

$$4\left(\frac{K}{\sqrt{3}}\right)^2 \left[(2K)^2 + \left(\frac{2K}{\sqrt{3}}\right)^2\right]$$

$$\Rightarrow \frac{4K^2}{3}\left[4K^2 + \frac{4K^2}{3}\right]$$

$$\Rightarrow \frac{64K^2}{9}$$

From option putting value of x, y and z
we get $(x^2 - y^2)^2$

$$= \left((2K)^2 - \frac{(2K)^2}{(3)}\right)^2 = \left[4K^2 - \frac{4K^2}{3}\right]$$

$$= \left(\frac{8K^2}{3}\right)^2 = \frac{64K^2}{9}$$

202. (a) When $\theta_1 = \theta_2 = \theta_3 = 0$ then $\cos\theta_1 + \cos\theta_2 + \cos\theta_3 = 3$ then $\sin \theta_2 + \sin \theta_2 + \sin\theta_3 = 0$

203. (a) From given option $\theta = 0°$ Satisfies the equation
$\cos 0° + \tan 0° = 1$
$1 + 0 = 1$

204. (a) $\sin x \sqrt{\dfrac{1}{1 + \cos x} + \dfrac{1}{1 - \cos x}}$

$$\Rightarrow \sin x \sqrt{\frac{1 + \cos x + 1 - \cos x}{1 - \cos^2 x}}$$

$$\Rightarrow \frac{\sin x}{\sin x}\sqrt{2} \Rightarrow \sqrt{2}$$

205. (c) $\dfrac{\cos^4 A - \sin^4 A}{\cos^2 A - \sin^2 A}$

$$\Rightarrow \frac{(\cos^2 A)^2 - (\sin^2 A)^2}{\cos^2 A - \sin^2 A}$$

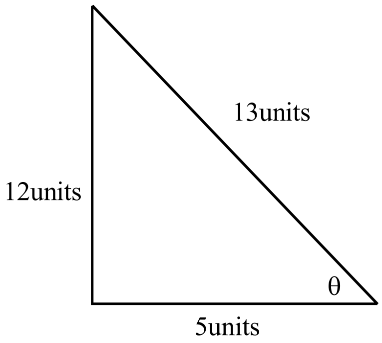
$$\frac{(\cos^2 A + \sin^2 A)(\cos^2 A - \sin^2 A)}{\cos^2 A - \sin^2 A} = 1$$

206. (d) $7\sin^2 x + 3 \cos^2 x = 4$
$7 \sin^2 x + 3 - 3 \sin^2 x = 4$

$4 \sin^2 x = \dfrac{4}{3}$

$\sin x = \dfrac{1}{\sqrt{3}}$

207. (d)



Since we know that 5, 13 and 12 forms a Pythagorean triplet, the side with 13 units is the longest side and the angle between the other two sides is 90°.

Therefore, $\sin \theta = \dfrac{P}{H} = \dfrac{12}{13}$ and $\cos \theta = \dfrac{B}{H} = \dfrac{5}{13}$

Thus, $\sin \theta + \cos \theta = \dfrac{12}{13} + \dfrac{5}{13} = \dfrac{12 + 5}{13} = \dfrac{17}{13}$

208. (c) Since we are given that $0 < x < \dfrac{\pi}{2}$, then

$$\sin 0 < x < \sin \frac{\pi}{2} \Rightarrow 0 < \sin x < 1 \quad ...(1)$$

Similarly,

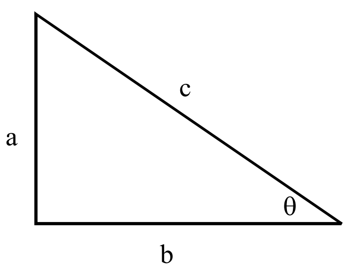
$$\operatorname{cosec} 0 > \operatorname{cosec} x > \operatorname{cosec} \frac{\pi}{2} \Rightarrow \infty > \operatorname{cosec} x > 1 \quad ...(2)$$

Adding (1) and (2), we get

$\infty > \sin x + \operatorname{cosec} x > 2$

Thus, we can say that the value of the sum of the two trigonometric functions would be ≥ 2.

209. (a) Suppose we have right angled triangle with sides *a*, *b* and *c* where *c* is the longest side.



Now, we can see that $\sin \theta = \dfrac{a}{c}$ and $\cos \theta = \dfrac{b}{c}$. Here,

we can see that in both the denominators we have the same hypotenuse which means from all the given options, only option (a) has the same hypotenuse as given in the question i.e. $m^2 + n^2$.
Thus, option (a) is the correct answer.

210. (a) $$A = \frac{\sin 45° - \sin 30°}{\cos 45° + \cos 60°} = \frac{\frac{1}{\sqrt{2}} - \frac{1}{2}}{\frac{1}{\sqrt{2}} + \frac{1}{2}} = \frac{\frac{2-\sqrt{2}}{2\sqrt{2}}}{\frac{2+\sqrt{2}}{2\sqrt{2}}}$$

$$= \frac{2-\sqrt{2}}{2+\sqrt{2}} \times \frac{2-\sqrt{2}}{2-\sqrt{2}} = \frac{(2-\sqrt{2})^2}{4-2} = \frac{4+2-4\sqrt{2}}{2}$$

$$= \frac{6-4\sqrt{2}}{2} = 3-2\sqrt{2}$$

$$B = \frac{\sec 45° - \tan 45°}{\operatorname{cosec} 45° + \cot 45°} = \frac{\sqrt{2}-1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$$

$$= \frac{(\sqrt{2}-1)^2}{2-1} = 2+1-2\sqrt{2} = 3-2\sqrt{2}$$

Hence, $A = B$

211. (c) In the first statement, we are given that $45° < \theta < 60°$. Therefore, if we consider $\theta < 60°$, then let us suppose $\theta = 45°$, so

$$\sec^2\theta + \operatorname{cosec}^2\theta = \sec^2 45° + \operatorname{cosec}^2 45°$$
$= 2 + 2 = 4 = \alpha^2 \Rightarrow \alpha = 2 > 1$ and if we consider $\theta > 45°$, then let us suppose $\theta = 60°$, so

$$\sec^2\theta + \operatorname{cosec}^2\theta = \sec^2 60° + \operatorname{cosec}^2 60°$$

$$= 4 + \frac{4}{3} = \frac{16}{3} = \alpha^2 \Rightarrow \alpha = 2.31 > 1$$

Hence, statement 1 is true.
In the second statement, we are given that $0° < \theta < 45°$. Therefore, if we consider $\theta < 45°$, then let us suppose $\theta = 0°$, so

$$\frac{1+\cos\theta}{1-\cos\theta} = \frac{1+\cos 0°}{1-\cos 0°} = \infty = x^2 \Rightarrow x = \infty > 2$$

and if we consider $\theta > 0°$, then let us suppose $\theta = 45°$, so

$$\frac{1+\cos\theta}{1-\cos\theta} = \frac{1+\cos 45°}{1-\cos 45°}$$

$$= \frac{\sqrt{2}+1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{(\sqrt{2}+1)^2}{2-1}$$

$$= \frac{2+1+2\sqrt{2}}{1} = 3+2\sqrt{2} = x^2 \Rightarrow x = 2.414 > 2$$

Hence, statement 2 is true.
In the third statement, we are given that $0° < \theta < 45°$. Therefore, if we consider $\theta < 45°$, then let us suppose $\theta = 0°$, so

$$\frac{\cos\theta}{1-\tan\theta} + \frac{\sin\theta}{1-\cot\theta} = \frac{\cos 0°}{1-\tan 0°} + \frac{\sin 0°}{1-\cot 0°}$$
$$= \frac{1}{1-0} + \frac{0}{-\infty} = 1 + 0 = 1$$

which is not ≥ 2 and if we consider $\theta > 0°$, then let us suppose $\theta = 45°$, so

$$\frac{\cos\theta}{1-\tan\theta} + \frac{\sin\theta}{1-\cot\theta} = \frac{\cos 45°}{1-\tan 45°} + \frac{\sin 45°}{1-\cot 45°}$$

$$= \frac{\frac{1}{\sqrt{2}}}{1-1} + \frac{\frac{1}{\sqrt{2}}}{1-1} = \infty \geq 2 \cdot$$

Hence, statement 3 is false.
Hence, only two statements are correct.

212. (a) $$\frac{\sin 1°}{\sin 1^c} = \frac{0.0174}{\sin\left(\frac{180}{\pi}\right)} = \frac{0.0174}{0.8415} = 0.0206 < 1$$

213. (d) $\sin^4\theta - \cos^4\theta$
$= (\sin^2\theta)^2 - (\cos^2\theta)^2 = (\sin^2\theta + \cos^2\theta)(\sin^2\theta - \cos^2\theta)$
$1(\sin^2\theta - \cos^2\theta) = 1 - \cos^2\theta - \cos^2\theta = 1 - 2\cos^2\theta$

214. (b) $\cot 1° \cot 23° \cot 45° \cot 67° \cot 89°$
$= \cot 1° \times \cot 23° \times \cot 45° \times \cot (90 - 23)° \times \cot (90 - 1)°$
$= \cot 1° \times \cot 23° \times \cot 45° \times \tan 23° \times \tan 1°$
$\cot 1° \times \tan 1° \times \cot 23° \times \tan 23° \times$ cat $45°$
$= 1 \times 1 \times 1 = 1$

215. (b) Checking statement 1
$(\sec^2\theta - 1)(1 - \operatorname{cosec}^2\theta) = 1$
L.H.S
$(\sec^2\theta - 1) \times (-1)(\operatorname{cosec}^2\theta - 1)$
$\tan^2\theta \times (-1) \times \cot^2\theta = -1$
Here L.H.S $\neq$ R.H.S
Hence, statement 1 is incorrect
checkin statement 2.
$\text{Sin}\theta (1 + \cos\theta)^{-1} + (1 + \cos\theta)(\sin\theta)^{-1} = 2 \operatorname{cosec}\theta$
L.H.S

$$\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = \frac{\sin^2\theta + (1+\cos\theta)^2}{(1+\cos\theta)\sin\theta}$$
$$= \frac{\sin^2\theta + 1 + \cos^2\theta + 2\cos\theta}{(1+\cos\theta)\sin\theta}$$

$$= \frac{1+1+2\cos\theta}{\sin\theta(1+\cos\theta)} = \frac{2(1+\cos\theta)}{\sin\theta(1+\cos\theta)} = \frac{2}{\sin\theta}$$

$= 2\operatorname{cosec}\theta = $ R.H.S

Hence, only statement 2 is correct.

216. (a) sec x cosec x = 2
This value is possible is x = 45°
$\tan^n x + \cot^n x = (\tan x)^n + (\cot x)^n$
$= (\tan 45°)^n + (\cot 45°)^n = (1)^n + (1)^n = 2$

217. (a) $\cos x + \cos^2 x = 1$
$\cos x = 1 - \cos^2 x = \sin^2 x$
$\sin^2 x + \sin^4 x = \sin^2 x + (\sin^2 x)^2$
$= \sin^2 x + (\cos x)^2 = \sin^2 x + \cos^2 x = 1$

218. (c) $\sin A + \cos A = p$ $\quad\quad \sin^3 A + \cos^3 A = q$

$\sin^3 A + \cos^3 A = (\sin A + \cos A)(\sin^2 A + \cos^2 A - \sin A \cos A)$

$q = p(1 - \sin A \cos A)$

$1 - \sin A \cos A = \frac{q}{p}$

$\sin A \cos A = 1 - \frac{q}{p} = \frac{p-q}{p}$

$(\sin A + \cos A)^3 = p^3$

$\sin^3 A + \cos^3 A + 3\sin A \cos A(\sin A + \cos A) = p^3$

$q + 3 \times \frac{p-q}{p} \times p = p^3$

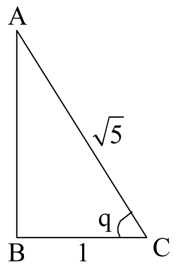
$p^3 = q + 3p - 3q$

$p^3 + 2q - 3p = 0$

219. (d) 220. (a)

221. (b) $\cos\theta = \frac{1}{\sqrt{5}}$

from ΔABC



$AB = \sqrt{(AB)^2 + (BC)^2}$

$= \sqrt{\left(\sqrt{5}\right)^2 - (1)^2} = \sqrt{5-1} = \sqrt{4} = 2$

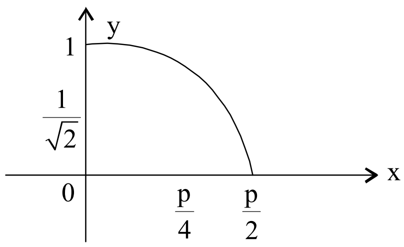
$\therefore \quad \tan\theta = \frac{AB}{BC} = \frac{2}{1} = 2$

Now, $\frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\times 2}{1-(2)^2} = -\frac{4}{3}$

222. (b) From question

$0 < (\theta, \phi) < 90°$

Graph of $\cos\theta$ and $\cos\phi$:



θ, ϕ	$\cos\theta$	$\cos\phi$
0	1	1
$\pi/6$	$\sqrt{3}/2$	$\sqrt{3}/2$
$\pi/4$	$1/\sqrt{2}$	$1/\sqrt{2}$
$\pi/3$	$1/2$	$1/2$
$\pi/2$	0	0

As $\cos\phi > \cos\theta$

thus $\theta > \phi$

223. (c) $\sin(A+B) = \frac{\sqrt{3}}{2} \Rightarrow \sin(A+B) = \sin\left(\frac{\pi}{3}\right)$

$\therefore \quad A + B = \frac{\pi}{3}$ ...(1)

and $\cos B = \frac{\sqrt{3}}{2} \Rightarrow \cos B = \cos\left(\frac{\pi}{6}\right)$

$\therefore \quad B = \frac{\pi}{6}$ ...(2)

From (1) and (2), we get $A = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$

Hence, $\tan(2A - B) = \tan\left(2 \times \frac{\pi}{6} - \frac{\pi}{6}\right)$

$= \tan\left(\frac{\pi}{3} - \frac{\pi}{6}\right) = \tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$

224. (c) **1.** $\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 4$

$\Rightarrow \frac{\cos\theta(1+\sin\theta) + \cos\theta(1-\sin\theta)}{1-\sin^2\theta} = 4$

$\Rightarrow \frac{\cos\theta + \sin\theta\cos\theta + \cos\theta - \sin\theta\cos\theta}{\cos^2\theta} = 4$

$\Rightarrow \frac{2}{\cos\theta} = 4$

$\Rightarrow \sec\theta = 2$

$\Rightarrow \sec\theta = \sec 60°$

$\Rightarrow \theta = 60°$

2. $3\tan\theta + \cot\theta = 5\operatorname{cosec}\theta$

$\frac{3}{\cot\theta} + \cot\theta = 5\operatorname{cosec}\theta.$

$3 + \cot^2\theta = 5\operatorname{cosec}\theta\cot\theta$

$2 + \operatorname{cosec}^2\theta = 5\operatorname{cosec}\theta\cot\theta$

$2\sin^2\theta + 1 = 5\cos\theta$

$2 - 2\cos^2\theta + 1 = 5\cos\theta$

$2\cos^2\theta + 5\cos\theta - 3 = 0.$

Solving quadratic equation for $\cos\theta$, we get

$\cos\theta = \frac{1}{2}$, $\cos\theta = -3$ (neglecting)

$\Rightarrow \quad \cos\theta = \frac{1}{2} = \cos 60°$

$\Rightarrow \quad \theta = 60°$

225. (c) We know that $\cos^2\theta$ is defined in [0, 1] so

$\cos^2\theta = 1 - \frac{p^2 + q^2}{2pq}$, where p, q are non zero real number, is possible only when $p = q$.

2. We know that $\tan^2\theta$ is defined in $[0, \infty]$. So $\tan^2\theta$

$= \frac{4pq}{(p+q)^2} - 1$, where p, q are non zero numbers, is possible only when p = q.

226. (c) **1.** If possible, let $\cos\theta + \sec\theta = \frac{3}{2}$

$\therefore \quad \cos\theta + \frac{1}{\cos\theta} = \frac{3}{2}$

$\Rightarrow \; 2\cos^2\theta + 2 = 3\cos\theta$

$\Rightarrow \; 2\cos^2\theta - 3\cos\theta + 2 = 0$

Solving it as a quadratic in $\cos\theta$, we get

$$\cos\theta = \frac{3 \pm \sqrt{9-16}}{4} = \frac{3 \pm \sqrt{-7}}{4}$$

Both of these values of $\cos\theta$ are imaginary and hence both these values are impossible as $\cos\theta$ is always real

$\therefore$ $\cos\theta + \sec\theta$ can never be equal to $\frac{3}{2}$.

2. $\sec^2\theta + \text{cosec}^2\theta$

$$\frac{1}{\sin^2\theta\cos^2\theta}$$

$$\frac{4}{4\sin^2\theta\cos^2\theta}$$

$$\frac{4}{(\sin 2\theta)^2}$$

This expression is less than 4 if $\sin 2\theta > 1$ but it not possible as. $\sin 2\theta$ lies in [0, 1]

227. (c) We have,

$\sin^2 x + \sin x = 1 \quad ...(1)$

$\therefore \sin x = 1 - \sin^2 x = \cos^2 x$

On cubing equation (1), we get

$\{\sin^2 x + \sin x\}^3 = \{1\}^3$

$\sin^6 x + \sin^3 x + 3\sin^2 x.\sin x(\sin^2 x + \sin x) = 1$

$\sin^6 x + \sin^3 x + 3\sin^5 x + 3\sin^4 x = 1$

$\therefore \quad \cos^{12} x + 3\cos^{10} x + 3\cos^8 x + \cos^6 x = 1$

228. (d) $(3\sin\theta + 5\cos\theta) = 4$

Squaring both sides,

$9\sin^2\theta + 25\cos^2\theta + 30\cos\theta\sin\theta = 16$

$9\sin^2\theta + 9\cos^2\theta + 16\cos^2\theta + 30\cos\theta\sin\theta = 16.$

$9 + 16\cos^2\theta + 30\cos\theta\sin\theta = 16.$

$30\cos\theta\sin\theta = 16 - 9 - 16\cos^2\theta \quad ...(1)$

Now,

$(3\cos\theta - 5\sin\theta)^2$

$= 9\cos^2\theta + 25\sin^2\theta - 30\sin\theta\cos\theta$

$= 9\cos^2\theta + 9\sin^2\theta + 16\sin^2\theta - 30\sin\theta\cos\theta$

$= 9 + 16\sin^2\theta - 16 + 9 + 16\cos^2\theta \quad$ [from (1)]

$= 18 + 16 - 16 = 18$

229. (b) Put $\theta = 45°$, we get

$m = \frac{\sqrt{2}+1}{4\sqrt{2}}$ and $n = \frac{\sqrt{2}-1}{4\sqrt{2}}$

R.H.S:

$$mn = \frac{(\sqrt{2}+1)}{4\sqrt{2}}\frac{(\sqrt{2}-1)}{4\sqrt{2}} = \frac{2-1}{32} = \frac{1}{32}$$

L.H.S: $(m^2 - n^2)^2$

$$\left[\left(\frac{\sqrt{2}+1}{4\sqrt{2}}\right)^2 - \left(\frac{\sqrt{2}-1}{4\sqrt{2}}\right)^2\right]^2 = \left[\frac{3+2\sqrt{2}}{32} - \frac{(3-2\sqrt{2})}{32}\right]^2$$

$$\left(\frac{3+2\sqrt{2}-3+2\sqrt{2}}{32}\right)^{32} = \left(\frac{4\sqrt{2}}{32}\right)^2 = \left(\frac{\sqrt{2}}{8}\right)^2$$

$$= \left(\frac{1}{4\sqrt{2}}\right)^2 = \frac{1}{32}$$

L.H.S = R.H.S.

230. (b) $\tan x = 1 = \tan 45^\circ$

$\therefore x = 45^\circ$

$2\sin x.\cos x = 2\sin(45^\circ).\cos(45^\circ)$

$$= 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 1$$

231. (c) From $\sin(90 - \theta) = \cos\theta$

$\sin 46^\circ.\cos 44^\circ + \cos 46^\circ.\sin 44^\circ$

$\Rightarrow \sin 46^\circ.\sin(90 - 44)^\circ + \cos 46^\circ.\cos(90 - 44)^\circ$

$= \sin^2 46^\circ + \cos^2 46^\circ = 1$

232. (b) Let $4\sin^2\theta + 1 \geq 4\sin\theta$

$4\sin^2\theta - 4\sin\theta + 1 \geq 0$

$(2\sin\theta - 1)^2 \geq 0$

$$\sin\theta \geq \frac{1}{2}$$

233. (b) $\tan 1^\circ.\tan 2^\circ.\tan 3^\circ....\tan 87^\circ.\tan 88^\circ, \tan 89^\circ$

$\tan 1^\circ.\tan 2^\circ.\tan 3^\circ....\tan(90^\circ - 3^\circ).\tan(90^\circ - 2^\circ).\tan(90^\circ - 1^\circ)$

$\tan 1^\circ.\tan 2^\circ.\tan 3^\circ....\cot 3^\circ.\cot 1^\circ$

$$\tan 1^\circ.\tan 2^\circ.\tan 3^\circ.....\frac{1}{\tan 3^\circ}.\frac{1}{\tan 2^\circ}.\frac{1}{\tan 1^\circ} = 1$$

234. (a) From question, we have

$3\tan\theta = \cot\theta$

$3\tan\theta = 1/\tan\theta$

$\tan^2\theta = 1/3$

$\tan\theta = 1\sqrt{3}$

$\theta = \pi/6$

235. (b) $\sin^2 25^o + \sin^2 65^o = \sin^2 25^o + \sin^2(90-25)^o = \sin^2 25^o + \cos^2 25^o = 1$

236. (a) $\sin^6\theta + \cos^6\theta + 3\sin^2\theta, \cos^2\theta - 1$

$(\sin^2\theta)^3 + (\cos^2\theta)^3 + 3\sin^2\theta.\cos^2\theta - 1$

$(\sin^2\theta + \cos^2\theta)^3 - 1$

$1 - 1 = 0$

237. (c) 1. As, $\sec\propto = (-\infty, -1] \cup [1, \infty)$

$\therefore \sec\propto \neq \pm\frac{1}{4}$

2. $\tan\beta = (-\infty, \infty)\ \tan\beta = 20$

3. As, $\operatorname{cosec}\propto = (-\infty, -1] \cup [1, \infty)$

$\therefore \operatorname{cosec}\propto \neq \pm\frac{1}{2}$

4. As $\cos\delta = [-1, 1]$

$\therefore \text{cas}\,\delta \neq \pm 2.$

238. (a) $\cos^2 x + \cos x = 1 \Rightarrow \quad \cos x = 1 - \cos^2 x = \sin^2 x$

$= \sin^{12}x + 3\sin^{10}x + 3\sin^8 x + \sin^6 x$

$= \sin^6 x[\sin^6 x + 3\sin^4 x + 3\sin^2 x + 1]$

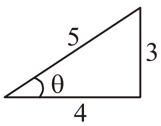
$= \sin^6 x[\sin^2 x + 1]^3$

$= [\sin^4 x + \sin^2 x]^3$

$(\therefore \quad \sin^4 x = \cos^2 x)$

$= (\sin^2 x + \cos^2 x) = 1$

239. (d) As $\sin\theta = \frac{3}{5} \Rightarrow \cot\theta = \frac{4}{3}$



$1 + 3x + 9x^2 + 27x^3 + 81x^4 + 243x^5$

$= 1 + 3\times\frac{4}{3} + 9\times\frac{16}{9} + 27\times\frac{64}{27} + 81\times\frac{256}{81} + 243\times\frac{1024}{243}$

$= 1 + 4 + 16 + 64 + 256 + 1024$

$= 5 + 30 + 1280 = 1365$

240. (c) $\frac{\sin 19°}{\cos 71°} + \frac{\cos 73°}{\sin 17°}$

$= \frac{\sin 19°}{\cos(90° - 19°)} + \frac{\cos 73°}{\sin(90° - 73°)}$

$= \frac{\sin 19°}{\sin 19°} + \frac{\cos 73°}{\cos 73°} = 1 + 1 = 2$

241. (c) Revolution in one minutes = 360

$\therefore$ revolution in one sec $= \frac{360}{60} = 6$

Number of radian it turns in one second $= 6 \times 2\pi$
$= 12\pi$ rad/s

242. (b) Min Vaule $= \left(\sqrt{25} + 1\right)^2 = 36$

243. (a) $100\,\theta = 90°$ $\quad\therefore$

$\theta = \frac{90}{100} = \frac{\pi}{2}\times\frac{1}{100}$

$\alpha = \cot\left(\frac{1}{100}.\frac{\pi}{2}\right)\cot\left(\frac{2}{100}.\frac{\pi}{2}\right)........\ \cot\left(\frac{99}{100}.\frac{\pi}{2}\right) = 1$

244. (a) $\tan 6\theta = \cot 2\theta$

$\Rightarrow \cot(90° - 6\theta) = \cot 2\theta$

$\Rightarrow 90° - 6\theta = 2\theta$

$\Rightarrow 8\theta = 90°$

$\therefore \quad 4\theta = 45°$

$\sec 4\theta = \sec 45° = \sqrt{2}$

245. (d) $\sin^2 6° + \sin^2 12° + \sin^2 18° + ...\sin^2 84° + \sin^2 90°$

Now, $\sin^2 84° = \sin^2(90° - 6°) = \cos^2 6°$

$\therefore \sin^2 6° + \sin^2 12° + \sin^2 18° + ...\cos^2 12° + \cos^2 6° + \sin^2 90°$

$\sin^2 6 + \cos^2 6 + ...+ \sin^2 42 + \cos^2 42 + \sin^2 90$

$= 7 + 1 = 8$

246. (b) $\frac{\cos\theta}{1+\sin\theta} + \frac{\sin\theta}{\cos\theta} = \frac{\cos^2 + \sin\theta^2 + \sin\theta}{(1+\sin\theta)\cos\theta}$

$= \frac{1}{\cos\theta} = \sec\theta$

247. (a) $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} - \frac{\sin\theta + 1}{\cos\theta}$

$= \frac{(\sin\theta - \cos\theta + 1)(\cos\theta) - (\sin\theta + 1)(\sin\theta + \cos\theta - 1)}{(\sin\theta + \cos\theta - 1)(\cos\theta)}$

$= \frac{\sin\theta.\cos\theta - \cos^2\theta + \cos\theta - \sin^2\theta \atop - \sin\theta - \sin\theta.\cos\theta - \cos\theta + \sin\theta + 1}{(\sin\theta + \cos\theta - 1)(\cos\theta)} = 0$

$= \frac{0}{(\sin\theta + \cos\theta - 1)(\cos\theta)} = 0$

248. (a) $\tan x = a;\ \tan y = b$

$(a+b)\left(1 - \frac{1}{ab}\right) + \left(\frac{1}{a} + \frac{1}{b}\right)(1 - ab)$

$= \frac{(a+b)(ab-1)}{ab} + \frac{(a+b)(1-ab)}{ab}$

$= \frac{(ab - 1 + 1 - ab)(a+b)}{ab} = 0$

249. (c) $\sqrt{\frac{\sec x-\tan x}{\sec x+\tan x}}=\sqrt{\frac{\sec^2 x-\tan^2 x}{(\sec x+\tan x)^2}}$

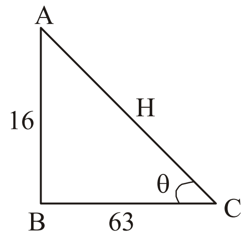
$\sqrt{\frac{1}{(\sec x+\tan x)^2}}=\frac{1}{\sec x+\tan x}$

$\because \sec^2 x-\tan^2 \times=1$

250. (c) As $\cot\theta=\frac{63}{16}$

From Δ ABC
$B=63, P=16$

$\therefore H=\sqrt{63^2+16^2}=65$



Now, $\sin\theta+\cos\theta$

$=\frac{P+B}{H}=\frac{63+16}{65}=\frac{79}{65}$

251. (d) $\frac{1-2\sin^2\theta\cos^2\theta}{\sin^4+\cos^4\theta}$

$=\frac{(\sin^2\theta+\cos^2\theta)^2-2\sin^2\theta\cos^2\theta}{\sin^4\theta+\cos^4\theta}$

$=\frac{\sin^4\theta+\cos^4\theta}{\sin^4\theta+\cos^4\theta}=1$

Hence, required sum $=1+4=5$

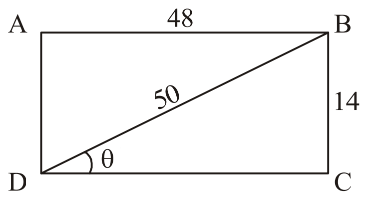
252. (a) Diagonal BD $=\sqrt{(AB)^2+(AD)^2}$

$=\sqrt{(48)^2+(14)^2}$

$=\sqrt{2500}=50$

Now $\sec\theta+\operatorname{cosec}\theta$

$=\frac{50}{48}+\frac{50}{14}$



$=\frac{50}{2}\left[\frac{24+7}{24\times7}\right]=\frac{775}{168}$

CHAPTER

18 Height and Distance

1. Two houses are collinear with the base of a tower and are at distance 3 m and 12 m from the base of the tower. The angles of elevation from these two houses of the top of the tower are complementary. What is the height of the tower? **[2007-I]**
(a) 4 m (b) 6 m (c) 7.5 m (d) 36 m

2. The angle of elevation from the bank of a river of the top of a tree standing on the opposite bank is 60º. The angle of elevation becomes 30º when observed from a point 40 m backwards in a direction perpendicular to the length of the river. What is the width of the river? **[2007-I]**
(a) 10 m (b) 20 m (c) 30 m (d) 40 m

3. Person standing at the end of the shadow of a pole measures that the length of the shadow is $\frac{1}{\sqrt{3}}$ times the length of the pole. At what angle of elevation will the man see the Sun? **[2007-II]**
(a) 60º (b) 30º (c) 45º (d) 15º

4. The angle of depression of vertex of a regular hexagon lying in a horizontal plane, from the top of tower of height 75 m located at the centre of the regular hexagon is 60º. What is the length of each side of the hexagon? **[2007-II]**
(a) $50\sqrt{3}$ m (b) 75 m (c) $25\sqrt{3}$ m (d) 25 m

5. A vertical stick 10 cm long casts a shadow 8 cm long. At the same time a tower casts a shadow 30 m long. What is the height of the tower? **[2008-I]**
(a) 37.5 m (b) 36 m (c) 32.5 m (d) 32 m

6. A round balloon of unit radius subtends an angle of 90º at the eye of an observer standing at a point, say A. What is the distance of the centre of the balloon from the point A? **[2008-II]**
(a) $1/\sqrt{2}$ (b) $\sqrt{2}$ (c) 2 (d) 1/2

7. The angle of elevation and angle of depression both are measured with **[2008-II]**
(a) the vertical only
(b) the horizontal line only
(c) both horizontal and vertical
(d) None of the above

8. A ladder 34 *m* long is placed in a lane so as to reach window 30 *m* high and on turning the ladder to the other side of the lane and keeping is foot at the same place, reaches a point 16 *m* height. What is the breadth of the lane? **[2008-II]**
(a) 18 *m* (b) 40 *m* (c) 46 *m* (d) 50 *m*

9. A radio transmitter antenna of height 100 m stands at the top of a tall building. At a point on the ground, the angle of elevation of bottom of the antenna is 45º and that of top of antenna is 60º. What is the height of the building? **[2009-I]**
(a) 100 m (b) 50 m
(c) $50(\sqrt{3}+1)$ m (d) $50(\sqrt{3}-1)$ m

10. The angle of elevation of the top of an unfinished pillar at a point 150 m from its base is 30º. If the angle of elevation at the same point is to be 45º, then the pillar has to be raised to a height of how many metres? **[2009-I]**
(a) 59.4 m (b) 61.4 m (c) 62.4 m (d) 63.4 m

11. A ladder 25 *m* long is leaning against a wall which is perpendicular to the level ground. The bottom of the ladder is 7 *m* from the base of the wall. If the top of the ladder slips down 4 *m*, how much will the bottom of the ladder slip? **[2009-I]**
(a) 7 *m* (b) 8 *m* (c) 10 *m* (d) 15 *m*

12. The length of the shadow of a person *s* cm tall when the angle of elevation of the Sun is α is *p* cm. It is *q* cm when the angle of elevation of the Sun is β. Which one of the following is correct when β = 3α? **[2009-II]**
(a) $p-q=s\left(\frac{\tan\alpha-\tan 3\alpha}{\tan 3\alpha\tan\alpha}\right)$
(b) $p-q=s\left(\frac{\tan 3\alpha-\tan\alpha}{3\tan 3\alpha\tan\alpha}\right)$
(c) $p-q=s\left(\frac{\tan 3\alpha-\tan\alpha}{\tan 3\alpha\tan\alpha}\right)$
(d) $p-q=s\left(\frac{\tan 2\alpha}{\tan 3\alpha\tan\alpha}\right)$

13. A man is watching from the top of a tower a boat speeding away from the tower. The boat makes an angle of depression of 45º with the man's eye when at a distance of 60 m from the bottom of tower. After 5 s, the angle of depression becomes 30º. What is the approximate speed of the boat assuming that it is running in still water? **[2010-I]**
(a) 31.5 km/h (b) 36.5 km/h
(c) 38.5 km/h (d) 40.5 km/h

14. Suppose the angle of elevation of the top of a tree at a point E due East of the tree is 60º and that at a point F due West of the tree is 30º. If the distance between the points E and F is 160 ft, then what is the height of the tree? **[2010-I]**
(a) $40\sqrt{3}$ ft (b) 60 ft
(c) $\frac{40}{\sqrt{3}}$ ft (d) 23 ft

15. A ladder of 17 ft length reaches a window which is 15 ft above the ground on one side of the street. Keeping its foot at the same point the ladder is turned to the other side of the street and now it reaches a window 8 ft high. What is the width of the street? **[2010-I]**
(a) 23 ft (b) 15 ft (c) 25 ft (d) 30 ft

16. The angle of elevation of the top of a tower from the bottom of a building is twice that from its top. What is the height of the building, if the height of the tower is 75 m and the angle to elevation of the top of the tower from the bottom of the building is 60º? **[2010-II]**
(a) 25 m (b) 37.5 m (c) 50 m (d) 60 m

17. The shadow of a tower is 15 m when the Sun's altitude is 30º. What is the length of the shadow when the Sun's altitude is 60º? **[2010-II]**
(a) 3 m (b) 4 m (c) 5 m (d) 6 m

18. Two poles of heights 6 m and 11 m stand vertically upright on a plane ground. If the distance between their feet is 12 m, what is the distance between their tops? **[2010-II]**
(a) 11 m (b) 12 m (c) 13 m (d) 14 m

19. The angle of elevation of the top of a tower at a point on level ground is 45º. When moved 20 m towards the tower, the angle of elevation becomes 60º. What is the height of the tower? **[2011-I]**
(a) $10(\sqrt{3}-1)$ m (b) $10(\sqrt{3}+1)$ m
(c) $10(3-\sqrt{3})$ m (d) $10(3+\sqrt{3})$ m

20. A telegraph post gets broken at a point against a storm and its top touches the ground at a distance 20 m from the base of the post making an angle 30º with the ground. What is the height of the post? **[2011-I]**
(a) $\frac{40}{\sqrt{3}}$ m (b) $20\sqrt{3}$ m (c) $40\sqrt{3}$ m (d) 30 m

21. The angles of elevation of the top of a tower from two points situated at distances 36 m and 64 m from its base and in the same straight line with it are complementary. What is the height of the tower? **[2011-II]**
(a) 50 m (b) 48 m (c) 25 m (d) 24 m

22. The angle of elevation of the top of an incomplete vertical pillar at a horizontal distance of 100 m from its base is 45º. If the angle of elevation of the top of complete pillar at the same point is to be 60º, then the height of the incomplete pillar is to be increased by **[2011-II]**
(a) $50\sqrt{2}$ m (b) 100 m
(c) $100(\sqrt{3}-1)$ m (d) $100(\sqrt{3}+1)$ m

23. The length of shadow of a tree is 16 m when the angle of elevation of the Sun is 60º. What is the height of the tree? **[2011-II]**
(a) 8 m (b) 16 m
(c) $16\sqrt{3}$ m (d) $\frac{16}{\sqrt{3}}$ m

24. From a lighthouse the angles of depression of two ships on opposite sides of the lighthouse are observed to 30º and 45º. If the height of lighthouse is h, what is the distance between the ships? **[2011-II]**
(a) $(\sqrt{3}+1)h$ (b) $(\sqrt{3}-1)h$
(c) $\sqrt{3}h$ (d) $\left(1+\frac{1}{\sqrt{3}}\right)h$

25. From the top of a cliff 200 m high, the angles of depression of the top and bottom of a tower are observed to be 30º and 45º, respectively. What is the height of the tower? **[2012-I]**
(a) 400 m (b) $400\sqrt{3}$ m
(c) $400/\sqrt{3}$ m (d) None of these

26. The angle of elevation of the tip of a tower from a point on the ground is 45º. Moving 21 m directly towards the base of the tower, the angle of elevation changes to 60º. What is the height of the tower, to the nearest meter? **[2012-I]**
(a) 48 m (b) 49 m (c) 50 m (d) 51 m

27. What is the angle of elevation of the Sum when the shadow of a pole is $\sqrt{3}$ times the length of the pole? **[2012-I]**
(a) 30º (b) 45º
(c) 60º (d) None of these

28. On walking 120 m towards a chimney in a horizontal line through its base the angle of elevation of tip of the chimney changes from 30º to 45º. The height of the chimney is. **[2012-II]**
(a) 120 m (b) $60(\sqrt{3}-1)$ m
(c) $60(\sqrt{3}+1)$ m (d) None of these

29. A ladder 20 m long is placed against a wall, so that the foot of the ladder is 10 m from the wall. The angle of inclination of the ladder to the horizontal will be **[2012-II]**
(a) 30º (b) 45º (c) 60º (d) 75º

30. The angles of elevation of the top of a tower from two points which are at distances of 10 m and 5 m from the base of the tower and in the same straight line with it are complementary. The height of the tower is. **[2012-II]**
(a) 5 m (b) 15 m (c) $\sqrt{50}$ m (d) $\sqrt{75}$ m

31. The angles of elevation of the top of an inaccessible tower from two points on the same straight line from the base of the tower are 30º and 60º, respectively. If the points are separated at a distance of 100 m, then the height of the tower is close to **[2012-II]**
(a) 86.6 m (b) 84.6 m
(c) 82.6 m (d) 80.6 m

32. Two poles of heights 6m and 11m stand on a plane ground. If the distance between their feet is 12 m, what is the distance between their tops? **[2012-II]**
(a) 13 m (b) 17 m
(c) 18 m (d) 23 m

DIRECTIONS (Qs. 33-36) : *Read the following information carefully to answer the questions that follow.*

As seen from the top and bottom of a building of height h m, the angles of elevation of the top of a tower of height $\frac{(3+\sqrt{3})h}{2}$ m are α and β, respectively.

33. If $\beta = 30°$, then what is the value of $\tan \alpha$? **[2013-I]**
(a) 1/2 (b) 1/3
(c) 1/4 (d) None of these

34. If $\alpha = 30°$, then what is the value of $\tan \beta$? **[2013-I]**
(a) 1 (b) 1/2
(c) 1/3 (d) None of these

35. If $\alpha = 30°$ and $h = 30$ m, then what is the distance between the base of the building and the base of the tower? **[2013-I]**
(a) $15+15\sqrt{3}$m (b) $30+15\sqrt{3}$m
(c) $45+15\sqrt{3}$ m (d) None of these

36. If $\beta = 30°$ and if θ is the angle of depression of the foot of the tower as seen from the top of the building, then what is the value of $\tan \theta$? **[2013-I]**
(a) $\frac{(3-\sqrt{3})}{3\sqrt{3}}$ (b) $\frac{(3+\sqrt{3})}{3\sqrt{3}}$
(c) $\frac{(2-\sqrt{3})}{3\sqrt{3}}$ (d) None of these

37. A spherical balloon of radius r subtends angle 60° at the eye of an observer. If the angle of elevation of its centre is 60° and h is the height of the centre of the balloon, then which one of the following is correct? **[2013-II]**
(a) $h = r$ (b) $h = \sqrt{2}r$
(c) $h = \sqrt{3}r$ (d) $h = 2r$

38. What is the angle of elevation of the Sun, when the shadow of a pole of height x m is $\frac{x}{\sqrt{3}}$ m? **[2013-II]**
(a) 30° (b) 45° (c) 60° (d) 75°

39. The heights of two trees are x and y, where $x > y$. The tops of the trees are at a distance z apart. If x is the shortest distance between the trees, then what is s^2 equal to? **[2013-II]**
(a) $x^2 + y^2 - z^2 - 2xy$
(b) $x^2 + y^2 - z^2$
(c) $x^2 + y^2 + z^2 - 2xy$
(d) $z^2 - x^2 - y^2 + 2xy$

40. The shadow of a tower standing on a level plane is found to be 50 m longer when the Sun's elevation is 30°, then when it is 60°. What is the height of the tower? **[2014-I]**
(a) 25m (b) $25\sqrt{3}$ m
(c) $\frac{25}{\sqrt{3}}$ m (d) 30m

41. The angle of elevation of the top of a tower 30 m high from the foot of another tower in the same plane is 60° and the angle of elevation of the top of the second tower from the foot of the first tower is 30°. The distance between the two towers in m times the height of the shorter tower. What is m equal to? **[2014-I]**
(a) $\sqrt{2}$ (b) $\sqrt{3}$
(c) $\frac{1}{2}$ (d) $\frac{1}{3}$

42. From a certain point on a straight road, a person observe a tower in the West direction at a distance of 200 m. He walks some distance along the road and finds that the same tower is 300 m South of him. What is the shortest distance of the tower from the road ? **[2014-II]**
(a) $\frac{300}{\sqrt{13}}m$ (b) $\frac{500}{\sqrt{13}}m$ (c) $\frac{600}{\sqrt{13}}m$ (d) $\frac{900}{\sqrt{13}}m$

43. The angles of elevation of the top of a tower from two points P and Q at distances m^2 and n^2 respectively, from the base and in the same straight line with it are complementary. The height of the tower is **[2015-I]**
(a) $(mn)^{1/2}$ (b) $mn^{1/2}$
(c) $m^{1/2}n$ (d) mn

44. The angle of elevation of a cloud from a point 200 m above a lake is 30° and the angle of depression of its reflection in the lake is 60°. The height of the cloud is **[2015-I]**
(a) 200 m (b) 300 m (c) 400 m (d) 600 m

45. From the top of a tower, the angles of depression of two objects P and Q (situated on the ground on the same side of the tower) separated at a distance of $100(3-\sqrt{3})m$ are 45° and 60° respectively. The height of the tower is **[2015-I]**
(a) 200 m (b) 250 m
(c) 300 m (d) None of the above

46. An aeroplane flying at a height of 3000 m passes vertically above another aeroplane at an instant when the angles of elevation of the two planes from some point on the ground are 60° and 45° respectively. Then the vertical distance between the two planes is **[2015-II]**
(a) $1000(\sqrt{3}-1)$m (b) $1000\sqrt{3}$m
(c) $1000(3-\sqrt{3})$m (d) $3000\sqrt{3}$m

47. A pole is standing erect on the ground which is horizontal. The tip of the pole is tied tight with a rope of length $\sqrt{12}$ m to a point on the ground. If the rope is making 30° with the horizontal, then the height of the pole is **[2015-II]**
(a) $2\sqrt{3}$m (b) $3\sqrt{2}$m (c) 3 m (d) $\sqrt{3}$m

48. Two observers are stationed due north of a tower (of height x metre) at a distance y metre from each other. The angles of elevation of the tower observed by them are 30° and 45° respectively. Then x/y is equal to **[2016-I]**
(a) $\frac{\sqrt{2}-1}{2}$ (b) $\frac{\sqrt{3}-1}{2}$ (c) $\frac{\sqrt{3}+1}{2}$ (d) 1

49. A man from the top of a 100 m high tower sees a car moving towards the tower at an angle of depression 30°. After some time, the angle of depression becomes 60°. What is the distance travelled by the car during this time? **[2016-II]**
(a) $100\sqrt{3}$m (b) $\frac{200\sqrt{3}}{3}$m
(c) $\frac{100\sqrt{3}}{3}$m (d) $200\sqrt{3}$m

50. Two men on either side of a tower 75 m high observe the angle of elevation of the top of the tower to be 30° and 60°. What is the distance between the two men ? **[2016-II]**

(a) $100\sqrt{3}$m (b) $75\sqrt{3}$m

(c) $\frac{100\sqrt{3}}{3}$m (d) $60\sqrt{3}$m

51. If the length of the shadow of a tower is equal to its height, then what is the Sun's altitude at that time ? **[2016-II]**

(a) 15° (b) 30° (c) 45° (d) 60°

52. A pole stands vertically inside a triangular park ABC. If the angle of elevation of the top of the pole from each corner of the park is same, then in the triangle ABC, the foot of the pole is at the **[2016-II]**

(a) Centroid (b) Circumference

(c) Incentre (d) Orthocentre

53. An aeroplane flying at a height of 300 m above the ground passes vertically above another plane at an instant when the angles of elevation of the two planes from the same point on the ground are 60° and 45° respectively. What is the height of the lower plane from the ground ? **[2017-I]**

(a) $100\sqrt{3}$ m (b) $\frac{100}{\sqrt{3}}$m

(c) $50\sqrt{3}$m (d) $150(\sqrt{3}+1)$m

54. From the top of a building 90 m high, the angles of depression of the top and the bottom of a tree are 30° and 45° respectively. What is the height of the tree ? **[2017-I]**

(a) $30\sqrt{3}$m (b) $90-30\sqrt{3}$m

(c) $90+30\sqrt{3}$m (d) $60+30\sqrt{3}$m

55. From an aeroplane vertically over a straight horizontal road, the angles of depression of two consecutive kilometre-stones on the opposite sides of the aeroplane are observed to be α and β. The height of the aeroplane above the road is **[2017-I]**

(a) $\frac{\tan\alpha+\tan\beta}{\tan\alpha\tan\beta}$ (b) $\frac{\tan\alpha\ \tan\beta}{\tan\alpha+\tan\beta}$

(c) $\frac{\cot\alpha\ \cot\beta}{\cot\alpha+\cot\beta}$ (d) $\frac{\cot\alpha+\cot\beta}{\cot\alpha\cot\beta}$

56. If angle A of triangle ABC is 30° and the circum-radius of the triangle is 10 cm, then what is the length of side BC? **[2017-II]**

(a) 5 cm (b) 10 cm

(c) $5\sqrt{3}$ cm (d) $10\sqrt{3}$ cm

57. Let AB represent a building of height h metre with A being its top, B being its bottom. Let A 'B' represent a tower of height (h + x) metre (x > 0) with A' being its top and B' being its bottom. Let BB' = d metre. Let the angle of elevation of A' as seen from A be 45°.

Consider the following statements :

Statement I : h + x > d

Statement II : The angle of depression of B as seen from A' is less than 45°.

Which one of the following is correct in respect of the above statements? **[2017-II]**

(a) Both Statement I and Statement II are true and Statement II is the correct explanation of Statement I

(b) Both Statement I and Statement II are true but Statement II is not the correct explanation of StatementI

(c) Statement I is true but Statement II is false

(d) Statement I is false but Statement II is true

58. A man, standing at a point X on the bank XY of a river that cannot be crossed, observes a tower to be N α° E on the opposite parallel bank. He then walks 200 m along the bank to the point Y towards East, and finds the tower to be N β° W. From these observations, the breadth of the river will be

(Given that tan α° = 2 and tan β° = 0.5) **[2017-II]**

(a) 60 m (b) 70 m (c) 80 m (d) 90 m

59. Each side of a square subtends an angle of 60° at the tip of a tower of height h metres standing at the centre of the square. If l is the length of each side of the square, then what is h^2 equal to? **[2018-I]**

(a) $2l^2$ (b) $\frac{l^2}{2}$ (c) $\frac{3l^2}{2}$ (d) $\frac{2l^2}{3}$

60. From a height of h units, a man observes the angle of elevation as α and angle of depression as β of the top and the bottom respectively of a tower of height H (> 4h). To what further height should he climb so that the values of angle of elevation and angle of depression get interchanged for the top and bottom of the tower? **[2018-I]**

(a) H – h units (b) H – 2h units

(c) H – 3h units (d) H – 4h units

61. On the top of a hemispherical dome of radius r, there stands a flag of height h. From a point on the ground, the elevation of the top of the flag is 30º. After moving a distance d towards the dome, when the flag is just visible, the elevation is 45º. The ratio of h to r is equal to **[2018-II]**

(a) $\sqrt{2}-1$ (b) $\frac{\sqrt{3}+1}{2\sqrt{2}}$

(c) $\frac{\sqrt{3}+1}{2\sqrt{2}}d$ (d) $\frac{(\sqrt{3}+1)(\sqrt{2}-1)}{2\sqrt{2}}d$

62. Consider a regular hexagon ABCDEF. Two towers are situated at B and C. The angle of elevation from A to the top of the tower at B is 30° and the angle of elevation to the top of the tower at C is 45°. What is the ratio of the height of towers at B and C ? **[2019-I]**

(a) $1:\sqrt{3}$ (b) 1 : 3 (c) 1 : 2 (d) $1:2\sqrt{3}$

63. There are two parallel streets each directed North to South. A person in the first street travelling from South to North wishes to take the second street which is on his right side. At some place, he makes a 150° turn to the right and he travels for 15 minutes at the speed of 20 km/hr. After that he takes a left turn of 60° and travels for 20 minutes at the speed of 30 km/hr in order to meet the second street. What is the distance between the two streets ? **[2019-I]**

(a) 7.5 km (b) 10.5 km (c) 12.5 km (d) 15 km

64. The angles of elevation of the tops of two pillars of heights h and $2h$ from a point P on the line joining the feet of the two pillars are complementary. If the distances of the foot of the pillars from the point P are x and y respectively, then which one of the following is correct ? **[2019-II]**

(a) $2h^2 = x^2y$ (b) $2h^2 = xy^2$
(c) $2h^2 = xy$ (d) $2h^2 = x^2y^2$

65. A tree of height 15 m is broken by wind in such a way that its top touches the ground and makes an angle 30° with the ground. What is the height from the ground to the point where tree is broken ? **[2020-I]**

(a) 10 m (b) 7 m (c) 5 m (d) 3 m

66. On a plane area there are two vertical towers separated by 100 feet apart. The shorter tower is 40 feet tall. A pole of length 6 feet stands on the line joining the base of two towers so that the tip of the towers and tip of the pole are also on the same line. If the distance of the pole from the shorter tower is 75 feet, then what is the height of the taller tower (approximately)? **[2020-I]**

(a) 85 feet (b) 110 feet
(c) 125 feet (d) 140 feet

67. The angles of elevation of the top of a tower from two points at distances p and q from the base and on the same straight line are 27° and 63° respectively. What is the height of the tower ? **[2020-I]**

(a) pq (b) $\sqrt{pq}$ (c) $\frac{pq}{2}$ (d) $\frac{\sqrt{pq}}{2}$

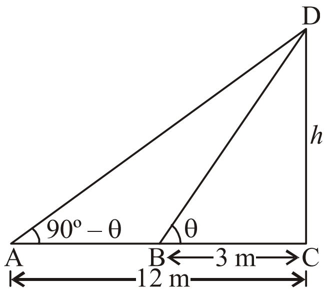
68. A ladder 5 m long is placed in a room so as to reach a point 4.8 m high on a wall and on turning the ladder over to the opposite side of the wall without moving the base, it reaches a point 1.4 m high. What is the breadth of the room ? **[2020-I]**

(a) 5.8 m (b) 6 m (c) 6.2 m (d) 7.5 m

HINTS & SOLUTIONS

1. (b) Let the height of the tower be h m and $\angle CBD = \theta$ then $\angle DAC = 90^\circ - \theta$

(Because both angles are complementary)



$\therefore$ In ΔBCD,

$\tan\theta = \frac{CD}{BC} \Rightarrow \tan\theta = \frac{h}{3}$

Now, in ΔACD

$\tan(90^\circ - \theta) = \frac{CD}{AC} \Rightarrow \cot\theta = \frac{h}{12}$

$\tan\theta = \frac{h}{12}$

$h\tan\theta = 12$

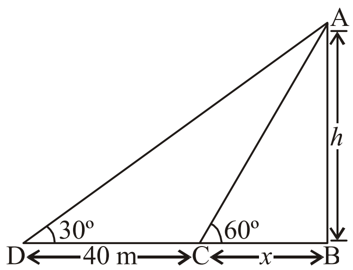
put the value of $\tan\theta$

$h \times \frac{h}{3} = 12$

$h^2 = 36 \quad \therefore h = 6$

Then, height of tower = 6 m.

2. (b) Let the height of the tower be h m and BC = x m



In ΔABC

$\tan 60^\circ = \frac{h}{x} \Rightarrow h = \sqrt{3}x$

Now in ΔADB,

$\tan 30^\circ = \frac{h}{40 + x}$

we put the volue of $h = \sqrt{3}x$

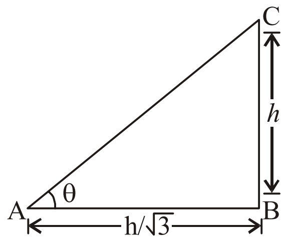
$h = \frac{40 + x}{\sqrt{3}}$

$\sqrt{3}x = \frac{40 + x}{\sqrt{3}} \Rightarrow 2x = 40 \Rightarrow x = 20\text{m}.$

3. (a) Let the height of pole be h then its shadow = $\frac{h}{\sqrt{3}}$

and θ be the angle of elevation.

In ΔABC

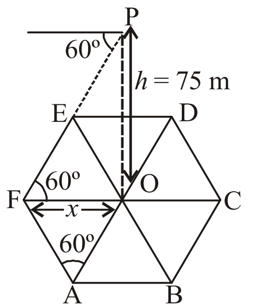


$$\tan\theta = \frac{h}{h/\sqrt{3}} \Rightarrow \tan\theta$$

$= \quad \sqrt{3} = \tan 60^\circ$

$\therefore \quad \theta = 60^\circ$

4. (c) Let OP be the height of the tower
x = Distance between O and F.



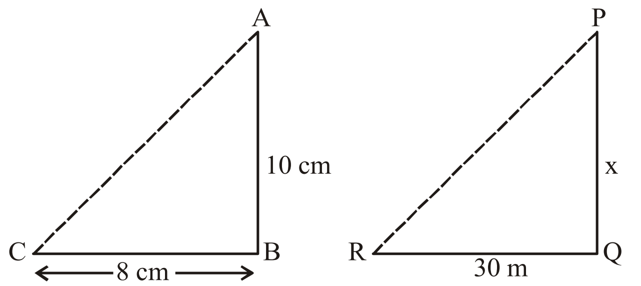
In ΔFOP, $\tan 60^\circ = \dfrac{75}{x} \Rightarrow x = \dfrac{75}{\sqrt{3}} = 25\sqrt{3}$ m

In regular hexagon ΔOEF, ΔOED ...etc, are equilateral triangles.

$\therefore$ OF $= AF = AB = BC = CD = DE = EF = 25\sqrt{3}$ m = Side of hexagon

$\therefore \quad$ Length of hexagon $= 25\sqrt{3}$ m

5. (a)



AB = Stick length $\qquad PQ$ = Tower height

BC = Shadow length $\qquad QR$ = Shadow length

Let height of a tower be x m

By given condition,

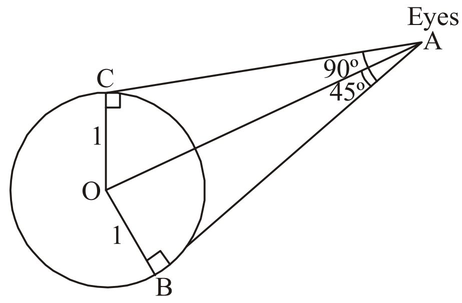
$$\frac{\text{Height of stick}}{\text{Length of shadow of stick}} = \frac{\text{Height of tower}}{\text{Length of shadow of tower}}$$

$$\Rightarrow \frac{10}{8} = \frac{x}{30}$$

$$\Rightarrow x = \frac{10 \times 30}{8} = 37.5\,m$$

Hence, length of the tower be $37.5\,m$.

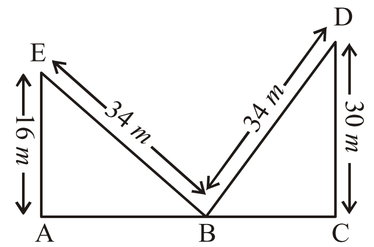
6. (b) Let O = Centre of the balloon
OB = OC = Radii of the balloon



In ΔOBA, $\sin 45^\circ = \dfrac{OB}{OA} \Rightarrow \dfrac{1}{\sqrt{2}} = \dfrac{1}{OA} \Rightarrow OA = \sqrt{2}$

7. (b) The angle of elevation and angle of depression are measured with the horizontal line only.

8. (c)



In $\Delta\, BCD$,

$BC \quad = \sqrt{(34)^2 - (30)^2}$

$= \sqrt{1156 - 900} = \sqrt{256} = 16\,m$

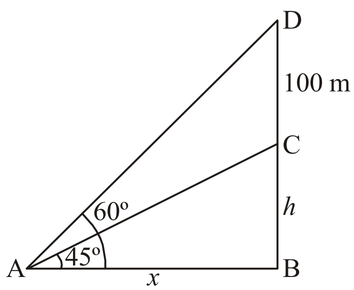
and In $\Delta\, BAE, AB = \sqrt{(34)^2 - (16)^2}$

$= \sqrt{1156 - 256} = 30\,m$

$\therefore$ Breadth of the lane $= AB + BC$

$= 30 + 16 = 46\,m$

9. (c) Let BC be a building of height h m and CD = 100 m be a height of antenna.
x = Distance between A and B



In ΔABC,

$$\tan 45^\circ = \frac{h}{x} \Rightarrow x = h \quad \text{...}(i)$$

Now, in ΔABD,

$$\tan 60^\circ = \frac{100+h}{x}$$

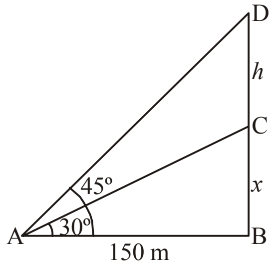
$$\Rightarrow \quad \sqrt{3} = \frac{100+h}{h} \quad \text{[from Eq. }(i)\text{]}$$

$$\Rightarrow \quad \sqrt{3}h = 100 + h \Rightarrow \left(\sqrt{3}-1\right)h = 100$$

$$\Rightarrow \quad h = \frac{100}{\sqrt{3}-1} \Rightarrow h = \frac{100}{\left(\sqrt{3}-1\right)} \times \frac{(\sqrt{3}+1)}{\left(\sqrt{3}+1\right)}$$

$$= \quad 50(\sqrt{3}+1)\,\text{m}$$

10. (d) Let BC = x m height of unfinished pillar and CD = h m = Raised height of pillar



In ΔABC,

$$\tan 30^\circ = \frac{x}{150} \Rightarrow x = \frac{150}{\sqrt{3}}$$

and in ΔABD,

$$\tan 45^\circ = \frac{h+x}{150} \Rightarrow 1 = \frac{h+x}{150}$$

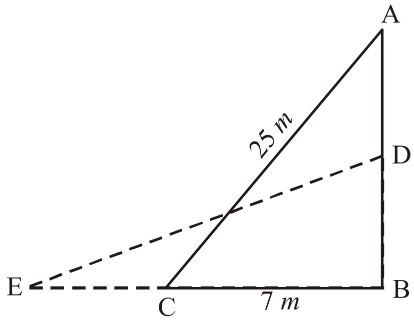
$$\Rightarrow \quad 150 = h + \frac{150}{\sqrt{3}} \quad \text{[from Eq. }(i)\text{]}$$

$$\Rightarrow \quad \frac{150(\sqrt{3}-1)}{\sqrt{3}} = \Rightarrow \quad h = 150 \times \frac{(1.732-1)}{1.732}$$

$$= \frac{150 \times 0.732}{1.732}$$

$$= 63.39 = 63.4\text{ m (approx)}$$

11. (b)



In $\Delta\,ABC$,

$AC^2 = AB^2 + BC^2$

$\Rightarrow \quad (25)^2 = (AB)^2 + (7)^2$

$\Rightarrow \quad (AB)^2 = 625 - 49 \therefore AB = 24\,m$

In $\Delta\,EBD$,

$ED^2 = (EB)^2 + (BD)^2$

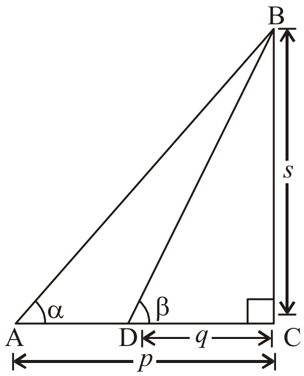
$\Rightarrow \quad (25)^2 = (EB)^2 + (20)^2$

$\Rightarrow \quad 625 - 400 = (EB)^2 \Rightarrow EB = 15\,m$

$\therefore \quad EC = 15 - 7 = 8\,m$

12. (c) In ΔBCA,

$$\tan\alpha = \frac{s}{p} \Rightarrow p = \frac{s}{\tan\alpha} \quad \text{...}(i)$$



In ΔBCD,

$$\tan\beta = \frac{s}{q} \Rightarrow q = \frac{s}{\tan 3\alpha}$$

($\because \beta = 3\alpha$, given) ...(ii)

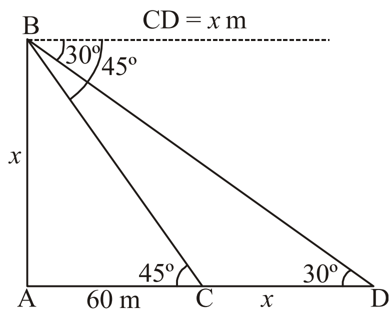
On subtracting Eq. (ii) from Eq. (i), we get

$$p - q = \frac{s}{\tan\alpha} - \frac{s}{\tan 3\alpha}$$

$$= s\left(\frac{\tan 3\alpha - \tan\alpha}{\tan 3\alpha\ \tan\alpha}\right)$$

13. (a) Let AB = x m

CD = x m



In ΔACB,

$$\tan 45^\circ = \frac{AB}{AC} \Rightarrow 1 = \frac{AB}{60}$$

$\Rightarrow \quad AB = 60$ m

Now, in ΔADB,

$$\tan 30^\circ = \frac{60}{60+x} \Rightarrow \frac{1}{\sqrt{3}} = \frac{60}{60+x}$$

$\Rightarrow \quad 60 + x = 60\sqrt{3}$

$\Rightarrow \quad x = 60(\sqrt{3}-1) = 60(1.73-1)$

$= \quad 60 \times 0.73 = 43.8$ m

$\therefore \quad$ Speed of boat $= \dfrac{43.8}{5} \times \dfrac{18}{5} = \dfrac{788.4}{25}$

$= \quad 31.5$ km/h

14\. (a) Let AC = h = Height of a tower

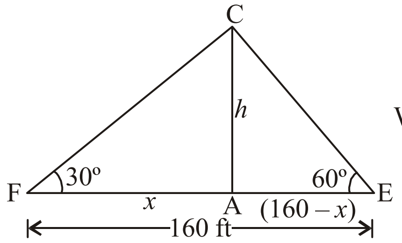
x = Distance between A and F

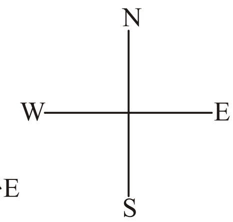
$\therefore \quad AE = 160 - x$

In ΔCAF,

$$\tan 30^\circ = \frac{h}{x}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x} \Rightarrow x = \sqrt{3}h \qquad ...(i)$$





And in ΔCAE,

$$\tan 60^\circ = \frac{h}{160-x}$$

$\Rightarrow \quad \sqrt{3}(160-x) = h$

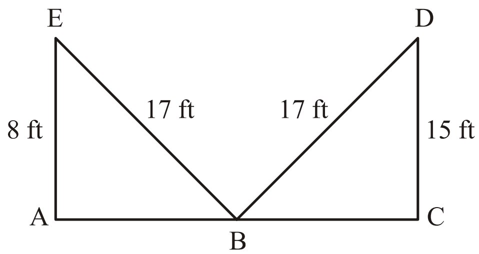
$\Rightarrow \quad \sqrt{3}(160-\sqrt{3}h) = h \qquad$ [from Eq. (i)]

$\Rightarrow \quad 160\sqrt{3} - 3h = h$

$\Rightarrow \quad 4h = 160\sqrt{3}$

$\therefore \quad h = 40\sqrt{3}$ ft

15\. (a)



In ΔABE,

$BE^2 = AE^2 + AB^2 \Rightarrow AB^2 = 17^2 - 8^2$

or $\quad AB^2$

$= \quad 289 - 64 = 225 \Rightarrow AB = \sqrt{225}$,

$AB = 15$ ft

In ΔBCD,

$BD^2 = BC^2 + CD^2 \quad \Rightarrow \quad BC^2 = 17^2 - 15^2$

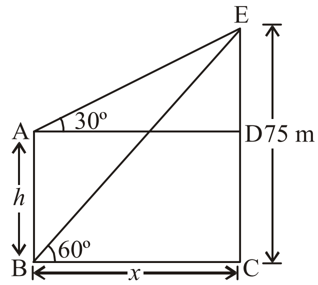
$= 289 - 225 = 64$

$\Rightarrow \quad BC = 8$ ft

$\therefore \quad$ Width of the street $= AB + BC = 15 + 8 = 23$ ft

16\. (c) Suppose height of building (AB) = h m

BC = x m



In ΔADE,

$$\tan 30^\circ = \frac{ED}{AD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{75-h}{x}$$

$$\Rightarrow \quad x = 75\sqrt{3} - h\sqrt{3} \qquad ...(i)$$

Now, in ΔBCE,

$$\tan 60^\circ = \frac{CE}{BC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{75}{x} \Rightarrow x\sqrt{3} = 75 \qquad \text{[from Eq. (ii)]}$$

$\Rightarrow \quad (75\sqrt{3} - h\sqrt{3})\sqrt{3} = 75 \Rightarrow 75 \times 3 - 3h = 75$

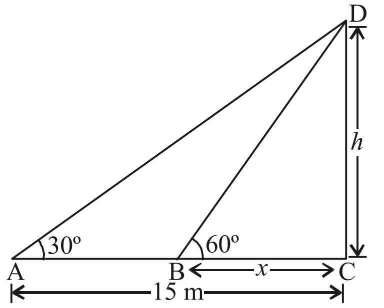
$\Rightarrow \quad 3h = 75 \times 3 - 75$

$$\Rightarrow \quad h = \frac{75 \times 2}{3}$$

$\therefore \quad h = 50$ m

17\. (c) In ΔACD,

$$\tan 30^\circ = \frac{CD}{AC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{15} \Rightarrow h = \frac{15}{\sqrt{3}}$$



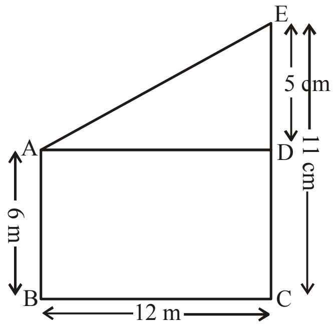
Now, in ΔBCD,

$$\tan 60^\circ = \frac{CD}{BC} \Rightarrow \sqrt{3} = \frac{h}{x}$$

put the value of $h = \dfrac{15}{\sqrt{3}}$

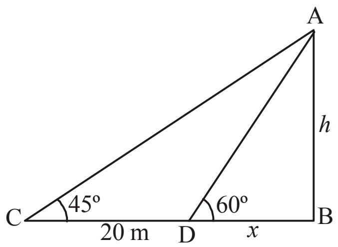
$$\sqrt{3} = \frac{15}{\sqrt{3}x} \Rightarrow x = 5 \text{ m.}$$

18. (c) $\because \quad AD = BC = 12$ m
and $\quad ED = 11 - 6 = 5$ m
Since, AE is distance between top point of AB and CE.



In ΔADE,
$AE^2 = AD^2 + ED^2 = 12^2 + 5^2$
$= 144 + 25 = 169$
$AE \quad = 13$ m
Hence, the distance between their tops = 13 m

19. (d) Let the height of the tower be h m and BD = xm.



In ΔACB,

$$\tan 45^\circ = \frac{h}{20+x} \Rightarrow h = 20 + x$$

Now, in ΔABD,

$$\tan 60^\circ = \frac{h}{x} \Rightarrow x = \frac{h}{\sqrt{3}}$$

Put the value of $h = 20 + x$

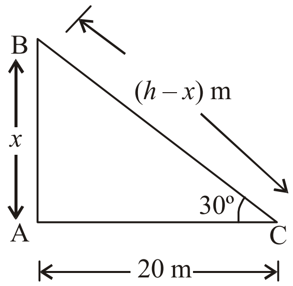
$$h = 20 + \frac{h}{\sqrt{3}} \Rightarrow h - \frac{h}{\sqrt{3}} = 20$$

$$\Rightarrow \quad h\left(1 - \frac{1}{\sqrt{3}}\right) = 20 \Rightarrow h\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right) = 20$$

$$\therefore \quad h = \frac{20\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{20\sqrt{3}(\sqrt{3}+1)}{2} = 10(3+\sqrt{3})\text{m}$$

20. (b) Let the height of the post be h m.



In ΔABC,
AC = 20 m
AB = x m

$$\tan 30^\circ = \frac{AB}{AC} = \frac{x}{20}$$

$$\frac{1}{\sqrt{3}} = \frac{x}{20} \Rightarrow x = \frac{20}{\sqrt{3}} \text{m}$$

$$\cos 30^\circ = \frac{20}{h-x}$$

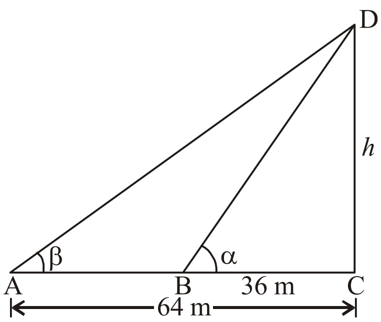
$$\frac{\sqrt{3}}{2} = \frac{20}{h-x} \Rightarrow h - x = \frac{40}{\sqrt{3}}$$

$$h = \frac{40}{\sqrt{3}} + x = \frac{40}{\sqrt{3}} + \frac{20}{\sqrt{3}} = \frac{60}{\sqrt{3}}$$

$$h = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{3} = \frac{60\sqrt{3}}{3} = 20\sqrt{3} \text{ m}$$

Thus, the height of the post is $20\sqrt{3}$ m.

21. (b) Let the height of the tower be h.



In ΔBCD,

$$\tan \alpha = \frac{h}{36}$$

In ΔACD,

$$\tan \beta = \frac{h}{64}$$

Since, angles of elevation are complementary.
So, $\angle DAC$ and $\angle DBC$ are complementary angle.

$$\beta = \frac{\pi}{2} - \alpha = \angle DAC$$

$$\tan\left(\frac{\pi}{2} - \alpha\right) = \frac{h}{64}$$

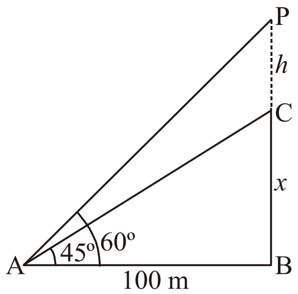
$$\Rightarrow \quad \cot\alpha = \frac{h}{64} \Rightarrow \frac{1}{\tan\alpha} = \frac{h}{64}$$

Now, put the value $\tan\alpha = \dfrac{h}{36}$

$$\Rightarrow \quad \frac{h}{36} \times h = 64 \Rightarrow h = \sqrt{64 \times 36}$$

$\therefore \quad h = 48$ m. (The height of the tower)

22. (c) Here, AB = 100 m
BC = x m
PC = h m



In ΔABC,

$$\tan 45^\circ = \frac{x}{100} \Rightarrow x = 100 \text{ m}$$

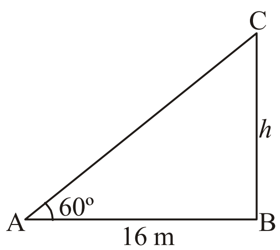
Now in ΔAPB,

$$\tan 60^\circ = \frac{x+h}{100} \Rightarrow x + h = 100\sqrt{3}$$

$$\Rightarrow \quad h = 100\sqrt{3} - x = 100\sqrt{3} - 100$$

$$\therefore \quad h = 100(\sqrt{3} - 1) \text{ m}$$

23. (c) Let the height of the tree is h m.



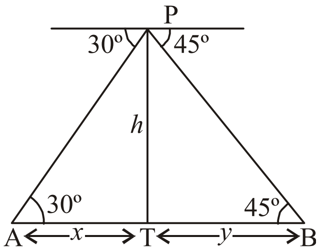
In ΔABC,

$$\tan 60^\circ = \frac{h}{16} \Rightarrow h = 16\sqrt{3} \text{ m}$$

24. (a) In ΔPBT,

$$\tan 45^\circ = \frac{h}{y} = 1$$

$$\therefore \quad y = h \qquad ...(i)$$



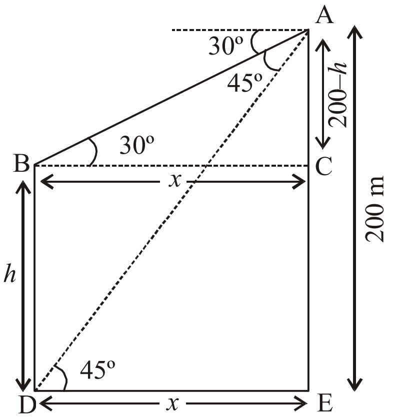
Now, in ΔPTA

$$\tan 30^\circ = \frac{h}{x} \Rightarrow x = \sqrt{3}h \qquad ...(ii)$$

$\therefore \quad$ Required distance $= x + 4$

$$= \sqrt{3}h + h = h(\sqrt{3} + 1) \text{ m}$$

25. (d)



$$\text{In } \Delta ACB, \tan 30^\circ = \frac{200-h}{x} = \frac{1}{\sqrt{3}} = \frac{500-h}{x}$$

$$\Rightarrow \quad x = (200 - h)\sqrt{3} \text{ and in } \Delta ADE,$$

$$\tan 45^\circ = \frac{200}{x}$$

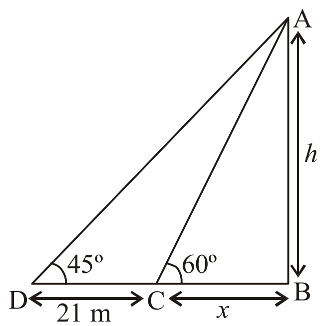
$$\Rightarrow \quad 1 = \frac{200}{x} \Rightarrow x = 200 \text{ m}$$

From Eq. (i)

$$200 = (200 - h)\sqrt{3}$$

$$\Rightarrow \quad h = 200\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right) \text{m}$$

26. (c) Let the height of the tower be h



In ΔABC,

$\tan C = \frac{AB}{BC}$

$\tan 60^\circ = \frac{h}{x}$

$\Rightarrow \quad \sqrt{3} = \frac{h}{x}$

$\Rightarrow \quad x = \frac{h}{\sqrt{3}}$

In ΔABD,

$\tan 45^\circ = \frac{h}{21+x} = 1$

$\Rightarrow \quad h = 21 + x$

Put the value of $x = \frac{h}{\sqrt{3}}$

$\Rightarrow \quad h = 21 + \frac{h}{\sqrt{3}}$

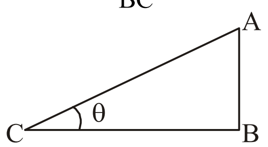
$\Rightarrow \quad h\left(1 - \frac{1}{\sqrt{3}}\right) = 21$

$\therefore \quad h = \frac{21\sqrt{3}}{(\sqrt{3}-1)} \cdot \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$

$= \quad \frac{21\sqrt{3}(\sqrt{3}+1)}{2} = 49.68 \approx 50\text{ m}$

27. (a) Let angle of elevation be θ.

$\therefore \tan\theta = \frac{AB}{BC}$



$BC = \sqrt{3}\ AB \Rightarrow \frac{AB}{BC} = \frac{1}{\sqrt{3}}$

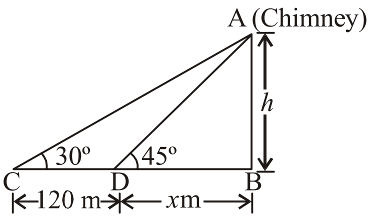
$\therefore \quad \tan\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad \theta = 30^\circ$

28. (c) Let the height of the Chemney be h

In ΔABD $\tan 45^\circ = \frac{AB}{BD}$

$\tan 45^\circ = \frac{h}{x} = 1 \Rightarrow h = x$



Now, in ΔABC

$\tan 30^\circ = \frac{h}{120+x} = \frac{1}{\sqrt{3}}$

Now put the value of $h = x$

$\Rightarrow \quad \frac{h}{120+h} = \frac{1}{\sqrt{3}}$

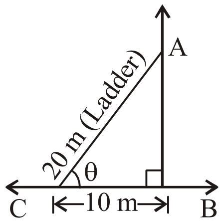
$\Rightarrow \quad \sqrt{3}h = 120 + h$

$\sqrt{3}h - h = 120 \Rightarrow h(\sqrt{3}-1) = 120$

$\therefore \quad h = \frac{120}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{120(\sqrt{3}+1)}{2}$

$\therefore$ Height of the chimney $(h) = 60(\sqrt{3}+1)$ m

29. (c) Let the inclination of the ladder angle be θ.



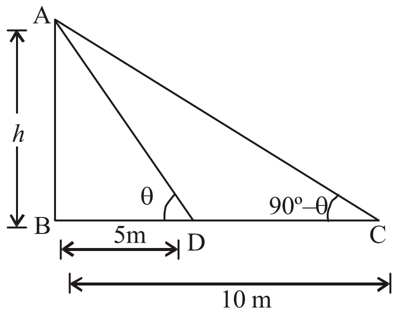
Now, in ΔABC

$\cos\theta = \frac{BC}{AC} = \frac{10}{20} = \frac{1}{2}$

$\Rightarrow \quad \cos\theta = \cos 60^\circ$

$\therefore \quad \theta = 60^\circ$

30. (c)



Here, given angles are comlementary.

$\angle ADB = \theta$

then, $\angle ACB = 90^\circ - \theta$

$AB = h$

$BD = 5\text{m}$

$BC = 10\text{m}$

In ΔABD,

$\tan\theta = \frac{h}{5}$ ...(*i*)

In ΔABC,

$\tan(90^\circ - \theta) = \frac{h}{10}$

$\Rightarrow \quad \cot\theta = \frac{h}{10}$ ...(*ii*)

From equation (*i*) and (*ii*),

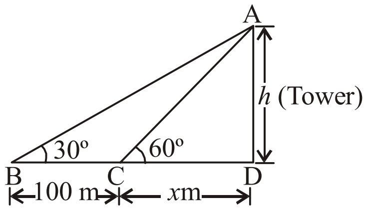
$\tan\theta \cdot \cot\theta = \frac{h}{5} \times \frac{h}{10} = \frac{h^2}{50} = 1$

$\therefore \quad h = \sqrt{50}$

Therefore, height of the tower is $\sqrt{50}$ m.

31. (a) Let the height of inaccessible tower be h.
In ΔACD

$$\tan 60^\circ = \frac{h}{x} = \sqrt{3} \Rightarrow x = \frac{h}{\sqrt{3}}$$



Now, in ΔABD,

$$\tan 30^\circ = \frac{h}{100+x} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \sqrt{3}h = 100 + x$$

Now put the value of $x = \frac{h}{\sqrt{3}}$

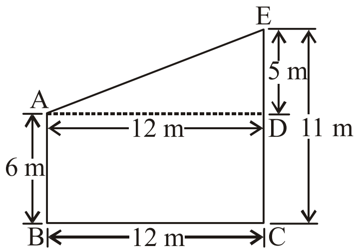
$$\Rightarrow \quad \left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)h = 100$$

$$\Rightarrow \quad \frac{2}{\sqrt{3}}h = 100 \Rightarrow h = 50\sqrt{3}$$

$\therefore \quad h = 50 \times 1.732 = 86.6$ m

Thus, the height of the tower (h) = 86.6m.

32. (a) Given that, AB = 6m and EC = 11m



$\Rightarrow \quad$ BC = 12 m

$\because \quad$ BC = AD = 12 m

and $\quad$ ED = EC – CD = EC – AB $\qquad (\because$ AB = CD)

$= \quad 11 - 6 = 5$m

In ΔAED, Using Pythagoras theorem

$\quad (AE)^2 = (AD)^2 + (ED)^2$

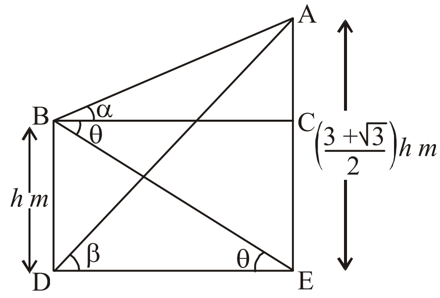
$= \quad (12)^2 + (5)^2 = 144 + 25$

$= \quad 169 = (13)^2$

$\because \quad$ AE = 13m

Them the distance between their tops is 13 m.

Solutions (33-36) :



33. (b) Given that, $\beta = 30^\circ$

In ΔAED, $\tan\beta = \tan 30^\circ = \frac{AE}{DE} = \frac{1}{\sqrt{3}}$

$$DE = \sqrt{3}\,AE = \sqrt{3}\left(\frac{3+\sqrt{3}}{2}\right)h$$

$$\Rightarrow \quad BC = DE = \frac{3}{2}(1+\sqrt{3})h \quad (\because BC = DE) \qquad ...(i)$$

Now, in ΔACB,

$$\Rightarrow \quad \tan\alpha = \frac{AC}{BC}$$

$$\Rightarrow \quad BC\tan\alpha = AE - CE = AE - BD \; (\because BD = CE)$$

$$\Rightarrow \quad BC\tan\alpha = \left(\frac{3+\sqrt{3}}{2}\right)h - h = h\left(\frac{3+\sqrt{3}-2}{2}\right)$$

$$\Rightarrow \quad \frac{3}{2}(1+\sqrt{3})h\tan\alpha = \left(\frac{1+\sqrt{3}}{2}\right)h \quad \text{[from Eq. } (i)]$$

$$\therefore \quad \tan\alpha = \frac{1}{3}$$

34. (a) Given that, $\alpha = 30^\circ$

In ΔACB, $\tan\alpha = \tan 30^\circ = \frac{AC}{BC} = \frac{1}{\sqrt{3}}$

$$\Rightarrow \quad BC = \sqrt{3}\,AC = \sqrt{3}\,(AE - CE)$$

$$= \quad \sqrt{3}\,(AE - BD) \qquad (\because BD = CE)$$

$$= \quad \sqrt{3}\left(\frac{3+\sqrt{3}}{2} - 1\right)h = \frac{\sqrt{3}}{2}(1+\sqrt{3})h \qquad ...(ii)$$

Now, in ΔAED,

$$\tan\beta = \frac{AE}{DE} \Rightarrow \tan\beta = \frac{AE}{BC}$$

$(\because$ DE = BC)

$$= \frac{\left(\frac{3+\sqrt{3}}{2}\right)h}{\frac{\sqrt{3}}{2}(1+\sqrt{3})h} = \frac{\frac{\sqrt{3}(1+\sqrt{3})}{2}h}{\frac{\sqrt{3}(1+\sqrt{3})}{2}h}$$

$$\therefore \quad \tan\beta = 1$$

35. (c) Given, $\alpha = 30^\circ$ and $h = 30$ m

In ΔACB,

$$\tan\alpha = \tan 30^\circ = \frac{AC}{BC} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{BC}{\sqrt{3}} = (AE - CE) = (AE - BD) \quad (\because BD = CE)$$

$$\Rightarrow BC = \sqrt{3}\left(\frac{3+\sqrt{3}}{2} - 1\right)h$$

$$\Rightarrow BC = \sqrt{3}\,\frac{(1+\sqrt{3})}{2}\cdot 30 = (\sqrt{3}+3)\cdot 15$$

$$\therefore DE = BC = (45 + 15\sqrt{3})\,\text{m} \qquad (\because DE = BC)$$

36. (a) Given that, $\beta = 30^\circ$

In ΔADE,

$$\tan\beta = \frac{AE}{DE} \Rightarrow \tan 30^\circ = \frac{\left(\frac{3+\sqrt{3}}{2}\right)h}{DE}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{\sqrt{3}\left(\frac{1+\sqrt{3}}{2}\right)h}{DE}$$

$$\Rightarrow DE = \frac{3}{2}(1+\sqrt{3})h \qquad ...(i)$$

In ΔBDE,

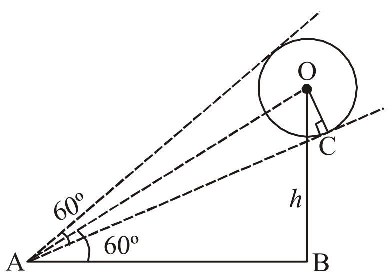
$$\tan\theta = \frac{BD}{DE} = \frac{h}{DE}$$

$$\Rightarrow \tan\theta = \frac{h}{\frac{3}{2}(1+\sqrt{3})h} \qquad \text{[from Eq. } (i)]$$

$$= \frac{2}{3}\,\frac{(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{2(\sqrt{3}-1)}{3.2}$$

$$= \frac{(\sqrt{3}-1)}{3}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{(3-\sqrt{3})}{3\sqrt{3}}$$

37. (c)



In ΔABO,

$$\sin 60^\circ = \frac{OB}{AO} \Rightarrow AO = \frac{OB}{\sin 60^\circ} \qquad ...(i)$$

Now, in ΔACO

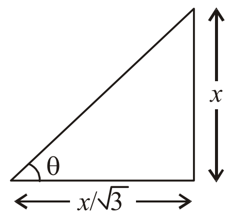
$$\sin\frac{60^\circ}{2} = \frac{OC}{AO} \Rightarrow AO = \frac{OC}{\sin 30^\circ} \qquad ...(ii)$$

Comparing equation (*i*) and equation (*ii*)

$$\frac{OB}{\sin 60^\circ} = \frac{OC}{\sin 30^\circ} \Rightarrow \frac{h}{\frac{\sqrt{3}}{2}} = \frac{r}{\frac{1}{2}}$$

$$\therefore \quad h = \sqrt{3}r$$

38. (c) Let angle of elevation be θ.



Height of the pole = xm

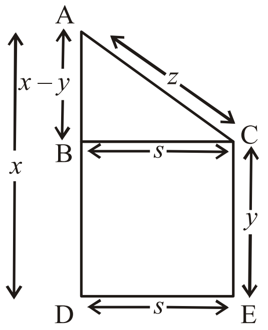
and length of shadow = $x/\sqrt{3}$ m

$$\tan\theta = \frac{x}{\frac{x}{\sqrt{3}}} = \frac{\sqrt{3}x}{x} = \sqrt{3}$$

$$\tan\theta = \sqrt{3} = \tan 60^\circ$$

$$\therefore \quad \theta = 60^\circ$$

39. (d)



Here, BC is the shortest distance

In ΔABC, $AB^2 + BC^2 = AC^2$ (Use Pythagoras theorem)

$$\Rightarrow z^2 = (x-y)^2 + s^2$$

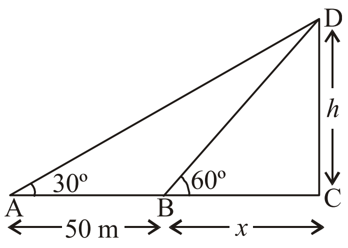
$$\Rightarrow z^2 = x^2 + y^2 - 2xy + s^2$$

$$\Rightarrow s^2 = z^2 - x^2 - y^2 + 2xy$$

40. (b) Let the height of tower be h and BC = xm.

In ΔBCD, $\tan 60^\circ = \frac{h}{x} \Rightarrow \sqrt{3} = \frac{h}{x}$

$$\Rightarrow \quad h = x\sqrt{3} \qquad ...(i)$$



In ΔACD, $\tan 30^\circ = \dfrac{h}{50+x}$

$\Rightarrow \quad \dfrac{1}{\sqrt{3}} = \dfrac{x\sqrt{3}}{50+x}$

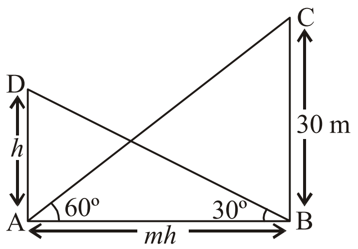
$\Rightarrow \quad 50 + x = 3x$

$\Rightarrow \quad x = 25\,m$

Now put the value of x in equation (i)

$\therefore \quad h = 25\sqrt{3}\,m$

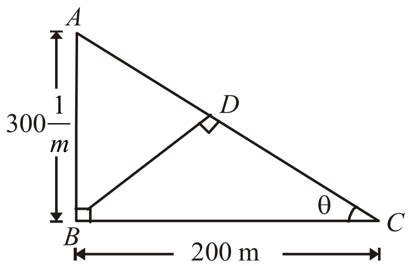
41. (b) Let the height of shorter tower be h then distance between two tower = hm.



In ΔABD, $\tan 30^\circ = \dfrac{h}{mh} \Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{1}{m}$

$\therefore \quad m = \sqrt{3}$

42. (c)



Let a person be at point C observe a tower in west direction at B. After walking some distance along road he observed same tower in south direction. Let angle C be = θ

In ΔABC

$\tan\theta = \dfrac{300}{200} = \dfrac{3}{2} = 1$

$h = \sqrt{(3)^2 + (2)^2}$

$= \sqrt{9+4} = \sqrt{13}$

$\sin\theta = \dfrac{3}{\sqrt{13}}$

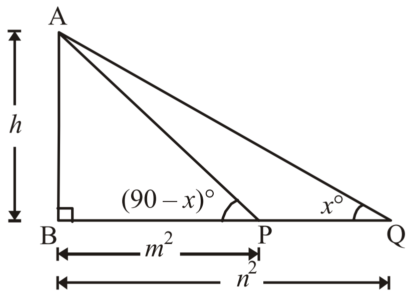
In ΔBDC

$\sin\theta = \dfrac{P}{h} = \dfrac{BD}{200}$

$\therefore \quad$ BD = 200 sin θ

$= 200 \times \dfrac{3}{\sqrt{13}} = \dfrac{600}{\sqrt{13}}$ m

43. (d)



Let h be the height of tower

In Δ ABQ

$\tan x = \dfrac{AB}{BQ} = \dfrac{h}{n^2}$ ...(i)

In Δ ABP

$\tan(90 - x) = \dfrac{AB}{BP} = \dfrac{h}{m^2}$

$\cot x = \dfrac{h}{m^2}\,;\ \dfrac{1}{\tan x} = \dfrac{h}{m^2}$

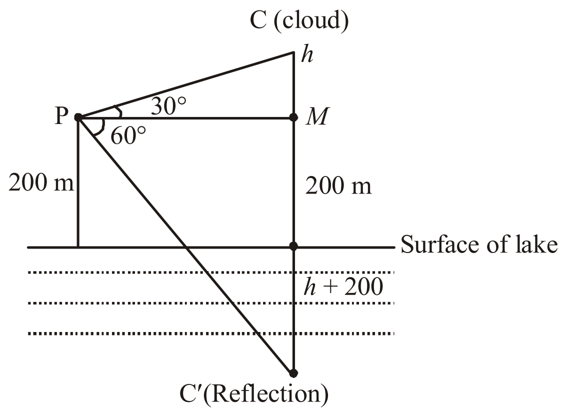
Now, from eqn (i),

$\Rightarrow \quad \dfrac{1}{\frac{h}{n^2}} = \dfrac{h}{m^2}$

$\Rightarrow \quad h^2 = m^2 n^2$

$\because \quad h = mn$

44. (c)



$\tan 30^\circ = \dfrac{h}{PM} \Rightarrow \text{PM} = \sqrt{3}h$

$\tan 60^\circ = \dfrac{h+400}{PM} \Rightarrow \text{PM} = \dfrac{h+400}{\sqrt{3}}$

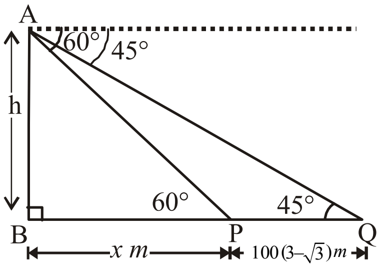
$\sqrt{3}h = \dfrac{h+400}{\sqrt{3}} = 3h - h = 400$

$\Rightarrow \quad 2h = 400$

$\Rightarrow$ So, height of the cloud = 200 + 200 = 400m

45. (c) Let AB is tower whose height is h.

Distance between objects P and Q are $100(3 - \sqrt{3})$ m and BP is x m.



In ΔABP

$$\tan 60° = \frac{AB}{x} = \frac{h}{x}$$

$$x = \frac{h}{\sqrt{3}} \quad ...(i)$$

In ΔABQ

$$\tan 45° = \frac{AB}{BQ} = \frac{h}{100(3-\sqrt{3})+x}$$

$$\Rightarrow \quad h = 100\,(3-\sqrt{3}\,)+x$$

Now, from eqn (i), we put value of x

$$h = 100\left(3-\sqrt{3}\right)+\frac{h}{\sqrt{3}}$$

$$h-\frac{h}{\sqrt{3}} = 100\left(3-\sqrt{3}\right)$$

$$h\frac{\left(\sqrt{3}-1\right)}{\sqrt{3}} = 100\left(3-\sqrt{3}\right)$$

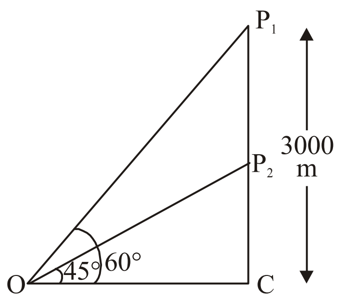
$$h = \frac{100\sqrt{3}\left(3-\sqrt{3}\right)}{\left(\sqrt{3}-1\right)}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{100\sqrt{3}\left(3\sqrt{3}+3-3-\sqrt{3}\right)}{3-1}$$

$$= \frac{100\times 2\sqrt{3}\sqrt{3}}{2}$$

$$\therefore \quad h = 100\times 3 = 300\text{ m}$$

46. (c)



$$CP_1 = 3000\text{ m}$$

$$OC = \frac{CP_1}{\tan 60°} = \frac{3000}{\sqrt{3}}\text{ m}$$

$$CP_2 = OC = \frac{3000}{\sqrt{3}}\text{ m}$$

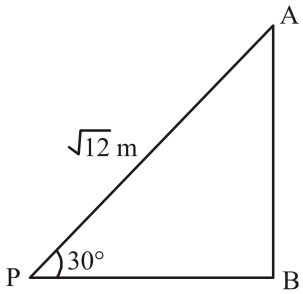
$$P_1P_2 = CP_1 - CP_2 = \left(3000-\frac{3000}{\sqrt{3}}\right)$$

$$= (3000 - 100\sqrt{3}\,)\text{ m}$$

$$= 1000\,(3-\sqrt{3}\,)\text{ m}$$

So, option (c) is correct.

47. (d) In ΔPAB



$$\sin 30° = \frac{AB}{AP}$$

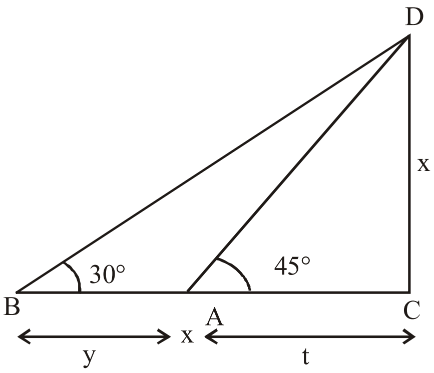
$$\frac{1}{2} = \frac{AB}{\sqrt{12}}$$

$$AB = \frac{\sqrt{12}}{2} = \frac{2\sqrt{3}}{2}$$

$$AB = \sqrt{3}\text{m}$$

So, option (d) is correct.

48. (c) Let two observers A and B are stationed at a distances y from each other.



Let CD be a tower of height x metre.

Also $\angle DAC = 45°$ and $\angle DBC = 30°$

In ΔACD, $\tan 45° = \frac{x}{t}$ and In ΔDBC, $\tan 30° = \frac{x}{y+t}$

$$\Rightarrow \quad 1 = \frac{x}{t} \quad ...(1)$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{x}{y+t} \quad ...(2)$$

From (1) and (2) we get

$$\frac{t}{y+t}=\frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \sqrt{3}t = y+t$$

$$\Rightarrow \quad \left(\sqrt{3}-1\right)t = y$$

$$\Rightarrow \quad t=\frac{1}{\sqrt{3}+1}y$$

From (1) we get

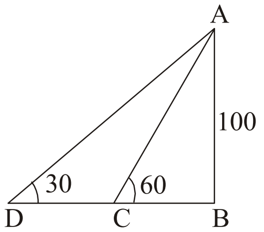
$x = t$

$$x=\frac{1}{\sqrt{3}-1}y\times\frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$=\frac{\sqrt{3}+1}{3-1}=\frac{\sqrt{3}+1}{2}$$

$\therefore$ Option (c) is correct.

49. (b) In ΔABD



$$\frac{AB}{DB}=\tan 30°$$

$$\frac{100}{DB}=\frac{1}{\sqrt{3}};\ DB=100\sqrt{3}$$

in ΔABC

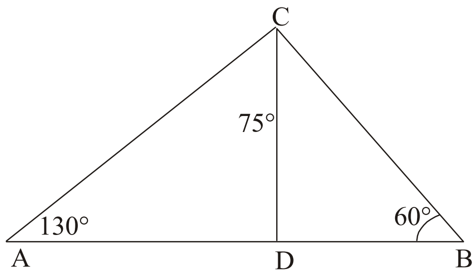
$$\frac{AB}{BC}=\tan 60°$$

$$\frac{100}{BC}=\sqrt{3};\ BC=\frac{100}{\sqrt{3}}$$

$$DC=100\sqrt{3}-\frac{100}{\sqrt{3}}$$

$$=\frac{300-100}{\sqrt{3}}=\frac{200}{\sqrt{3}}=\frac{200\sqrt{3}}{3}$$

50. (a) in ΔADC



$$\frac{DC}{AC}=\tan 30°$$

$$\frac{75}{AC}=\frac{1}{\sqrt{3}};\ AC=75\sqrt{3}$$

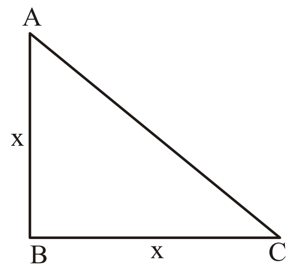
in ΔBDC

$$\frac{DC}{BD}=\tan 60°$$

$$\frac{75}{BD}=\sqrt{3};\ BD=\frac{75}{\sqrt{3}}=\frac{75\sqrt{3}}{3}$$

$$AB=75\sqrt{3}+25\sqrt{3}=100\sqrt{3}$$

51. (c)



When object height is equals to its shadow than sun is at 45° altitude

$$\frac{AB}{BC}=\tan\theta$$

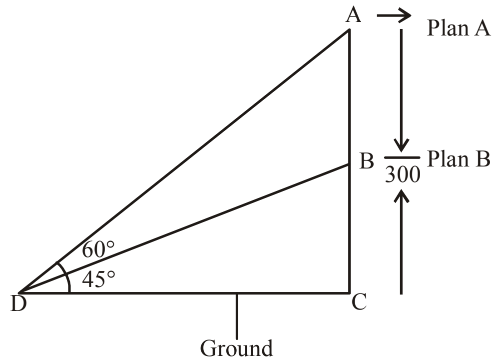
$$\tan\theta=\frac{AB}{AB}=1$$

$\theta = 45°$

52. (a) Centroid

53. (a) in ΔACD

$$\frac{AC}{DC}=\tan 60°$$



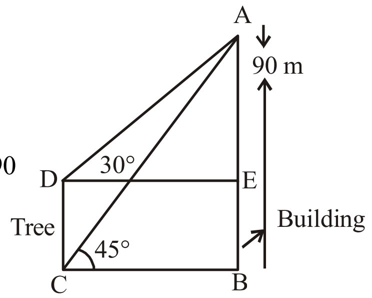
$$\frac{300}{DC}=\sqrt{3};\ DC=\frac{300}{\sqrt{3}}=100\sqrt{3}$$

in ΔBCD

$$\frac{BC}{DC}=\tan 45°$$

$$BC=\frac{300}{\sqrt{3}}=100\sqrt{3}$$

54. (b) AB in Δ ABC



$$\frac{AB}{BC}=\tan 45°$$

$$\frac{AB}{90}=1; AB=90$$

in ΔAED

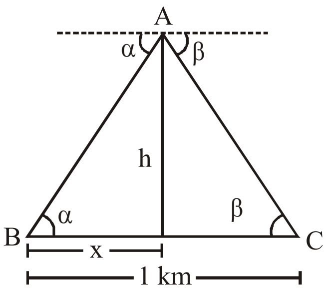
$$\frac{AE}{90}=\tan 30°$$

$$\frac{AE}{90}=\frac{1}{\sqrt{3}}; AE=\frac{90}{\sqrt{3}}$$

$$BC=90-\frac{90}{\sqrt{3}}$$

$BC=90-30\sqrt{3}=DC$ (Tree)

55. (b)



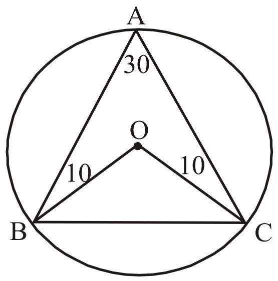
$$\tan\alpha=\frac{h}{x} \quad ...(i)$$

$$\tan\beta=\frac{h}{1-x} \quad ...(ii)$$

$$\tan\beta=\frac{h\tan\alpha}{\tan\alpha-h}$$

$$h=\frac{\tan\alpha.\tan\beta}{\tan\alpha+\tan\beta}$$

56. (b)



Since $\angle A$ = 30° and we know that the angle subtended by an arc at the center of a circle is double the angle subtended by it at any point on the remaining part of the circle, therefore in the center, $\angle O = 2\times 30° = 60°$

Also, since ΔOBC is an isosceles triangle so, its base angles will be equal i.e. $\angle B$ and $\angle C$ are equal.

Let these angles be x.

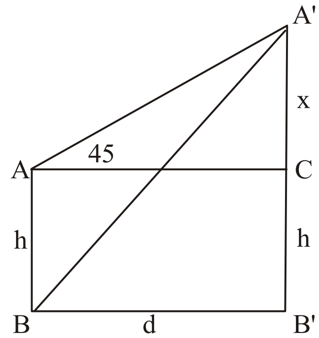
Therefore, by angle sum property of a triangle,

$$\angle O+\angle B+\angle C=180° \Rightarrow 60°+x+x=180°$$

$$\Rightarrow 60°+2x=180° \Rightarrow 2x=120° \Rightarrow x=60°$$

Thus, we say that ΔOBC is an equilateral triangle and hence, BC is also equal to 10 cm.

57. (c)



In ΔAA'C, we have

$$\tan 45°=\frac{A'C}{AC}=\frac{x}{AC}\Rightarrow 1=\frac{x}{AC}$$

$$\Rightarrow AC=x \Rightarrow AC=BB' \Rightarrow x=d$$

Adding h on both sides, we get $h+x=h+d \Rightarrow h+x>d$

Hence, statement 1 is true.

Now, in ΔA'BB', we have

$$\tan 45°=\frac{A'B'}{BB'}=\frac{h+x}{d}\Rightarrow 1=\frac{h+x}{d}\Rightarrow d=h+x$$

But, by statement 1, this is not possible. Therefore, $\theta \neq 45°$.

Now, we can say that either $\theta<45°$ or $\theta>45°$. If $\theta>45°$, let us suppose $\theta=60°$.

So, in ΔA'BB', we have

$$\tan 60°=\frac{A'B'}{BB'}=\frac{h+x}{d}\Rightarrow \sqrt{3}=\frac{h+x}{d}$$

$$\Rightarrow d\sqrt{3}=h+x \quad ...(1)$$

If $\theta>45°$, let us suppose $\theta=30°$.

So, in ΔA'BB', we have

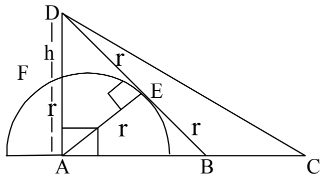
$$\tan 30°=\frac{A'B'}{BB'}=\frac{h+x}{d}\Rightarrow \frac{1}{\sqrt{3}}=\frac{h+x}{d}\Rightarrow d$$

$$=\sqrt{3}(h+x) \quad ...(2)$$

From equation (1), we can conclude that either LHS = RHS or LHS > RHS but from equation (2), we conclude that LHS < RHS. Hence, we cannot have surety that angle of depression of B as seen from A' is less than 45° which states that statement 2 is false.

58. (c) 59. (b) 60. (c)

61. (a)



Given that after moving a distance d toward the dome, when the flag is just visible, the elevation is 45°

$\because$ DB is tangent

And $\angle DBA = 45°$

$\therefore$ DB is tangent and AE is radius

$\therefore$ $\angle EAB = 45° \Rightarrow AE = BE = r$

$[\angle EAB = \angle EBA = 45°]$

$\because$ $\angle DAB = 90°$ & $\angle EAB = 45°$

$\Rightarrow$ $\angle DAE = 45° \Rightarrow \angle ADE = 45°$

$\Rightarrow$ $AE = DE = r$

In ΔDAB

$$\tan 45° = \frac{(h+r)}{AB}$$

$\Rightarrow$ $AB = h + r$

Again in triangle DAB, by Pythagoras theorem.

$(2r)^2 = (h + r)^2 + (h + r)^2$

$4r^2 = 2(h + r)2$

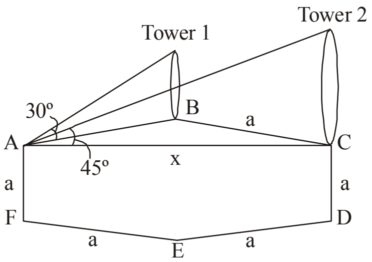
$2r^2 = (h + r)2$

$h + r = \sqrt{2}r$

$h = (\sqrt{2} - 1)r$

$h/r = \sqrt{2} - 1$

62. (b)



Let the side of regular hexagon be 'a'

Let height of the tower 1 be h_1 and tower 2 be h_2

Height of tower 1= h1 = (distance between A and B)* (tan 30º) $= a.\frac{1}{\sqrt{3}}$

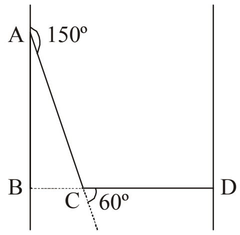
Distance between A and C $= \frac{2\sqrt{3}a}{2} = \sqrt{3}a$

Height of tower 2 = h2 = (distance between A and C)* (tan 45º) $= \sqrt{3}a.1 = \sqrt{3}a$

Ratio of height of towers at B and C respectively

$$= \frac{\frac{a}{\sqrt{3}}}{\sqrt{3}a} = \frac{1}{3}$$

63. (c)



Initially person is travelling from street BA and at point A took 150º turn toward his right and after 15 min. reached at point C.

So, distance $AC = \frac{15}{60} \times 20 = 5\text{km}$

From point C person took 60º turn to word his left and after walking for 20 min. reached at point D.

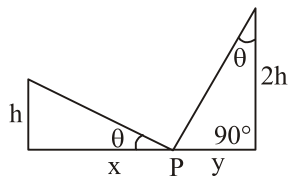
$\therefore$ $CD = \frac{20}{60} \times 30 = 10\text{km}$

From, ΔABC, $BC = AC. \cos 60º$

$= 5 \times \frac{1}{2} = 2.5\text{km}$

Distance between two road = 10 + 2.5 = 12.5 km.

64. (c) $\tan\theta = \frac{h}{x}$ ...(i)



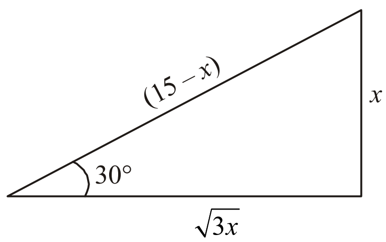
$\tan(90 - \theta)$

$= \cot\theta = \frac{2h}{y}$

$\Rightarrow$ $\tan\theta = \frac{y}{2h}$ ...(ii)

from (i) and (ii) we get $\frac{h}{x} = \frac{y}{2h}$ $\Rightarrow$ $\boxed{2h^2 = xy}$

65. Let the tree is broken at height x from ground



then, length of tree broken $= (15 - x)$

From above triangle, $\sin 30° = \frac{x}{15 - x}$

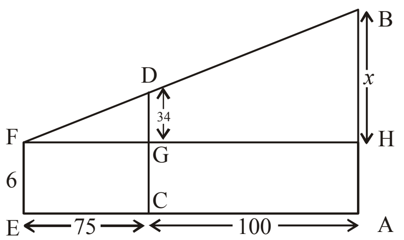
$\Rightarrow \frac{1}{2} = \frac{x}{15 - x}$

$\Rightarrow 2x = 15 - x$

$3x = 15$

$x = 5$ m

66. (a)



Let AB, CD and EF are Lenght of longer, shorter towers and a pole respectively then, from question, we have

AC = 100 feet

CD = 40 feet

EF = 6 feet

CE = 75 feet

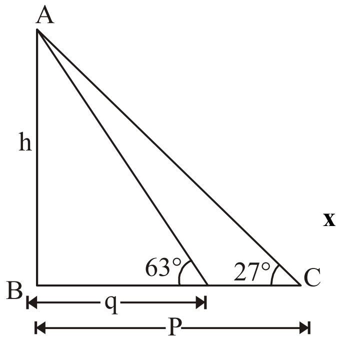
Then, from similar $\triangle$DFG and $\triangle$BFH

$$\frac{DG}{BH} = \frac{FG}{GH} \Rightarrow \frac{34}{BH} = \frac{75}{175}$$

$$BH = \frac{175 \times 34}{75} = 79.3 \text{ feet}$$

AB = 79.3 + 6 ≈ 85 feet.

67. (b)



$$\tan \quad 63° = \frac{h}{q} \qquad ...(i)$$

$$\tan 27° = \frac{h}{p}$$

$$\tan (90 - 63°) = \frac{h}{p}$$

$$\cot 63° = \frac{h}{p}$$

$$\tan 63° = \frac{p}{h} \qquad ...(ii)$$

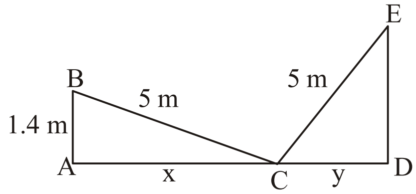
Fromm equation (i) and (ii) we get :-

$$\frac{h}{q} = \frac{p}{h}$$

$$h^2 = pq$$

$$\therefore \quad h = \sqrt{pq}$$

68. (c)



Let breadth of the room = x + y

$$x = \sqrt{25 - 1.96} = \sqrt{23.04} = 4.8 \text{ m}$$

$$y = \sqrt{25 - 23.04} = \sqrt{1.96} = 1.4 \text{ m}$$

$$\therefore \quad x + y = (4.8 + 1.4) \text{ m} = 6.2 \text{ m}$$

CHAPTER

19 Area and Perimeter

1. Consider a circle C of radius 6 cm with centre at O. What is the difference in the area of the circle C and the area of the sector of C subtending an angle of 80º at O? **[2007-I]**
(a) $26\pi cm^2$ (b) $16\pi cm^2$
(c) $28\pi cm^2$ (d) $30\pi cm^2$

2. A rectangle carboard is 18 cm × 10 cm. From the four corners of the rectangle, quarter circles of radius 4 cm are cut. What is the perimeter (approximate) of the remaining portion? **[2007-II]**
(a) 47.1 cm (b) 49.1 cm (c) 51.0 cm (d) 53.0 cm

3. A cycle wheel makes 1000 revolutions is moving 440 m. What is the diameter of the wheel? **[2007-II]**
(a) 7 cm (b) 14 cm (c) 28 cm (d) 21 cm

4. A circle is inscribed in an equilateral triangle of side a. What is the area of any square inscribed in this circle? **[2007-II]**
(a) $\frac{a^2}{3}$ (b) $\frac{a^2}{4}$ (c) $\frac{a^2}{6}$ (d) $\frac{a^2}{8}$

5. The perimeter of a square S_1 is 12 m more than perimeter of the square S_2. If the area of S_1 equals three times, the area of S_2 minus 11, then what is the perimeter of S_1? **[2008-1]**
(a) 24 m (b) 32 m (c) 36 m (d) 40 m

6. From a rectangular sheet of cardboard of size 5 cm × 2 cm, the greatest possible circle is cut-off. What is the area of the remaining part? **[2008-I]**
(a) $(25-\pi)$ cm² (b) $(10-\pi)$ cm²
(c) $(4-\pi)$ cm² (d) $(10-2\pi)$ cm²

7. A chord AB of a circle of radius 20 cm makes a right angle at the centre of the circle. What is the area of the minor segment in cm²? (take $\pi = 3.14$) **[2008-I]**
(a) 31.4 cm² (b) 57 cm²
(c) 62.8 cm² (d) 114 cm²

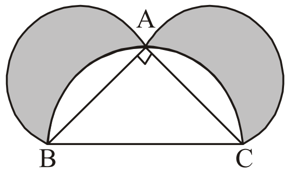
8. The minute hand of a clock is 14 cm long. How much distance does the end of the minute hand travel in 15 min? $\left(\text{take } \pi = \frac{22}{7}\right)$ **[2008-I]**
(a) 11 cm (b) 22 cm (c) 33 cm (d) 44 cm

9. A square of side x is taken. A rectangle is cut out from this square such that one side of the rectangle is half that of the square and the other is $\frac{1}{3}rd$ the first side of the rectangle. What is the area of the remaining portion? **[2008-I]**
(a) $\left(\frac{3}{4}\right)x^2$ (b) $\left(\frac{7}{8}\right)x^2$
(c) $\left(\frac{11}{12}\right)x^2$ (d) $\left(\frac{15}{16}\right)x^2$

10. If x and y are respectively the areas of a square and a rhombus of sides of same length, then what is $x : y$? **[2008-II]**
(a) 1 : 1 (b) $2:\sqrt{3}$
(c) $4:\sqrt{3}$ (d) 3 : 2

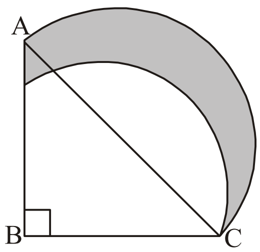
11. If the area of a circle, inscribed in an equilateral triangle is 4π cm², then what is the area of the triangle? **[2008-II]**
(a) $12\sqrt{3}$ cm² (b) $9\sqrt{3}$ cm²
(c) $8\sqrt{3}$ cm² (d) 18 cm²

12. In the given figure, ΔABC is a right angled triangle, right angled at A. Semi-circles are drawn on the sides AB, BC and AC. Then, the area of shaded portion is equal to which one of the following? **[2008-II]**



(a) Area of ΔABC
(b) 2 times the area of ΔABC
(c) Area of semi-circle ABC
(d) None of the above

13. In the given figure, ABC is a right angled triangle, right angled at B. BC = 21 cm and AB = 28 cm. Width AC as diameter of a semi-circle and width BC as radius a quarter circle are drawn. What is the area of the shaded portion? **[2008-II]**



(a) 425 cm² (b) 425.75 cm²
(c) 428 cm² (d) 428.75 cm²

14. If a man walking at the rate 3 km/h crosses a square field diagonally in 1 min, then what is the area of the field? **[2009-I]**

(a) 1000 m^2 (b) 1250 m^2
(c) 2500 m^2 (d) 5000 m^2

15. From a rectangular metal sheet of sides 25 cm and 20 cm, a circular sheet as large as possible is cut-off. What is the area of the remaining sheet? **[2009-I]**

(a) 186 cm^2 (b) 144 cm^2
(c) 93 cm^2 (d) 72 cm^2

16. What is the area of a right angled isosceles triangle whose hypotenuse is $6\sqrt{2}$ cm? **[2009-I]**

(a) 12 cm^2 (b) 18 cm^2
(c) 24 cm^2 (d) 36 cm^2

17. If A is the area of a triangle in cm^2, whose sides are 9 cm, 10 cm and 11 cm, then which one of the following is correct? **[2009-I]**

(a) A < 40 cm^2 (b) 40 cm^2 < A < 45 cm^2
(c) 45 cm^2 < A < 50 cm^2 (d) A > 50 cm^2

18. The difference between the area of a square and that of an equilateral triangle on the same base is 1/4 cm^2. What is the length of side of triangle? **[2009-II]**

(a) $\left(4-\sqrt{3}\right)^{1/2}$ cm (b) $\left(4+\sqrt{3}\right)^{1/2}$ cm
(c) $\left(4-\sqrt{3}\right)^{-1/2}$ cm (d) $\left(4+\sqrt{3}\right)^{-1/2}$ cm

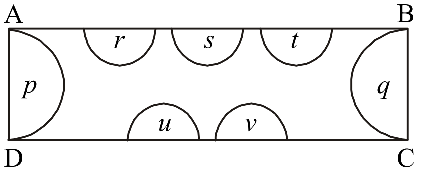
19. A horse is tied to a pole fixed at one corner of a 50 m × 50 m square field of grass by means of a 20 m long rope. What is the area of that part of the field which the horse can graze? **[2009-II]**

(a) 1256 m^2 (b) 942 m^2
(c) 628 m^2 (d) 314 m^2

20. A wire is in the form of a circle of radius 42 cm. If it is bent into a square, then what is the side of the square? **[2009-II]**

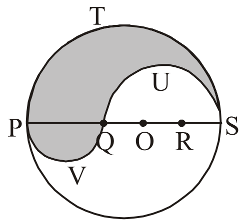
(a) 66 cm (b) 42 cm (c) 36 cm (d) 33 cm

21. Seven semi-circular areas are revoved from the rectangle ABCD as shown in the figure above, in which AB = 2 cm and AD = 0.5 cm. The radius of each semi-circle, *r*, *s*, *t*, *u* and *v* is half of that of semi-circle *p* or *q*. What is the area of the remaining portion?



(a) (128 – 13π)/128 cm^2 (b) (125 – 13π)/125 cm^2
(c) (128 – 15π)/128 cm^2 (d) None of these

22. PQRS is a diameter of a circle of radius 6cm as shown in the figure above. The lengths PQ, QR and RS are equal semi circles are drawn on PQ and QS as diameters.What is the peremeters of the shaded region ? **[2009-II]**

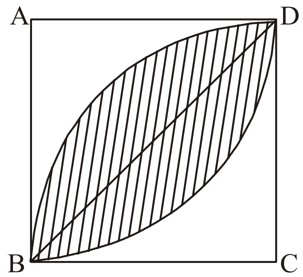


(a) 12π cm (b) 14π cm
(c) 16π cm (d) 18π cm

23. A person rides a bicycle round a circular path of radius 50 m. The radius of the wheel of the bicycle is 50 cm. The cycle comes to the starting point for the first time in 1 h. What is the number of revolutions of the wheel in 15 min? **[2009-II]**

(a) 20 (b) 25 (c) 30 (d) 35

24. In the given figure, the side of square ABCD is 7 cm. What is the area of the shaded portion, formed by the arcs BD of the circles with centre at C and A? **[2010-I]**



(a) 7 cm^2 (b) 28 cm^2 (c) 14 cm^2 (d) 21 cm^2

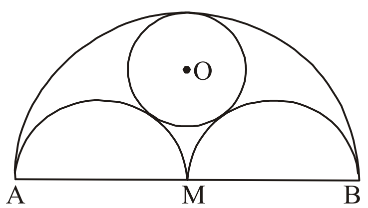
25. What is the maximum area of a rectangle, the perimeter of which is 18 cm? **[2010-I]**

(a) 20.25 cm^2 (b) 20.00 cm^2
(c) 19.75 cm^2 (d) 19.60 cm^2

26. Three circular laminas of the same radius are cut out from a larger circular lamina. When the radius of each lamina cut out is the largest possible, then what is the ratio (approximate) of the area of the residual piece of the original lamina to its original total area? **[2010-I]**

(a) 0.30 (b) 0.35
(c) 0.40 (d) 0.45

27. In the figure given below, AB is a line of length 2*a*, with M as mid-point. Semi-circles are drawn on one side with AM, MB and AB as diameters. A circle with centre O and radius *r* is drawn such that this circle touches all the three semi-circles. What is the value of *r*? **[2010-II]**



(a) $\frac{2a}{3}$ (b) $\frac{a}{2}$ (c) $\frac{a}{3}$ (d) $\frac{a}{4}$

28. A circle and a square have the same perimeter. Which one of the following is correct? **[2010-II]**
(a) The area of the circle is equal to that of square
(b) The area of the circle is larger than that of square
(c) The area of the circle is less than that of square
(d) No conclusion can be drawn

29. What is the radius of the circle inscribed in a triangle having side lengths 35 cm, 44 cm and 75 cm? **[2010-II]**
(a) 3 cm (b) 4 cm (c) 5 cm (d) 6 cm

30. A rectangular area of 6 sq m is to be painted on a 3m × 4 m board leaving a border of uniform width on all sides. What should be the width of the border? **[2010-II]**
(a) 0.25 m (b) 0.5 m (c) 1 m (d) 3 m

31. A wheel of a bicycle has inner diameter 50 cm and thickness 10 cm. What is the speed of the bicycle, if it makes 10 revolutions in 5 s? **[2010-II]**
(a) 5.5 m/s (b) 4.4 m/s
(c) 3.3 m/s (d) 2.2 m/s

32. If a wire of length 36 cm is bent in the form of a semi-circle, then what is the radius of the semi-circle? **[2010-II]**
(a) 9 cm (b) 8 cm (c) 7 cm (d) 6 cm

33. Three congruent circles each of radius 4 cm touch one another. What is the area (in cm^2) of the portion included between them? **[2011-I]**
(a) 8π (b) $16\sqrt{3}-8\pi$
(c) $16\sqrt{3}-4\pi$ (d) $16\sqrt{3}-2\pi$

34. The two diagonals of a rhombus of lengths 55 cm and 48 cm. If p is the perpendicular height of the rhombus, then which one of the following is correct? **[2011-I]**
(a) 36 cm < p < 37 cm (b) 35 cm < p < 36 cm
(c) 34 cm < p < 35 cm (d) 33 cm < p < 34 cm

35. The perimeter of a triangular field is 240 m. If two of its sides are 78 m and 50 m, then what is the length of the perpendicular on the side of length 50 m from the opposite vertex? **[2011-I]**
(a) 43 m (b) 52.2 m (c) 67.2 m (d) 70 m

36. A piece of wire 78 cm long is bent in the form of an isosceles triangle. If the ratio of one of the equal sides to the base is 5 : 3, then what is the length of the base? **[2011-I]**
(a) 16 cm (b) 18 cm (c) 20 cm (d) 30 cm

37. The length of a minute hand of a wall clock is 9 cm. What is the area swept (in cm^2) by the minute hand in 20 min? (take $\pi = 3.14$) **[2011-I]**
(a) 88.78 (b) 84.78 (c) 67.74 (d) 57.78

38. If the outer and inner diameters of a stone parapet around a well are 112 cm and 70 cm respectively. Then, what is the area of the parapet? **[2011-II]**
(a) 264 sq cm (b) 3003 sq cm
(c) 6006 sq cm (d) 24024 sq cm

39. If the area of a ΔABC is equal to area of square of side length 6 cm, then what is the length of the altitude of AB, where AB = 9 cm? **[2011-II]**
(a) 18 cm (b) 14 cm (c) 12 cm (d) 8 cm

40. What is the area of an equilateral triangle having altitude equal to $2\sqrt{3}$ cm? **[2011-II]**
(a) $\sqrt{3}$ sq cm (b) $2\sqrt{3}$ sq cm
(c) $3\sqrt{3}$ sq cm (d) $4\sqrt{3}$ sq cm

41. If a lawn 30 m long and 16 m wide is surrounded by a path 2 m wide, then what is the area of the path? **[2011-II]**
(a) 200 m^2 (b) 280 m^2 (c) 300 m^2 (d) 320 m^2

42. If a circle circumscribes a rectangle with side 16 cm and 12 cm, then what is the area of the circle? **[2011-II]**
(a) 48 π sq cm (b) 50 π sq cm
(c) 100 π sq cm (d) 200 π sq cm

43. The lengths of two sides of a right angled triangle which contain the right angle are a and b, respectively. Three squares are drawn on the three sides of the triangle on the outer side. What is the total area of the triangle and the three squares? **[2011-II]**
(a) $2(a^2+b^2)+ab$ (b) $2(a^2+b^2)+2.5\,ab$
(c) $2(a^2+b^2)+0.5ab$ (d) $2.5(a^2+b^2)$

44. A wall is of the form of a trapezium with height 4 m and parallel sides being 3 m and 5 m. What is the cost of painting the wall, if the rate of painting is Rs. 25 per sq m? **[2011-II]**
(a) ₹240 (b) ₹400 (c) ₹480 (d) ₹800

45. A grassy field has the shape of an equilateral triangle of side 6 m. A horse is tied to one of its vertices with a rope of length 4.2 m. The percentage of the total area of the field which is available for grazing is best approximated by **[2011-II]**
(a) 50% (b) 55% (c) 59% (d) 62%

46. The areas of two circles are in the ratio 1 : 2. If the two circles are bent in the form of squares, then what is the ratio of their areas? **[2011-II]**
(a) 1 : 2 (b) 1 : 3
(c) 1 : $\sqrt{2}$ (d) 1 : 4

47. If the area of a rectangle whose length is 5 units more than twice its width is 75 sq units, then what is the width? **[2011-II]**
(a) 3 units (b) 5 units (c) 7 units (d) 10 units

48. If the diameter of a circle circumscribing a square is $15\sqrt{2}$ cm, then what is the length of the side of the square? **[2011-II]**
(a) 15 cm (b) 12 cm (c) 10 cm (d) 7.5 cm

49. In the ΔABC, the base BC is trisected at D and E. The line through D, parallel to AB, meets AC at F and the line through E parallel to AC meets AB at G. If EG and DF intersect at H, then what is the ratio of the sum of the area of parallelogram AGHF and the area of the ΔDHE to the area of the ΔABC? **[2012-I]**
(a) 1/2 (b) 1/3 (c) 1/4 (d) 1/6

50. If the area of a circle is equal to the area of a square with side $2\sqrt{\pi}$ units, then what is the diameter of the circle? **[2012-I]**
(a) 1 unit (b) 2 units (c) 4 units (d) 8 units

51. A square, a circle and an equilateral triangle have same perimeter. **[2012-I]**
Consider the following statements
I. The area of square is greater than the area of the triangle.
II. The area of circle is less than the area of triangle.
Which of the above statement is/are correct?
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

52. If the area of a rectangle whose length is 5 more than twice its width is 75 sq units. What is the perimeter of the rectangle? **[2012-I]**
(a) 40 units (b) 30 units
(c) 24 units (d) 20 units

53. If the altitude of an equilateral triangle is $\sqrt{3}$ cm, then what is its perimeter? **[2012-I]**
(a) 3 cm (b) $3\sqrt{3}$ cm
(c) 6 cm (d) $6\sqrt{3}$ cm

54. The area of a rectangle, whose one side is a and other side is $2a^2$. What is the area of a square having one of the diagonals of the rectangle as side? **[2012-I]**
(a) $2a^2$ (b) $3a^2$ (c) $4a^2$ (d) $5a^2$

55. Consider the following statements **[2012-II]**
I. Area of a segment of a circle is less than area of its corresponding sector.
II. Distance travelled by a circular wheel of diameter $2d$ cm in one revolution is greater than $6d$ cm.
Which of the above statements is/are correct?
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

56. The perimeter of a rectangle is 82 m and its area is 400 sq m. What is the breadth of the rectangle? **[2012-II]**
(a) 18 m (b) 16 m (c) 14 m (d) 12 m

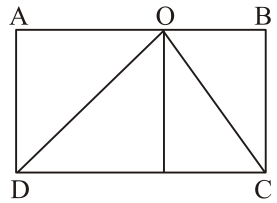
57. The area enclosed between the circumferences of two concentric circles is 16 π sq cm and their radii are in the ratio 5 : 3. What is the area of the outer circle? **[2012-II]**
(a) 9π sq cm (b) 16π sq cm
(c) 25π sq cm (d) 36π sq cm

58. If the circumference of a circle is equal to the perimeter of square, then which one of the following is correct? **[2012-II]**
(a) Area of circle = Area of square
(b) Area of circle ≥ Area of square
(c) Area of circle > Area of square
(d) Area of circle < Area of square

59. If the circumferences of two circles are in the ratio 2 : 3, then what is the ratio of their areas? **[2012-II]**
(a) 2 : 3 (b) 4 : 9 (c) 1 : 3 (d) 8 : 27

60. If the area of a circle inscribed in an equilateral triangle is 154 sq cm, then what is the perimeter of the triangle? **[2012-II]**
(a) 21 cm (b) $42\sqrt{3}$ cm
(c) $21\sqrt{3}$ cm (d) 42 cm

61. In the figure given below, the area of rectangle ABCD is 100 sq cm. O is any point on AB and CD = 20 cm. Then, the area of ΔCOD is **[2013-I]**



(a) 40 sq cm (b) 45 sq cm
(c) 50 sq cm (d) 80 sq cm

62. If an isosceles right angled triangle has area 1sq unit, then what is its perimeter? **[2013-I]**
(a) 3 units (b) $2\sqrt{2}+1$ units
(c) $(\sqrt{2}+1)$ units (d) $2(\sqrt{2}+1)$ units

63. A circular water fountain 6.6 m in diameter is surrounded out side by a path of width 1.5 m. The area of this path (in sq m) is **[2013-I]**
(a) 13.62 π (b) 13.15 π
(c) 12.15 π (d) None of these

64. The area of a rectangular field is 4500 sq m. If its length and breadth are in the ratio 9 : 5, then its perimeter is **[2013-I]**
(a) 90 m (b) 150 m
(c) 280 m (d) 360 m

65. The area of a square inscribed in a circle of radius 8 cm is **[2013-I]**
(a) 32 sq cm (b) 64 sq cm
(c) 128 sq cm (d) 256 sq cm

66. The short and long hands of a clock are 4 cm and 6 cm long, respectively. Then, the ratio of distances travelled by tips of short hand in 2 days and long hand in 3 days is **[2013-I]**
(a) 4 : 9 (b) 2 : 9
(c) 2 : 3 (d) 1 : 27

67. The arc AB of the circle with centre at O and radius 10 cm has length 16 cm. What is the area of the sector bounded by the radii OA, OB and the arc AB? **[2013-I]**
(a) 40π sq cm (b) 40 sq cm
(c) 80 sq cm (d) 20π sq cm

68. The minute hand of a watch is 2.5 cm long. The distance its extreme end transverses in 40 min is **[2013-I]**
(a) $10\pi/3$ cm (b) $3\pi/10$ cm
(c) 10/3 cm (d) 10 cm

69. If the area of a regular hexagon is $96\sqrt{3}$ sq cm, then its perimeter is **[2013-I]**
(a) 36 cm (b) 48 cm (c) 54 cm (d) 64 cm

70. What is the area of a circle whose area is equal to that of a triangle with sides 7 cm, 24 cm and 25 cm? **[2013-II]**
(a) 80 cm^2 (b) 84 cm^2 (c) 88 cm^2 (d) 90 cm^2

71. If the area of an equilateral triangle is x and its perimeter is y, then which one of the following is correct? **[2013-II]**
(a) $y^4 = 432x^2$ (b) $y^4 = 216x^2$
(c) $y^2 = 432x^2$ (d) None of these

72. A rectangular field is 22 m long and 10 m wide. Two hemispherical pitholes of radius 2 m are dug from two places and the mud is spread over the remaining part of the field. The rise in the level of the field is. **[2013-II]**

(a) $\frac{8}{93}$ m (b) $\frac{13}{93}$ m (c) $\frac{16}{93}$ m (d) $\frac{23}{93}$ m

73. The area of an isosceles ΔABC with AB = AC and altitude AD = 3 cm is 12 sq cm. What is its perimeter? **[2013-II]**

(a) 18 cm (b) 16 m (c) 14 cm (d) 12 cm

74. A hospital room is to accommodate 56 patients. It should be done in such a way that every patient gets 2.2 m^2 of floor and 8.8 m^3 of space. If the length of the room is 14 m, then breadth and the height of the room are respectively **[2013-II]**

(a) 8.8 m, 4 m (b) 8.4 m, 4.2 m
(c) 8 m, 4 m (d) 7.8 m, 4.2 m

75. How many 200 mm lengths can be cut from 10 m of ribbon? **[2013-II]**

(a) 50 (b) 40 (c) 30 (d) 20

76. What is the area between a square of side 10 cm and two inverted semi-circular, cross-sections each of radius 5 cm inscribed in the square? **[2013-II]**

(a) 17.5 cm^2 (b) 18.5 cm^2
(c) 20.5 cm^2 (d) 21.5 cm^2

77. The perimeter of a rectangle having area equal to 144 cm^2 and sides in the ratio 4 : 9 is **[2013-II]**

(a) 52 cm (b) 56 cm (c) 60 cm (d) 64 cm

78. One side of a parallelogram is 8.06 cm and its perpendicular distance from opposite side is 2.08 cm. What is the approximate area of the parallelogram? **[2013-II]**

(a) 12.56 cm^2 (b) 14.56 cm^2
(c) 16.76 cm^2 (d) 22.56 cm^2

79. If the diagonals of a rhombus are 4.8 cm and 1.4 cm, then what is the perimeter of the rhombus? **[2013-II]**

(a) 5 cm (b) 10 cm
(c) 12 cm (d) 20 cm

80. How many circular plates of diameter d be taken out of a square plate of side $2d$ with minimum loss of material? **[2014-I]**

(a) 8 (b) 6 (c) 4 (d) 2

81. What is the total area of three equilateral triangles inscribed in a semi-circle of radius 2 cm? **[2014-I]**

(a) 12 cm^2 (b) $\frac{3\sqrt{3}}{4}$ cm^2

(c) $\frac{9\sqrt{3}}{4}$ cm^2 (d) $3\sqrt{3}$ cm^2

82. The area of sector of a circle of radius 36 cm is 72π cm^2. The length of the corresponding arc of the sector is **[2014-I]**

(a) π cm (b) 2π cm (c) 3π cm (d) 4π cm

83. A square is inscribed in a circle of diameter $2a$ and another square is circumscribing circle. The difference between the areas of outer and inner squares is **[2014-I]**

(a) a^2 (b) $2a^2$ (c) $3a^2$ (d) $4a^2$

84. ABC is a triangle right angled at A. AB = 6 cm and AC = 8 cm. Semi-circles drawn (outside the triangle) on AB, AC and BC as diameters which enclose areas x, z square units, respectively. What is $x + y - z$ equal to? **[2014-I]**

(a) 48 cm^2 (b) 32 cm^2
(c) 0 (d) None of these

85. Consider an equilateral triangle of a side of unit length. A new equilateral triangle is formed by joining the mid-points of one, then a third equilateral triangle is formed by joining the mid-points of second. The process is continued. The perimeter of all triangles, thus formed is **[2014-I]**

(a) 2 units (b) 3 units (c) 6 units (d) Infinity

86. What is the area of the larger segment of a circle formed by a chord of length 5 cm subtending an angle of 90º at the centre? **[2014-I]**

(a) $\frac{25}{4}\left(\frac{\pi}{2}+1\right)$ cm^2 (b) $\frac{25}{4}\left(\frac{\pi}{2}-1\right)$ cm^2

(c) $\frac{25}{4}\left(\frac{3\pi}{2}+1\right)$ cm^2 (d) None of these

87. A rectangle of maximum area of drawn inside a circle of diameter 5 cm. What is the maximum area of such a rectangle? **[2014-I]**

(a) 25 cm^2 (b) 12.5 cm^2
(c) 12 cm^2 (d) None of these

88. If AB and CD are two diameters of a circle of radius r and they are mutually perpendicular, then what is the ratio of the area of the circle to the area of the ΔACD? **[2014-I]**

(a) $\frac{\pi}{2}$ (b) π (c) $\frac{\pi}{4}$ (d) 2π

89. The area of a rectangle lies between 40 cm^2 and 45 cm^2. If one of the sides is 5 cm, then its diagonal lies between **[2014-I]**

(a) 8 cm and 10 cm (b) 9 cm and 11 cm
(c) 10 cm and 12 cm (d) 11 cm and 13 cm

90. The areas of the three adjacent faces of a cuboidal box are x, $4x$ and $9x$ sq unit. What is the volume of the box ? **[2014-II]**

(a) $6x^2$ cu unit (b) $6x^{3/2}$ cu unit
(c) $3x^{3/2}$ cu unit (d) $2x^{3/2}$ cu unit

91. The sides of a triangular field are 41 m, 40 m and 9 m. The number of rose beds that can be prepared in the field if each rose bed, on an average, needs 900 square cm space, is **[2015-I]**

(a) 2000 (b) 1800 (c) 900 (d) 800

92. The ratio of the outer and inner perimeters of a circular path is 23 : 22. If the path is 5 m wide, the diameter of the inner circle is **[2015-I]**

(a) 55 m (b) 110 m
(c) 220 m (d) 230 m

93. Four equal-sized maximum circular plates are cut off from a square paper sheet of area 784 square cm. The circumference of each plate is **[2015-I]**
(a) 11 cm (b) 22 cm (c) 33 cm (d) 44 cm

94. A square and an equilateral triangle have equal perimeter. If the diagonal of the square is $12\sqrt{2}$ cm, then the area of the triangle is **[2015-II]**
(a) $24\sqrt{2}$ cm^2 (b) $24\sqrt{3}$ cm^2
(c) $48\sqrt{3}$ cm^2 (d) $64\sqrt{3}$ cm^2

95. A boy is cycling such that the wheels of the cycle are making 140 revolutions per minute. If the radius of the wheel is 30 cm, the speed of the cycle is **[2015-II]**
(a) 15.5 km/hour (b) 15.84 km/hour
(c) 16 km/hour (d) 16.36 km/hour

96. There are 437 fruit plants in an orchard planted in rows. The distance between any two adjacent rows is 2 m and the distance between any two adjacent plants is 2m. Each row has the same number of plants. There is 1 m clearance on all sides of the orchard. What is the cost of fencing the area at the rate of ₹100 per metre **[2015-II]**
(a) ₹ 15,600
(b) ₹16,800
(c) ₹ 18,200
(d) More information is required

97. The circumference of a circle is 100 cm. The side of the square inscribed in the circle is **[2015-II]**
(a) $50\sqrt{2}$cm (b) $\frac{100}{\pi}$cm
(c) $\frac{50\sqrt{2}}{\pi}$cm (d) $\frac{100\sqrt{2}}{\pi}$cm

98. The diameter of a wheel that makes 452 revolutions to move 2 km and 26 decametre is equal to **[2015-II]**
(a) $1\frac{9}{22}$m (b) $1\frac{13}{22}$m (c) $2\frac{5}{11}$m (d) $2\frac{7}{11}$m

99. A square is inscribed in a right triangle with legs x and y and has common right angle with the triangle. The perimeter of the square is given by **[2015-II]**
(a) $\frac{2xy}{x+y}$ (b) $\frac{4xy}{x+y}$
(c) $\frac{2xy}{\sqrt{x^2+y^2}}$ (d) $\frac{4xy}{\sqrt{x^2+y^2}}$

100. The area of a trapezium is 336 cm^2. If its parallel sides are in the ratio 5 : 7 and the perpendicular distance between them is 14 cm, then the smaller of the parallel sides is **[2015-II]**
(a) 20 cm (b) 22 cm (c) 24 cm (d) 26 cm

101. The number of rounds that a wheel of diameter 7/11 metre will make in traversing 4 km will be **[2016-I]**
(a) 500 (b) 1000 (c) 1700 (d) 2000

102. The base of an isosceles triangle is 300 unit and each fo tis equal sides is 170 units. Then the area of the triangle is **[2016-I]**
(a) 9600 square units (b) 10000 square units
(c) 12000 square units (d) None of the above

103. Four equal discs are placed such that each one touches two others. If the area of empty space enclosed by them is 150/847 square centimetre, then the radius of each disc is equal to **[2016-I]**
(a) 7/6 cm (b) 5/6 cm (c) 1/2 cm (d) 5/11 cm

104. If a square of side x and an equilateral triangle of side y are inscribed in a circle, then what is the ratio of x to y ? **[2016-II]**
(a) $\sqrt{\frac{2}{3}}$ (b) $\sqrt{\frac{3}{2}}$ (c) $\frac{3}{\sqrt{2}}$ (d) $\frac{\sqrt{2}}{3}$

105. A circle and a square have the same perimeter. Which one of the following is correct ? **[2016-II]**
(a) Their areas are equal
(b) The area of the circle is larger
(c) The area of the square is $\frac{\pi}{2}$ times area of circle
(d) The area of the square is π times area of circle

106. What is the area of a triangle with sides of length 12 cm, 13 .cm and 5 cm ? **[2016-II]**
(a) 30 cm^2 (b) 35 cm^2 (c) 40 cm^2 (d) 42 cm^2

107. If the perimeter of a rectangle is 10 cm and the area is 4 cm^2, then its length is **[2017-I]**
(a) 6 cm (b) 5 cm (c) 4.5 cm (d) 4 cm

108. The areas of two circular fields are in the ratio 16 : 49. If the radius of the bigger field is 14 m, then what is the radius of the smaller field ? **[2017-I]**
(a) 4m (b) 8m (c) 9m (d) 10m

109. The area of a regular hexagon of side 'a' is equal to **[2017-I]**
(a) $\frac{\sqrt{2}}{3}a^2$ square units (b) $\frac{3\sqrt{3}}{2}a^2$ square units
(c) $\frac{1}{3}a^2$ square units (d) $\frac{\sqrt{3}}{2}a^2$ square units

110. An isosceles triangle is drawn outside on one of the sides of a square as base in such a way that the perimeter of the complete figure is $\frac{7}{6}$ times the perimeter of the original square. What is the ratio of area of the triangle to the area of the original square? **[2017-II]**
(a) 1 : 1 (b) 2 : 3 (c) 1 : 2 (d) 1 : 3

111. What is the area of the triangle whose sides are 51 cm, 37 cm and 20 cm? **[2017-II]**
(a) 300 square cm (b) 305 square cm
(c) 306 square cm (d) 307 square cm

112. If the length of a side of a square is increased by 8 cm, its area increases by 120 square cm. What is the length of a side of the square? **[2018-I]**
(a) 2.5 cm (b) 3.5 cm (c) 4.5 cm (d) 5.5 cm

113. A rectangular pathway having width 4.5 m and length 10 m will have to be tiled using square tiles of side 50 cm. Each packet of such tiles contains 20 pieces and costs ₹100. What will be the total cost of tiles for the pathway? **[2018-I]**

(a) ₹1,200 (b) ₹1,100 (c) ₹1,000 (d) ₹900

114. The product of the lengths of the diagonals of a square is 50 square units. What is the length of a side of the square? **[2018-I]**

(a) $5\sqrt{2}$ units (b) 5 units
(c) 10 units (d) $2\sqrt{5}$ units

115. In a rectangle, length is three times its breadth. If the length and the breadth of the rectangle are increased by 30% and 10% respectively, then its perimeter increases by **[2019-I]**

(a) $\frac{40}{3}\%$ (b) 20% (c) 25% (d) 27%

116. A rectangular red carpet of size 6 ft × 12 ft has a dark red border 6 inches wide. What is the area of the dark red border? **[2019-I]**

(a) 9 square feet (b) 15 square feet
(c) 17 square feet (d) 18 square feet

117. If one side of a right-angled triangle (with all sides integers) is 15 cm, then what is the maximum perimeter of the triangle? **[2019-II]**

(a) 240 cm (b) 225 cm (c) 113 cm (d) 112 cm

118. The length and breadth of a rectangle are increased by 20% and 10% respectively. What is the percentage increase in the area of the rectangle ? **[2019-II]**

(a) 32% (b) 30% (c) 25% (d) 15%

119. If the area of a square is $2401x^4 + 196x^2 + 4$, then what is its side length ? **[2020-I]**

(a) $49x^2+3x+2$ (b) $49x^2-3x+2$
(c) $49x^2+2$ (d) $59x^2+2$

120. What is the area of the triangle having side lengths $\frac{y}{z}+\frac{z}{x}, \frac{z}{x}+\frac{x}{y}, \frac{x}{y}+\frac{y}{z}$? **[2020-I]**

(a) $\frac{(x+y+z)^2}{xyz}$ (b) $\frac{\sqrt{xyz}}{x+y+z}$

(c) $\sqrt{\frac{x}{y}+\frac{y}{z}+\frac{z}{x}}$ (d) $\sqrt{\frac{xy+yz+zx}{xyz}}$

121. A square and a rectangle have equal areas. If one side of the rectangle is of length numerically equal to the square of the length of the side of the square then the other side of the rectangle is **[2020-I]**

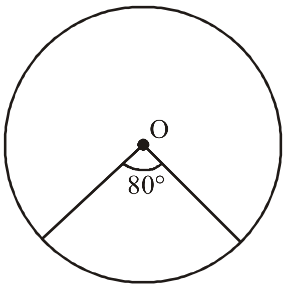
(a) square root of the side of the square
(b) half the side of the square
(c) of unit length
(d) double the side of the square

122. The length and breadth of a rectangle are in the ratio 4:3. Then what is the ratio of the area of the triangle formed by the parts of the diagonals with a long side to the area of the triangle formed by the parts of diagonals with a short side? **[2020-I]**

(a) 3 : 4 (b) 4 : 3 (c) 16 : 9 (d) 1 : 1

HINTS & SOLUTIONS

1. (c)



Radius of circle, $r = 6$ cm

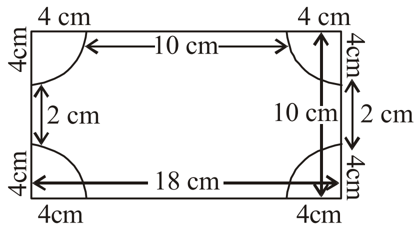
$\therefore$ Area of circle $= \pi r^2 = \pi \times 6^2 = 36\pi$ cm²

and Area of sector subtending an angle of 80° at O

$$= \frac{\pi r^2 \theta}{360^\circ} = \frac{\pi \times 6^2 \times 80^\circ}{360^\circ} = 8\pi \text{ cm}^2$$

$\therefore$ Required difference $= 36\pi - 8\pi = 28\pi$ cm²

2. (b)



Remaining perimeter

$$= \left(\frac{2\pi r}{4}\right)4 + 10 + 2 + 10 + 2 = 2 \times 3.14 \times 4 + 24$$

$= 25.12 + 24 = 49.12$ cm

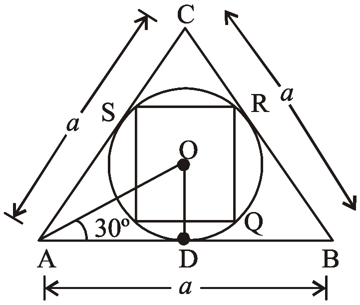
$= 49.1$ cm (approx)

3. (b) Distance travel in 1 revolution = $\frac{440}{1000}$ m

and circumference = $\pi \times d = \frac{44000}{1000}$ cm

$\therefore \quad d = \frac{44000 \times 7}{1000 \times 22} = 14$ cm

4. (c)



In ΔAOD,

$\tan 30^\circ = \frac{OD}{AD} = \frac{1}{\sqrt{3}}$

$\Rightarrow OD = \frac{1}{\sqrt{3}} AD$

$= \frac{a}{2\sqrt{3}}$ ...(*i*)

Now, OD is radius of circle. Therefore diagonal of square $= 2 \cdot r$

$= \frac{a}{2\sqrt{3}} = \frac{a}{\sqrt{3}}$

Let side of a square = y

$\therefore \quad \left(\frac{a}{\sqrt{3}}\right)^2 = y^2 + y^2$

$\Rightarrow \quad \frac{a^2}{3} = 2y^2$

$y^2 = \frac{a^2}{6}$ = Area of square.

5. (b) Suppose side of $S_1 = a$

Side of $S_2 = b$

Perimeter of $S_1 = 4a$

Perimeter of $S_2 = 4b$

Area of $S_1 = a^2$

Area of $S_2 = b^2$

According to Question,

$4a = 4b + 12$

$\Rightarrow a = b + 3$

and $a^2 = 3(b^2) - 11 \Rightarrow (b+3)^2 = 3(b^2) - 11$

$\Rightarrow b^2 + 9 + 6b = 3b^2 - 11$

$\Rightarrow 2b^2 - 6b - 20 = 0$

$\Rightarrow b^2 - 3b - 10 = 0$

$\Rightarrow b^2 - 5b + 2b - 10 = 0$

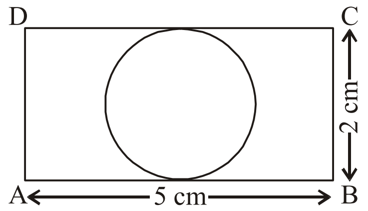
$\Rightarrow b(b-5) + 2(b-5) = 0$

$\Rightarrow b = 5, b \neq -2$

$\therefore$ Perimeter of $S_1 = 4 \times 5 + 12$

$= 32$ M

6. (b) From a rectangular sheet of cardboard of size 5 × 2 cm², a cirle of radius 1 cm, can be cut-off.



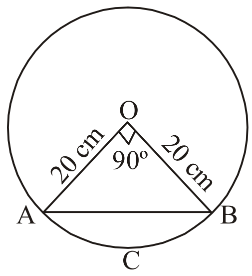
Area of rectangular sheet = 5 × 2 = 10 cm²

and area of circle = $\pi(1)^2 = \pi$ cm²

$\therefore$ Required area = Area of sheet – Area of circle

$= (10 - \pi)$ cm²

7. (d) Area of $\Delta AOB = \frac{1}{2} \times OA \times OB$



$= \frac{1}{2} \times 20 \times 20 = 200$ cm²

and area of sector OACBO

$= \frac{\pi r^2 \theta}{360^\circ} = \frac{3.14 \times 20 \times 20 \times 90^\circ}{360^\circ}$

$= \frac{3.14 \times 400}{4} = 314$ cm²

$\therefore$ Area of minor segment

= Area of sector OACBO – Area of ΔAOB

$= 314 - 200 = 114$ cm²

8. (b) $\because$ Angle made in 60 min by minute hand of a clock $= 360^\circ$

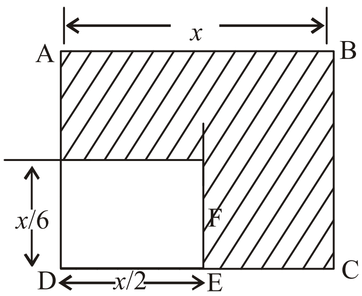
and angle made in 15 min by minute hand of a clock

$= \frac{360^\circ}{60^\circ} \times 15^\circ = 90^\circ$

$\therefore$ Required distance

$= \frac{2\pi(14)90^\circ}{360^\circ} = \frac{22}{7} \times \frac{14 \times 2}{4} = 22$ cm

9. (c) Let the length of rectangle = $\frac{x}{2}$



Here, ABCD is a square and DEFG is a rectangle
Where length = $x/2$
breadth = $x/6$

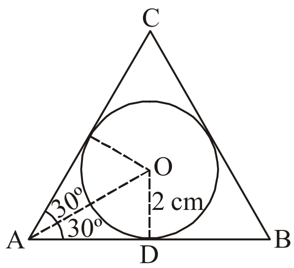
Therefore, Area of rectangle = $\frac{x}{2}\times\frac{x}{6}=\frac{x^2}{12}$

and area of square = x^2
Hence, area of remaining portion

$= x^2-\frac{x^2}{12}=\frac{11x^2}{12}$

10. (a) As we know that, if the length of square and rhombus are same, then the area should be same
$\therefore\ x : y = 1 : 1$

11. (a) Since, area of circle = 4p cm² (given)
$\Rightarrow \pi r^2 = 4\pi \Rightarrow r = 2$ cm



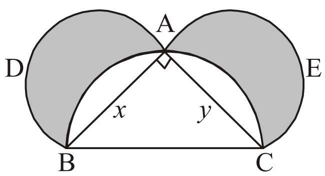
In ΔOAD, $\tan 30° = \frac{OD}{AD} \Rightarrow AD = 2\sqrt{3}$ cm

Now, AB = 2 AD = $4\sqrt{3}$ cm
$\therefore$ Area of equilateral ΔABC

$=\frac{\sqrt{3}}{4}(AB)^2=\frac{\sqrt{3}}{4}(4\sqrt{3})^2$

$= 12\sqrt{3}$ cm²

12. (a) In ΔABC,



BC = $\sqrt{x^2+y^2}$

$\therefore$ Area of $\Delta ABC = \frac{1}{2}\times x\times y=\frac{1}{2}xy$

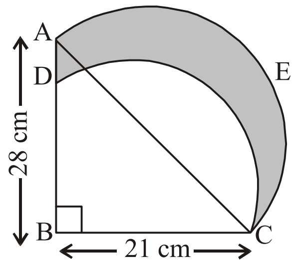
Area of semi-circle BACB = $\frac{\pi(x^2+y^2)}{4}$

$\therefore$ Area of shaded portion = Semi-circle ABDA + Area of semi-circle AECA – (Area of semi-circle BACB – Area of ΔABC)

$=\frac{\pi x^2}{4}+\frac{\pi y^2}{4}-\pi\left(\frac{x^2+y^2}{4}\right)+$ Area of ΔABC

= Area of ΔABC

13. (d) In ΔABC,
$AC^2 = \sqrt{28^2+21^2}=\sqrt{784+441}=\sqrt{1225}$
$\Rightarrow$ AC = 35 cm



Area of shaded portion = Area of semi-circle ACE + Area of ΔABC – Area of quadrant circle BCD

$=\frac{\pi r^2}{2}+\frac{1}{2}\times BC\times BA-\frac{\pi}{4}\times r_1^2$

$=\frac{22}{7}\times\frac{1}{2}\times\frac{35}{2}\times\frac{1}{2}\times 21\times 28-\frac{22}{7\times 4}\times 21\times 21$

$=\frac{5\times 11\times 35}{4}+\frac{1}{2}(21\times 28-33\times 21)$

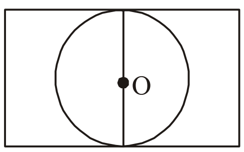
$=\frac{1925}{4}+\frac{1}{2}(-105)=481.25-52.50=428.75\text{ cm}^2$

14. (b) Since, the distance covered by a man diagonally is

$d=\frac{3\times 100}{60}\times 1=50$ m

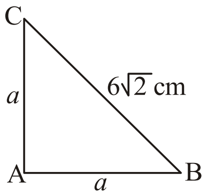
$\therefore$ Area of field = $\frac{1}{2}d^2=\frac{1}{2}\times(50)^2=1250\text{ m}^2$

15. (a) Here width of sheet is 20 cm, which is the maximum diameter of the circular sheet.



$\therefore$ Remaining area of sheet = Area of rectangle sheet – Area of circular sheet
$= 25\times 20-\pi(10)^2=500-314=186\text{ cm}^2$

16. (b) Let the other sides of a right isosceles triangle be a cm.
In ΔABC,



$\sqrt{a^2+a^2} = 6\sqrt{2}$

$\Rightarrow \quad a\sqrt{2} = 6\sqrt{2}$

$\Rightarrow \quad a = 6$ cm

$\therefore$ Area of $\Delta ABC = \frac{1}{2}\times a^2$

$= \frac{1}{2}\times 6\times 6 = 18$ cm²

17. (b) Given A = 9 cm, $b = 10$ cm and $c = 11$ cm

$\because \quad s = \frac{9+10+11}{2} = 15$ cm

$\therefore \quad A = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{15(15-9)(15-10)(15-11)}$

$= \sqrt{15\times 6\times 5\times 4} = 30\sqrt{2}$

$= 42.3$ cm²

18. (c) Let the side of an square be a cm.
By given condition,

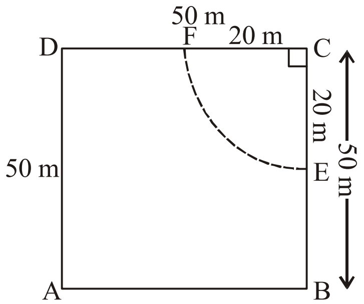
Area of square – Area of an equilateral triangle $= \frac{1}{4}$

$\Rightarrow \quad a^2 - \frac{\sqrt{3}}{4}a^2 = \frac{1}{4} \Rightarrow a^2\left(1-\frac{\sqrt{3}}{4}\right) = \frac{1}{4}$

$\Rightarrow \quad a^2(4-\sqrt{3}) = 1 \Rightarrow a^2 = \frac{1}{4-\sqrt{3}}$

$\therefore \quad a = (4-\sqrt{3})^{-1/2}$ cm

19. (d) Suppose a pole is fixed at a point C.
$\therefore$ Area of field in which the horse can graze
= Area of curve CFE



$= \frac{1}{4}(\pi r^2) = \frac{3.14\times 20\times 20}{4} = 314$ m²

20. (a) Circumference of circle $= 2\pi\times 42$

$= 2\times\frac{22}{7}\times 42 = 264$ cm

Perimeter of square $= 4x \Rightarrow 264 = 4x$
$x = 66$ cm

21. (a) Area of 2 bigger semi-cirles $= 2\times\frac{\pi r^2}{2}$

$= 2\pi\left(\frac{0.5}{2}\right)^2\times\frac{1}{2} = \frac{0.25\pi}{4}$ cm²

and area of 5 smaller semi-circles

$= \frac{5\pi r^2}{2} = 5\times\pi\times\frac{1}{2}\times\left(\frac{0.5}{4}\right)^2$

$= \frac{5\pi}{2}\times\frac{0.25}{16} = \frac{1.25\pi}{32}$ cm²

Area of rectangle ABCD $= 2\times 0.5 = 1$ cm²

Area of remaining portion $= 1 - \frac{0.25\pi}{4} - \frac{1.25\pi}{32}$

$= 1 - \frac{\pi}{16} - \frac{5\pi}{128}$

$= \frac{128-8\pi-5\pi}{128}$

$= \frac{128-13\pi}{128}$ cm²

22. (a) Given, OS = 6 cm
$\therefore$ PQ = QR = RS = 4 cm
$\therefore$ Perimeter of shaded region
= Perimeter of semi-circle PTS
+ Perimeter of semi-circle QUS
+ Perimeter of semi-circle PVQ

$= \pi(6) + \pi(4) + \pi(2) = 12\pi$ cm

23. (b) $\because$ Circumference of circular path $= 2\pi\times 50$ m
$= 10000\pi$ cm
and circumference of wheel $= 2\pi\times 50 = 100\pi$ cm
$\therefore$ Distance covered in 60 min $= 10000\pi$ cm

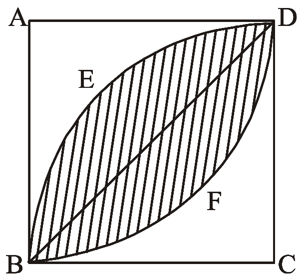
Distance covered in 1 min $= \frac{10000}{60}\pi$

Distance covered in 15 min

$= \frac{10000}{60}\pi\times 15 = 2500\pi$ cm

$\therefore$ Number of revolutions $= \frac{2500\pi}{100\pi} = 25$

24. (b)



The above figure is symmetrical about BD

Area of shaded part

$= 2 \times$ Area of BEDB

$= 2 \times$ (Area of BC DEB – Area of ΔBCD)

$$= 2\left(\frac{\pi r^2}{4} - \frac{1}{2} \times BC \times CD\right)$$

$$= 2\left(\frac{22}{7 \times 4} 7 \times 7 - \frac{1}{2} \times 7 \times 7\right)$$

$$= 2 \times \frac{28}{2} = 28 \text{ cm}^2$$

25. (a) Let sides of a rectangle be l and b.

Then, $2(l + b) = 18 \Rightarrow l + b = 9$

$\therefore$ Area of rectangle $= l \times b$

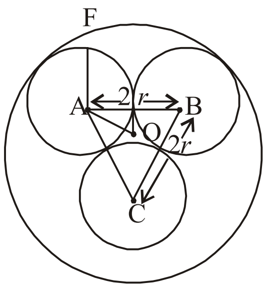
For maximum, area of rectangle, $l = b$

$\therefore$ $2l = 9 \Rightarrow l = 4.5$

$\therefore$ Maximum area of rectangle $= l \times b = (4.5)^2$

$=$ 20.25 cm²

26. (b)



Let O be the centre of a larger circular lamina. Three circular lamina of same radius are cut out from a larger lamina. Here ABC is equilateral triangle. Radius of larger lamina = OA + OF.

Let r be radius of smaller circle.

$$OA = \frac{2r}{\sqrt{9}}, \ OF = r$$

$$\text{Radius of larger lamina} = \frac{2r}{\sqrt{3}} + r$$

$$= \frac{r(2+\sqrt{3})}{\sqrt{3}}$$

Area of 3 laminas $= 3\pi r^2$

$$\text{Area of larger lamina} = \pi\left(\frac{r(2+\sqrt{3})}{\sqrt{3}}\right)^2$$

$$\text{Area of larger lamina} = \frac{\pi r^2(7+\sqrt{5})}{3}$$

Area of Resideal part of larger lamina

$$= \frac{\pi r^2(7+4\sqrt{5})}{3} - 3\pi r^2$$

$$= \frac{\pi r^2\left(7+4\sqrt{5}-9\right)}{3}$$

$$= \frac{\pi r^2(4\sqrt{3}-2)}{3}$$

$$\text{Required Ratio} = \frac{\frac{\left(4\sqrt{5}-2\right)}{3}\pi r^2}{\frac{(7+4\sqrt{5})}{3}\pi r^2}$$

$$= \frac{4\sqrt{3}-2}{7+4\sqrt{5}} \times \frac{7-4\sqrt{5}}{7-4\sqrt{5}}$$

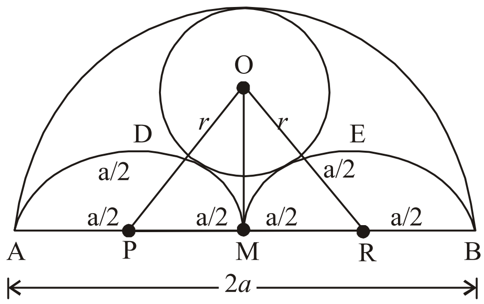
$$= 28\sqrt{5} - 14 - 16 \times 3 + 8\sqrt{5}$$

$$= 36\sqrt{3} - 62$$

$$= 36 \times 1.732 - 62$$

$$= 0.352 = 0.35$$

27. (c)



$$\text{Here, } AP = PM = MR = RB = \frac{a}{2}$$

It is clear from figure that,

$$OP = OR = r + \frac{a}{2}$$

In ΔOPR, $OP^2 = PM^2 + OM^2$

$$\Rightarrow \left(r + \frac{a}{2}\right)^2 = \left(\frac{a}{2}\right)^2 + (a-r)^2$$

$$\Rightarrow r^2 + \frac{a^2}{4} + ra = \frac{a^2}{4} + a^2 + r^2 - 2ar$$

$$\Rightarrow ra = a^2 - 2ar$$

$\Rightarrow \quad a^2 - 2ar - ar = 0$

$\Rightarrow \quad a^2 - 3ar = 0$

$\Rightarrow \quad a^2 = 3ar$

$\Rightarrow \quad a = 3r$

$$\boxed{\therefore r = \frac{a}{3}}$$

28. (b) Let r be the radius of circle and a be the side of square.
By given condition,
$2\pi r = 4a$

$\Rightarrow \quad a = \frac{\pi r}{2}$

$\therefore$ Area of square $= \left(\frac{\pi r}{2}\right)^2 = \frac{\pi^2 r^2}{4} = \frac{9.86 r^2}{4} = 2.46 r^2$

and area of circle $= \pi r^2 = 3.14 r^2$
Hence, area of the circle is larger than that of square.

29. (d) Let side of triangle a, b, c are 35 cm, 44 cm and 75 respectively.

$\therefore \quad s = \frac{a+b+c}{2} = \frac{35+44+75}{2} = 77$

Area of triangle

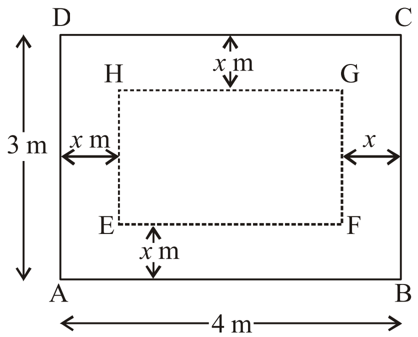
$\Delta \quad = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{77 \times 42 \times 33 \times 2}$

$= \sqrt{7 \times 11 \times 2 \times 3 \times 7 \times 3 \times 11 \times 2}$

$= 7 \times 11 \times 2 \times 3 = 462$ cm^2

radius of incircle $= \frac{\Delta}{s} = \frac{462}{77} = 6$ cm

30. (b) Width of the border $= x$ m



Given, area of EFGH $= 6$ m^2

$\Rightarrow \quad (4 - 2x)(3 - 2x) = 6$

$\Rightarrow \quad 12 - 8x - 6x + 4x^2 = 6$

$\Rightarrow \quad 4x^2 - 14x + 12 = 6$

$\Rightarrow \quad 4x^2 - 14x + 6 = 0$

$\Rightarrow \quad 2x^2 - 7x + 3 = 0$

$\Rightarrow \quad 2x^2 - 6x - x + 3 = 0$

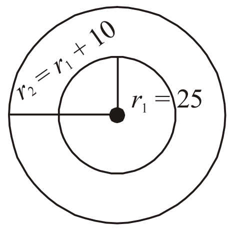
$\Rightarrow \quad 2x(x-3) - 1(x-3) = 0$

$\Rightarrow \quad (x-3)(2x-1) = 0$

$\therefore \quad x = 3, \frac{1}{2}$

$\therefore \quad x = \frac{1}{2} = 0.5$ m $\qquad (\because x \neq 3)$

31. (b) $\because$ Inner radius, $r_1 = 25$ cm



External radius, $r_2 = 25 + 10 = 35$ cm
Distance covered in 1 revolution

$= 2\pi \times 35 = 70 \times \frac{22}{7} = 220$ cm

Distance covered in 10 revolutions $= 2200$ cm

$\therefore$ Speed of bicycle $= \frac{\text{Covered distance}}{\text{Time}} = \frac{2200}{5}$ cm/s

$= \frac{22}{5}$ m/s $= 4.4$ m/s

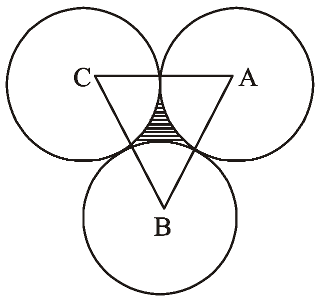
32. (c) Length of wire $= 36$ cm
$\therefore$ Perimeter of semi-circle $= \pi r + 2r$

$\Rightarrow \quad 36 = r\left(\frac{22}{7} + 2\right)$

$\Rightarrow \quad r = \frac{36 \times 7}{36} = 7$ cm

$\therefore$ Radius of semi-circle $= 7$ cm

33. (b) Here, each circle has radius 4 cm.
Then sides of triangle CA=AB = BC = 4 + 4 = 8 cm.



As, all sides of triangle are equal then triangle formed a equilateral triangle and in which each angle is 60º.
Area of shaded part or the area of portion included between circles
= Area of triangle – Area of 3 sectors of circle.

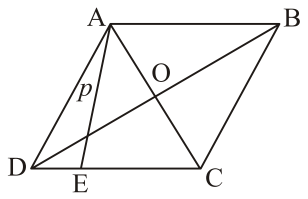
$= \frac{\sqrt{3}}{4}a^2 - \frac{3 \times \pi r^2}{360} \times 60$

$= \frac{\sqrt{3}}{4}(8)^2 - 3 \times \frac{60}{360}\pi(4)^2$

$= (16\sqrt{3} - 8\pi)$ cm^2

34. (a) Area of rhombus

$= \frac{1}{2} \times d_1 \times d_2 = \frac{1}{2} \times 55 \times 48 = 1320 \text{ cm}^2$



$\therefore$ Area of rhombus = Base × Height = DC × AE

$\Rightarrow$ DC × AE = 1320 $\Rightarrow p \times \sqrt{OD^2 + OC^2} = 1320$

$\Rightarrow \quad p \times \sqrt{\left(\frac{55}{2}\right)^5 + \left(\frac{48}{2}\right)^2} = 1320$

$\left(\begin{array}{l} \because OD = \frac{1}{2} BD \\ \text{and } DC = \frac{1}{2} AC \end{array} \right)$

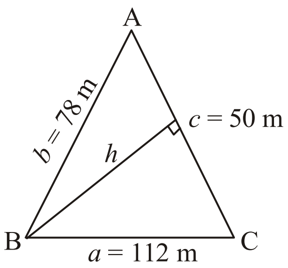
$\Rightarrow \quad p \times \sqrt{\frac{5329}{4}} = 1320$

$\Rightarrow \quad p = \frac{1320}{36.5} = 36.16$

$36 \text{ cm} < p < 37 \text{ cm}$

35. (c) Given, $2s = 240 \Rightarrow s = 120$ and $c = 50\text{m}, b = 78 \text{ m}, a = 112\text{m}$

$\therefore$ Area of triangle $= \frac{1}{2}$ × Base × Height



and also, $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$\therefore \quad = \sqrt{120(120-112)(120-78)(120-50)}$

$= \sqrt{120 \times 8 \times 42 \times 70} = 1680 \text{ m}^2$

$\because$ Area of triangle

$= \frac{1}{2}$ × Base × Height

$\Rightarrow \quad 1680 = \frac{1}{2} \times 50 \times \text{h}$

$\therefore \quad h = \frac{2 \times 1680}{50} = 67.2\text{m}$

36. (b) Let the sides of isosceles triangle be $5x$, $5x$ and $3x$ cm respectively

According to question,

Perimeter of isosceles triangle = Length of wire

$5x + 5x + 3x = 78 \Rightarrow 13x = 78$

$\Rightarrow \quad x = 6 \text{ cm}$

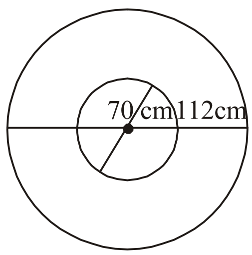
$\therefore$ Length of base = $3 \times 6 = 18$ cm

37. (b) The angle made by the minute hand in 20 min = 120º

$\therefore$ The area swept by the minute hand in 20 min

$= \frac{\theta}{360^\circ} \times \pi r^2 = \frac{120^\circ}{360^\circ} \times 3.14 \times 9 \times 9 = 84.78 \text{ cm}^2$

38. (c) Outer diameter = 112 cm and inner diameter = 70 cm



Area of parapet

$a = \frac{1}{4}\pi(112^2 - 70^2) = \frac{1}{4}(12544 - 4900)\pi$

$= \frac{1}{4} \times 7644 \times \frac{22}{7} = \frac{1}{4} \times 24024 = 6006 \text{ cm}^2$

39. (d) Let the length of altitude AB = h

According to question

Area of ΔABC = Area of square

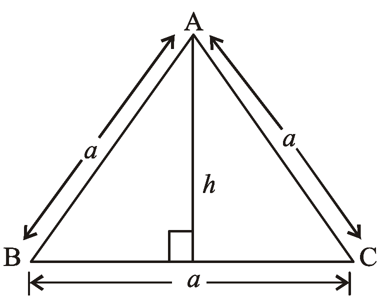
$\therefore \quad \frac{1}{2}$ × Base × Altitude = (Side length)^2

$\Rightarrow \quad \frac{1}{2} \times 9 \times h = 36$

$\Rightarrow \quad h = \frac{36 \times 2}{9}$

$\therefore \quad h = 8 \text{ cm}$

40. (a) Area of equilateral triangle $= \frac{\sqrt{3}}{4} a^2$



$\Rightarrow \quad \frac{1}{2} \times a \times h = \frac{\sqrt{3}}{4} a^2$

$\Rightarrow \quad h = \frac{\sqrt{3}}{2}a^2$

$\Rightarrow \quad 2\sqrt{3} = \frac{\sqrt{3}}{2}a^2$

$\Rightarrow \quad a^2 = 2 \times 2 = 4$

$a = 2$ cm

Area of equilateral triangle

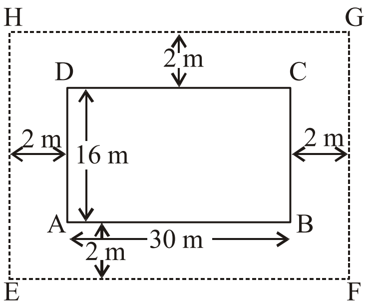
$= \frac{\sqrt{3}}{4}a^2$

$= \frac{\sqrt{3}}{4}(a)^2 = \frac{\sqrt{3}}{4}(2)^2$

$= \sqrt{3}$ cm^2

41. (a) $\therefore$ Required area of the path,

EF = 30 + 4 = 34 m, GF = 16 + 4 = 20 m

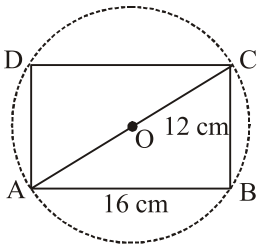


$\therefore$ Area of path = Area of EFGH – Area of ABCD

$= 34 \times 20 - 30 \times 16 = 680 - 480 = 200$ m^2

42. (c) Diagonal of rectangle passes through its. centre So, radius of circle

$(r) = \frac{AC}{2}$



Using Pythagoras theorem

In ABC, AC^2 = AB^2 + BC^2

AC^2 = (16)^2 + (12)^2 = 256 + 144 = 400

AC = 20 cm, AO = $\frac{AC}{2}$

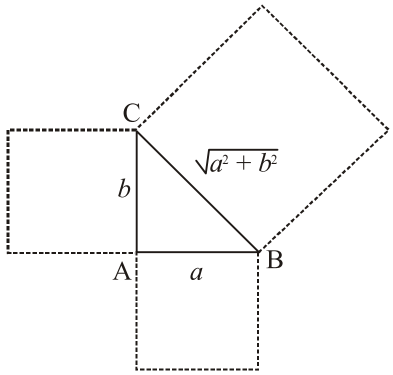
$\therefore$ AO = 10 cm (radius)

Area of circumcircle

$= \pi r^2 = \pi \times (10)^2 = 100\pi$ cm^2

43. (c) In ΔABC, Use Pythagorias theorem,

$BC = \sqrt{AB^2 + AC^2} = \sqrt{a^2 + b^2}$



$\therefore$ Required total area

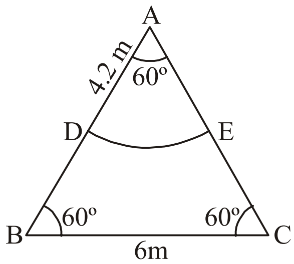
$= a^2 + b^2 + \left(\sqrt{a^2+b^2}\right)^2 + \frac{1}{2}ab$

$= 2(a^2 + b^2) + 0.5ab$

44. (b) $\because$ Area of trapezium $= \frac{1}{2}(3+5) \times 4 = 16$ m^2

$\therefore$ Total cost of painting ₹ 25 per sq m = 16 × 25 = ₹ 400

45. (c) Suppose a horse is tied at vertex A. Then, area available grazing field is ADE.



Now, area of curve

$ADE = \frac{\pi r^2 \theta}{360°} = \frac{22 \times (4.2)^2 \times 60°}{7 \times 360°} = 9.24$ m^2

and area of equilateral ΔABC = $\frac{\sqrt{3}}{4}$ (side)^2

$= \frac{\sqrt{3}}{4} \times (6)^2 = 15.57$

$\therefore$ Required percentage

$= \frac{9.24}{15.57} \times 100 = 59.34\% = 59\%$ (approx)

46. (a) According to question

$\frac{A_1}{A_2} = \frac{\pi r_1^2}{\pi r_2^2} = \frac{1}{2}$

$\Rightarrow \left(\frac{r_1}{r_2}\right)^2 = \frac{1}{2}$ ...(*i*)

As, Circles are bent in the form of square then their perimeter will become equal.

$2\pi r_1 = 4a_1$

$\Rightarrow a_1 = \frac{\pi r_1}{2}$

Similarly, $a_2 = \frac{\pi r_2}{2}$

$\therefore \quad \frac{A_1}{A_2} = \frac{a_1^2}{a_2^2} = \frac{\left(\frac{\pi r_1}{2}\right)^2}{\left(\frac{\pi r_2}{2}\right)^2} = \frac{r_1^2}{r_2^2} = \frac{1}{2}$ [from Eq. (*i*)]

47. (b) Let width of the rectangle = x unit, length of the rectangle = $(2x + 5)$ unit

$\therefore$ Area of rectangle = $(2x + 5) \times x$

$\Rightarrow 75 = 2x^2 + 5x$

$\Rightarrow 2x^2 + 5x - 75 = 0$

$\Rightarrow 2x^2 + 15x - 10x - 75 = 0$

$\Rightarrow x(2x + 15) - 5(2x + 15) = 0$

$\Rightarrow (x - 5)(2x + 15) = 0$

$\Rightarrow x = 5$ units

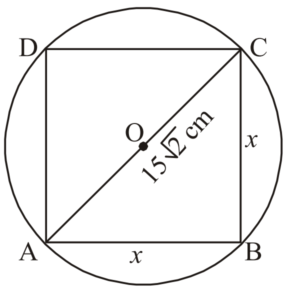
$x = -\frac{15}{2}$

x cannot be negative

So, width of the rectangle = 5 units

48. (b) Let the sides of a square = x cm.

In ΔABC, $AC^2 = AB^2 + BC^2$



Use Pythagoras theorem

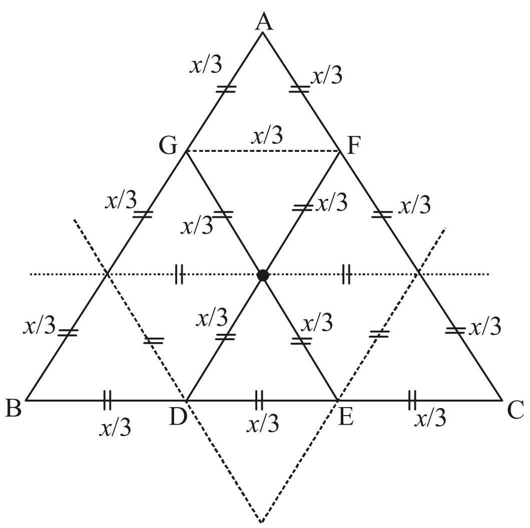
$AC^2 = x^2 + x^2$

$\Rightarrow (15\sqrt{2})^2 = 2x^2 \Rightarrow 2x^2 = 225 \times 2$

$\Rightarrow x^2 = 225 \Rightarrow x = 15$ cm

So, length of the side of the square = 15 cm.

49. (b) Here, ΔABC forms an equilateral triangle.



where, AGFH form a rhombus and ΔHDE is also an equilateral triangle.

$\therefore$ Area of rhombus

= (Area of ΔAGF + Area of ΔGFH)

$= \frac{\sqrt{3}}{4}\left(\frac{x}{3}\right)^2 + \frac{\sqrt{3}}{4}\left(\frac{x}{3}\right)^2 = 2 \times \frac{\sqrt{3}}{4}\left(\frac{x}{3}\right)^2$

Now, area of $\Delta HDE = \frac{\sqrt{3}}{4}\left(\frac{x}{3}\right)^2$

and area of $\Delta ABC = \frac{\sqrt{3}}{4}x^2$

By given condition,

$$\frac{\text{Area of rhombus AGHF} + \text{Area of } \Delta \text{ HDE}}{\text{Area of } \Delta \text{ ABC}}$$

$$= \frac{2 \times \frac{\sqrt{3}}{4}\left(\frac{x}{3}\right)^2 + \frac{\sqrt{3}}{4}\left(\frac{x}{3}\right)^2}{\frac{\sqrt{3}}{4}x^2}$$

$$= \frac{3 \times \frac{\sqrt{3}}{4}\left(\frac{x}{3}\right)^2}{\frac{\sqrt{3}}{4}x^2} = \frac{3}{9} = \frac{1}{3}$$

50. (c) Area of the circle = Area of the square = $(\text{Side})^2$

$\pi r^2 = (2\sqrt{\pi})^2 \Rightarrow \pi r^2 = 4\pi$

$\Rightarrow r^2 = \frac{4\pi}{\pi} = 4$

$\therefore \quad r = \sqrt{4} = 2$ units

$\therefore$ Diameter of circle (d) = $2 \times r = 2 \times 2 = 4$ units

51. (c) Let the radius of circle is r and the side of a square is a, then by given condition.

$2\pi r = 4a \Rightarrow a = \frac{\pi r}{2}$

$\therefore$ Area of square

$= \left(\frac{\pi r}{2}\right)^2 = \frac{\pi^2 r^2}{4} = \frac{9.86r^2}{4} = 2.46r^2$

and area of circle = $\pi r^2 = 3.14r^2$

and let the side of equilateral triangle is x.

Then, by given condition,

$3x = 2\pi r \Rightarrow x = \frac{2\pi r}{3}$

$\therefore$ Area of equilateral triangle

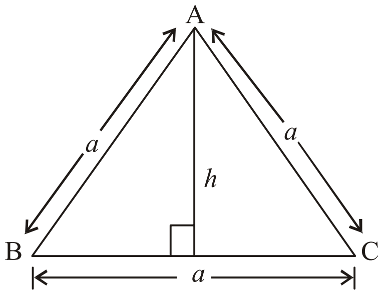
$= \frac{\sqrt{3}}{4}x^2 = \frac{\sqrt{3}}{4} \times \frac{4\pi^2 r^2}{9}$

$= \frac{\pi^2}{3\sqrt{3}}r^2 = 1.89r^2$

Hence, Area of circle > Area of square > Area of equilateral triangle

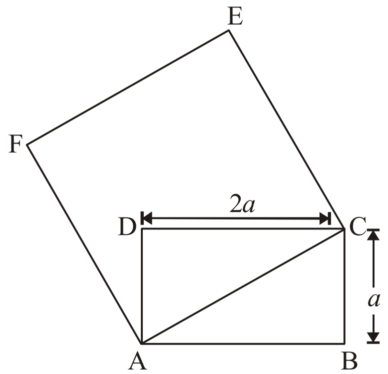
52. (a) Let the width of the rectangle = x unit
Therefore, Length = $(2x + 5)$ unit
According to the question,
Area = $x(2x + 5) \Rightarrow 75 = 2x^2 + 5x$
$\Rightarrow \quad 2x^2 + 5x - 75 = 0$
$\Rightarrow \quad 2x^2 + 15x - 10x - 75 = 0$
$\Rightarrow \quad x(2x + 15) - 5(2x + 15) = 0$
$\Rightarrow \quad (2x + 15)(x - 5) = 0$
$\Rightarrow \quad x = 5 \text{ and } \frac{-15}{2}$
Since, width cannot be negative
$\therefore$ width = 5 units and length = $2x + 5$
$= 2 \times 5 + 5 = 15$ units
$\therefore$ Perimeter of the rectangle = $2(15 + 5) = 40$ units

53. (c) Area of equiateral triangle = $\frac{\sqrt{3}}{4}a^2$



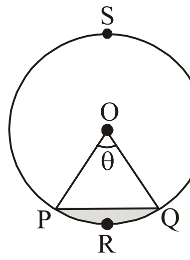
According to Question
$\Rightarrow \quad \frac{1}{2} \times a \times h = \frac{\sqrt{3}}{4} \times a^2$
$\Rightarrow \quad h = \frac{\sqrt{3}}{2}a \qquad [\because h = \sqrt{3}]$
$\Rightarrow \quad \sqrt{3} = \frac{\sqrt{3}}{2}a$
$a = 2$ cm
Perimeter of triangle = $3 \times a = 3 \times 2 = 6$ cm.

54. (d) Given that,



Area of rectangle = $2a^2 = l \times b$
$\Rightarrow \quad l \times b = 2a^2 = l \times a \Rightarrow l = 2a$
Now, in ΔACD,
$AC^2 = AD^2 + CD^2 = a^2 + 4a^2 = 5a^2$
$\therefore \quad AC = \sqrt{5}a$ unit
Hence, area of square
$= \quad (a\sqrt{5})^2$
$= \quad 5a^2$ sq units

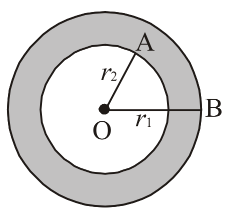
55. (c) I. We know that, Area of segment (PRQP)
= Area of sector (OPRQO) – Area of ΔOPQ
$= \quad \frac{\pi r^2\theta}{360} - \frac{1}{2}r^2 \sin\theta$



So, the area of a segment of a circle is always less than area of its corresponding sector.
II. Distance travelled by a circular wheel of diameter 2 d cm in one revolution = $2\pi \frac{(2d)}{2}$
$= 2 \times 3.14 \times d$
$= 6.28\,d$
which is greater than $6d$ cm.
Therefore, statement I and II both are correct.

56. (b) Perimeter of a rectangle = 82 m
$\therefore \quad 2(\text{Length} + \text{Breadth}) = 82\text{ m}$
$\Rightarrow \quad \text{Length} + \text{Breadth} = 41\text{ m}$
$\Rightarrow \quad l + b = 41\text{ m} \qquad ...(i)$
Also, its area = 400 m^2
$\Rightarrow \quad l \cdot b = 400 \qquad ...(ii)$
Now, $(l - b)^2 = (l + b)^2 - 4lb$
$= \quad (41)^2 - 4(400)$
$= \quad 1681 - 1600 = 81$
$\therefore \quad l - b = 9 \qquad ...(iii)$
From Eqs. (i) and (iii)
$2l = 50 \Rightarrow l = 25$ m and $b = 16$ m
$\therefore$ Required breadth (b) = 16 m

57. (c) Given that, ratio of their radii = 5 : 3
i.e. $r_1 : r_2 = 5 : 3$



$\Rightarrow \quad \frac{r_1}{r_2} = \frac{5}{3} \qquad ...(i)$
Let $r_1 = 5x$ and $r_2 = 3x$
Also, given that, area enclosed between the circumferences of two concentric circles = 16π cm^2
$\therefore \quad \pi(r_1^2 - r_2^2) = 16\pi$

$\Rightarrow \quad (5x)^2 - (3x)^2 = 16$

$\Rightarrow \quad 25x^2 - 9x^2 = 16$

$\Rightarrow \quad 16x^2 = 16$

$\Rightarrow \quad x^2 = 1$

$\Rightarrow \quad x = 1$

$\therefore \quad r_1 = 5$ and $r_2 = 3$

$\therefore$ Area of the outer circle $= \pi r_1^2 = \pi(5)^2$

$= 25\,\pi\,\text{cm}^2$

58. (c) Let the radius of a circle is r and a be the length of the side of a square.

Given, circumference of a circle = Perimeter of a square

$\Rightarrow \quad 2\pi r = 4a$

$\Rightarrow \quad a = \dfrac{\pi}{2} r = 1.57r$

Now, area of the circle $(A_c) = \pi r^2 = 3.14r^2$

and are of the square $(A_s) = a^2 = 2.4649r^2$

$\therefore$ Area of circle > Area of square

59. (b) Let the radii of two circles are r_1 and r_2, respectively.

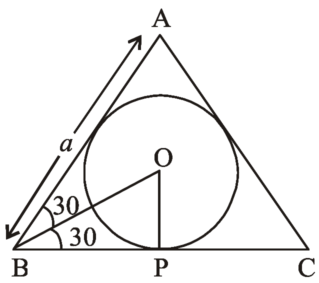
Given, $\dfrac{\text{Circumference of Ist circle}}{\text{Circumference of IInd circle}} = \dfrac{2}{3}$

$\Rightarrow \quad \dfrac{2\pi r_1}{2\pi r_2} = \dfrac{2}{3} \Rightarrow \dfrac{r_1}{r_2} = \dfrac{2}{3}$

$\Rightarrow \quad \left(\dfrac{r_1}{r_2}\right)^2 = \dfrac{4}{9} \qquad ...(i)$

$\therefore \quad \dfrac{\text{Area of Ist circle}}{\text{Area of IInd circle}} = \dfrac{\pi r_1^2}{\pi r_2^2} = \left(\dfrac{r_1}{r_2}\right)^2 = \dfrac{4}{9}$

60. (b)



Here OP is radius of circle and given triangle is equilateral.

$\therefore \quad BP = \dfrac{a}{2}$

In ΔBOP,

$\tan 30^\circ = \dfrac{OP}{BP}$

$OP = BP \tan 30$

$= \dfrac{a}{2} \times \dfrac{1}{\sqrt{3}} = \dfrac{a}{2\sqrt{3}} = \text{radius.}$

Now, Area of circle $= \pi r^2$

$154 = \pi \times \left(\dfrac{a}{2\sqrt{3}}\right)^2$

$\Rightarrow \quad 154 = \pi \dfrac{a^2}{12}$

$\Rightarrow \quad 154 = \dfrac{22}{7} \times \dfrac{a^2}{12}$

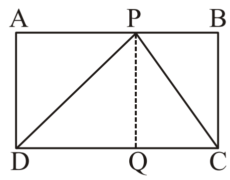
$a = \sqrt{7 \times 7 \times 12}$

$= \quad 7 \times 2\sqrt{3} = 14\sqrt{3}$

Perimeter of triangle = 3

$= \quad 3 \times 14\sqrt{3} = 42\sqrt{3}$

61. (c) Given that, CD = 20 cm and area of rectangle ABCD = 100 m²



$\Rightarrow \quad AD \times CD = 100\ \text{cm}^2 \Rightarrow AD \times 20 = 100$

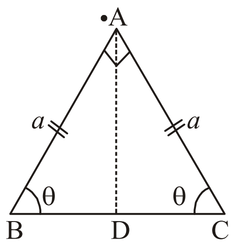
$\therefore \quad AD = 5\ \text{cm} \begin{pmatrix} \text{In the given rectangle,} \\ AB = CD = 20\ \text{cm and} \\ AD = BC = PQ = 5\ \text{cm} \end{pmatrix}$

$\therefore \quad$ Area of $\Delta PDC = \dfrac{1}{2} \times PQ \times CD = \dfrac{1}{2} \times 5 \times 20$

$= 5 \times 10 = 50\ \text{cm}^2$

62. (d) Let AB = AC = a

$\therefore \quad BC^2 = AB^2 + AC^2$ (by Pythagoras theorem)



In ΔABC, $= a^2 + a^2 = 2a^2 \Rightarrow BC = a\sqrt{2}$

$90^\circ + \theta + \theta = 180^\circ$

(since, sum of all interior angles of any triangle is 180°)

$\Rightarrow \quad 2\theta = 90^\circ$

$\therefore \quad \theta = 45^\circ$

Now, in ΔABD,

$\sin 45^\circ = \frac{AD}{a} \Rightarrow AD = \frac{a}{\sqrt{2}}$

$\therefore$ Area of $\Delta ABC = \frac{1}{2} \times AD \times BC$

$= \frac{1}{2} \times a\sqrt{2} \times \frac{a}{\sqrt{2}} = 1$ sq unit (given)

$\Rightarrow \frac{a^2}{2} = 1$

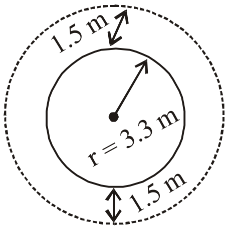
$\therefore a = \sqrt{2}$

$\therefore$ Perimeter of ΔABC

$= 2a + a\sqrt{2} = 2\sqrt{2} + \sqrt{2} \times \sqrt{2}$

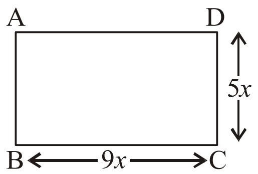
$= 2(1+\sqrt{2})$ units

63. (c) Area of path = Area of (fountain + path) – Area of fountain



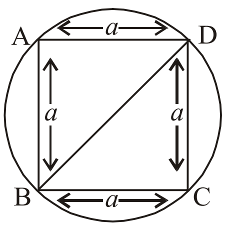
$= \pi(3.3 + 1.5)^2 - \pi(3.3)^2 = [(4.8)^2 - (3.3)^2]\pi$
$= (23.04 - 10.89)\pi = 12.15\,\pi\,m^2$

64. (c) Let length and breadth of a rectangular field are $9x$ and $5x$.



$\therefore$ Area of a rectangular field = 4500 m^2
$\Rightarrow 9x \times 5x = 4500$
$\Rightarrow x^2 = 100 = (10)^2$
$\therefore x = 10$
So, the length and breadth of a rectangular field are 90 m and 50 m.
$\therefore$ Perimeter of rectangular field
= 2 (Length + Breadth)
= 2 (90 + 50) = 2 × 140 = 280 m

65. (c) Given, radius of a circle = 8 cm
and diameter of circle = 16 cm



$\therefore$ Length of diagonal of a square = Diameter of a circle

$\Rightarrow a\sqrt{2} = 16$

$\therefore a = 8\sqrt{2}$ cm
$\therefore$ Area of square ABCD
$= a^2 = (8\sqrt{2})^2$
$= 64 \times 2 = 128$ sq cm

66. (d) Given that, length of hour hand = 4 cm
and length of minute hand = 6 cm
$\therefore$ Complete Rotation of hour-hard in a day
$= 2 \times 360^\circ = 720^\circ$
$\therefore$ Rotation of hour-hand in 2 days $= 2 \times 720^\circ$

$= 1440 \times \frac{\pi}{180}$ rad

Similarly,
Rotation of Minute hand in a day $= 24 \times 360^\circ$
$\therefore$ Rotation of minute hand in 3 days

$= 72 \times 360^\circ \times \frac{\pi}{180}$ rad

$\therefore$ Distance travelled by hour hand

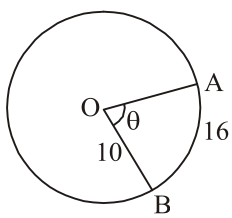
$= 4 \times 1440^\circ \times \frac{\pi}{180^\circ} = 32\pi$

and distance travelled by minute hand

$= 6 \times 72^\circ \times 360^\circ \times \frac{\pi}{180} = 6 \times 144\pi$ cm

$\therefore$ Required ratio $= 32\pi : 6 \times 144\pi = 1 : 27$

67. (c) Arc of length $= 2\pi r \times \frac{\theta}{360^\circ}$



$\Rightarrow 16 = 2\pi r \times \frac{\theta}{360^\circ}$

$\Rightarrow \frac{\theta}{360^\circ} = \frac{16}{2\pi r}$

Now, area of sector OAB

$= \pi r^2 \times \frac{\theta}{360^\circ}$

$= \pi r^2 \times \frac{16}{2\pi r} = 8r = 8 \times 10 = 80$ sq cm

68. (a) $\because$ Angle made in 60 min by minute hand of a clock
$= 360^\circ$
and Angle made in 40 min by minute hand of a clock

$= \frac{360^\circ}{60^\circ} \times 40 = 240^\circ$

$\therefore$ Required distance $= \frac{2\pi(2.5) \times 240^\circ}{360^\circ} = \frac{10\pi}{3}$ cm

69. (b) Area of regular polygon

$$= \frac{na^2}{4}\cot\frac{180^\circ}{n}$$

here polygon is hexagon so $n = 6$

Now, $\frac{6\times a^2}{4}\cot\frac{180^\circ}{6} = 96\sqrt{3}$

$$\Rightarrow \frac{6a^2}{4}\times\cot 30^\circ = 96\sqrt{3}$$

$$\Rightarrow \frac{6a^2}{4}\times\sqrt{3} = 96\sqrt{3}$$

$a^2 = 64$

$\boxed{\because a = 8}$

Perimeter of a regular hexagon

$= 6\times \text{side} = 6\times 8 = 48$ cm

70. (b) Semi-perimeter of triangle

$$= \frac{a+b+c}{2} = \frac{7+24+25}{2} = \frac{56}{2} = 28\text{ cm}$$

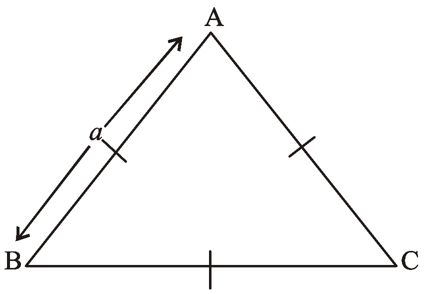
Area of circle = Area of triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{28(28-7)(28-24)(28-25)}$$

$$= \sqrt{28\times 21\times 4\times 3} = \sqrt{7056} = 84\ \text{cm}^2$$

71. (a) Let side of equilateral triangle $= a$



Perimeter = 3a = y

$$a = \frac{y}{3}\text{a}$$

Area of equilateral trangle $= \frac{\sqrt{3}}{4}a^2$

$$\Rightarrow \frac{\sqrt{3}}{4}\left(\frac{y}{3}\right)^2 = x$$

$$\Rightarrow \frac{\sqrt{3}}{4}\times\frac{y^2}{9} = x$$

$$y^2 = \frac{36}{\sqrt{3}}x$$

$$y^2 = \frac{36}{\sqrt{3}}x$$

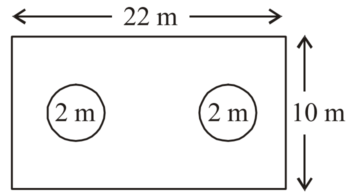
$$y^2 = \frac{36}{3}\sqrt{3}x = 12\sqrt{3}x$$

Squaring both sides

$y^4 = 144\times 3x^2$

$\therefore\ y^4 = 432x^2$

72. (c) Volume of mud dug out in two hemispherical pitholes



$$= 2\times\frac{2}{3}\pi r^3 = 2\times\frac{2}{3}\times\frac{22}{7}\times 2^3 = \frac{2\times 2\times 22\times 8}{21} = \frac{704}{21}\ \text{m}^3$$

Area on which the mud is spread over

= Area of field – Area of pitholes

$$= l\times b - 2\times\pi r^2 = 22\times 10 - 2\times\frac{22}{7}\times 2^2$$

$$= 220 - \frac{176}{7} = \frac{1540-176}{7} = \frac{1364}{7}\ \text{m}^2$$

Now, let the rise in level by h m, then

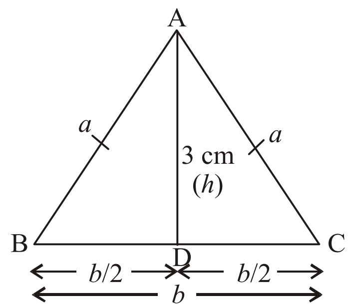
Area of remaining field $\times h$

= Volume of mud dugged out

$$\Rightarrow \frac{1364}{7}\times h = \frac{704}{21}$$

$$\therefore\ h = \frac{704\times 7}{1364\times 21} = \frac{16}{93}\ \text{m}$$

73. (a) Area of the $\Delta ABC = \frac{1}{2}\times b\times h$



$$\Rightarrow 12 = \frac{1}{2}\times b\times 3$$

$$\therefore\ b = \frac{12\times 2}{3} = 8\text{ cm}$$

Here, $BD = CD = \frac{b}{2} = \frac{8}{2} = 4$ cm

In right angled ΔABD, by Pythagoras theorem,

$AB = \sqrt{BD^2 + AD^2}$

$\Rightarrow \quad a = \sqrt{4^2+3^2} = \sqrt{16+9} = \sqrt{25} = 5$ cm

Now, perimeter of an isosceles triangle

$= \quad 2a + b = 2 \times 5 + 8 = 10 + 8 = 18$ cm

74. (a) Let the breadth and height of room be b and h m, respectively.

Then, according to the question,

$\Rightarrow \quad l \times b = n \times$ Area occupied by one patient

$\Rightarrow \quad 14 \times b = 56 \times 22$

$\Rightarrow \quad b = \dfrac{56 \times 2.2}{14} = 8.8$ m

Now, total volume of the room is equal to total patients multiplied by volume occupied by each patient.

Then, $14 \times 8.8 \times h = 8.8 \times 56$

$\therefore \quad h = \dfrac{8.8 \times 56}{14 \times 8.8} = 4$ m

75. (a) As, 1 m = 1000 mm

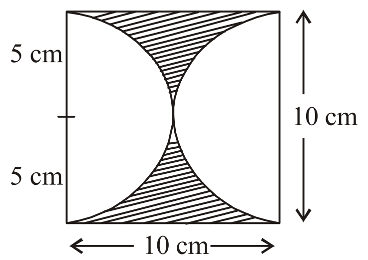
$\therefore \quad$ 10 m = 10000 mm

Number of 200 mm lengths that can be out from 10 m of ribbon

$= \dfrac{10000}{200} = 50$

76. (a) Area between square and semi-circles

= Area of square – 2x Area of semi-circle



$= (10)^2 - 2x\dfrac{22}{7} \times \dfrac{(5)^2}{2} = 100 - 78.5 = 21.5 \text{ cm}^2$

77. (a) Let $l = 4x$ and $b = 9x$

$\therefore \quad$ Area of rectangle $= l \times b$

$144 = 4x \times 9x$

$\Rightarrow \quad x^2 = \dfrac{144}{36} \Rightarrow x^2 = 4$

$\therefore \quad x = 2$

$l = 4 \times 2 = 8$ cm and $b = 9 \times 2 = 18$ cm

$\therefore \quad$ Perimeter of rectangle

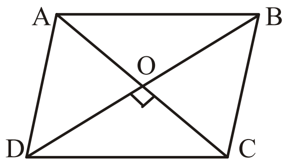
$= \quad 2(l + b) = 2(8 + 18)$

$= \quad 2 \times 26 = 52$ cm

78. (c) Area of parallelogram = Base × Height

$= \quad 8.06 \times 2.08 = 16.76 \text{ cm}^2$

79. (b)



Here, $OD = \dfrac{BD}{2} = \dfrac{4.8}{2} = 2.4;\ OC = \dfrac{AC}{2} = \dfrac{1.4}{2} = 0.7$

Since, in rhombus diagonal bisect at 90°. Then, in ΔODC.

$OD^2 + OC^2 = CD^2$

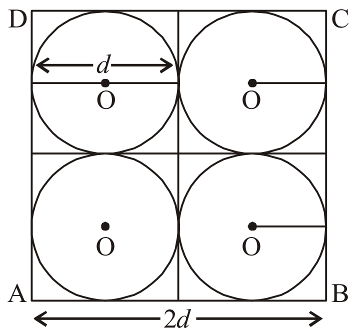
$\Rightarrow CD = \sqrt{OD^2 + OC^2}$

$= \sqrt{(2.4)^2 + (0.7)^2}$

$CD = \sqrt{6.25} \Rightarrow CD = 2.5 = \dfrac{5}{2}$

$\Rightarrow \quad$ Perimeter of rhombus = 4(side) $= 4 \times \dfrac{5}{2} = 10$ cm

80. (c) From the figure it is clear that, 4 circular plates of diameter can be made of a Square plate of side 2d with minimum loss of material.



Let ABCD be square

Diameter of circle $= d$

Radius of circle $= \dfrac{d}{2}$

Here from figure it is clear that side of the square is equal to diameter of two circle

Side of square $= d + d = 2d$

$= \quad AB = BC = CD = DA$

Therefore, perimeter of square = no. of circular plates × sum of diameter two circular plates

$\Rightarrow$ (2d + 2d + 2d + 2d) no. of circular plates × 2d

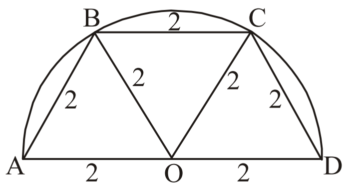
no. of circular plates $= \dfrac{8d}{2d} = 4$

81. (d) Here, $\Delta AOB, \Delta BOC$ and ΔCOD are equilateral triangles.

$\therefore \quad$ Side = 2 cm

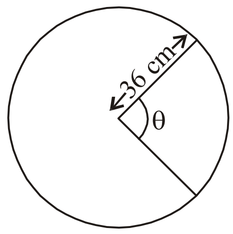
Total area of three equilateral triangles

$= \quad 3 \times \dfrac{\sqrt{3}}{4}\ (\text{Side})^2$



$= \quad 3\times\frac{\sqrt{3}}{4}\times 4 = 3\sqrt{3}\ \text{cm}^2$

82. (d) Area of sector = $72\,\pi\ \text{cm}^2$

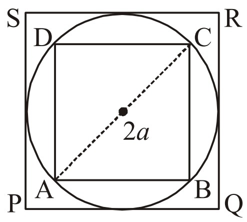


$\Rightarrow \quad \frac{\pi r^2\theta}{360^o} = 72\,\pi$

$\therefore \quad \theta = \frac{72\times 360}{36\times 36} = 20^o$

Length of arc = $\frac{\pi r\,\theta}{180^o} = \frac{\pi\times 36\times 20}{180} = 4\pi$ cm

83. (b)



For inscribed square.
Diameter of circle = Diagonal of square
using Pythagoras theorem,

$\Rightarrow \quad AB^2 + BC^2 = AC^2$

$\Rightarrow \quad 2AB^2 = (2a)^2 \qquad [\therefore AB = BC]$

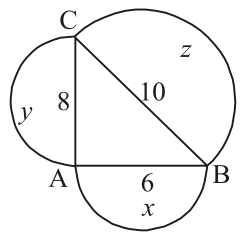
$\quad 2AB^2 = 4a^2$

$\therefore \quad AB^2 = 2a^2$

$\because \quad AB = \sqrt{2}a$

Area of square ABCD = $(\sqrt{2}a)^2 = 2a^2$
For circumscribed circle,
Diameter of circle = Side of square = $2a$
Area of square PQ RS = $(2a)^2 = 4a^2$
Difference between area of outer square and inner square
$= \quad 4a^2 - 2a^2 = 2a^2$

84. (c) In ΔABC, by Pythagoras theorem,



$BC^2 = AB^2 + AC^2 = 36 + 64 = 100$ cm

Now, area of semi-circle $x = \frac{\pi(3)^2}{2} = \frac{9\pi}{2}\ \text{cm}^2$

$\therefore$ BC = 10 cm.

Area of semi-circle $y = \frac{16\pi}{2}\ \text{cm}^2$

Area of semi-circle $z = \frac{25\pi}{2}\ \text{cm}^2$

Now, value of $x + y - z = \left(\frac{9\pi}{2} + \frac{16\pi}{2}\right) - \frac{25\pi}{2} = 0$

85. (c) Sides of equelateral triangle are follows:

$3, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}$ ... so on

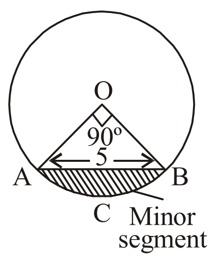
These sequence formed a GP serves.
So sum of GP for Infinite terms.

$S = \frac{a}{1-r}$

Here $a = 3, r = \frac{1}{2}$

$S = \frac{3}{1-\frac{1}{2}} = 6$ units

86. (c) In ΔAOB, AO = OB = r



Using Pythagoras theorem,
$AB^2 = OA^2 + OB^2 \Rightarrow (5)^2 = r^2 + r^2$

$\therefore \quad r^2 = \frac{25}{2}$ cm

Now, area of sector

$AOB = \frac{\theta}{360^o}\times\pi r^2 = \frac{90^o}{360^o}\times\pi\times\frac{25}{2} = \frac{25\pi}{8}\ \text{cm}^2$

Now, area of minor segment
= Area of sector – Area of triangle

$= \frac{25\pi}{8} - \frac{r^2}{2} = \frac{25\pi}{8} - \frac{25}{4} = \left(\frac{25\pi - 50}{8}\right)$

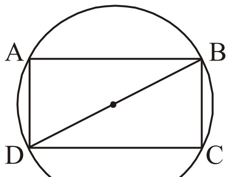
Area of major segment = Area of circle – Area of minor segment

$= \pi r^2 - \left(\frac{25\pi - 50}{8}\right) = \frac{25\pi}{2} - \frac{(25\pi - 50)}{8}$

$$= \frac{100\pi - 25\pi + 50}{8} = \frac{75\pi + 50}{8}$$

$$= \frac{25}{8}(3\pi + 2) = \frac{25}{4}\left(\frac{3\pi}{2} + 1\right) \text{cm}^2$$

87. (c) ABCD be the rectangle inscribed in the circle of diameter 5 cm.



$\therefore$ Diameter = Diagonal of rectangle

Now, let x and y be the lengths and breadths of rectangle, respectively.

Now in ΔABD,

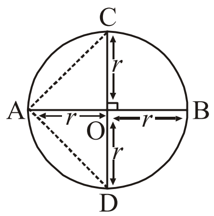
$AB^2 + AD^2 = (5)^2 \Rightarrow x^2 + y^2 = 25$

Since, they form Pythagoras triplet,

$\therefore$ $x = 4$ and $y = 3$

So, area of rectangle = 3 × 4 = 12 cm²

88. (b) Required ratio = $\frac{\text{Area of circle}}{\text{Area of } \Delta\text{ACD}}$



$$= \frac{\pi r^2}{\frac{1}{2} \times 2r \times r} = \pi$$

89. (b) Here, Area of Rectangle lies between 40cm^2 and 45cm^2

Given that one side = 5cm.

Area of Rectangle = 5 × second side

Now, If Area = 40cm^2

then, 40 = 3 × second side

$\because$ second side = 8cm.

Again, If Area = 45cm^2

45 = 3 × second side

$\because$ Second side = 9cm.

It means that second side varies between 8 cm to 9 cm.

Let diagonal = d

$$\Rightarrow \sqrt{8^2 + 5^2} < d < \sqrt{5^2 + 9^2}$$

$$\Rightarrow \sqrt{64 + 25} < d < \sqrt{25 + 81}$$

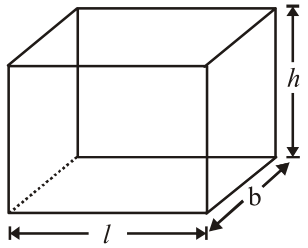
$$\Rightarrow \sqrt{87} < d < \sqrt{106}$$

$$\Rightarrow \sqrt{81} < \sqrt{89} < d < \sqrt{106} < \sqrt{121}$$

$$\Rightarrow \sqrt{81} < d < \sqrt{121}$$

$\Rightarrow$ 9cm < d < 11cm.

90. (b) Let length, breadth and height of a cuboidal box be l, b and h, respectively



Areas of three adjacent faces are x, $4x$ and $9x$ sq unit respectively. So $lb = x$

[$\because$ area of rectangular face = length × breadth]

Similarly, $bh = 4x$ and $lh = 9x$

Now, $(lb) . (bh) . (lh) = (x) . (4x) . (9x)$

$\Rightarrow$ $(lbh)^2 = 36x^3$

$\Rightarrow$ $lbh = \sqrt{36x^3}$

$\therefore$ $lbh = 6x^{3/2}$

Therefore, volume of cuboidal box = lbh

$= 6x^{3/2}$ cu unit

91. (a) Area of triangular field

$= \sqrt{s(s-a)(s-b)(s-c)}$

$$s = \frac{a+b+c}{2} = \frac{41+40+9}{2} = 45\text{m}$$

$$\text{Area} = \sqrt{45 \times (45-41) \times (45-40) \times (45-9)}$$

$= \sqrt{45 \times 4 \times 5 \times 36}$

$= 180 \text{ m}^2 = 1800000 \text{ cm}^2$

Number of rose bed = $\frac{1800000}{900} = 2000.$

92. (c) $\frac{r_2}{r_1} = \frac{23}{22}$

$$\Rightarrow \frac{r_2 - r_1}{r_1} = \frac{23-22}{22}$$

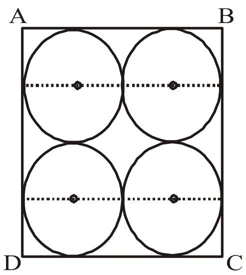
$$\Rightarrow \frac{r_2 - r_1}{r_1} = \frac{1}{22}$$

$$\Rightarrow \frac{5}{r_1} = \frac{1}{22}$$

$r_1 = 110$ m

Diameter of inner circle = 110 × 2 m = 220 m

93. (d)



Let ABCD is square

Side of square $(a) = \sqrt{784} = 28$ cm

Diameter of single circle $= \frac{28}{2} = 14$ cm

Radius of single circle $= \frac{14}{2} = 7$ cm

Circumference of each plate $= 2\pi r$

$$= 2\times\frac{22}{7}\times 7 = 44 \text{ cm}$$

94. (d) Diagonal of square $= 12\sqrt{2}$cm

Side of square $= \frac{12\sqrt{2}}{\sqrt{2}} = 12$cm

Perimeter of square $= 4 \times 12 = 48$ cm

Perimeter of equilateral triangle $= 48$ cm

Side of the equilateral triangle $= \frac{48}{3} = 16$cm

Area of triangle $\frac{\sqrt{3}}{4}\times(16)^2$

$= \frac{\sqrt{3}}{4}\times 256$

$= 64\sqrt{3}\text{cm}^2$

So, option (d) is correct.

95. (b) Distance covered in one revolution

$= 2\pi \times 30 = 60\pi$ cm

Distance covered in one minute by 140 revolutions

$= 140 \times 60\,\pi$ cm

Distance covered in one hour

$= 140 \times 60\,\pi \times 60$ cm $= 15.82560$ cm $= 15.825$ km

So, option (b) is correct.

96. (b) $437 = 19\,(23)$

There are 19 rows with 23 trees in each row.

The distance between any two adjacent plants is 2m and the distance between any two adjacent rows is 2m. $\therefore$ The dimensions of the orchard are

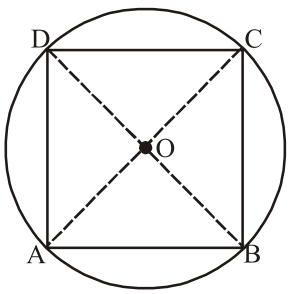
Length $= [\,1 + 22 \times 2 + 1] = 46$m

Breadth $= [\,1 + 18 \times 2 + 1] = 38$m

Perimeter $= 2(46 + 38) = 168$ m

Cost of fencing $= 100 + 168 =$ ₹16800

97. (c)



Radius of circle $= \frac{100}{2\pi}$

$OC = OD = \frac{100}{2\pi}$

In ΔOCD

$\angle DOC = 90°$

$OC^2 + OD^2 = CD^2$

$\Rightarrow CD^2 = 2OC^2$

$\Rightarrow CD^2 = 2 \times \left(\frac{100}{2\pi}\right)^2$

$\Rightarrow CD = \sqrt{2}\times\frac{100}{2\pi}$

$\Rightarrow CD = \frac{50\sqrt{2}}{2\pi}$ cm

So, option (c) is correct.

98. (b) Let the diameter be D.

According to question-

$452 \times \pi.D = 2$ km $+ 26$ decameter

$452 \times \pi.D = (2000 + 260)$ metre

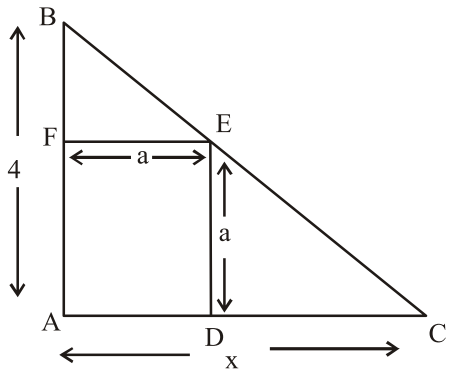
$452 \times \pi.D = 2260$

$D = \frac{2260}{452}\times\frac{7}{22}$

$= \frac{35}{22} = 1\frac{13}{22}$ m

So, option (b) is correct.

99. (b)



Let the side of square be ‘a’.

$AD = ED = EF = FA = a$

$CD = x - a$

$ED = y - a$

as Δ CED and Δ CBA are similar

$\frac{CD}{CA} = \frac{ED}{BA}$

$\Rightarrow \frac{x-a}{x} = \frac{a}{y}$

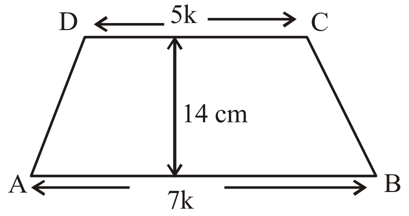
$\Rightarrow xy - ay = ax \Rightarrow ax + ay = xy$

$\Rightarrow a = \frac{xy}{(x+y)}$

Perimeter of square $4a = \frac{4xy}{(x+y)}$

So, option (b) is correct.

100. (a)



Area of trapezium

$= \frac{1}{2}(AB + CD) \times \text{height}$

$336 = \frac{1}{2}(5k + 7k) \times 14$

$12k = 48$

$k = 4$

Smaller parallel side = $5k = 5 \times 4 = 20$ cm

101. (d) Total distance travelled = 4 km = 4000 m

Distance travelled in one round.

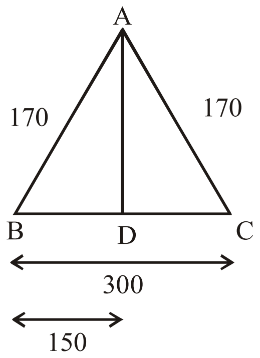
= Circumference of the wheel

$= 2\pi r = 2 \times \frac{22}{7} \times \frac{7}{11 \times 2} = 2m$

$\therefore$ Number of rounds in 4 km $= \frac{4000}{2} = 2000$ rounds

$\therefore$ Option (d) is correct.

102. (c) Let ABC be an isosceles triangle.



$\text{Area} = \frac{1}{2} \times AD \times BC$

$= \frac{1}{2} \times \sqrt{(170)^2 - (150)^2} \times 300$

$= \frac{1}{2} \times \sqrt{28900 - 22500} \times 300$

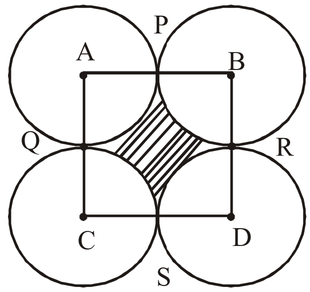
($\because \Delta ADC$ is a right angled triangle then by pythagoras theorem, we find AD)

$= 150 \times \sqrt{6400} = 150 \times 80 = 12000$ units.

$\therefore$ Option (c) is correct.

103. (d) Given Area of shaded region $= \frac{150}{847} \text{cm}^2$

We have to find radius of each disc.



Since these discs are equal then their radius are also equal.

$\Rightarrow$ AC = AB = BD = DC = Diameter of disc.

Let r be radius of disc.

Area of square ABCD = $(AB)^2 = (2r)^2 = 4r^2$

Area of shaded region = Area of square – 4 (Area of sector APQ)

Area of sector APQ $= \frac{\theta}{360} \times \pi r^2$

$[\because \angle A = \angle B = \angle C = \angle D = 90°]$

$\frac{150}{847} = 4r^2 - 4 \times \frac{90}{360} \times \frac{22}{7} \times (r)^2$

$\frac{150}{847} = 4r^2 - \frac{22}{7}r^2 \Rightarrow \frac{6r^2}{7} = \frac{150}{847}$

$r^2 = \frac{25}{121} \Rightarrow r = \frac{5}{11}$ cm

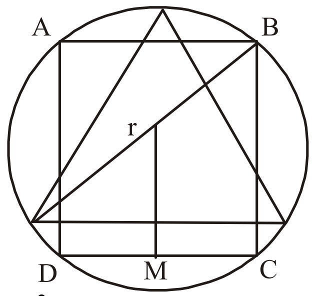
$\therefore$ Option (d) is correct.

104. (a) Let Radius of circle be r

Case I (Square)

side = x

$x^2 + x^2 = (2r)^2$



$2x^2 = 4r^2$

$x = \sqrt{2}r$

Case II (equilateral triangle)

$y \cos 30° = h$

$y \times \frac{\sqrt{3}}{2} = h$ $\quad \begin{bmatrix} \text{if } h = 3 \\ r = 2 \end{bmatrix}$

$r = \frac{2}{3} \times \frac{\sqrt{3}}{2} y$

$r = \frac{y}{\sqrt{3}}$

$$\frac{y}{2r}=\frac{\sqrt{3}}{2}$$

$$y=\frac{2\sqrt{3}r}{2}=\sqrt{3}r$$

$$\frac{x}{y}=\frac{\sqrt{2}r}{\sqrt{3}r}=\sqrt{\frac{2}{3}}$$

105. (b) Let perimeter of square = 4x
then $2\pi r=4x$

$$r=\frac{2x}{\pi}$$

area of square = x^2

$$\text{area of circle}=\pi\left(\frac{2x}{\pi}\right)^2=\pi 4x=\frac{28}{22}x^2$$

thus $\frac{28}{22}x^2$ is $> x^2$

106. (a) $\because$ triangle with side 12cm, 13cm and 5 cm is a right triangle

$$\text{Area}=\frac{1}{2}\,b\times h=\frac{1}{2}\times 12\times 5=30$$

107. (d) Perimeter P = 10 cm
Area A = 4 cm^2
$2(\ell+B)=10$
$B=5-\ell$
$\ell\times B=4\text{ cm}^2$
$l(5-l)=4$
$5\ell-l^2=4$
$l^2-5\ell+4=0$
$l^2-l-4l+4=0$
$(\ell-1)(\ell-4)=0$
$l=4, l\neq 1$
$B=5-4=1$

108. (b) $R_1=14$ $R_2=x$

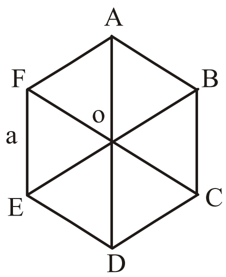
$$\frac{\text{Area } C_1}{\text{Area } C_2}=\frac{\Pi(14)^2}{\Pi x^2}$$

$$\frac{49}{16}=\frac{(14)^2}{x^2}$$

$$x^2=\frac{14\times 14\times 16}{49}=8^2$$

$R_2=8$

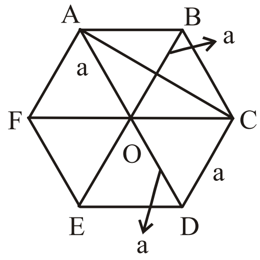
109. (b)



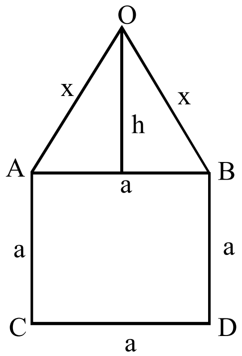
let ABCDEF be a regular hexagon which can be devided into 6 equilateral triangle

$$\text{as } \Delta=\frac{\sqrt{3}}{4}a^2$$

$$\text{as } 6\times\Delta=6\times\frac{\sqrt{3}}{4}a^2=\frac{3\sqrt{3}}{2}a^2$$



110. (d)



Let the side of each side of the square be a and the other two sides of the triangle be x.

Perimeter of the complete figure = $\frac{7}{6}$ perimeter of the original square (given)

$$3a+2x=\frac{7}{6}(4a)\Rightarrow 9a+6x=14a\Rightarrow 6x=5a \quad ...(1)$$

(by given and Pythagoras theorem) Height of the triangle,

$$h=\sqrt{x^2-\left(\frac{a}{2}\right)^2}=\sqrt{x^2-\frac{a^2}{4}}=\sqrt{\left(\frac{5a}{6}\right)^2-\frac{a^2}{4}}$$

$$=\sqrt{\frac{25a^2}{36}-\frac{a^2}{4}}=\sqrt{\frac{25a^2-9a^2}{36}}$$

$$=\sqrt{\frac{16a^2}{36}}=\frac{4a}{6}=\frac{2a}{3}$$

Now, ratio of the area of triangle to area of the original square $=\dfrac{\frac{1}{2}\times a\times\frac{2a}{3}}{a^2}=\frac{a^2}{3}\times\frac{1}{a^2}=\frac{1}{3}=1:3$

111. (c) Calculating area of triangle using heron's formula, we get
Semi perimeter,

$$s=\frac{a+b+c}{2}=\frac{51+37+20}{2}=\frac{108}{2}=54$$

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{54(54-51)(54-37)(54-20)}$

$= \sqrt{54 \times 3 \times 17 \times 34}$

$= \sqrt{3.3.3.2.3.17.17.2} = 3.3.2.17 = 306$ sq. cm

112. (b) Let length of side of square be a cm
after increase side of square = (a + 8) cm
According to the question
$(a+8)^2 = a^2 + 120$
$\Rightarrow a^2 + 16a + 64 = a^2 + 120$
$\Rightarrow 16a = 120 - 64$
$\Rightarrow 16a = 56 \quad \therefore \; a = \frac{56}{16} = 3.5$ cm

113. (d) Area of the pathway $= 10 \times 4.5 = 45\ m^2$

Area of each tile $= \frac{50 \times 50}{100 \times 100} = 0.25\ m^2$

Required number of tiles $= \frac{45}{0.25} = 180$

Cost of 1 tile $= \frac{100}{20} = 5$

Total cost $= 180 \times 5 = 900$

114. (b) Area of square $= \frac{1}{2} \times d_1 \times d_2 = a^2$

According to the question

$a^2 = \frac{1}{2} \times 50$

$a^2 = 25 \quad \therefore \; a = 5$ units

115. (c) Let length and breadth of the rectangle are l and b;
According to the question $l = 3b$.
then, perimeter $P = 2(l + b) = 2(3b + b) = 8b$

Changed length $l' = l + \frac{30l}{100} = 1.3l = 3.9b$

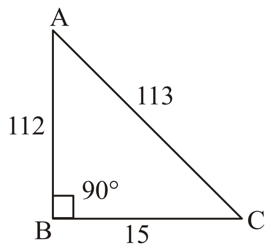
breadth $b' = b + \frac{10b}{100} = 1.1b$

New perimeter $P' = 2(l' + b')$
$= 2(3.9b + 1.1b) = 10b$
Percent increase in perimeter

$= \left(\frac{P'-P}{P}\right) \times 100 = \left(\frac{10b - 8b}{8b}\right) \times 100 = 25\%$

116. (c) Width of border = 6 inches $= \frac{6}{12}$ feets = 0.5 feet
Area of border $= 2(12 \times 0.5) + (6-1) \times 0.5 \times 2$
$= 12 + 5 = 17$ square feet.

117. (a)



$\sqrt{(112)^2 + (15)^2} = \sqrt{12544 + 225}$

$= \sqrt{12769} = 113$

$\therefore$ Perimeter of the triangle $= (113 + 112 + 25)$cm
$= 240$ cm

118. (a) Let length and breadth of as recgtangle is x and y.
$\therefore$ Area $= x \times y$
After increase length 20% and breadth 10% Change in Area $= (1.32\,xy - xy) = 0.32xy$

$\therefore$ % increase $= \frac{0.32}{1} \times 100 = 32\%$

119. (c) Area of the square
$2401\,x^4 + 196x^2 + 4$
$= (7x)^4 + 2 \times 2\,(7x)^2 + 2^2$
$= ((7x)^2 + 2)^2 = (\text{side})^2$
$\Rightarrow \therefore$ Side $= (7x)^2 + 2 = 49x^2 + 2$

120. (c) $S = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$

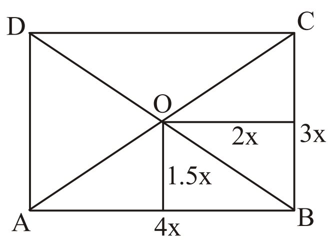
Area $= \sqrt{S(S-a)(S-b)(S-c)}$

$= \sqrt{\left(\frac{x}{y} + \frac{y}{z} + \frac{z}{x}\right)\left(\frac{x}{y}\right)\left(\frac{y}{z}\right)\left(\frac{z}{x}\right)}$

$= \sqrt{\frac{x}{y} + \frac{y}{z} + \frac{z}{x}}$

121. (c) Area of square of side length a $= a^2$
Area of rectangle = xy
by questions
$x = a^2$
also $a^2 = xy$
$\therefore y = 1$

122. (d)



Let length and breadth of a rectangle is 4x and 3x
Area of triangle BOC $= \frac{1}{2} \times 3x \times 2x = 3x^2$
Area of triangle AOB $= \frac{1}{2} \times 4x \times 1.5x = 3x^2$
$\therefore$ ratio = 1 : 1

CHAPTER 20

Volume and Surface Area

1. If the diameter of a wire is decreased by 10%, by how much per cent (approximately) will the length be increased to keep the volume constant? **[2007-II]**
(a) 5% (b) 17%
(c) 20% (d) 23%

2. A cone is inscribed in a hemisphere such that their bases are common. If C is the volume of the cone and H that of the hemisphere, then what is the value of C : H? **[2007-II]**
(a) 1 : 2 (b) 2 : 3
(c) 3 : 4 (d) 4 : 5

3. S_1, S_2 and S_3 are three rectangular sheets of identical areas with their lengths in the ratio 1 : 2 : 3. If each is converted into a right circular cylinder open at both ends by joining its shorter parallel sides, then what is the ratio of the volumes of the three cylinders, S_1, S_2 and S_3, respectively, so formed? **[2007-II]**
(a) 1 : 1 : 1 (b) 4 : 2 : 3
(c) 1 : 2 : 3 (d) 6 : 3 : 2

4. The base diameter of a right circular cylinder is 3 cm. There is a section making an angle of 30º with the cross-section. What is its area? **[2007-II]**
(a) $\frac{9\pi}{4}$ sq cm (b) $\frac{3\sqrt{3}\pi}{2}$ sq cm
(c) $\frac{9\pi}{8}$ sq cm (d) $\frac{9\sqrt{3}\pi}{8}$ sq cm

5. 27 drops of water form a big drop of water. If the radius of each smaller drop is 0.2 cm, then what is the radius of the biggeer drop? **[2007-II]**
(a) 0.4 cm (b) 0.6 cm
(c) 0.8 cm (d) 1.0 cm

6. The total surface area of a cone, whose stant height is equal to the radius R of its base, is S. If A is the area of a circle of radius 2R, then which one of the following is correct? **[2007-II]**
(a) A = S (b) A = 2S
(c) A = S/2 (d) A = 4S

7. An iron block is in the form of a cylinder of 1.5 m diameter and 3.5 m length. The block is to be rolled into the form of a bar, having a square section of side 5 cm. What will be the length of the bar? **[2007-II]**
(a) 2375 m (b) 2475 m
(c) 2575 m (d) 2600 m

8. If the circumference of the inner edge of a hemispherical bowl is $\frac{132}{7}$ cm, then what is its capacity? **[2008-I]**
(a) 12π cu cm (b) 18π cu cm
(c) 24π cu cm (d) 36π cu cm

9. What is the number of spherical balls of 2.5 mm diameter that can be obtained by melting a semi-circular disc of 8 cm diameter and 2 cm thickness? **[2008-I]**
(a) 6144 (b) 3072
(c) 1536 (d) 768

10. From a wooden cylindrical block, whose diameter is equal to its height, a sphere of maximum possible volume is carved out. What is the ratio of the volume of the utilised wood to that of the wasted wood? **[2008-I]**
(a) 2 : 1 (b) 1 : 2
(c) 2 : 3 (d) 3 : 2

11. A conical flask of base radius r and height h is full of milk. The milk is now poured into a cylinderical flask of radius $2r$. What is the height to which the milk will rise in the flask? **[2008-I]**
(a) $\frac{h}{3}$ (b) $\frac{h}{6}$
(c) $\frac{h}{9}$ (d) $\frac{h}{12}$

12. From a solid cylinder whose height is 4 cm and radius 3 cm a conical cavity of height 4 cm and base radius 3 cm is hollowed out. What is the volume of the remaining solid? **[2008-I]**
(a) 9π cu cm (b) 15π cu cm
(c) 21π cu cm (d) 24π cu cm

13. The diameters of two right circular cones are equal. If their slant heights are in the ratio 3 : 2, then what is the ratio of their curved surface areas? **[2008-I]**
(a) 9 : 4 (b) $\sqrt{3} : \sqrt{2}$
(c) 3 : 2 (d) 2 : 3

14. In which one of the following pairs does the first number represent the perimeter of one face of a cube and the second number represent the volume of the cube? **[2008-I]**
(a) (16, 32) (b) (20, 125)
(c) (9, 27) (d) (10, 100)

15. Three cubes of metal whose edges are 6 cm, 8 cm and 10 cm, respectively are melted and a single cube is formed. What is the length of the edge of the newly formed cube? **[2008-I]**
(a) 10 cm (b) 12 cm
(c) 16 cm (d) None of these

16. Consider the following
The length of a side of a cube is 1 cm. Which of the following can be the distance between any two vertices?
I. 1 cm II. $\sqrt{2}$ cm
III. $\sqrt{3}$ cm

Select the correct answer using the codes given below. **[2008-I]**

(a) Only I (b) Only II
(c) Only III (d) I, II and III

17. A sphere is cut into two equal halves and both the halves are painted from all the sides. The radius of the sphere is r unit and the rate of painting is ₹ 8 per sq unit. What is the total cost of painting the two halves of the sphere in rupees? **[2008-II]**

(a) $6\pi r^2$
(b) $32\pi r^2$
(c) $48\pi r^2$
(d) Insufficient data to answer

18. If the number of square centimetres on the surface area of a sphere is three times the number of cubic centimetres in its volume, then what is its diameter? **[2008-II]**

(a) 1 cm (b) 2 cm
(c) 3 cm (d) 6 cm

19. The ratio of the surface areas of two hemispheres is 4 : 1. What is the ratio of their volumes? **[2008-II]**

(a) 8 : 1 (b) 4 : 1
(c) 3 : 1 (d) 2 : 1

20. A container is in the form of a right circular cylinder surmounted by a hemisphere of the same radius 15 cm as the cylinder. If the volume of the container is 32400π cm^3, then the height h of the container satisfies which one of the following? **[2008-II]**

(a) 135 cm < h < 150 cm (b) 140 cm < h < 147 cm
(c) 145 cm < h < 148 cm (d) 139 cm < h < 145 cm

21. A hollow cylindrical iron pipe of length 1.4 m has base radius 2.5 cm and thickness of the metal is 1 cm. What is the volume of the iron used in the pipe? **[2008-II]**

(a) 2640 cu cm (b) 2604 cu cm
(c) 2460 cu cm (d) None of these

22. A right circular cone is cut by a plane parallel to its base in such a way that the slant heights of the original and the smaller cone thus obtained are in the ratio 2 : 1. If V_1 and V_2 are respectively the volumes of the original cone and of the new cone, then what is $V_1 : V_2$? **[2008-II]**

(a) 2 : 1 (b) 3 : 1
(c) 4 : 1 (d) 8 : 1

23. If C_1 is a right circular cone with base radius r_1 cm and height h_1 cm and C_2 is a right circular cylinder with base radius r_2 cm and height h_2 cm and if $r_1 : r_2 = 1 : n$ (where, n is a positive integer) and their volumes are equal, then which one of the following is correct? **[2008-II]**

(a) $h_1 = 3nh_2$ (b) $h_1 = 3n^2h_2$
(c) $h_1 = 3\,h_2$ (d) $h_1 = n^2\,h_2$

DIRECTIONS: *The following two questions consists of two statements, one labelled as the 'Assertion (A)' and the other as 'Reason (R)'. You are to examine these two statements carefully and select the answers to these items using the codes given below.*

Codes: **2008-II]**

(a) Both A and R are individually true and R is the correct explanation A
(b) Both A and R are individually true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

24. **Assertion (A):** The curved surface area of a right circular cone of base radius r and height h is given by $\pi r\left(\sqrt{h^2+r^2}\right)$.

Reason (R): The right circular cone of base radius r and height h, when cut opened along the slant height forms a rectangle of length πr and breadth $\sqrt{h^2+r^2}$.

25. **Assertion (A)**: The volume of a cuboid is the product of the lengths of its coterminous edges.
Reason (R): The surface area of a cuboid is twice the sum of the products of lengths of its coterminous edges taken two at a time. **[2008-II]**

26 The radius and height of a right circular cone are in the ratio 3 : 4 and its volume is 96π cm^3. What is its lateral surface area? **[2008-II]**

(a) 24π cm^2 (b) 36π cm^2
(c) 48π cm^2 (d) 60π cm^2

27. From a solid cube of edge 3 m, a largest solid sphere is curved out. What is the volume of solid left? **[2008-II]**

(a) $(27 - 2.25\pi)$ cu m (b) $(27 - 4.5\pi)$ cu m
(c) 2.25π cu m (d) 4.5π cu m

28. The dimensions of a rectangular box are in the ratio 2 : 3 : 4 and the difference between the cost of covering it with a sheet of some metal at ₹ 8 and ₹ 10 per sq m is ₹ 234. What are the length, breadth and height of the box, respectively? **[2008-II]**

(a) 2 m, 3 m, 4 m (b) 3 m, 4.5 m, 6 m
(c) 4 m, 6 m, 8 m (d) 5 m, 7.5 m, 10 m

29. A solid metallic cube of edge 4 cm is melted and recast into solid cubes of edge 1 cm. If x is the surface area of the melted cube and y is the total surface area of all the cubes recast, then what is $x : y$? **[2008-II]**

(a) 2 : 1 (b) 1 : 2
(c) 1 : 4 (d) 4 : 1

30. What is the total surface area of a one-side open cubical box of outer side of length 5 cm and thickness 0.5 cm? **[2008-II]**

(a) 125 sq cm (b) 214 sq cm
(c) 180 sq cm (d) None of these

31. A field is 125 m long and 15 m wide. A tank 10m × 7.5m × 6m was dug in it and the Earth thus dug out was spread equally on the remaining field. The level of the field thus raised is equal to which one of the following? **[2008-II]**

(a) 15 cm (b) 20 cm
(c) 25 cm (d) 30 cm

32. A lead pencil is in the shape of a cylinder. The pencil is 21 cm long with radius 0.4 cm and its lead is of radius 0.1 cm. What is the volume of wood in the pencil? **[2009-I]**

(a) 9 cu cm (b) 9.4 cu cm
(c) 9.9 cu cm (d) 10.1 cu cm

33. A cylindrical vessel of base radius 14 cm is filled with water to some height. If a rectangular solid of dimensions 22 cm × 7 cm × 5 cm is immersed in it, what is the rise in water level ? **[2009-I]**

(a) 0.5 cm (b) 1.0 cm
(c) 1.25 cm (d) 1.5 cm

34. A cylindrical rod of length h is melted and cast into a cone of base radius twice that of the cylinder. What is the height of the cone? **[2009-I]**

(a) $3h/4$ (b) $4h/3$
(c) $2h$ (d) $h/2$

35. A roller of diameter 70 cm and length 2 m is rolling on the ground. What is the area covered by the roller in 50 revolutions? **[2009-I]**

(a) 180 sq m (b) 200 sq m
(c) 220 sq m (d) 240 sq m

36. The diameter of the Moon is approximately one-fourth of that of the Earth. What is the (approximate) ratio of the volume of the Moon to that of Earth? **[2009-I]**

(a) 1/16 (b) 1/32
(c) 1/48 (d) 1/64

37. Three cubes each of side 5 cm are joined end to end. What is the surface area of the resulting cuboid? **[2009-I]**

(a) 300 sq cm (b) 350 sq cm
(c) 375 sq cm (d) 400 sq cm

38. A solid cone of height 8 cm and base radius 6 cm is melted and recast into identical cones, each of height 2 cm and radius 1 cm. What is the number of cones formed? **[2009-II]**

(a) 36 (b) 72
(c) 144 (d) 180

39. From a cylindrical log whose height is equal to its diameter, the greatest possible sphere has been taken out. What is the fraction of the original log which is cut away? **[2009-II]**

(a) 1/2 (b) 1/3
(c) 1/4 (d) 2/3

40. If a sphere of radius 10 cm is intersected by a plane at a distance 8 cm from its centre, what is the radius of the curve of intersection of the plane and the sphere? **[2009-II]**

(a) 8 cm (b) 6 cm
(c) 5 cm (d) 4 cm

41. A cylindrical can of internal diameter 24 cm contains water. A solid sphere of radius 6 cm is completely immersed in water in the cylinder. The water level increases by **[2009-II]**

(a) 0.25 cm (b) 0.5 cm
(c) 2 cm (d) 3 cm

42. A bucket which is in the form of a frustum of a cone, has radii 3 and 5 unit and vertical height 6 unit. How much water can the bucket hold? **[2009-II]**

(a) 33π cu unit (b) 45π cu unit
(c) 48π cu unit (d) None of these

43. How many litres of water (approximately) can a hemispherical container of radius 21 cm hold? **[2009-II]**

(a) 19.4 L (b) 38.8 L
(c) 194 L (d) 388 L

44. How many litres of water flow out of a pipe having an area of cross-section of 5 cm^2 in one minute, if the speed of water in the pipe is 30 cm/s? **[2009-II]**

(a) 90 L (b) 15 L
(c) 9 L (d) 1.5 L

45. A right ΔABC with sides 5 cm, 12 cm and 13 cm is revolved about the side 12 cm. What is the volume of the solid so obtained? **[2009-II]**

(a) 50π cu cm (b) 100π cu cm
(c) 125π cu cm (d) 150π cu cm

46. The ratio of volumes of two cones is 4 : 5 and the ratio of the radii of their bases is 2 : 3. What is the ratio of their vertical heights? **[2009-II]**

(a) 5 : 6 (b) 6 : 5
(c) 9 : 5 (d) 5 : 9

47. The paint in a certain container is sufficient to paint an area equal to 5.875 m^2. How many bricks of dimensions 12.5 cm × 10 cm × 7.5 cm can be painted out of this container? **[2009-II]**

(a) 225 (b) 180
(c) 150 (d) 100

48. The inner and outer radii of a 7 m long hollow iron right circular cylindrical pipe are 2 cm and 4 cm, respectively. If 1000 cm^3 of iron weight 5 kg, then what is the weight of the pipe? **[2010-I]**

(a) 264 kg (b) 132 kg
(c) 396 kg (d) None of these

49. A sphere and a cube have same surface area. What is the ratio of the square of volume of the sphere to the square of volume of the cube? **[2010-I]**

(a) $\pi : 6$ (b) $1 : 1$
(c) $6 : \pi$ (d) $3 : \pi$

50. A hemisphere is made of a sheet of a metal 1 cm thick. If the outer radius is 5 cm. What is the weight of the hemisphere (1 cm^3 of the metal weight 9 g)? **[2010-I]**

(a) 54π g (b) 366π g
(c) 122π g (d) 108π g

51. A hemispherical bowl of internal radius 20 cm contains sauce. The sauce is to be filled in conical shaped bottles of radius 5 cm and height 8 cm. What is the number of bottles required? **[2010-I]**

(a) 100 (b) 90
(c) 80 (d) 75

52. Smaller lead shots are to be prepared by using the material of a spherical lead shot of radius 1 cm. Some possibilities are listed in the statements given below **[2010-I]**

I. The material is just sufficient to prepare 8 shots each of radius 0.5 cm.

II. A shot of radius 0.75 cm and a second shot of radius 0.8 cm can be prepared from the available matereial.

Which of the above statements is/are correct?

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

53. A semicircular thin sheet of a metal of diameter 28 cm is bent and an open conical cup is made. What is the capacity of the cup? **[2010-I]**

(a) $\frac{1000}{3}\sqrt{3}$ cu cm (b) $300\sqrt{3}$ cu cm
(c) $\frac{700}{3}\sqrt{3}$ cu cm (d) $\frac{1078}{3}\sqrt{3}$ cu cm

54. The volume of a cone is equal to that of a sphere. If the diameter of base of cone is equal to the diameter of the sphere, then what is the ratio of height of cone to the diameter of the sphere? **[2010-I]**
(a) 2 : 1 (b) 1 : 2
(c) 3 : 1 (d) 4 : 1

55. A cylinder having base of circumference 60 cm is rolling without sliding at a rate of 5 rounds per second. How much distance will the cylinder roll in 5 *s* ? **[2010-I]**
(a) 15 m (b) 1.5 m
(c) 30 m (d) 3 m

56. What is volume of the frustum of a cone with height h and radii r_1, r_2? **[2010-I]**
(a) $\frac{1}{3}\pi h\left(r_1^2+r_2^2\right)$ (b) $\frac{1}{3}\pi h\left(r_1^2+r_2^2+r_1r_2\right)$
(c) $\frac{1}{3}\pi h\left(r_1^2+r_2^2-r_1r_2\right)$ (d) $\frac{1}{3}\pi h\left(r_1^2-r_2^2\right)$

57. A rectangular tank whose length and breadth are 2.5 m and 1.5 m, respectively is half full of water. If 750 L more water is poured into the tank, then what is the height through which water level further goes up? **[2010-I]**
(a) 20 cm (b) 18 cm
(c) 15 cm (d) 200 cm

58. The length, breadth and height of a rectangular parallelopiped are in ratio 6 : 3 : 1. If the surface area of a cube is equal to the surface area of this parallelopiped, then what is the ratio of the volume of the cube to the volume of the parallelopiped? **[2010-I]**
(a) 1 : 1 (b) 5 : 4
(c) 7 : 5 (d) 3 : 2

59. In order to fix an electric pole along a roadside, a pit with dimensions 50 cm × 50 cm is dug with the help of a spade. The pit is prepared by removing Earth by 250 strokes of spade. If one stroke of spade removes 500 cm^3 of Earth, then what is the depth of the pit? **[2010-II]**
(a) 2 m (b) 1 m
(c) 0.75 m (d) 0.5 m

60. If three cubic biscuits having edges 0.3 m, 0.4 m and 0.5 m respectively are melted and formed into a single cubic biscuit, then what is the total surface area of the cubic biscuit? **[2010-II]**
(a) 1.08 sq m (b) 1.56 sq m
(c) 1.84 sq m (d) 2.16 sq m

61. Half of a large cylindrical tank open at the top is filled with water and identical heavy spherical balls are to be dropped into the tank without spilling water out. If the radius and the height of the tank are equal and each is four times the radius of a ball, then what is the maximum number of balls that can be dropped? **[2010-II]**
(a) 12 (b) 24
(c) 36 (d) 48

62. The diagonals of the three faces of a cuboid are x, y and z, respectively. What is the volume of the cuboid? **[2010-II]**
(a) $\frac{xyz}{2\sqrt{2}}$
(b) $\frac{\sqrt{(y^2+z^2)(z^2+x^2)(x^2+y^2)}}{2\sqrt{2}}$
(c) $\frac{\sqrt{(y^2+z^2-x^2)(z^2+x^2-y^2)(x^2+y^2-z^2)}}{2\sqrt{2}}$
(d) None of the above

63. A figure is formed by revolving a rectangular sheet of dimensions 7 cm × 4 cm about its length. What is the volume of the figure, thus formed? **[2010-II]**
(a) 352 cu cm (b) 296 cu cm
(c) 176 cu cm (d) 616 cu cm

64. A solid cylinder of height 9 m has its curved surface area equal to one-third of the total surface area. What is the radius of the base? **[2010-II]**
(a) 9 m (b) 18 m
(c) 27 m (d) 30 m

65. The volume of a sphere is 8 times that of another sphere. What is the ratio of their surface areas? **[2010-II]**
(a) 8 : 1 (b) 4 : 1
(c) 2 : 1 (d) 4 : 3

66. What is the maximum length of rod that can be placed inside a box having the shape of a cuboid of langth 30 cm, breadth 24 cm and height 18 cm? **[2010-II]**
(a) 30 cm (b) $30\sqrt{2}$ cm
(c) 24 cm (d) $18\sqrt{5}$ cm

DIRECTIONS (Qs. 67-69) : *Read the following information carefully to answer the questions that follow.*

Let C be a right circular cone. It is given that the two ends of a frustum of C are of radii 3 cm and 6 cm and the height of the frustum is 9 cm. **[2010-II]**

67. What is the total surface area of the given frustum?
(a) $9\pi(2\sqrt{10}+5)$ sq cm
(b) $9\pi(3\sqrt{10}+5)$ sq cm
(c) $9\pi(3\sqrt{10}+4)$ sq cm
(d) $27\pi(\sqrt{10}+1)$ sq cm

68. What is the height of the cone?
(a) 9 cm (b) 12 cm
(c) 13.5 cm (d) 18 cm

69. What is the slant height of the given frustum?
(a) $3\sqrt{10}$ cm (b) $6\sqrt{10}$ cm
(c) 12 cm (d) 15 cm

70. The diagonal of a cube is $4\sqrt{3}$ cm. What is its volume? **[2011-I]**
(a) 16 cu cm (b) 32 cu cm
(c) 64 cu cm (d) 192 cu cm

71. A cylindrical rod of iron whose radius is one-fourth of its height is melted and cast into spherical balls of the same radius as that of the cylinder. What is the number of spherical balls? **[2011-I]**
(a) 2 (b) 3
(c) 4 (d) 5

72. The outer and inner diameters of a circular pipe are 6 cm and 4 cm, respectively. If its length is 10 cm, then what is the total surface area in sq cm? **[2011-I]**
(a) 35π (b) 110π
(c) 150π (d) None of these

73. A cistern 6 m long and 4 m wide contains water to a depth of 1.25 m. What is the area of wetted surface? **[2011-I]**
(a) 40 sq m (b) 45 sq m
(c) 49 sq m (d) 73 sq m

74. A toy is in the form of a cone mounted on a hemisphere such that the diameter of the base of the cone is equal to that of the hemisphere. If the diameter of the base of the cone is 6 cm and its height is 4 cm, what is the surface area of the toy in sq cm? (take $\pi = 3.14$) **[2011-I]**
(a) 93.62 (b) 103.62
(c) 113.62 (d) 115.50

75. What is the volume (in cu cm) of a spherical shell with 8 cm and 10 cm as its internal and external diameters respectively? **[2011-I]**
(a) $\frac{61\pi}{3}$ (b) $\frac{122\pi}{3}$
(c) $\frac{244\pi}{3}$ (d) $\frac{250\pi}{3}$

76. The curved surface of a cylinder is 1000 sq cm. A wire of diameter 5 mm is wound around it, so as to cover it completely. What is the length of the wire used? **[2011-I]**
(a) 22m (b) 20m
(c) 18m (d) None of these

77. The material of a solid cone is converted into the shape of a solid cylinder of equal radius. If the height of the cylinder is 5 cm, what is the height of the cone? **[2011-I]**
(a) 15 cm (b) 20 cm
(c) 25 cm (d) 30 cm

78. The surface area of a sphere is 616 sq cm. If its radius is changed so that the area gets reduced by 75%, then the radius becomes **[2011-II]**
(a) 1.6 cm (b) 2.3 cm
(c) 2.5 cm (d) 3.5 cm

79. A cylindrical vessel of height 10 cm has base radius 60 cm. If d is the diameter of a spherical vessel of equal volume, then what is d equal to? **[2011-II]**
(a) 30 cm (b) 60 cm
(c) 90 cm (d) 120 cm

80. A sphere is inscribed in a cubical box such that the sphere is tangent to all six faces of the box. What is the ratio of the volume of the cubical box to the volume of sphere? **[2011-II]**
(a) 6π (b) 36π
(c) $4\pi/3$ (d) $6/\pi$

81. If the diameter of a sphere is doubled, then how does its surface area change? **[2011-II]**
(a) It increases two times
(b) It increases three times
(c) It increases four times
(d) It increases eight times

82. From a solid cylinder of height 4 cm and radius 3 cm, a conical cavity of height 4 cm and of base radius 3 cm is hollowed out. What is the total surface area of the remaining solid? **[2011-II]**
(a) 15π sq cm (b) 22π sq cm
(c) 33π sq cm (d) 48π sq cm

83. A hollow sphere of internal and external diameters 4 cm and 8 cm respectively is melted into a cone of base diameter 8 cm. What is the height of the cone? **[2011-II]**
(a) 12 cm (b) 14 cm
(c) 17 cm (d) 18 cm

84. The radii of two cylinders are in the ratio 2 : 3 and their curved surface areas are in the ratio 5 : 3. What is the ratio of their volumes? **[2011-II]**
(a) 20 : 27 (b) 10 : 9
(c) 9 : 10 (d) 27 : 20

85. A cylindrical tank 7 m in diameter, contains water to a depth of 4 m. What is the total area of wetted surface? **[2011-II]**
(a) 110.5 sq m (b) 126.5 sq m
(c) 131.5 sq m (d) 136.5 sq m

86. If S is the total surface area of a cube and V is its volume then which one of the following is correct? **[2011-II]**
(a) $V^3 = 216\,S^2$ (b) $S^3 = 216\,V^2$
(c) $S^3 = 6\,V^2$ (d) $S^2 = 36\,V^3$

87. The radii of the circular ends of a bucket of height 40 cm are of lengths 35 cm and 14 cm. What is the volume of the bucket? **[2011-II]**
(a) 60060 cu cm (b) 70040 cu cm
(c) 80080 cu cm (d) 80160 cu cm

88. 10 cylindrical pillars of a building have to be painted. The diameter of each pillar is 70 cm and the height is 4 m. What is the cost of painting at the rate of ₹ 5 per sq m? **[2011-II]**
(a) ₹ 400 (b) ₹ 440
(c) ₹ 480 (d) ₹ 500

89. What is the number of wax balls, each of radius 1 cm, that can be molded out of a sphere of radius 8 cm? **[2011-II]**
(a) 256 (b) 512
(c) 768 (d) 1024

90. What will be the cost to plaster the inner surface of a well 14 m deep and 4 m in diameter at the rate of ₹ 25 per sq m? **[2012-I]**
(a) ₹ 4000 (b) ₹ 4200
(c) ₹ 4400 (d) ₹ 5400

91. What is the length of the uniform wire of diameter 0.4 cm that can be drawn from a solid sphere of radius 9 cm? **[2012-I]**
(a) 243 m (b) 240 m
(c) 60.75 cm (d) 60 m

92. The total surface area of a cube is 150 sq cm. What is its volume? **[2012-I]**
(a) 64 cu cm (b) 81 cu cm
(c) 125 cu cm (d) 160 cu cm

93. If the volume of a cube is 729 cu cm, then what is the length of its diagonal? **[2012-I]**
(a) $9\sqrt{2}$ cm (b) $9\sqrt{3}$ cm
(c) 18 cm (d) $18\sqrt{3}$ cm

94. The curved surface area of a right circular cone of radius 14 cm is 440 sq cm. What is the slant height of the cone? **[2012-I]**
(a) 10 cm (b) 11 cm
(c) 12 cm (d) 13 cm

95. A cardboard sheet in the form of a circular sector of radius 30 cm and central angle 144º is folded to make a cone. What is the radius of the cone? **[2012-I]**
(a) 12 cm (b) 18 cm
(c) 21 cm (d) None of these

96. A large solid metallic cylinder whose radius and height are equal to each other is to be melted and 48 identical solid balls are to be recast from the liquid metal, so formed. What is the ratio of the radius of a ball to the radius of the cylinder? **[2012-I]**
(a) 1 : 16 (b) 1 : 12
(c) 1 : 8 (d) 1 : 4

97. What are the dimensions (length, breadth and height, respectively) of a cuboid with volume 720 cu cm, surface area 484 sq cm and the area of the base 72 sq cm? **[2012-I]**
(a) 9, 8 and 10 cm (b) 12, 6 and 10 cm
(c) 18, 4 and 10 cm (d) 30, 2 and 12 cm

98. If the surface area of a sphere is 616 sq cm, then what is its volume? **[2012-I]**
(a) 4312/3 cu cm (b) 4102/3 cu cm
(c) 1257 cu cm (d) 1023 cu cm

DIRECTIONS (Qs. 99-100) : *The areas of the ends of a frustum of a cone are P and Q, where P < Q and H is its thickness.*

99. What is the volume of the frustum?
(a) $3H(P+Q+\sqrt{PQ})$ (b) $H(P+Q+\sqrt{PQ})$
(c) $H(P+Q+\sqrt{PQ})/3$ (d) $2H(P+Q+\sqrt{PQ})/3$

100. What is the difference in radii of the ends of the frustum?
(a) $\frac{\sqrt{Q}-\sqrt{P}}{\sqrt{\pi}}$ (b) $\frac{\sqrt{Q}-\sqrt{P}}{\pi}$
(c) $\sqrt{Q}-\sqrt{P}$ (d) None of these

101. 10 circular plates each of thickness 3 cm, each are placed one above the other and a hemisphere of radius 6 cm is placed on the top just to cover the cylindrical solid. What is the volume of the solid so formed? **[2012-II]**
(a) 264 π cu cm (b) 252π cu cm
(c) 236 π cu cm (d) None of these

102. Let the largest possible right circular cone and largest possible sphere be fitted into two cubes of same length. If C and S denote the volume of cone and volume of sphere, respectively. Then, which one of the following is correct? **[2012-II]**
(a) C = 2S (b) S = 2C
(c) C = S (d) C = 3S

103. A right circular metal cone (solid) is 8 cm high and the radius is 2 cm. It is melted and recast into a sphere. What is the radius of the sphere? **[2012-II]**
(a) 2 cm (b) 3 cm
(c) 4 cm (d) 5 cm

104. The volume of a cube is numerically equal to sum of its edges. What is the total surface area in square units? **[2012-II]**
(a) 12 (b) 36
(c) 72 (d) 144

105. The diameter of base of a right circular cone is 7 cm and slant height is 10 cm, then what is its lateral surface area? **[2012-II]**
(a) 110 sq cm (b) 100 sq cm
(c) 70 sq cm (d) 49 sq cm

106. What is the height of a solid cylinder of radius 5 cm and total surface area is 660 sq cm? **[2012-II]**
(a) 10 cm (b) 12 cm
(c) 15 cm (d) 16 cm

107. If the ratio of the diameters of two spheres is 3 : 5, then what is the ratio of their surface areas? **[2012-II]**
(a) 9 : 25 (b) 9 : 10
(c) 3 : 5 (d) 27 : 125

108. What is the volume of the largest sphere that can be carved out from a cube of edge 3 cm? **[2012-II]**
(a) 9π cu cm (b) 6π cu cm
(c) 4.5π cu cm (d) 3π cu cm

109. What is the quantity of cloth required to roll up to form a right circular tent whose base is of radius 12 m and height 5 m? **[2013-I]**
(a) 40π sq m (b) 60π sq m
(c) 78π sq m (d) 156π sq m

110. The length, breadth and height of a box are respectively 14 m, 12 m and 13 m. The length of the greatest rod that can be put in it is **[2013-I]**
(a) 22.31 m (b) 22.56 m
(c) 20 m (d) 19.5 m

111. The volume of the material of a hemispherical shell with outer and inner radii 9 cm and 7 cm, respectively is approximately **[2013-I]**
(a) 808 cu cm (b) 800 cu cm
(c) 816 cu cm (d) 824 cu cm

112. The ratio of surface area to diameter of a sphere whose volume is 36 π cu cm is **[2013-I]**
(a) 3π (b) 6π
(c) 6 (d) None of these

113. A cylindrical tube open at both ends is made of metal. The internal diameter of the tube is 6 cm and length of the tube is 10 cm. If the thickness of the metal used is 1 cm, then the outer curved surface area of the tube is **[2013-I]**
(a) 140π sq cm (b) 146.5π sq cm
(c) 70π sq cm (d) None of these

114. The volume of a right circular cone of height 3 cm and slant height 5 cm is **[2013-I]**
(a) 49.3 cu cm (b) 50.3 cu cm
(c) 52 cu cm (d) 53 cu cm

115. A bucket is of a height 25 cm. Its top and bottom radii are 20 cm and 10 cm, respectively. Its capacity (in L) is **[2013-I]**
(a) 17.5π/3 (b) 17.5π
(c) 20 π (d) 25π

116. The height of a cylinder is 15 cm. The lateral surface area is 660 sq cm. Its volume is **[2013-I]**
(a) 1155 cu cm (b) 1215 cu cm
(c) 1230 cu cm (d) 2310 cu cm

117. From a solid wooden right circular cylinder, a right circular cone whose radius and height are same as the radius and height of the cylinder, respectively is carved out. What is the ratio of the volume of the utilised wood to that of the wasted wood? **[2013-I]**
(a) 1 : 2 (b) 2 : 1
(c) 2 : 3 (d) 1 : 3

118. A solid spherical ball of iron of radius 4 cm is melted to form spheres of radius 2 cm each. The number of spheres, so formed is **[2013-I]**
(a) 8 (b) 9
(c) 10 (d) 16

119. If the heights and the areas of the base of a right circular cone and a pyramid with square base are the same, then they have **[2013-I]**
(a) same volume and same surface area
(b) same surface area but different volumes
(c) same volume but different surface areas
(d) different volumes and different surface areas

120. The diameter of the Moon is approximately one-fourth of the diameter of the Earth. What is the ratio (approximate) of their volumes? **[2013-II]**
(a) 1 : 16 (b) 1 : 64
(c) 1 : 4 (d) 1 : 128

121. If the total surface area of a cube is 6 sq units, then what is the volume of the cube? **[2013-II]**
(a) 1 cu unit (b) 2 cu units
(c) 4 cu units (d) 6 cu units

122. Let A be a pyramid on a square base and B be a cube. If a, b and c denote the number of edges, number of faces and number of corners, respectively. Then, the result $a = b + c$ is true for **[2013-II]**
(a) Only A (b) Only B
(c) Both A and B (d) Neither A nor B

123. What is the volume of a cone having a base of radius 10 cm and height 21 cm? **[2013-II]**
(a) 2200 cm^3 (b) 3000 cm^3
(c) 5600 cm^3 (d) 6600 cm^3

124. A conical cap has base diameter 24 cm and height 16 cm. What is the cost of painting the surface of the cap at the rate of 70 paise per sq cm? **[2013-II]**
(a) ₹ 520 (b) ₹ 524
(c) ₹ 528 (d) ₹ 532

125. What is the whole surface area of a cone of base radius 7 cm and height 24 cm? **[2013-II]**
(a) 654 sq cm (b) 704 sq cm
(c) 724 sq cm (d) 964 sq cm

126. A tent is in the form of a right circular cylinder surmounted by a cone. The diameter of the cylinder is 24 m. The height of the cylindrical portion is 11 m, while the vertex of the cone is 16 m above the ground. What is the area of the curved surface for conical portion? **[2013-II]**
(a) 3434/9 sq m (b) 3431/8 sq m
(c) 3432/7 sq m (d) 3234/7 sq m

127. If x is the curved surface area and y is the volume of a right circular cylinder, then which one of the following is correct? **[2013-II]**
(a) Only the ratio of the height to radius of the cylinder is independent of x
(b) Only the ratio of height to radius of the cylinder is independent of y
(c) Either (a) or (b)
(d) Neither (a) nor (b)

128. A cylinder is surmounted by a cone at one end, a hemisphere at the other end. The common radius is 3.5 cm, the height of the cylinder is 6.5 cm and the total height of the structure is 12.8 cm. The volume V of the structure lies between **[2014-I]**
(a) 370 cm^3 and 380 cm^3 (b) 380 cm^3 and 390 cm^3
(c) 390 cm^3 and 400 cm^3 (d) None of these

129. The diameter of the base of a cone is 6 cm and its altitude is 4 cm. What is the approximate curved surface area of the cone? **[2014-I]**
(a) 45 cm^2 (b) 47 cm^2
(c) 49 cm^2 (d) 51 cm^2

130. A drainage tile is a cylindrical shell 21 cm long. The inside and outside diameters are 4.5 cm and 5.1 cm, respectively. What is the volume of the clay required for the tile? **[2014-I]**
(a) $6.96\pi\, cm^3$ (b) $6.76\pi\, cm^3$
(c) $5.76\pi\, cm^3$ (d) None of these

131. For a plot of land of 100 m × 80 m, the length to be raised by spreading the earth from stack of a rectangular base 10 m × 8 m and vertical section being a trapezium of height 2 m. The top of the stack is 8 m × 5 m. How many centimeters can the level raised? **[2014-I]**
(a) 3 cm (b) 2.5 m
(c) 2 cm (d) 1.5 cm

132 A cube has each edge 2 cm and a cuboid is 1 cm long, 2 cm wide and 3 cm high. The paint in a certain container is sufficient to paint an area equal to 54 cm^2.
Which one of the following is correct? **[2014-I]**
(a) Both cube and cuboid can be painted
(b) Only cube can be painted
(c) Only cuboid can be painted
(d) Neither cube nor cuboid can be painted

133. A cone of radius r cm and height h cm is divided into two parts by drawing a plane through the middle point of its height and parallel to the base. What is the ratio of the volume of the original cone to the volume of the smaller cone? **[2014-I]**
(a) 4 : 1 (b) 8 : 1
(c) 2 : 1 (d) 6 : 1

134. If 64 identical small spheres are made out of big sphere of diameter 8 cm, then what is surface area of each small sphere? **[2014-I]**
(a) $\pi\, cm^2$ (b) $2\pi\, cm^2$
(c) $4\pi\, cm^2$ (d) $8\pi\, cm^2$

135. The dimensions of a field are 15 m by 12 m. A pit 8 m long, 2.5 m wide and 2 m deep is dug in one corner of the field and the earth removed is evenly spread over the remaining area of the field. The level of the field is raised by **[2014-I]**
(a) 15 cm (b) 20 cm
(c) 25 cm (d) $\frac{200}{9}$ cm

136.. What is the diameter of the largest circle lying on the surface of a sphere of surface area 616 sq cm? **[2014-I]**
(a) 14 cm (b) 10.5 cm
(c) 7 cm (d) 3.5 cm

137. The volume of a hollow cube is $216x^3$. What surface area of the largest sphere which be enclosed in it? **[2014-I]**
(a) $18\pi x^2$ (b) $27\pi x^2$
(c) $36\pi x^2$ (d) $72\pi x^2$

138. A cylinder circumscribes a sphere. What is the ratio of volume of the sphere to that of the cylinder? **[2014-II]**
(a) 2 : 3 (b) 1 : 2
(c) 3 : 4 (d) 3 : 2

139. Consider the following statements :
1. The volume of the cone generated when the triangle is made to revolve about its longer leg is same as the volume of the cone generated when the triangle is made to revolve about its shorter leg.
2. The sum of the volume of the cone generated when the triangle is made to revolve about its longer leg and the volume of the cone generated when the triangle is made to revolve about its shorter leg is equal to the volume of the double cone generated when the triangle is made to revolve about its hypotenuse.

Which of the above statements is/are correct ? **[2014-II]**
(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

140. If the side of a cube is increased by 100%, then by what percentage is the surface area of the cube increased ? **[2014-II]**
(a) 150% (b) 200%
(c) 300% (d) 400%

141. Consider the following statements in respect of four spheres *A*,*B*, *C* and *D* having respective radii 6, 8, 10 and 12 cm.
1. The surface area of sphere *C* is equal to the sum of surface areas of sphere *A* and *B* .
2. The volume of sphere *D* is equal to the sum of volumes of sphere *A, B* and *C.*

Which of the above statements is / are correct ? **[2014-II]**
(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

142. The diameter of a metallic sphere is 6 cm. The sphere is melted and drawn into a wire of uniform circular cross-section. If the length of the wire is 36m, then what is its radius equal to ? **[2014-II]**
(a) 0.1 cm (b) 0.01 cm
(c) 0.0001 cm (d) 1.0 cm

143. A cylindrical vessel of radius 4 cm contains water. A solid sphere of radius 3 cm is lowered into the water until it is completely immersed. The water level in the vessel will rise by **[2015-I]**
(a) 1.5 cm (b) 2 cm
(c) 2.25 cm (d) 4.5 cm

144. If the radius of a sphere is increased by 10%, then the volume will be increased by **[2015-I]**
(a) 33.1% (b) 30%
(c) 50% (d) 10%

145. A sphere and a cube have same surface area. The ratio of square of their volumes is **[2015-I]**
(a) $6 : \pi$ (b) $5 : \pi$
(c) 3 : 5 (d) 1 : 1

146. The radius of a sphere is equal to the radius of the base of a right circular cone, and the volume of the sphere is double the volume of the cone. The ratio of the height of the cone to the radius of its base is **[2015-I]**
(a) 2 : 1 (b) 1 : 2
(c) 2 : 3 (d) 3 : 2

147. Water flows through a cylindrical pipe of internal diameter 7 cm at the rate of 5 m/s. The time, in minutes, the pipe would take to fill an empty rectangular tank 4m × 3m × 2.31m is **[2015-I]**
(a) 28 (b) 24 (c) 20 (d) 12

148. The total outer surface area of a right circular cone of height 24 cm with a hemisphere of radius 7 cm upon its base is **[2015-I]**
(a) 327π square cm (b) 307π square cm
(c) 293π square cm (d) 273π square cm

149. A rectangular block of wood having dimensions 3m × 2m × 1.75m has to be painted on all its faces. The layer of paint must be 0.1 mm thick. Paint comes in cubical boxes having their edges equal to 10 cm. The minimum number of boxes of paint to be purchased is **[2015-I]**
(a) 5 (b) 4 (c) 3 (d) 2

150. The diagonals of three faces of a cuboid are 13, $\sqrt{281}$ and 20 linear units. Then the total surface area of the cuboid is **[2015-I]**
(a) 650 square units (b) 658 square units
(c) 664 square units (d) 672 square units

151. A rectangular paper of 44 cm long and 6 cm wide is rolled to form a cylinder of height equal to width of the paper. The radius of the base of the cylinder so rolled is **[2015-I]**
(a) 3.5 cm (b) 5 cm
(c) 7 cm (d) 14 cm

152. If three metallic spheres of radii 6 cm, 8 cm and 10 cm are melted to form a single sphere, then the diameter of the new sphere will be **[2015-I]**
(a) 12 cm (b) 24 cm
(c) 30 cm (d) 36 cm

153. If the height of a right circular cone is increased by 200% and the radius of the base is reduced by 50%, then the volume of the cone **[2015-I]**
(a) remains unaltered (b) decreases by 25%
(c) increases by 25% (d) increases by 50%

154. A pipe with square cross–section is supplying water to a cistern which was initially empty. The area of cross–section is 4 cm^2 and the nozzle velocity of water is 40 m/s. The dimensions of the cistern are 10 m × 8 m × 6 m. Then the cistern will be full in **[2015-II]**
(a) 9.5 hours (b) 9 hours
(c) 8 hours 20 minutes (d) 8 hours

155. A hollow cylindrical drum has internal diameter of 30cm and a height of 1 m. What is the maximum number of cylindrical boxes of diameter 10 cm and height 10 cm each that can be packed in the drum ? **[2015-II]**
(a) 60 (b) 70 (c) 80 (d) 90

156. Consider the following statements : **[2015-II]**
1. If the height of a cylinder is doubled, the area of the curved surface is doubled.
2. If the radius, of a hemispherical solid is doubled, its total surface area becomes fourfold.

Which of the above statements is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

157. A large water tank has the shape of a cube. If 128 m^3 of water is pumped out, the water level goes down by 2 m. Then the maximum capacity of the tank is **[2015-II]**
(a) 512 m^3 (b) 480 m^3
(c) 324 m^3 (d) 256 m^3

158. From the solid gold in the form of a cube of side length 1 cm, spherical solid balls each having the surface area $\frac{1}{\pi^3}$ cm^2 are to be made. Assuming that there is no loss of the material in the process of making the balls, the maximum number of balls made will be **[2015-II]**
(a) 3 (b) 4
(c) 6 (d) 9

159. Water is filled in a container in such a manner that its volume doubles every 5 minutes. If it takes 30 minutes for the container to be full, in how much time will it be one–fourth full? **[2015-II]**
(a) 7.5 minutes (b) 15 minutes
(c) 20 minutes (d) 17.5 minutes

160. 30 metallic cylinders of same size are melted and cast in the form of cones having the same radius and height as those of the cylinders. **[2015-II]**
Consider the following statements :
Statement I : A maximum of 90 cones will be obtained.
Statement II : The curved surface of the cylinder can be flattened in the shape of a rectangle but the curved surface of the cone when flattened has the shape of triangle.
Which one of the following is correct in respect of the above?
(a) Both Statement I and Statement II are correct and Statement II is the correct explanation of Statement I
(b) Both Statement I and Statement II are correct and Statement II is not the correct explanation of StatementI
(c) Statement I is correct but Statement II is not correct
(d) Statement I is not correct but Statement II is correct

161. A water tank, open at the top, is hemispherical at the bottom and cylindrical above it. The radius is 12m and the capacity is 3312π m^3. The ratio of the surface areas of the spherical and cylindrical portions is **[2015-II]**
(a) 3 : 5 (b) 4 : 5
(c) 1 : 1 (d) 6 : 5

162. The areas of three mutually perpendicular faces of a cuboid are x, y, z. If V is the volume, then xyz is equal to **[2015-II]**
(a) V (b) V^2
(c) 2V (d) $2V^2$

163. Let V be the volume of an inverted cone with vertex at origin and the axis of the cone is along positive y–axis. The cone is filled with water up to half of its height. The volume of water is **[2015-II]**
(a) $\frac{V}{8}$ (b) $\frac{V}{6}$
(c) $\frac{V}{3}$ (c) $\frac{V}{2}$

164. If the surface area of a cube is 13254 cm^2, then the length of its diagonal is **[2015-II]**
(a) $44\sqrt{2}$ cm (b) $44\sqrt{3}$ cm
(c) $47\sqrt{2}$ (d) $47\sqrt{3}$ cm

165. How many spherical bullets each of 4 cm in diameter can be made out of a cube of lead whose edge is 44 cm ? **[2015-II]**
(a) 2541 (b) 2551 (c) 2561 (d) 2571

166. A river 2.5 m deep and 45 m wide is flowing at the speed of 3.6 km/hour. The amount of water that runs into the sea per minute is **[2015-II]**
(a) 6650 m^3 (b) 6750 m^3
(c) 6850 m^3 (d) 6950 m^3

167. The area of four walls of a room is 120m^2. The length of the room is twice its breadth. If the height of the room is 4 m, what is area of the floor ? **[2016-II]**
(a) 40 m^2 (b) 50 m^2
(c) 60 m^2 (d) 80 m^2

168. The ratio of the curved surface area to the total surface area of a right circular cylinder is 1 : 2. If the total surface area is 616 cm^2, what is the volume of the cylinder ? **[2016-II]**
(a) 539 cm^3 (b) 616 cm^3
(c) 1078 cm^3 (d) 1232 cm^3

169. A cubic metre of copper weighing 9000 kg is rolled into a square bar 9 m long. An exact cube is cut off from the bar; How much does the cube weigh ? **[2016-II]**
(a) 1000 kg (b) $\frac{1000}{3}$ kg
(c) 300 kg (d) $\frac{500}{3}$ kg

170. Into a conical tent of radius 8.4m and vertical height 3.5 m, how many full bags of wheat can be emptied, if space required for the wheat in each bag is 1.96 m^3 ? . **[2016-II]**
(a) 264 (b) 201
(c) 132 (d) 105

171. A building is in the form of a cylinder surmounted by a hemispherical dome on the diameter of the cylinder. The height of the building is three times the radius of the base of the cylinder. The building contains $67\frac{1}{21}m^3$ of air. What is the height of the building ? **[2016-II]**
(a) 6m (b) 4m
(c) 3m (d) 2m

172. The radius of the base and the height of a solid right circular cylinder are in the ratio 2:3 and its volume is 1617 cm^3. What is the total surface area of the cylinder ? **[2016-II]**
(a) 462 cm^2 (b) 616 cm^2
(c) 770 cm^2 (d) 786 cm^2

173. The cost of painting a spherical vessel of diameter 14 cm is ₹8008. What is the cost of painting per square centimetre? **[2016-II]**
(a) ₹8 (b) ₹9
(c) ₹13 (d) ₹14

174. A drinking glass of height 24 cm is in the shape of frustum of a cone and ' diameters of its bottom and top circular ends are 4 cm and 18 cm respectively. If we take capacity of the glass as πx cm^3, then what is the value of x ? **[2016-II]**
(a) 824 (b) 1236
(c) 1628 (d) 2472

175. Rain water from a roof 22m × 20m drains into a cylindrical vessel having diameter of base 2 m and height 3.5 m. If the vessel is just full, what is the rainfall? **[2016-II]**
(a) 3.5 cm (b) 3 cm
(c) 2.5 cm (d) 2 cm

176. The height of a cone is 60 cm. A small cone is cut off at the top by a plane parallel to the base and its volume is $\frac{1}{64}$ the volume of original cone. What is the height from the base at which the section is made ? **[2016-II]**
(a) 15 cm (b) 20 cm
(c) 30 cm (d) 45 cm

177. What is the volume of a sphere of radius 3 cm? **[2016-II]**
(a) $36\pi\, cm^3$ (b) $18\pi\, cm^3$
(c) $9\pi\, cm^3$ (d) $6\pi\, cm^3$

DIRECTIONS (Qs. 178-180): *A tent of a circus is made of canvas and is in the form of right circular cylinder and right circular cone above it. The height and diameter of the cylindrical part of the tent are 5 m and 126 m respectively. The total height of the tent is 21 m.*

[2016-II]

178. What is the slant height of the cone ?
(a) 60 m (b) 65 m
(c) 68 m (d) 70 m

179. What is the curved surface area of the cylinder ?
(a) $1980\, m^2$ (b) $2010\, m^2$
(c) $2100\, m^2$ (d) $2240\, m^2$

180. How many square metres of canvas are used ?
(a) 14450 (b) 14480
(c) 14580 (d) 14850

181. If the perimeter of a circle is equal to that of a square, then what is the ratio of area of circle to that of square ? **[2016-II]**
(a) 22 : 7 (b) 14 : 11
(c) 7 : 22 (d) 11 : 14

182. If the radius of a right circular cone is increased by p% without increasing its height, then what is the percentage increase in the volume of the cone ? **[2017-I]**
(a) p2 (b) $2P^2$
(c) $\frac{p^2}{100}$ (d) $p\left(2+\frac{p}{100}\right)$

183. If the surface area of a sphere is reduced to one-ninth of the area, its radius reduces to **[2017-I]**
(a) One-fourth (b) One-third
(c) One-fifth (d) One-ninth

184. Ice-cream, completely filled in a cylinder of diameter 35 cm and height 32 cm, is to be served by completely filling identical disposable cones of diameter 4 cm and height 7 cm. The maximum number of persons that can be served in this way is **[2017-I]**
(a) 950 (b) 1000
(c) 1050 (d) 1100

185. If the HCF of polynomials **[2017-II]**
$f(x) = (x - 1)(x^2 + 3x + a)$ and
$g(x) = (x + 2)(x^2 + 2x + b)$ is $(x^2 + x - 2)$,
then what are the values of a and b respectively?
(a) 2, 2 (b) 2, –3
(c) –1, –3 (d) –2, –1

186. A cylinder of height 2x is circumscribed by a sphere of radius 2x such that the circular ends of the cylinder are two small circles on the sphere. What is the ratio of the curved surface area of the cylinder to the surface area of the sphere? **[2017-II]**
(a) $\sqrt{3}:4$ (b) $\sqrt{3}:3$
(c) $\sqrt{3}:2$ (d) $\sqrt{3}:1$

187. A cylindrical vessel 60 cm in diameter is partially filled with water. A sphere 30 cm in diameter is gently dropped into the vessel and is completely immersed. To what further height will the water in the cylinder rise? **[2017-II]**
(a) 20 cm (b) 15 cm
(c) 10 cm (d) 5 cm

188. The vertical angle of a right circular cone is $\frac{\pi}{2}$ and the slant height is $\sqrt{2}$ r cm. What is the volume of the cone in cubic cm? **[2017-II]**
(a) πr^3 (b) $9\pi r^3$
(c) $\frac{\pi r^3}{3}$ (d) $3\pi r^3$

189. The radii of the frustum of a right circular cone are in the ratio 2 : 1. What is the ratio of the volume of the frustum of the cone to that of the whole cone? **[2017-II]**
(a) 1 : 8 (b) 1 : 4
(c) 3 : 4 (d) 7 : 8

190. From a solid cylinder whose height is 8 cm and of base radius 6 cm, a conical cavity of height 8 cm and of base radius 6 cm is formed by hollowing out. What is the inner surface area of the cavity. **[2017-II]**
(a) 6π square cm (b) 8π square cm
(c) 10π square cm (d) 60π square cm

191. A tent has been constructed which is in the form of a right circular cylinder surmounted by a right circular cone whose axis coincides with the axis of the cylinder. If the radius of the base of the cylinder is 50 m, the height of the cylinder is 10 m and the total height of the tent is 15 m, then what is the capacity of the tent in cubic metres? **[2017-II]**
(a) 37500π (b) $\frac{87500\pi}{3}$
(c) $\frac{26500\pi}{3}$ (d) 25000π

192. Two rectangular sheets of sizes $2\pi \times 4\pi$ and $\pi \times 5\pi$ are available. A hollow right circular cylinder can be formed by joining a pair of parallel sides of any sheet. What is the maximum possible volume of the circular cylinder that can be formed this way? **[2017-II]**
(a) $4\pi^2$ (b) $8\pi^2$ (c) $1.25\pi^2$ (d) $6.25\pi^2$

193. The radius and slant height of a right circular cone are 5 cm and 13 cm respectively. What is the volume of the cone? **[2018-I]**
(a) $100\pi\,cm^3$ (b) $50\pi\,cm^3$
(c) $65\pi\,cm^3$ (d) $169\pi\,cm^3$

194. If the ratio of the radius of the base of a right circular cone to its slant height is 1 : 3, what is the ratio of the total surface area to the curved surface area? **[2018-I]**
(a) 5 : 3 (b) 3 : 1 (c) 4 : 1 (d) 4 : 3

195. A right circular cone is sliced into a smaller cone and a frustum of a cone by a plane perpendicular to its axis. The volume of the smaller cone and the frustum of the cone are in the ratio 64 : 61. Then their curved surface areas are in the ratio **[2018-I]**
(a) 4 : 1 (b) 16 : 9 (c) 64 : 61 (d) 81 : 64

196. In a room whose floor is a square of side 10 m, an equilateral triangular table of side 2 m is placed. Four book-shelves of size 4m × 1m × 9m are also placed in the room. If half of the rest of the area in the room is to be carpeted at the rate of ₹100 per square metre, what is the cost of carpeting (approximately)? **[2018-I]**
(a) ₹ 7,600 (b) ₹ 5,635
(c) ₹ 4,113 (d) ₹ 3,200

197. A region of area A bounded by a circle C is divided into n regions, each of area A/n, by drawing circles of radii $r_1, r_2, r_3, ... r_{n-1}$ such that $r_1 < r_2 < r_3 < .... r_{n-1}$ concentric with the circle C. If $p_m = \frac{r_{m+1}}{r_m}$ where m = 1, 2, 3, ... (n – 2), then which one of the following is correct? **[2018-I]**
(a) p increases as m increases
(b) p decreases as m increases
(c) p remains constant as m increases
(d) p increases for some values of m as m increases and then decreases thereafter

198. What is the volume of a cone of maximum volume cut out from a cube of edge 2a such that their bases are on the same plane? **[2018-I]**
(a) πa^3 (b) $\frac{\pi a^3}{3}$ (c) $\frac{2\pi a^3}{3}$ (d) $\frac{3\pi a^3}{4}$

199. There are as many square centimetres in the surface area of a sphere as there are cubic centimetres in its volume. What is the radius of the sphere? **[2018-I]**
(a) 4 cm (b) 3 cm
(c) 2 cm (d) 1 cm

200. The curved surface area of a right circular cone is $1.76\ m^2$ and its base diameter is 140 cm. What is the height of the cone? **[2018-I]**
(a) 10 cm (b) $10\sqrt{2}$ cm
(c) $20\sqrt{2}$ cm (d) $10\sqrt{15}$ cm

201. A cube of maximum volume (each corner touching the surface from inside) is cut from a sphere. What is the ratio of the volume of the cube to that of the sphere? **[2018-I]**
(a) $3:4\pi$ (b) $\sqrt{3}:2\pi$
(c) $2:\sqrt{3}\pi$ (d) $4:3\pi$

202. If the ratio of the circumference of the base of a right circular cone of radius r to its height is 3 : 1, then what is the area of the curved surface of the cone? **[2018-I]**
(a) $3\pi r^2$ (b) $\frac{2\pi r^2\sqrt{4\pi^2+9}}{3}$
(c) $\frac{\pi r^2\sqrt{\pi^2+1}}{3}$ (d) $\frac{\pi r^2\sqrt{4\pi^2+9}}{3}$

203. A wire is in the form of a circle of radius 98 cm. A square is formed out of the wire. What is the length of a side of the square? (Use $\pi = 22/7$) **[2018-I]**
(a) 146 cm (b) 152 cm
(c) 154 cm (d) 156 cm

204. What is the area of the largest circular disc cut from a square of side $\frac{2}{\sqrt{\pi}}$ units? **[2018-I]**
(a) π square units (b) 1 square unit
(c) π^2 square units (d) 2 square units

205. The surface area of closed cylindrical box is 352 square cm. If its height is 10 cm, then what is its diameter? (Use $\pi = \frac{22}{7}$) **[2018-I]**
(a) 4 cm (b) 8 cm
(c) 9.12 cm (d) 19.26 cm

206. Walls (excluding roofs and floors) of 5 identical rooms having length, breadth and height 6 m, 4 m and 2.5 m respectively are to be painted. Out of five rooms, two rooms have one square window each having a side of 2.5 m. Paints are available only in cans of litre; and 1 litre of paint can be used for painting 20 square metres. The number of cans required for painting is **[2018-II]**
(a) 10 (b) 12 (c) 13 (d) 14

207. The lateral surface area of a cone is $462\ cm^2$. Its slant height is 35 cm. The radius of the base of the cone is **[2018-II]**
(a) 8.4 cm (b) 6.5 cm (c) 4.2 cm (d) 3.2 cm

208. A semi-circular plate is rolled up to form a conical surface. The angle between the generator and the axis of the cone is **[2018-II]**
(a) 60° (b) 45° (c) 30° (d) 15°

209. A solid right cylinder is of height π cm. If its lateral surface area is half its total surface area, then the radius of its base is ` **[2018-II]**
(a) $\pi/2$ cm (b) π cm (c) $1/\pi$ cm (d) $2/\pi$ cm

210. A rectangular block of length 20 cm, breadth 15 cm and height 10 cm is cut up into exact number of equal cubes. The least possible number of cubes will be **[2018-II]**
(a) 12 (b) 16 (c) 20 (d) 24

211. If the diagonal of a cube is of length l, then the total surface area of the cube is **[2018-II]**
(a) $3\,l^2$ (b) $\sqrt{3}l^2$ (c) $\sqrt{2}l^2$ (d) $2\,l^2$

212. Two cones have their heights in the ratio 1 : 3. If the radii of their bases are in the ratio 3 : 1, then the ratio of their volumes will be **[2018-II]**
(a) 1 % 1 (b) 2 % 1 (c) 3 % 1 (d) 9 % 1

213. The volume of a spherical balloon is increased by 700%. What is the percentage increase in its surface area ? **[2019-I]**
(a) 300% (b) 400% (c) 450% (d) 500%

Consider the following for the next three (03) items:
A cube is inscribed in a sphere. A right circular cylinder is within the cube touching all the vertical faces. A right circular cone is inside the cylinder. Their heights are same and the diameter of the cone is equal to that of the cylinder. **[2019-I]**

214. What is the ratio of the volume of the sphere to that of the cone?
(a) $6\sqrt{3}:1$ (b) 7 : 2 (c) $3\sqrt{3}:1$ (d) $5\sqrt{3}:1$

215. What is the ratio of the volume of the cube to that of the cylinder ?
(a) 4 : 3 (b) 21 : 16 (c) 14 : 11 (d) 45 : 32

216. Consider the following statements :
1. The surface area of the sphere is $\sqrt{5}$ times the curved surface area of the cone.
2. The surface area of the cube is equal to the curved surface area of the cylinder.

Which of the above statements is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

Consider the following for the next two (02) items:
The sum of length, breadth and height of a cuboid is 22 cm and the length of its diagonal is 14 cm. **[2019-I]**

217. What is the surface area of the cuboid ?
(a) 288 cm^2
(b) 216 cm^2
(c) 144 cm^2
(d) Cannot be determined due to insufficient data

218. If S is the sum of the cubes of the dimensions of the cuboid and V is its volume, then what is (S – 3V) equal to? **[2019-I]**
(a) 572 cm^3 (b) 728 cm^3
(c) 1144 cm^3 (d) None of the above

219. A hollow sphere of external and internal diameters 6 cm and 4 cm respectively is melted into a cone of base diameter 8 cm. What is the height of the cone? **[2019-II]**
(a) 4.75 cm (b) 5. 50 cm
(c) 6. 25 cm (d) 6. 75 cm

220. A solid metallic cylinder of height 10 cm and radius 6 cm is melted to make two cones in the ratio of volume 1 : 2 and of same height as 10 cm. What is the percentage increase in the flat surface area ? **[2019-II]**
(a) 25% (b) 50% (c) 75% (d) 100%

221. A thin rod of length 24 feet is cut into rods of equal size and joined so as to form a skeleton cube. What is the area of one of the faces of the largest cube thus constructed ? **[2019-II]**
(a) 25 square feet (b) 24 square feet
(c) 9 square feet (d) 4 square feet

222. Three copper spheres of radii 3 cm, 4 cm and 5 cm are melted to form a large sphere. What is its radius? **[2019-II]**
(a) 12 cm (b) 10 cm (c) 8 cm (d) 6 cm

223. The volume of a hemisphere is 155232 cm^3. What is the radius of the hemisphere ? **[2019-II]**
(a) 40 cm (b) 42 cm (c) 38 cm (d) 36 cm

224. A bucket is in the form of a truncated cone. The diameters of the base and top of the bucket are 6 cm and 12 cm respectively. If the height of the bucket is 7 cm, what is the capacity of the bucket ? **[2019-II]**
(a) 535 cm^3 (b) 462 cm^3 (c) 234 cm^3 (d) 166 cm^3

225. A right circular cone has height 8 cm. If the radius of its base is 6 cm, then what is its total surface area? **[2019-II]**
(a) 96π cm^2 (b) 69π cm^2
(c) 54π cm^2 (d) 48π cm^2

226. Six cubes, each with 12 cm edge are joined end to end. What is the surface area of resulting cuboid? **[2019-II]**
(a) 3000 cm^2 (b) 3600 cm^2
(c) 3744 cm^2 (d) 3777 cm^2

227. The areas of three adjacent faces of a cuboid are x, y and z. If V is the volume of the cuboid, then which one of the following is correct ? **[2019-II]**
(a) $V = xyz$ (b) $V^2 = xyz$
(c) $V^3 = xyz$ (d) $V = (xyz)^2$

228. Two cylinders of equal volume have their heights in the ratio 2 : 3. What is the ratio of their radii ? **[2019-II]**
(a) $\sqrt{3}:1$ (b) $\sqrt{3}:\sqrt{2}$
(c) $2:\sqrt{3}$ (d) $\sqrt{3}:2$

229. A right circular cylinder has a diameter of 20 cm and its curved surface area is 1000 cm^2. What is the volume of the cylinder? **[2019-II]**
(a) 4000 cm^3 (b) 4500 cm^3
(c) 5000 cm^3 (d) 5200 cm^3

230. A hollow right circular cylindrical vessel of volume V whose diameter is equal to its height, is completely filled with water. A heavy sphere of maximum possible volume is then completely immersed in the vessel. What volume of water remains in the vessel ? **[2019-II]**
(a) $\frac{V}{2}$ (b) $\frac{V}{3}$ (c) $\frac{2V}{3}$ (d) $\frac{V}{4}$

231. The thickness of a cylinder is 1 foot, the inner radius of the cylinder is 3 feet and height is 7 feet. To paint the inner surface it requires one litre of a particular colour. How much quantity of the same colour is required to paint all the surfaces of the cylinder ? **[2020-I]**
(a) $\frac{7}{3}$ litre (b) $\frac{3}{2}$ litre
(c) $\frac{8}{3}$ litre (d) $\frac{10}{3}$ litre

232. How many cubic metre of earth is to be dug out to dig a well of radius 1.4 m and depth 5m? **[2020-I]**
(a) 30.2 cubic metre (b) 30.4 cubic metre
(c) 30.6 cubic metre (d) 30.8 cubic metre

233. A right circular cylinder just encloses a sphere. If p is the surface area of the sphere and q is the curved surface area of the cylinder, then which one of the following is correct? **[2020-I]**
(a) $P = q$ (b) $p = 2q$
(c) $2p = q$ (d) $2p = 3q$

HINTS & SOLUTIONS

1. (d) Volume of wire $= \pi r^2 h$

New radius of the wire $= \dfrac{r \times 90}{100} = \dfrac{9r}{10}$

Let new length of the wire be L.

$\therefore$ Volume of new wire $= \pi\left(\dfrac{9r}{10}\right)^2 \times L = \dfrac{81}{100}\pi r^2 L$

According to question,

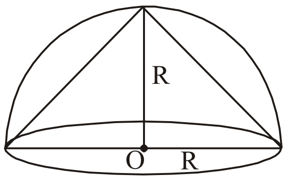
$\pi r^2 h = \dfrac{81}{100}\pi r^2 L \Rightarrow L = \dfrac{100}{81}h$

Increase in length $= \dfrac{100}{81}h - h = \dfrac{19}{81}h$

Per cent increase $= \dfrac{19h/81h}{h} \times 100\% = 23.46\%$

$= 23\%$ (approx)

2. (a) Volume of cone, $C = \dfrac{1}{3}\pi R^2 H$



$= \dfrac{1}{3}\pi R^3 \qquad (\because H = R)$

Volume of hemisphere, $H = \dfrac{2}{3}\pi R^3$

$\therefore\ C : H = \dfrac{1}{3}\pi R^3 : \dfrac{2}{3}\pi R^3 = 1 : 2$

3. (d) Let $l_1 : l_2 : l_3 = 1 : 2 : 3$

$\therefore\ b_1 : b_2 : b_3 = \dfrac{x}{1} : \dfrac{x}{2} : \dfrac{x}{3} = 6 : 3 : 2$

$\therefore$ Ratio of volumes $= \pi r_1^2 h : \pi r_2^2 h_2 : \pi r_3^2 h_3$

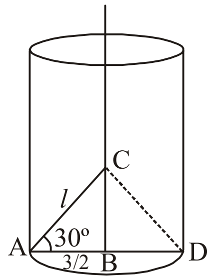
$= 3^2 \times 1 : \dfrac{9}{4} \times 2 : 1 \times 3 = 9 : \dfrac{9}{2} : 3$

$= 3 : \dfrac{3}{2} : 1 = 6 : 3 : 2$

4. (b) In ΔABC,

$\cos 30^\circ = \dfrac{3/2}{l}$

$\Rightarrow\ l = \dfrac{3/2}{\sqrt{3}/2} = \sqrt{3}$ cm



$\therefore$ Area of cone ACD $= \pi r l$

$= \pi \times \dfrac{3}{2} \times \sqrt{3}$

$= \dfrac{3\sqrt{3}\pi}{2}$ sq cm

5. (b) According to question,

27 × Volume of smaller drops = Volume of bigger drop

$\therefore\ 27 \times \dfrac{4}{3}\pi r^3 = \dfrac{4}{3}\pi R^3$

$\Rightarrow\ 27 \times (0.2)^3 = R^3$

$\Rightarrow\ (3 \times 0.2)^3 = R^3$

$R = 0.6$ cm

6. (b) Given, $l = R$

$\therefore$ Total surface area of cone,

$S = \pi r(r + l) = \pi R(R + R) = 2\pi R^2$

$\therefore$ Area of circle, $A = \pi(2R)^2 = 4\pi R^2$

$\Rightarrow\ A = 2S$

7. (b) According to question,

Volume of cylinder = Volume of bar

$\Rightarrow\ \pi r^2 h =$ base area of block × length

$\Rightarrow\ \dfrac{22}{7} \times \left(\dfrac{1.5}{2}\right)^2 \times 3.5 = \dfrac{5}{100} \times \dfrac{5}{100} \times L$

$\Rightarrow\ \dfrac{22}{7} \times \dfrac{2.25}{4} \times 3.5 \times \dfrac{100}{5} \times \dfrac{100}{5} = L$

$\therefore\ L = 2475$ m

8. (b) Let r be the radius of hemispherical bowl.

$\therefore\ 2\pi r = \dfrac{132}{7}$

$\Rightarrow\ r = \dfrac{132}{7} \times \dfrac{7}{2 \times 22} = 3$ cm

$\therefore$ Capacity of hemispherical bowl = Volume of hemispherical bowl

$= \dfrac{2}{3}\pi(3)^3 = 18\pi$ cu cm

9. (d) Volume of semi-circular disc

$= \frac{1}{2}\pi \times 4 \times 4 \times 2$ cu cm ($\because r = 4$ cm and $h = 2$ cm)

and volume of spherical ball

$= \frac{4}{3}\pi\left(\frac{2.5}{10}\right)^3$ cu cm

$\therefore$ Number of balls $= \dfrac{\frac{1}{2}\pi \times 4 \times 4 \times 2}{\frac{4}{3}\pi\left(\frac{2.5}{10}\right)^3} = 768$

10. (a) Let r be the radius of cylindrical block, then height will be $2r$.

Volume of block $= \pi(r^2)(2r) = 2\pi r^3$

A sphere of maximum possible volume is carved out whose radius will be r.

$\therefore$ Volume of sphere $= \frac{4}{3}\pi r^3$

$\therefore$ Volume of utilised wood $= \frac{4}{3}\pi r^3$

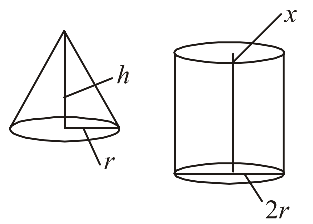
and volume of wasted wood $= 2\pi r^3 - \frac{4}{3}\pi r^3$

$= \dfrac{6\pi r^3 - 4\pi r^3}{3} = \dfrac{2\pi r^3}{3}$

$\therefore$ Required ratio $= \frac{4}{3}\pi r^3 : \frac{2}{3}\pi r^3 = 2 : 1$

11. (d) Radius of conical flask $= r$

Height of conical flask $= h$



$\therefore$ Volume of conical flask $= \frac{1}{3}\pi r^2 h$

Radius of cylindirical flask $= 2r$

Height of cylindirical flask $= x$

$\therefore$ Volume of cylindirical flask $= \pi(2r)^2 x$

$= \pi 4r^2 x$

According to question,

$\frac{1}{3}\pi r^2 h = 4\pi r^2 x$

$x = \dfrac{1.\pi r^2 h}{3 \times 4\pi r^2} = \dfrac{h}{12}$

$\therefore$ Height of the cylinderical flask is $\frac{h}{12}$.

12. (d) Volume of solid cylinder $= \pi(3)^2 4 = 36\pi$ cm^3

and volume of conical cavity

$= \frac{1}{3}\pi(3)^2(4) = 12\pi$ cm^3

$\therefore$ Volume of remaining solid

$= 36\pi - 12\pi = 24\pi$ cm^3

13. (c) Let the radii and slant height of two right circular cones are r_1, l_1 and r_2, l_2, respectively.

$\therefore$ Ratio of their curved surface areas

$= \dfrac{\pi r_1 l_1}{\pi r_2 l_2} = \dfrac{l_1}{l_2}$ ($\because r_1 = r_2$)

$= \frac{3}{2} = 3 : 2$

14. (b) Let perimeter $4a = 20$

$\Rightarrow a = 5$

Volume of cube $= a^3 = 125$

Hence, pairs (20, 125), represents the perimeter of one face of a cube and volume of cube.

15. (b) Volume of the new cube = Sum of all the three cubes.

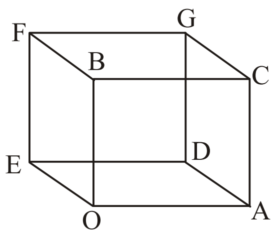
$= 6^3 + 8^3 + 10^3 = 216 + 512 + 1000 = 1728$ cm^3

let Length of the edge of new cube $= a$ cm

$a^3 = 1728$ cm^3

$a = \sqrt[3]{1728} = 12$ cm.

16. (d) The distance between vertices B and C is 1 cm.



The distance between A and B is $\sqrt{1^2 + 1^2} = \sqrt{2}$ cm

The distance between diagonal B and D is

$\sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$ cm

17. (c) Total surface area $= 3\pi r^2 + 3\pi r^2 = 6\pi r^2$

$\therefore$ Painting cost of two halves $= 6\pi r^2 \times 8$

$= 48\pi r^2$

18. (b) According to question

Surface area of sphere = 3 (Volume of sphere)

$\Rightarrow 4\pi r^2 = 3 \times \frac{4}{3}\pi r^3 \Rightarrow r = 1$

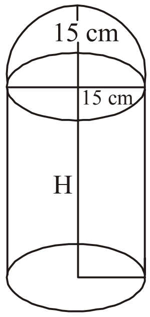
$\therefore$ Diameter $= 2r = 2$ cm

19. (a) According to question

$\dfrac{3\pi r_1^2}{3\pi r_2^2} = \dfrac{4}{1} \Rightarrow \dfrac{r_1}{r_2} = \dfrac{2}{1}$

$\therefore$ Required ratio $= \dfrac{\frac{2}{3}\pi r_1^3}{\frac{2}{3}\pi r_2^3} = \dfrac{8}{1}$ or $8 : 1$

20. (a) Let height of the cylinder be h.



According to question,
Volume of hemisphere + Volume of cylinder
= Volume of container

$\Rightarrow \frac{2}{3}\pi r^3 + \pi r^2 h = 32400\pi$

$\Rightarrow \frac{2}{3}\pi \times 3375 + \pi \times 225\,h = 32400\pi$

$\Rightarrow 2\pi \times 1125 + \pi \times 225\,h = 32400\pi$

$\Rightarrow 10 + h = 144$

$\Rightarrow h = 134$

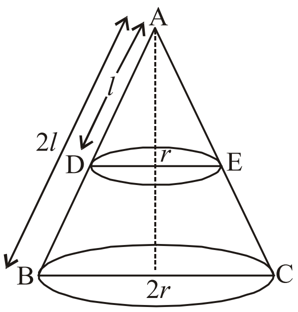
$\therefore$ Height of container (h) = 15 + 134 = 149

21. (a) $\therefore$ Volume of pipe, $V = \pi\left(r_1^2 - r_2^2\right) \times h$

$= \frac{22}{7}[(3.5)^2 - (2.5)^2] \times 140 = \frac{22}{7}(12.25 - 6.25) \times 140$

$= 22 \times 6 \times 20 = 2640$ cu cm

22. (d) Here similar triangle
$\Delta ABC \sim$ In ΔADE



$\therefore \frac{V_1}{V_2} = \frac{\frac{1}{3}\pi \times 4r^2 \times \sqrt{(2l)^2 - (2r)^2}}{\frac{1}{3}\pi \times r^2 \times \sqrt{l^2 - r^2}} = \frac{8}{1} = 8:1$

23. (b) Let $r_1 = 1k$ and $r_2 = nk$
According to question,
$V_1 = V_2$

$\therefore \frac{1}{3}\pi r_1^2 h_1 = \pi r_2^2 h_2 \Rightarrow \frac{1}{3}\pi k^2 \times h_1 = \pi n^2 k^2 h_2$

$\Rightarrow h_1 = 3n^2 h_2$

24. (c) A: it is a true statement.
R. It is a false statement.
Because the length may be πr but the breadth is always less than $\sqrt{h^2 + r^2}$ of formed rectangle.

25. (b) Both statements are true but (R) is not a correct explanation of (A).

26. (d) Let radius and height of *a* cone be *r* and *h*

$\because \frac{r}{h} = \frac{3}{4}$

Volume of cone = $\frac{1}{3}\pi r^2 h$

$\therefore 96\pi = \frac{1}{3}\pi \times r^2 \times \frac{4r}{3}$

$\Rightarrow r^3 = \frac{96 \times 3 \times 3}{4} = 216 \Rightarrow r = 6$ cm

and $h = 8$ cm

$\therefore$ Lateral surface area

$= \pi r\sqrt{r^2 + h^2}$

$= \pi \times 6\sqrt{36 + 64}$

$= 60\pi$ sq cm

27. (b) The maximum diameter of a sphere in a cube = 3 m.
$\therefore$ Radius of the sphere = 1.5 m

$\therefore$ Volume of sphere, $V_1 = \frac{4}{3}\pi(1.5)^3 = 4.5\pi$ cu m

$\therefore$ Volume of cube, $V_2 = (3)^3 = 27\ m^3$

$\therefore$ Volume of solid left $= V_2 - V_1 = (27 - 4.5\pi)\ m^3$

28. (b) Suppose ratios of dimensions of a rectangular box be $2x$, $3x$ and $4x$.
According to question,

$= (10-8) \times 2\left[2x \times 3x + 3x \times 4x + 4x \times 2x\right] = 234$

$\Rightarrow 2 \times 2 \times 26x^2 = 234$

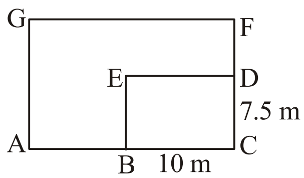
$\therefore x^2 = \frac{234}{104} = 2.25 \quad \therefore x = 1.5$

Thus, the dimensions are,
$2x = 2 \times 1.5 = 3.0$
$3x = 3 \times 1.5 = 4.5$
$4x = 4 \times 1.5 = 6.0$

29. (c) Volume of solid cube = $(4)^3 = 64\ cm^3$
Volume of recast cube = $(1)^3 = 1\ cm^3$
$\therefore$ Total surface area of cube : Total surface area of recast cube
$= x : y$
$\Rightarrow x : y = 6(4)^2 : 6(1)^2 \times 64 = 1 : 4$

30. (d) Since, the outer edges of a cubical box is 5 cm.
$\therefore$ Surface area of the outer cubical box
= 5 (edge)^2
= 5 $(5)^2$ = 125 sq cm
Surface area of the inner cubical box
= 5 × $(4)^2$
$\Rightarrow$ 80sq cm
$\therefore$ Total surface area = 125 + 80 =205
= 205 sq cm

31. (c) Area of tank, BCDE = 10 × 7.5 = 75 m^2



Area of remaining field ABEDFGA
$= 125 \times 15 - 75 = 1800$ sq m
Volume of Earth dug $= 10 \times 7.5 \times 6 = 450$ cu m
By given condition, $1800 \times h = 450$

$\Rightarrow \quad h = \frac{1}{4} \text{m} = \frac{1}{4} \times 100 \text{ cm} = 25 \text{ cm}$

32. (c) Volume of wood = Volume of lead pencil – Volume of lead

$= \pi(0.4)^2 21 - \pi(0.1)^2 \times 21$

$= 21 \times \frac{22}{7}(0.16 - 0.01)$

$= 66\,(0.15) = 9.9$ cu cm

33. (c) Volume of soild $= l \times b \times h$
$= 22 \times 7 \times 5 = 770$ cu cm
Let the water rise in height be h.
$\because$ Volume of water rise in vessel = Volume of solid
$\Rightarrow \pi r^2 h = 770$

$\Rightarrow \frac{22}{7} \times 14 \times 14 \times h = 770$

$\Rightarrow h = \frac{770 \times 7}{22 \times 14 \times 14} = \frac{5}{4} = 1.25$ cm

34. (a) $R = 2r$
(given)
According to question
Volume of cylinder = Volume of cone

$\therefore \quad \pi r^2 h = \frac{1}{3} \pi R^2 H$

$\Rightarrow \quad r^2 h = \frac{1}{3}(2r)^2 H$

$\therefore \quad H = \frac{3h}{4}$

35. (c) Radius of roller = 0.35 m.
The area covered in one revolution curved surface area of roller

$= 2 \times \frac{22}{7} \times 0.35 \times 2$

$= 4.4$ sq m
$\therefore$ Total area covered in 50 revolutions
$= 4.4 \times 50 = 220$ sq m

36. (d) Let x be the diameter of moon.

$$\text{Required Ratio} = \frac{\text{Volume of Moon}}{\text{Volume of Earth}}$$

$$= \frac{\frac{4}{3}\pi\left(\frac{x}{8}\right)^3}{\frac{4}{3}\pi\left(\frac{x}{2}\right)^3} = \frac{1}{64}$$

37. (b) When we join three cubes to form a cuboid. The length of the cuboid becomes 15 cm and height, width remains the 5 cm each.
$\therefore$ Surface area of cuboid $= 2\,(lb + bh + hl)$
$= 2\,(15 \times 5 + 5 \times 5 + 15 \times 5)$
$= 2\,(75 + 25 + 75)$
$= 350$ sq cm

38. (c) Volume of bigger cone

$= \frac{1}{3}\pi(6)^2 \times 8 = 96\pi \text{ cm}^3$

Volume of smaller cone $= \frac{1}{3}\pi(1)^2 \times 2$

$= \frac{2\pi}{3} \text{cm}^3$

Number of cones $= \frac{96\pi}{\frac{2\pi}{3}} = 144$

39. (d) Diameter of cylindrical log $= d$
Then, height of cylindrical log $= d$
Diameter of greatest possible sphere $= d$

Radius of sphere $= \frac{d}{2}$

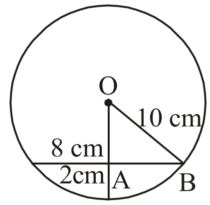
Volume of cylindrical log $= \pi r^2 h$

$= \pi\left(\frac{d}{2}\right)^2 d = \frac{\pi d^3}{4}$

Volume of sphere $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3 = \frac{\pi d^3}{6} = \frac{\pi d^3}{4} \times \frac{4}{6}$

$= \frac{2}{3}$ volume of cylinder

40. (b) In ΔOAB, $OA^2 + AB^2 = OB^2$



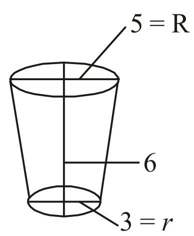
$AB^2 = 10^2 - 8^2 = 36$
$AB = 6$ cm

41. (c) Let water level increase by x cm.
$\therefore$ Volume of cylindrical can =
$\pi\,(12)^2 \times x = 144\pi x$

Volume of sphere $= \frac{4}{3}\pi(6)^3 = 288\pi$ cu cm

According to question
$144\pi x = 288\pi$
$x = 2$ cm

42. (d) Volume of frustum of cone



$= \frac{\pi h}{3}[R^2 + r^2 + Rr]$

$= \frac{\pi \times 6}{3}[5^2 + 3^2 + 5 \times 3]$

$= \pi \times 2 \times 49 = 98\pi$ cu unit

43. (a) Volume of hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times 21 \times 21 \times 21$

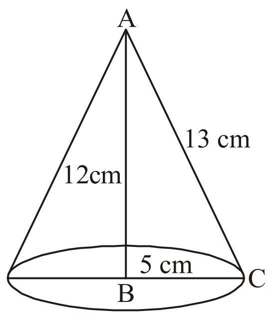
$= 44 \times 441 = 19404$ cu cm

$= \frac{19404}{1000}$ L $= 19.4$ L

44. (c) Height of water in a second = 30 cm
Height of water in 60 $s = 30 \times 60$
$h = 1800$ cm
Area of cross section, $\pi r^2 = 5$ sq cm
Volume of water flow in one minute $= \pi r^2 h$
$= 5 \times 1800 = 9000$ cu cm

$= \frac{9000}{1000}$ L $= 9$ L

45. (b) Volume of cone $= \frac{1}{3}\pi r^2 h$



$= \frac{1}{3}\pi \times 5^2 \times 12 = 100\pi$ cu cm

46. (c) According to question

$\frac{\text{Volume of first cone}}{\text{Volume of second cone}} = \frac{4}{5}$

$\therefore \quad \frac{\frac{1}{3}\pi(2)^2 h_1}{\frac{1}{3}\pi(3)^2 h_2} = \frac{4}{5}$

$\Rightarrow \quad \frac{4h_1}{9h_2} = \frac{4}{5} \Rightarrow \frac{h_1}{h_2} = \frac{9}{5}$

47. (d) The dimensions of a bricks are
$l = 12.5$ cm
$b = 10$ cm
$h = 7.5$ cm

$\therefore$ Area of brick $= 2\,(lb + bh + hl)$
$= 2\,(12.5 \times 10 + 10 \times 7.5 + 12.5 \times 7.5)$
$= 2\,(125 + 75 + 93.75)$
$= 2 \times 293.75$
$= 587.50$ sq cm
Area to be painted by the available paint = 5.875 sq m
$= 5.875 \times 10^4$ sq cm
$= 58750$ sq cm

No. of bricks can be painted $= \frac{58750}{587.50} = 100$ bricks.

48. (b) Volume of hollow cylindrical pipe

$= \pi\left(r_2^2 - r_1^2\right) \times h = \frac{22}{7}\{(4)^2 - (2)^2\} \times 700$

$= \frac{22}{7} \times 12 \times 700 = 26400$ cu cm

$= 26400 \times \frac{5}{1000}$ kg $\quad (\because 1000$ cu cm $= 5$ kg$)$

$= 132$ kg

49. (c) According to question,
Surface area of sphere = Surface area of cube

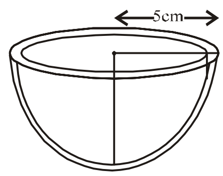
$\Rightarrow \quad 4\pi r^2 = 6a^2 \Rightarrow \left(\frac{r}{a}\right)^2 = \frac{3}{2\pi}$

Required Ratio $= \frac{(\text{Volume of sphere})^2}{(\text{Volume of cube})^2}$

$= \frac{\left(\frac{4}{3}\pi r^3\right)^2}{(a^3)^2} = \frac{16}{9}\pi^2\left[\left(\frac{r}{a}\right)^2\right]^3$

$= \frac{16}{9}\pi^2\left(\frac{3}{2\pi}\right)^3 = \frac{16}{9}\pi^2 \times \frac{27}{8\pi^3} = \frac{6}{\pi}$

50. (b) Volume of hemisphere $= \frac{2}{3}\pi(5^3 - 4^3)$



$= \frac{2}{3}\pi(125 - 64)$

$= \frac{2}{3}\pi \times 61$ cu cm

$= \frac{2}{3}\pi \times 61 \times 9g \qquad$ (Given, 1 cu cm = 9 g)

$= 366\pi$ g

51. (c) Volume of hemispherical bowl

$= \frac{2}{3}\pi(20)^3$

$= \frac{16000}{3}\pi$ cu cm

Volume of conical shape bottle

$= \frac{1}{3}\pi(5)^2 8 = \frac{200\pi}{3}$ cu cm

$\therefore$ Required number of bottles

$= \frac{16000\pi/3}{200\pi/3} = 80$

52. (a) Volume of spherical lead shot

$= \frac{4}{3}\pi(1)^3 = \frac{4}{3}\pi$ cu cm

I. Volume of shots $= \frac{4}{3}\pi(0.5)^3 \times 8 = \frac{4}{3}\pi$ cu cm

II. Volume of both shots

$= \frac{4}{3}\pi(0.75)^3 + \frac{4}{3}\pi(0.8)^3$

$= \frac{4}{3}\pi\left[\left(\frac{3}{4}\right)^3 + \left(\frac{4}{5}\right)^3\right] = \frac{4}{3}\pi\left[\frac{27}{64} + \frac{64}{125}\right]$

$= \frac{4}{3}\pi\left[\frac{3375+4096}{8000}\right] = \frac{4}{3}\pi\left(\frac{7471}{8000}\right)$

$= \frac{4}{3}\pi(0.93)$ cu cm

Thus, only Statement I is true.

53. (d) Let r of sheet = 14 cm

it bent and form a conical cup of length l = 14 cm

According to question

Circumference of base of the cone = Circumferecne of semi-circle

$\Rightarrow 2\pi R = \pi r$

$\Rightarrow 2R = r \Rightarrow 2R = 14$

$\Rightarrow R = 7$ cm

$\because l^2 = R^2 + h^2$

$\Rightarrow (14)^2 = (7)^2 + h^2$

$\Rightarrow h^2 = 196 - 49 = 147$

$\Rightarrow h = 7\sqrt{3}$

$\therefore$ Capacity of cup

$= \frac{1}{3}\pi R^2 h = \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 7\sqrt{3}$

$= \frac{1078}{3}\sqrt{3}$ cu cm

54. (a) Let the radius of cone and sphere be r,

Height = h

diameter = $2r$

According to question

Volume of cone = Volume of sphere

$\Rightarrow \frac{1}{3}\pi r^2 h = \frac{4}{3}\pi r^3$

$\Rightarrow \frac{h}{r} = \frac{4}{1}$

$\Rightarrow \frac{h}{2r} = \frac{2}{1} = \frac{h}{d} = \frac{2}{1}$

55. (a) In one round, distance covered by cylinder = 60 cm

In one second, distance covered by cylinder = $60 \times 5 = 300$ cm

In five seconds, distance covered by cylinder = $300 \times 5 = 1500$ cm = 15 m

56. (b) Volume of frustum of a cone

$= \frac{\pi h}{3}\left(r_1^2 + r_2^2 + r_1 r_2\right)$

57. (a) Increase in the height of water level

$= \frac{0.75}{2.5 \times 1.5}$ m = 0.2 m = 20 cm

58. (d) Let Length, breadth and height are $6x$, $3x$ and x respectively.

Let the side of a cube be a.

According to question,

Surface area of a cube = Surface area of rectangular parallelopiped

$\Rightarrow 6(a)^2 = 2(6x \times 3x + 3x \times x + x \times 6x)$

$\Rightarrow 6a^2 = 2(18x^2 + 3x^2 + 6x^2)$

$\Rightarrow 6a^2 = 54x^2 \Rightarrow a^2 = 9x^2$

$\therefore a = 3x$

Required ratio $= \frac{\text{Volume of cube}}{\text{Volume of rectangular parallelopiped}}$

$= \frac{a^3}{6x \times 3x \times x} = \frac{(3x)^3}{18x^3} = \frac{3}{2}$

59. (d) Let h be the depth of the pit.

$\therefore$ Volume of Earth dug $= 500 \times 250\ \text{cm}^3 = 125000$ cu cm

Volume of pit $= 50 \times 50 \times h$

$\therefore h = \frac{125000}{50 \times 50} = 50$ cm = 0.5 m

60. (d) Given $a_1 = 0.3$m, $a_2 = 0.4$ m and $a_3 = 0.5$ m

$\therefore$ Side of new cubic biscuit

$= \sqrt[3]{a_1^3 + a_2^3 + a_3^3}$

$= \sqrt[3]{(0.3)^3 + (0.4)^3 + (0.5)^3}$

$= \sqrt[3]{0.027 + 0.064 + 0.125}$

$= \sqrt[3]{0.216} = 0.6$ m

Thus, the total surface area of cubic biscuit

$= 6(0.6)^2$

$= 2.16$ sq m

61. (b) Let the radius of ball = r

$\therefore$ Radius of base of cylinder = $4r$

Height of cylinder = $4r$

$\therefore$ Volume of spherical ball $= \frac{4}{3}\pi r^3$

and volume of water $= \pi(4r)^2(2r)$

$= 32\pi r^3$

Volume of remaining portion of cylinder $= 32\pi r^3$

Let number of spherical balls $= n$

$\therefore \quad 32\pi r^3 = n \times \frac{4}{3}\pi r^3$

$\Rightarrow \quad n = 8 \times 3 = 24$

62. (c) Let l, b and h be the sides of cuboid.

$l^2 + b^2 = x^2$ ...(i)

$b^2 + h^2 = y^2$ ...(ii)

$h^2 + l^2 = z^2$ ...(iii)

On adding equation (*i*), (*ii*) and (*iii*)

$2(l^2 + b^2 + h^2) = x^2 + y^2 + z^2$

$\Rightarrow \quad l^2 + b^2 + h^2 = \frac{1}{2}(x^2 + y^2 + z^2)$ ...(iv)

On solving Eqs. (i), (ii), (iii) and (iv);

$h = \sqrt{\frac{y^2 + z^2 - x^2}{2}}, l = \sqrt{\frac{z^2 + x^2 - y^2}{2}}$

and $b = \sqrt{\frac{x^2 + y^2 - z^2}{2}}$

Volume of cuboid $= l\,b\,h$

$= \sqrt{\frac{(y^2 + z^2 - x^2)(z^2 + x^2 - y^2)(x^2 + y^2 - z^2)}{2 \times 2 \times 2}}$

$= \frac{1}{2\sqrt{2}}\sqrt{(y^2 + z^2 - x^2)(z^2 + x^2 - y^2)(x^2 + y^2 - z^2)}$

63. (a) Sheet is revolved about its length

$\therefore \quad h = 7$ cm and $r = 4$ cm

$\therefore \quad$ Then, Volume of the figure

$= \pi r^2 h = \frac{22}{7} \times 4 \times 4 \times 7 = 352$ cu cm

64. (b) Let radius and height of cylinder be r and h respectively.

According to question

$2\pi r \times h = \frac{2\pi r}{3}(h + r)$

$\Rightarrow \quad 9 = \frac{1}{3}(9 + r)$

$\Rightarrow \quad 27 = 9 + r$

$\therefore \quad r = 18$ m

65. (b) Let r_1 and r_2 be the radii of these spheres.

Given Ratio $\frac{v_1}{v_2} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{8}{1}$

$\Rightarrow \quad \frac{r_1}{r_2} = \frac{2}{1}$

$\therefore \quad$ Ratio of their surface areas

$= \frac{4\pi r_1^2}{4\pi r_2^2} = \left(\frac{2}{1}\right)^2 = 4:1$

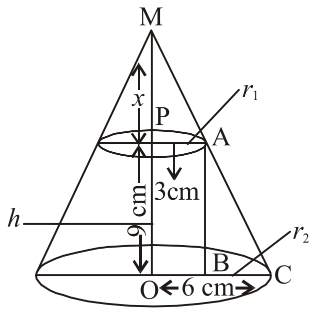
66. (b) Given, $l = 30$ cm, $b = 24$ cm and $h = 18$ cm

Here, Maximum length of the rod can be placed in the cuboid is equal to diagonal of box

$= \sqrt{30^2 + 24^2 + 18^2} = \sqrt{900 + 576 + 324} = \sqrt{1800}$

$= 30\sqrt{2}$ cm

Sol. (67-69):



Here, $r_1 = 3$ cm, $r_2 = 6$ cm and $h = 9$ cm

67. (b) Total surface area of the frustum

$= \pi\,[(R + r)\,l + r^2 + R^2]$

where, $l = \sqrt{h^2 + (R - r)^2}$

$= \pi[(6 + 3)\sqrt{81 + 9} + 9 + 36]$

$= \pi[9\sqrt{90} + 45] = 9\pi[3\sqrt{10} + 5]$ sq cm

68. (d) By using properties at similar triangle in ΔMPA and ΔMOC,

$\frac{MP}{MO} = \frac{PA}{OC}$

$\Rightarrow \frac{x}{9 + x} = \frac{3}{6} \Rightarrow x = 9$

Height of the cone= MO $= x + 9$

$= (9 + 9)$ cm $= 18$ cm

69. (a) In ΔABC,

$AC^2 = AB^2 + BC^2$

$\Rightarrow \quad AC^2 = 9^2 + (6 - 3)^2 = 81 + 9 = 90$

$\Rightarrow \quad AC = 3\sqrt{10}$ cm

70. (c) Side of cube $= \frac{\text{Diagonal}}{\sqrt{3}} = \frac{4\sqrt{3}}{\sqrt{3}} = 4$ cm

$\therefore$ Volume of cube $= (\text{Side})^3 = 4^3 = 64$ cu cm

71. (b) Let the radius of cylindrical rod $= r$ and height $= 4r$

$\therefore \quad$ Required number of spherical balls

$= \frac{\text{Volume of cylindrical rod}}{\text{Volume of spherical balls}} = \frac{\pi r^2(4r)}{\frac{4}{3}\pi r^3} = 3$

72. (b) Given, $R = 3$ cm, $r = 2$ cm, $h = 10$ cm

Total surface area $= 2\pi R\eta + 2\pi rh + 2\pi(R^2 - r^2)$

$= 60\pi + 40\pi + 10\pi$

$= 110\pi$ sq cm

73. (c) Given $l = 6$ m, $b = 4$ m and $h = 1.25$ m

Area of wetted surface

$= 2(l \times h + b \times h) + 6 \times 4$

$= 2(7.5 + 5) + 24$

$= 25 + 24 = 49$ sq m

74. (b) For conical part,

$r = \frac{6}{2} = 3$ cm, $h = 4$ cm,

$l = \sqrt{h^2 + r^2} = 5$ cm

Surface area of conical part $= \pi rl$

$= 3.14 \times 3 \times 5 = 47.1\text{ cm}^2$

For hemispherical part, $r = \frac{6}{2} = 3$ cm

Surface area of hemispherical part $= 2\pi r^2$

$= 2 \times 3.14 \times 3 \times 3 = 56.52$ sq cm

$\therefore$ Surface area of toy $= 47.1 + 56.52 = 103.62$ sq cm

75. (c) Volume of a spherical shell

$= \frac{4}{3}\pi(R^3 - r^3)$

$= \frac{4}{3}\pi(5^3 - 4^3)$

$= \frac{4}{3}\pi \times 61$

$= \frac{244\pi}{3}$

76. (b) Curved surface of a cylinder $= 1000\text{ cm}^2$

$\therefore 2\pi rh = 1000$

Length of wire used in a round = Perimeter of cylinder's base

$= 2\pi r$

Number of rounds $= \frac{\text{Height of cylinder}}{\text{Diameter of wire}} = \frac{h}{0.5}$

$\therefore$ Required length of wire

$= 2\pi r \times \frac{h}{0.5} \Rightarrow \frac{2\pi rh}{0.5} = \frac{1000}{0.5}$

$= 2000$ cm or 20 m

77. (a) According to question,

Volume of cone = Volume of cylinder

$\therefore \frac{1}{3}\pi r^2 h = \pi r^2 \times 5$

$\Rightarrow h = 5 \times 3 = 15$ cm

78. (d) According to question,

Surface area of sphere = 25% of 616

$4\pi r^2 = 154$

$\Rightarrow r^2 = \frac{154}{\frac{22}{7} \times 4} = \left(\frac{7}{2}\right)^2 \Rightarrow r = 3.5$ cm

79. (b) According to question

Volume of cylindrical vessel = Volume of sphere

$\therefore \pi r^2 h = \frac{4}{3}\pi R^3$

$\Rightarrow (60)^2 \times 10 = \frac{4}{3}\left(\frac{d}{2}\right)^3$

$\Rightarrow (60)^2 \times 10 = \frac{4}{3} \times \frac{d^3}{8}$

$\Rightarrow d^3 = (60)^2 \times (60)$

$\Rightarrow d = 60$ cm

80. (d) Let side of a cube $= a$.

Radius of sphere is $\frac{a}{2}$ unit.

Required Ratio $= \frac{\text{Volume of cube}}{\text{Volume of sphere}} = \frac{(a)^3}{\frac{4\pi}{3}\left(\frac{a}{2}\right)^3} = \frac{6}{\pi}$

81. (c) Surface area of sphere, $S_1 = 4\pi r^2$

If radius is 2r, then surface area of sphere,

$S_2 = 4\pi(2r)^2 = 16\pi r^2$

$\therefore S_2 = 4S_1$

Hence, it increases four times.

82. (d) Total surface area = Curved surface area of cylinder + Curved surface area of cone + Top surface area of cylinder

$= 2\pi rh + \pi rl + \pi r^2$

$= \pi(2 \times 3 \times 4 + 3\sqrt{3^2 + 4^2} + 3^2)$

$(\because l = \sqrt{r^2 + h^2})$

$= \pi(24 + 15 + 9) = 48\pi$ sq cm

83. (b) External Radius(R) $= \frac{8}{2} = 4$ cm and Internal Radius(r)

$= \frac{4}{2} = 2$cm

$\therefore$ Volume of hollow sphere

$= \frac{4}{3}\pi(R^3 - r^3)$

$= \frac{4}{3}\pi(4^3 - 2^3)$

$= \frac{4\pi}{3} \times 56$

Let h = Height of the cone

According to question

Volume of cone = Volume of hollow sphere

$\therefore \frac{1}{3}\pi r_1^2 h = \frac{4}{3}\pi \times 56$

$\Rightarrow (4)^2 h = 4 \times 56$

$\Rightarrow h = \frac{4 \times 56}{16}$

$\therefore h = 14$ cm

84. (b) Let height h_1, radius r_1, area S_1 and volume V_1 of first cylinder.

Similarly, for second cylinder, height h_2, radius r_2, area S_2 and volume V_2

According to question

$\frac{r_1}{r_2} = \frac{2}{3}$ ...(i)

$\therefore \quad \dfrac{S_1}{S_2} = \dfrac{5}{3} \Rightarrow \dfrac{2\pi r_1 h_1}{2\pi r_2 h_2} = \dfrac{5}{3}$

$\Rightarrow \quad \dfrac{h_1}{h_2} \times \dfrac{2}{3} = \dfrac{5}{3} \Rightarrow \dfrac{h_1}{h_2} = \dfrac{5}{2}$ ...(*ii*)

$\therefore \quad \dfrac{V_1}{V_2} = \dfrac{\pi r_1^2 h_1}{\pi r_2^2 h_2} = \left(\dfrac{r_1}{r_2}\right)^2 \left(\dfrac{h_1}{h_2}\right) = \left(\dfrac{2}{3}\right)^2 \left(\dfrac{5}{2}\right) = \dfrac{10}{9}$

85. (b) Total area of wetted surface = Curved surface area of cylinder + Area of base of cylinder

$= 2\pi rh + \pi r^2 = \pi[2 \times 3.5 \times 4 + (3.5)^2]$

$= \dfrac{22}{7}(28 + 12.25) = \dfrac{22}{7} \times 40.25$

= 126.5 sq m

86. (b) Let side of a cube be *a* unit.

$\therefore$ Volume of cube, $V = a^3$

and total surface area of cube, $S = 6\,(a)^2$

$V = a^3$

Squaring on both sides

$V^2 = a^6 \Rightarrow V^3 = (a^2)^3$

$V^2 = \left(\dfrac{S}{6}\right)^3 \Rightarrow 216 V^2 = S^3$

87. (c) Given, R = 35 cm, *r* = 14 cm and *h* = 40 cm

$\therefore$ Volume of the bucket $= \dfrac{\pi h}{3}(R^2 + r^2 + Rr)$

$= \dfrac{22 \times 40}{7 \times 3}(35^2 + 14^2 + 35 \times 14)$

$= \dfrac{880}{21}(1225 + 196 + 490)$

$= \dfrac{880}{21} \times 1911 = 80080$ cu cm

88. (b) Given that $r = \dfrac{70}{2}$ cm = 0.35 cm. *h* = 4 m

$\therefore$ Surface area of cylinder

$= 10\,(2\pi rh) = 10\left(2 \times \dfrac{22}{7} \times 0.35 \times 4\right) = 88$ m

$\therefore$ Total cost of painting at the rate of ₹ 5 per sq m.

= 88 × 5 = ₹ 440

89. (b) $\because$ Volume of wax balls $= \dfrac{4}{3}\pi(1)^3 = \dfrac{4}{3}\pi$ cm^3

Volume of sphere $= \dfrac{4}{3}\pi(8)^3$

$\therefore$ Required number of balls

$= \dfrac{\frac{4}{3}\pi(8)^3}{\frac{4}{3}\pi} = 512$

90. (c) Curved surface area of the well = $2\pi rh$

$= 2 \times \dfrac{22}{7} \times 2 \times 14 = 176$ m^2

$\therefore$ Expense of getting per square metre plastered = ₹ 25

$\therefore$ Expense of 176 m^2 = 176 × 25

= ₹ 4400

91. (a) Radius of sphere (*r*) = 9 cm = 0.09 m

Diameter of wire (d) = 0.4 cm

$\Rightarrow$ R = 0.2 cm = 0.002 m

According to question,

Volume of sphere = Volume of wire

$\Rightarrow \quad \dfrac{4}{3}\pi r^3 = \pi R^2 h$

$\therefore \quad h = \dfrac{4}{3} \times \dfrac{r^3}{R^2}$

$= \dfrac{4}{3} \times \dfrac{0.09 \times 0.09 \times 0.09}{0.002 \times 0.002} = 81 \times 3 = 243$ m

92. (c) Total surface area of cube = 6 × (Side)^2

$\therefore$ 150 = 6 × (Side)^2

$\Rightarrow$ Side$^2 = \dfrac{150}{6} = 25$

$\therefore$ Side $= \sqrt{25} = 5$ cm

$\therefore$ Volume of cube = (Side)^3

= 5 × 5 × 5 = 125 cm^3

93. (b) Volume of cube = (Side)^3

$\therefore$ $729 = a^3$

$\Rightarrow$ $a = 9$ cm

$\therefore$ Diagonal of cube = Side $\times\sqrt{3} = 9 \times \sqrt{3} = 9\sqrt{3}$ cm

94. (a) Curved surface area of right circular cone = $\pi r l$

$\therefore \quad 440 = \dfrac{22}{7} \times 14 \times l$

$\Rightarrow \quad l = \dfrac{440 \times 7}{22 \times 14} = 10$ cm

95. (a) length of the are $= 2\pi r\left(\dfrac{\theta}{360^\circ}\right)$

Radius of arc(r) = 30 am

Length of the arc $= 2\pi r = 2\pi \times 30 \times \dfrac{144}{360} = 24\pi$

Let the radius of the cone = R $\therefore$ $2\pi R = 24\pi$

$\Rightarrow \quad R = \dfrac{30 \times 144}{360} = 12$ cm

96. (d) Volume of cylinder $= \pi r_1^2 h$

Volume of ball $= \dfrac{4}{3}\pi r_2^3$

Number of balls = 48

$\therefore$ Number of balls $= \dfrac{\text{Volume of cylinder}}{\text{Volume of balls}} = \dfrac{\pi r_1^2 h}{\frac{4}{3}\pi r_2^3}$

$$\Rightarrow \quad \frac{\pi r_1^2 h}{\frac{4}{3}\pi r_2^3} = 48 \Rightarrow \frac{\pi r_1^3}{\frac{4}{3}\pi r_2^3} = 48$$

$(\because r_1 = h)$

$$\Rightarrow \quad \frac{3}{4}\left(\frac{r_1}{r_2}\right)^3 = 48 \Rightarrow \left(\frac{r_1}{r_2}\right)^3 = \frac{48 \times 4}{3}$$

$$\Rightarrow \quad \frac{r_1}{r_2} = (16 \times 4)^{1/3} \Rightarrow \frac{r_2}{r_1} = \frac{1}{4}$$

Thus, the ratio of radius of ball to cylinder is 1 : 4.

97. (a) Volume of the cuboid = 720 cm^3

$$\text{Height of the cuboid} = \frac{\text{Volume of the cuboid}}{\text{Base area of the cuboid}}$$

$$= \frac{lbh}{lb} = \frac{720}{72} = 10 \text{ cm}$$

Surface area of cuboid = $2(lb + bh + hl)$

$484 = 2(72 + 10(l + b))$

$484 - 144 = 20(l + b)$

$$\frac{340}{20} = (l + b) \Rightarrow 17 = l + b$$

By checking options, we get option (a) is the correct dimensions of the cuboid.

98. (a) Curved surface area of the sphere = $4\pi r^2$

$\Rightarrow \quad 616 = 4\pi r^2$

$$\Rightarrow \quad \pi r^2 = \frac{616}{4} = 154 \Rightarrow r^2 = \frac{154 \times 7}{22} = 49$$

$\therefore \quad r = \sqrt{49} = 7$ cm

$$\therefore \quad \text{Volume of the sphere} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7 = \frac{4312}{3} \text{ cm}^3.$$

99. (c) Volume of frustum

$$= \frac{\pi H}{3}(R^2 + r^2 + Rr)$$

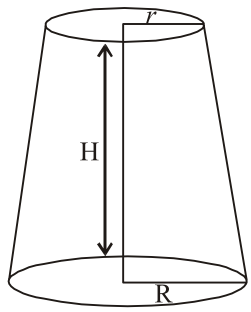
$$= \frac{\pi}{3} H \left\{ \left(\sqrt{\frac{Q}{\pi}}\right)^2 + \left(\sqrt{\frac{P}{\pi}}\right)^2 + \sqrt{\frac{Q}{\pi}}\sqrt{\frac{P}{\pi}} \right\}$$

$$= \frac{\pi H}{3}\left\{ \frac{Q}{\pi} + \frac{P}{\pi} + \frac{\sqrt{PQ}}{\pi} \right\}$$

$$= \frac{H}{3}(P + Q + \sqrt{PQ})$$

100. (a) Area of first end $P = \pi r^2 \Rightarrow r = \sqrt{\frac{P}{\pi}}$

Area of second end $Q = \pi R^2 \Rightarrow R = \sqrt{\frac{Q}{\pi}}$



According to question P < Q

$\therefore$ Difference in radii of the ends of the frustum $= R - r$

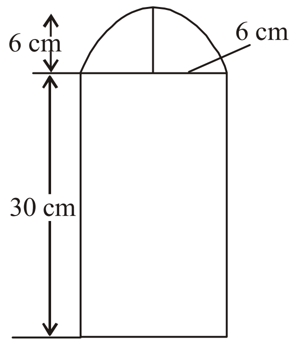
$$= \sqrt{\frac{Q}{\pi}} - \sqrt{\frac{P}{\pi}} = \frac{\sqrt{Q} - \sqrt{P}}{\sqrt{\pi}}$$

101. (d) Thickness of 10 circular plates are 3 cm cach.

$\therefore$ Height of the cylindrical $(h) = 3 \times 10 = 30$ cm

Radius of the hemisphere $(r) = 6$ cm

$\therefore$ Volume of the solid



= Volume of cylinder + Volume of hemisphere.

$$= \pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \pi\left[(6)^2 \times 30 + \frac{2}{3} \times 6 \times 6 \times 6\right]$$

$= \pi[36 \times 30 + 2 \times 72]$

$= \pi[1080 + 144] = 1224\,\pi \text{cm}^3$.

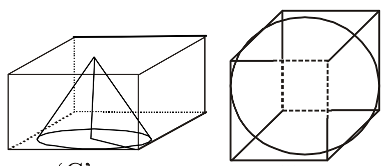
Therefore, the volume of the solid = $1224\,\pi$ cm^3.

102. (b) Let the side of cube = a

According to question

Height = a, radius = $\frac{a}{2}$

$r = \frac{a}{2}$, $h = a$



'C' 'S'

Radius of sphere (R) = $\frac{a}{2}$

$\therefore$ Volume of cone (C)

$$= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi\left(\frac{a}{2}\right)^2 a = \frac{\pi a^3}{12} \quad ...(i)$$

$$\text{Volume of sphere (S)} = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi\left(\frac{a}{2}\right)^3$$

$$= \frac{\pi a^3}{6} \quad ...(ii)$$

On solving eqs. (*i*) and (*ii*),

S = 2C

103. (a) Given that, the height and radius of a right circular metal cone (solid) are 8 cm and 2 cm, respectively.
i.e., $h = 8$ cm and $r = 2$ cm
Let the radius of the sphere is R.

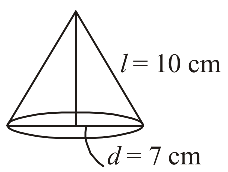
Then, by condition, $\frac{1}{3}\pi r^2 h = \frac{4}{3}\pi R^3$

$\Rightarrow$ $4 \times 8 = 4R^3$
$\Rightarrow$ $R^3 = (2)^3$
$\therefore$ R = 2
$\therefore$ Radius of the sphere = 2 cm

104. (c) Let the edge of a square x. Then, its volume = x^3 and sum of its edges = $12x$
According to question
$x^3 = 12x \Rightarrow x(x^2 - 12) = 0$
$\Rightarrow$ $x^2 = 12$ $(\because x \neq 0)$
$\therefore$ Total surface area = $6x^2 = 6(12) = 72$ sq units

105. (a) Diameter of a cone = 7 cm

$\therefore$ Radius of cone = $\frac{7}{2}$ cm



Slant height of a right circular cone (l) = 10 cm
$\therefore$ Lateral surface area of a cone = $\pi r l$

$$= \frac{22}{7} \times \frac{7}{2} \times 10 = 11 \times 10 = 110 \text{ cm}^2$$

106. (d) Radius (r) = 5 cm
Total surface area = 660 cm^2
$\Rightarrow$ $2\pi rh + 2\pi r^2 = 660$
$\Rightarrow$ $2\pi r(h + r) = 660$

$$\Rightarrow (h+5) = \frac{330}{5\pi} = \frac{330}{5} \times \frac{7}{22}$$

$$\Rightarrow h = \frac{66 \times 7}{22} - 5 = 21 - 5 = 16 \text{ cm.}$$

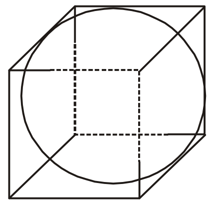
107. (a) Given that, let the diameters of two spheres are d_1 and d_2, respectively.
$\therefore$ $d_1 : d_2 = 3:5$
$\therefore$ Ratio of their surface areas

$$= \frac{4\pi r_1^2}{4\pi r_2^2} = \frac{(2r_1)^2}{(2r_2)^2} = \frac{d_1^2}{d_2^2}$$

$$= \left(\frac{d_1}{d_2}\right)^2 = \left(\frac{3}{5}\right)^2 = \frac{9}{25} = 9:25$$

108. (c) The diameter of largest sphere that is carved out from cube is equal to side of the cube.
Diameter of a sphere = Side of the cube = 3 cm

$\therefore$ Radius = $\frac{3}{2}$ cm



$\therefore$ Volume of the largest sphere

$$= \frac{4}{3}\pi\,(\text{radius})^3 = \frac{4}{3}\pi\left(\frac{3}{2}\right)^3$$

$$= \frac{4}{3}\pi\frac{27}{8} = \frac{9}{2}\pi = 4.5\pi \text{ cm}^3$$

109. (d) Radius of circular cone (r) = 12 m
Height of a circular cone (h) = 5 m

$\therefore$ Slant height (l) = $\sqrt{r^2 + h^2} = \sqrt{144 + 25} = \sqrt{169}$
= 13 m
$\therefore$ Required quantity of cloth to roll up to form a circulartent = $\pi r l = \pi(12)(13) = 156\pi$ sq m

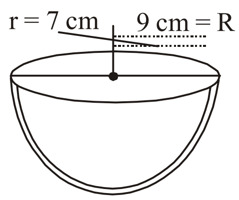
110. (b) Given that,
Length of a box (l) = 14 m
Breadth of a box (b) = 12 m
Height of a box (h) = 13 m
The greatest rod in a box is equal to diagonal of a box

$$= \sqrt{l^2 + b^2 + h^2} = \sqrt{(14)^2 + (12)^2 + (13)^2}$$

$$= \sqrt{196 + 144 + 169}$$

$$= \sqrt{509} = 22.56 \text{ m}$$

111. (a) Given that,
Outer radius of hemispherical shell (R) = 9 cm



and inner radius of hemispherical shell (r) = 7 cm
$\therefore$ Volume of a hemispherical shell

$$= \frac{2}{3}\pi(R^3 - r^3) = \frac{2}{3} \times \frac{22}{7} \times (729 - 343)$$

$$= \frac{2}{3} \times \frac{22}{7} \times 386 = \frac{16984}{21} = 808.76$$

$\approx$ 808 cu cm (approx)

112. (b) Let sphere radius = r
Volume of sphere = 36π

$\Rightarrow \quad \frac{4}{3}\pi r^3 = 36\pi$

$\Rightarrow \quad r^3 = 27 = (3)^3$

$\therefore \quad r = 3$ cm

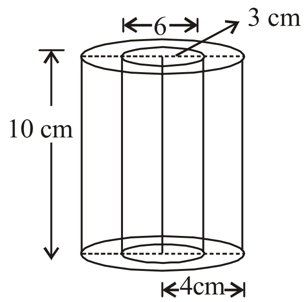
$\therefore$ Diameter of sphere

$= 2r = 2(3) = 6$ cm

Surface area of sphere $= 4\pi r^2 = 4\pi(3)^2 = 36\pi$ sq cm

$$\text{Ratio} = \frac{\text{Surface area of sphere}}{\text{Diameter of sphere}} = \frac{36\pi}{6} = 6\pi$$

113. (d) Internal diameter of the tube = 6 cm



$\therefore$ Internal radius (r) = 3 cm

Height of the tube (h) = 10 cm

Thickness of the metal = 1 cm

$\therefore$ Outer radius (R) = Thickness of the metal + Internal radius = 1 + 3 = 4 cm

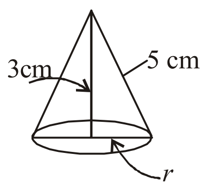
$\therefore$ Outer curved surface area

$= 2\pi rh + 2\pi(R^2 - r^2)$

$= 2\pi(3)(10) + 2\pi(16 - 9)$

$= 60\pi + 14\pi = 74\pi$ sq cm

114. (b) Given that, height of cone (h) = 3 cm



Slant height of cone (l) = 5 cm

$l = \sqrt{r^2 + h^2} = 5$

$\Rightarrow \quad r^2 + h^2 = 25 \Rightarrow r^2 = 25 - 9 = 16$

$\therefore \quad r = 4$ cm

$\therefore \quad$ Volume of cone $= \frac{1}{3}\pi r^2 h$

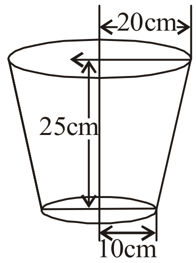
$= \frac{1}{3} \times \frac{22}{7} \times 16 \times 3 = \frac{352}{7} = 50.3$ cu cm

115. (a) Given that,

Height of bucket = 25 cm

Radii of top (R) = 20 cm

radii of bottom (r) = 10 cm



$\therefore$ Capacity of bucket

$= \frac{\pi}{3} h(R^2 + r^2 + Rr)$

$= \frac{\pi}{3} \times 25\,(400 + 100 + 200)\ \text{cm}^3$

$= \frac{\pi}{3} \times 25 \times 700\ \text{cm}^3$

$= \frac{\pi}{3} \times \frac{175 \times 100}{1000} = \frac{17.5\pi}{3}$ L

116. (d) Given that, $h = 15$ cm

and lateral surface = 660 cm^2

Let radius of cylinder = r

Lateral surface area of cylinder = $2\pi rh$

$\Rightarrow \quad 2\pi rh = 660$

$\Rightarrow \quad \pi rh = 330 \Rightarrow \frac{22}{7} \times r \times 15 = 330$

$\Rightarrow \quad \frac{22}{7} \times r = 22$

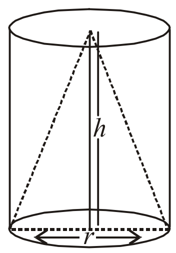
$\therefore \quad r = 7$ cm

$\therefore$ Volume of cylinder

$= \pi r^2 h = \frac{22}{7} \times 49 \times 15 = 22 \times 7 \times 15 = 2310$ cu cm.

117. (a) Let height and radius of cylinder is h and r respectively.

Volume of cylinder = $\pi r^2 h$



Volume of circular cone $= \frac{1}{3}\pi r^2 h$

$$\therefore \text{Required ratio} = \frac{\text{Volume of utilised wood}}{\text{Volume of wasted wood}}$$

$$= \frac{\text{Volume of right circular cone}}{(\text{Volume of right circular cylinder} - \text{Volume of right circular cone})}$$

$$= \frac{\frac{1}{3}\pi r^2 h}{\pi r^2 h - \frac{1}{3}\pi r^2 h} \Rightarrow \frac{\frac{1}{3}\pi r^2 h}{\frac{2}{3}\pi r^2 h} = \frac{1}{2} = 1:2$$

118. (a) Number of spheres

$$= \frac{\text{Volume of a solid sphere with radius 4 cm}}{\text{Volume of a solid sphere with radius 2 cm}}$$

$$= \frac{\frac{4}{3}\pi(4)^3}{\frac{4}{3}\pi(2)^3} = \frac{(2)^6}{(2)^3} = (2)^3 = 8$$

119. (c) Volume of cone and pyramid $= \frac{1}{3} \times$ Base area $\times$ Height

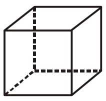
Since, volume of cone and pyramid are same but their surface area are not same because of their slant height.

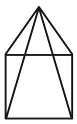
120. (b) The diameter of Moon is approximately one-fourth of the diameter of Earth.
Let radius of Moon $= r$,
then radius of Earth $= 4r$
Required ratio =

$$\frac{\text{Volume of Moon}}{\text{Volume of Earth}} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi(4r)^3} = \frac{r^3}{64r^3} = \frac{1}{64} = 1:64$$

121. (a) Total surface area of a cube $= 6a^2$
$\Rightarrow \quad 6 = 6a^2 \Rightarrow a^2 = 1$
$\therefore \quad a = 1$ unit
Volume of the cube $= a^3 = 1^3 = 1$ cu unit

122. (d) **Cube figure** **Pyramid figure**





Cube figure	Pyramid figure
Edges, $a = 12$	Edges, $a = 8$
Faces, $b = 6$	Faces, $b = 5$
Corner, $c = 8$	Corner, $c = 5$

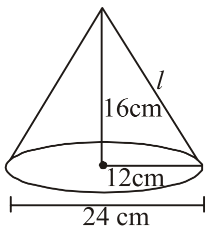
Therefore, the result $a = b + c$ is neither true for cube nor for the pyramid.

123. (a) Volume of cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 10 \times 10 \times 21$

$= 2200\ \text{cm}^3$

124. (c) $l = \sqrt{h^2 + r^2} = \sqrt{16^2 + 12^2} = \sqrt{256 + 144}$

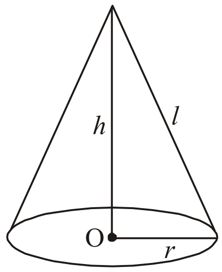
$= \sqrt{400} = 20$ cm



Curved surface area $= \pi r l = \frac{22}{7} \times 12 \times 20$

Cost of painting $= \frac{22}{7} \times 12 \times 20 \times 0.70 =$ ₹ 528

125. (b) Slant height, $l = \sqrt{h^2 + r^2}$



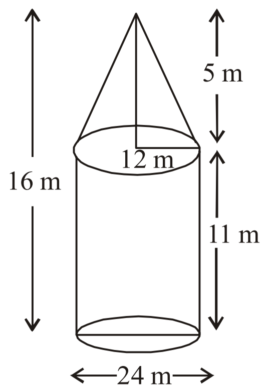
$= \sqrt{(24)^2 + (7)^2} = \sqrt{576 + 49} = \sqrt{625} = 25$

Total surface area $= \pi r l + \pi r^2$

$= \frac{22}{7} \times 7 \times 25 + \frac{22}{7} \times 7 \times 7 = 704$ sq cm.

126. (c) Slant height of the cone $= \sqrt{5^2 + 12^2}$

$= \sqrt{25 + 144} = \sqrt{169} = 13$ m



Curved surface area for conical portion $= \pi r l$

$= \frac{22}{7} \times 12 \times 13 = \frac{3432}{7}$ sq m

127. (d) According to question,
Curved surface area of cylinder $= 2\pi rh = x$
Volume of cylinder $= \pi r^2 h = y$

$\Rightarrow \quad \frac{2\pi rh}{\pi r^2 h} = \frac{x}{y}$

$\Rightarrow \quad r = \frac{2y}{x}$

Now, Curved surface area of cylinder
$\Rightarrow \quad 2\pi rh = x$

$\therefore \quad h = \frac{x}{2\pi r} = \frac{x^2}{4\pi y}$

$\therefore$ Required ratio =

So, Neither (A) nor (B) is correct.

128. (a) Let common radius = r cm
Height of cylinder = h_1
Height of cone = h_2
$\therefore$ Volume of the complete structure

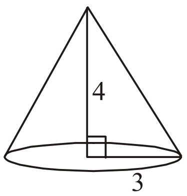
$$= \frac{1}{3}\pi r^2 h_2 + \pi r^2 h_1 + \frac{2}{3}\pi r^3$$

$$= \pi r^2\left(\frac{h_2}{3} + h_1 + \frac{2}{3}r\right)$$

$$= \pi(3.5)^2\left(\frac{2.8}{3} + 6.5 + \frac{2}{3}\times 3.5\right)$$

$= \pi \times 3.5 \times 3.5 \times 9.76 = 375.86\ \text{cm}^3$
Volume (V) of the structure lies between 370 cm^3 and 380 cm^3.

129. (b) Radius of the cone = $\frac{6}{2} = 3$ cm
Height of the cone = 4 cm



$l = \sqrt{r^2 + h^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = 5$ cm
Curved surface area = $\pi r l$
$\therefore$ Curved surface area = $\pi \times 3 \times 5 = 15\pi \approx 47$ sq cm

130. (d) Volume of clay required

$$= \pi\left[\left(\frac{5.1}{2}\right)^2 - \left(\frac{4.5}{2}\right)^2\right] \times 21$$

$= \pi\,[(2.55)^2 - (2.25)^2] \times 21$
$= \pi\,(0.3 \times 4.8) \times 21 = 30.24\,\pi\ \text{cm}^3$

131. (d) The stack is in the form having vertical cross section of trapezium.
$\therefore$ Volume of Earth in the stack = Area of cross section of trapezium × Height

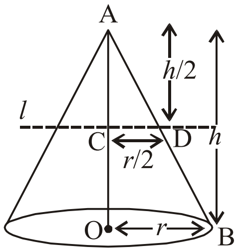
$\therefore$ Volume $= \frac{1}{2} \times (10 + 5) \times 2 \times 8 = 15 \times 8\ \text{m}^2$

According to the question,
Volume of Earth to be spread = (Area of field) × Level raised

$\therefore$ Level raised $= \frac{15 \times 8}{100 \times 80} = \frac{15}{1000}\ \text{m} = 1.5$ cm

132. (a) Surface area of cube = 6 $(\text{Side})^2$
$= 6(2)^2 = 24\ \text{cm}^2$
Surface area of cuboid
$= 2(lb + bh + lh)$
$= 2(2 + 6 + 3) = 22\ \text{cm}^2$
Total surface area of both cube and cuboid
$= 24 + 22 = 46\ \text{cm}^2$
Give area to point is 54 cm^2
But total surface area which is need to be painted is 46 cm^2. So both, cube and cuboid painted.

133. (b) Let the cone is divided into two parts by a line l.



Now triangle ACD and AOB are similar.
(According to proportionality theorem)

$\text{CD} = \frac{r}{2}$, since $\text{AC} = \frac{h}{2}$

Required ratio = $\frac{\text{Volume of original cone}}{\text{Volume of smaller cone}}$

$$= \frac{\frac{1}{3}\pi r^2 h}{\frac{1}{3}\pi\left(\frac{r}{2}\right)^2\left(\frac{h}{2}\right)} = \frac{8}{1}$$

$\therefore$ Required ratio = 8 : 1

134. (c) Volume of each small sphere

$$= \frac{\text{Volume of bigger sphere}}{\text{Number of small spheres}}$$

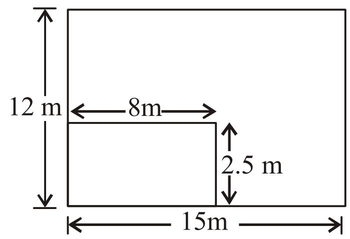
$$= \frac{\frac{4}{3}\pi(4)^3}{64} = \frac{4}{3} \times \frac{\pi \times 4 \times 4 \times 4}{64} = \frac{4}{3}\pi\ \text{cm}^3$$

Let radius of small sphere = R

$\therefore \frac{4}{3}\pi R^3 = \frac{4}{3}\pi \Rightarrow R = 1$ cm

Surface area of small sphere = $4\pi R^2 = 4\pi\ \text{cm}^2$

135. (c)



Volume of pit = $l\,b\,h = 8 \times 2.5 \times 2 = 40\ \text{m}^3$.
Let the label of the earth spread over remaining area = h.
Volume of the earth spread = Volume of a pit
$\Rightarrow [(12 \times 15) - (8 \times 2.5)] \times h = 40$

$\therefore h = \frac{40}{180 - 20} = \frac{40}{160} = \frac{1}{4}\ \text{m} = 25$ cm

136. (a) Surface area of sphere = 616 cm^2
$4\pi r^2 = 616$

$\Rightarrow r^2 = \frac{616 \times 7}{4 \times 22}$

$\Rightarrow r^2 = 7 \times 7$
$\therefore r = 7$ cm

$\therefore$ Diameter of largest circle which lying on sphere $= 2 \times r = 14$ cm

137. (c) Volume of the cube $= 216x^3$

$(\text{Side})^3 = 216x^3 \Rightarrow \text{Side} = 6x$

Largest sphere which is enclosed in cube the diameter of sphere is equal to side of the cube.

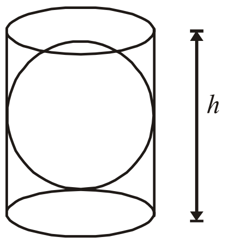
$\therefore$ Diameter of sphere $= 6x$

Surface area of the sphere $= 4\pi r^2$

$$= 4\pi\left(\frac{6x}{2}\right)^2 = 36\pi x^2$$

138. (a) Let radius of the sphere be r.

Cylinder circumscribes a sphere it means that radius of the base of cylinder $= r =$ sphere radius. and height of cylinder $= 2r =$ Diameter of sphere



Volume of sphere $= \frac{4}{3}\pi r^3$

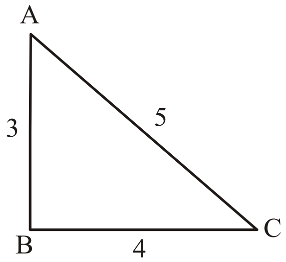
Volume fo cylinder $= \pi r^2 h = \pi r^2 (2r) = 2\pi r^3$

$\therefore$ Required ratio $= \dfrac{\frac{4}{3}\pi r^3}{2\pi r^3} = \dfrac{4}{3\times 2} = \dfrac{2}{3} = 2 : 3$

139. (d) ΔABC is right angled triangle.

$AB = 3$cm, $BC = 4$cm and $AC = 5$cm

1. When the triangle revolves about its longer leg, $BC =$ 4 cm.



Volume of cone $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(3)^2 \times 4$

$= 12\pi\text{ cm}^3$ .... (i)

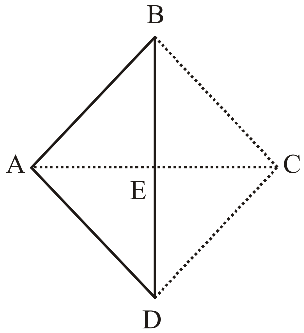
Now triangle revolve about its shorter leg, $AB = 3$ cm

Volume of cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(4)^2 \times 3$

$= 16\pi\text{ cm}^3$ .... (ii)

From equations (i) and (ii), it is clear that volume of both cones are not same. So, statement 1 is not correct.

2. The triangle revolve about hypotenus, then we get double cones ABD and BCD.



$\because$ $\Delta BEA \sim \Delta BAC$

$\therefore$ $\frac{BE}{BC} = \frac{AB}{AC} \Rightarrow \frac{\text{BE}}{4} = \frac{3}{5}$

$BE = 2.4$ cm

Radius of the base of cone, $BE = 2.4$ cm

In right angled ΔBEA,

By Pythagoras theorem,

$AE = \sqrt{(AB)^2 - (BE)^2} = \sqrt{9-(2.4)^2} = 1.8$ cm

Height of cone $ABD = AE = 1.8$ cm

$\therefore$ Height of cone $BCD = AC - AE = 5 - 1.8 = 3.2$ cm

Now, volume of cone $ABD = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(2.4)^2 \times 1.8 = 3.456\pi\text{ cm}^3$

volume of cone $BCD = \frac{1}{3}\pi(2.4)^2 \times 3.2 = 6.144\pi\text{ cm}^3$

Volume of double cone $= 3.456\pi + 6.144\pi$

$= 9.6\pi\text{ cm}^3$ ... (iii)

From equations (i) and (ii), we get

Volume of both cones $= 12\pi + 16\pi = 28\pi\text{ cm}^3$ ...(iv)

From equations (iii) and (iv), we get

Volume of double cone Volume of both cones

So, Statement 2 is also not correct.

140. (c) $x + y + \frac{xy}{100}$

$= 100 + 100 + \frac{100\times 100}{100}$

$= 200 + 100 = 300\%$

141. (c) $\therefore$ Surface area of sphere $A = 4\pi r^2 = 4\pi(6)^2 = 144\pi\text{ cm}^2$

Surface area of sphere $B = 4\pi(8)^2 = 256\pi\text{ cm}^2$

Surface area of sphere $C = 4\pi(10)^2 = 400\pi\text{ cm}^2$

and Surface area of sphere $D = 4\pi(12)^2 = 576\pi\text{ cm}^2$

Sum of surface area of spheres A and B

$= 144\pi + 256\pi = 400\pi\text{ cm}^2$

= Surface area of sphere, C

Hence, Statements 1 is correct.

2. $\because$ Volume of sphere $D = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(12)^3$

$= 2304\pi\text{ cm}^3$

Volume of sphere $A = \frac{4}{3}\pi(6)^3 = 288\pi\text{ cm}^3$

Volume of sphere $B = \frac{4}{3}\pi(8)^3 = \frac{2048}{3} = \pi\text{ cm}^3$

and Volume of sphere $C = \frac{4}{3}\pi(10)^3 = \frac{4000}{3} = \pi \text{ cm}^3$

According to question
sum of volumes of sphere A, B and C

$$= \left(288\pi + \frac{2048\pi}{3} + \frac{4000}{3}\pi\right)\text{cm}^3$$

$$= \frac{864 + 2048 + 4000}{3}\pi\text{cm}^3 = \frac{6912}{3}\pi\text{cm}^3$$

$= 2304\pi \text{ cm}^3 =$ Volume of sphere D

Hence, Statement 2 is also correct.

142. (a) Diameter of a sphere, $d = 6$ cm

$\therefore$ Radius of a sphere, $r = \frac{d}{2} = \frac{6}{2} = 3\,cm$

Let the radius of wire be R cm.
Also, given the length of wire, H = 36 m = 3600 cm
According to question
Volume of sphere = Volume of wire

$$\Rightarrow \frac{4}{3}\pi r^3 = \pi R^2 H$$

$$\Rightarrow \frac{4}{3}\times(3)^3 = R^2 \times 3600$$

$$\Rightarrow R^2 = \frac{4\times 3^2}{3600} = \frac{(6)^2}{(60)^2}$$

$$\therefore R = \frac{6}{60} = \frac{1}{10} = 0.1\,cm$$

143. (c) Volume of sphere $= \frac{4}{3}\pi r^3$

Here,
Volume of Sphere = Volume displaced in cylinder

$$\Rightarrow \frac{4}{3}\pi r_S{}^3 = \pi r_C{}^2 (h - h')$$

$$\Rightarrow \frac{4}{3}\pi \times 27 = \pi \times 16\,(h - h')$$

$$h - h' = \frac{9}{4} = 2.25\,\text{cm}$$

144. (a) $x + y + z + \frac{xy + yz + zx}{100} + \frac{xyz}{(100)^2}$

Increase in volume of sphere

$$= 10 + 10 + 10 + \frac{10\times 10 + 100 + 100}{100} + \frac{10\times 10\times 10}{(100)^2}$$

$= 30 + 3 + .1 = 33.1\%$

145. (a) According to question

$$4\pi r^2 = 6a^2$$

$$\frac{r^2}{a^2} = \frac{6}{4\pi}$$

Ratio of their volume $= \dfrac{\frac{4}{3}\pi r^3}{a^3}$

$$= \frac{4}{3}\pi\left(\frac{r}{a}\right)^3 = \frac{4\pi}{3}\cdot\frac{6}{4\pi}\sqrt{\frac{6}{4\pi}} = \sqrt{\frac{6}{\pi}}$$

Square of their volume ratio $= \frac{6}{\pi} = 6 : \pi$

146. (a) Radius of sphere = Radius of right circular cone
Now, Volume of sphere = 2 × Volume of cone

$$\Rightarrow \frac{4}{3}\pi r^3 = 2\times\frac{1}{3}\pi r^2 h$$

$$\Rightarrow 2r = h$$

$$\frac{h}{r} = \frac{2}{1} = 2{:}1$$

147. (b) Area of cross-section of cylindrical pipe

$$= \pi\times\left(\frac{7}{2}\right)^2 = \frac{49\pi}{4}\text{cm}^2$$

Volume of water flow per minute

$$= \frac{49\pi}{4} \times 5 \times 100 \times 60\,\text{cm}^3$$

Volume of the tank $= 3 \times 4 \times 231 \times 10000\,\text{cm}^3$

Now, time taken to fill up tank

$$= \frac{3\times 4\times 231\times 10000}{\frac{49}{4}\times\frac{22}{7}\times 5\times 100\times 60} = 24\,\text{min.}$$

148. (d) Surface area of cone $= \pi rl$

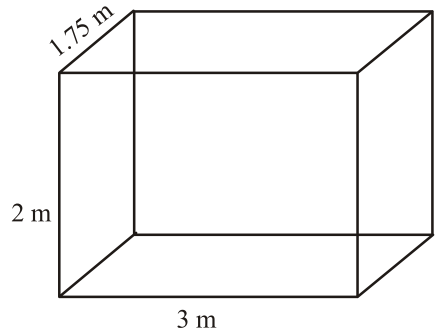
$$l = \sqrt{r^2 + h^2}$$

$$= \sqrt{(7)^2 + (24)^2}$$

$$= \sqrt{49 + 576} = \sqrt{625} = 25\text{ cm}$$

Surface area of cone $= \pi rl$
$= \pi \times 7 \times 25$
$= 175\pi$
Total surface area with hemisphere
$= 175\pi + 2\pi r^2$
$= 175\pi + 2 \times \pi \times 49$
$= 273\pi\,\text{cm}^2$

149. (c)



Surface Area of rectangular blocks

$= 2\,(3 \times 2 + 2 \times 1.75 + 3 \times 1.75) = 29.5\ \text{m}^2$

Paint required for 0.1 mm thickness

$= 29.5 \times \dfrac{1}{10{,}000} = 0.00295\text{m}^3$

Volume of cubical boxes

$= \dfrac{10}{100} \times \dfrac{10}{100} \times \dfrac{10}{100} = \dfrac{1}{1000}\text{cm}^3$

So boxes required $= \dfrac{0.00295}{0.001} = 2.95 \approx 3$

150. (c) Diagonals of the three faces are 13, $\sqrt{281}$ and 20

Let the sides of cuboid be l, b and h respectively

$l^2 + b^2 = (13)^2 = 169$ ... (i)

$b^2 + h^2 = (\sqrt{281})^2 = 281$ ... (ii)

$h^2 + l^2 = (20)^2 = 400$ ... (iii)

On subtracting equation (ii) from equation (iii)

$l^2 - b^2 = 400 - 281 = 119$ ... (iv)

Now add equations (iv) and (i)

$2l^2 = 119 + 169$

$l^2 = \dfrac{288}{2} = 144 \Rightarrow l = 12$

Put $l = 12$ in equation (i)

$b^2 = 169 - 144$

$b^2 = 25, b = 5$

Now, put value of b in equation (ii)

$b^2 + h^2 = 281$

$25 + h^2 = 281$

$h^2 = 256$

$\because \quad h = 16$

Total surface area of cuboid

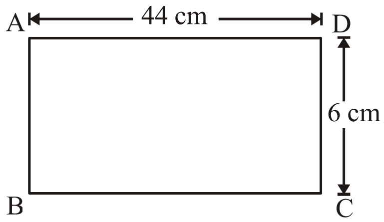
$= 2(lb + bh + hl)$

$= 2(12 \times 5 + 5 \times 16 + 16 \times 12)$

$= 2(60 + 80 + 192)$

$= 664$ square units

151. (c)



Radius of rolled cylinder

$\Rightarrow \quad 2\pi r = 44$

$\Rightarrow \quad 2 \times \dfrac{22}{7} \times r = 44 \Rightarrow r = 7\text{ cm}$

152. (a) Volume of new sphere

$V = V_1 + V_2 + V_3$

Formula for volume of sphere is $\dfrac{4}{3}\pi r^3$

$\Rightarrow \quad V = \dfrac{4}{3}\pi r_1^3 + \dfrac{4}{3}\pi r_2^3 + \dfrac{4}{3}\pi r_3^3$

$= \dfrac{4}{3}\pi\,(216 + 512 + 1000)$

$\Rightarrow \quad \dfrac{4}{3}\pi r^3 = \dfrac{4}{3}\pi(1728)$

$r^3 = 1728$

$\therefore \quad r = 12$

153. (b) Volulme of cone

$= x + y + z + \dfrac{xy + yz + zx}{100} + \dfrac{xyz}{(100)^2} \begin{pmatrix} x = 200 \\ y = -50 \\ z = -50 \end{pmatrix}$

$= 200 - 50 - 50 + \dfrac{\{200 \times (-50)\} + \{200 \times (-50)\} + 50 \times 50}{100}$

$+ \dfrac{\{200 \times (-50) \times (-50)\}}{100 \times 100}$

$= 100 + (-100 - 100 + 25) + 50$

$= 100 - 100 - 100 + 25 + 50$

$= -25\%$

Decrease by 25%

154. (c) Area of cross-section of nozzle $= 4\text{ cm}^2$

Velocity of water = 40 m/s = 4000 cm/sec

Water coming from nozzle in 1 sec

$= 4 \times 4000 = 16000\text{ cm}^3$

Dimension of cistern $= 10 \times 8 \times 6\text{ m}^3$

$= 480 \times 10^6\text{ cm}^3$

Time taken to fill the cistern $= \dfrac{480 \times 10^6}{16000}$ sec

$= \dfrac{480 \times 10^6}{16000} \times \dfrac{1}{3600}$ hours

$= \dfrac{4800}{16 \times 36}$ hours

$= \dfrac{25}{3}$ hours

= 8 hours 20 minutes.

So, option (c) is correct.

155. (d) Number of cylindrical boxes that can be packed in the drum

$= \dfrac{\pi \times (15)^2 \times 100}{\pi \times (5)^2 \times 10} = 90$

So, option (d) is correct.

156. (c) Curved surface area of cylinder $= 2\pi rh$

$S = 2\pi rh$

If height is doubled-

$S' = 2\pi r\,(2h)$

$S' = 4\pi rh$

$\boxed{S' = 2S}$

Total surface area of hemisphere $= 3\pi R^2$

If radius is doubled-

total Surface Area $= 3\pi(2R)^2$

$= 12\pi R^2 = 4 \times 3\pi R^2$

$= 4 \times$ (Initial Surface Area)

So option (c) is correct.

157. (a) Let side of cubical water tank be 'x' meter.
Capacity of tank = x^3
According to question-
$\Rightarrow x^3 - 128 = (x-2).x^2$
$\Rightarrow x^3 - 128 = x^3 - 2x^2$
$\Rightarrow 2x^2 = 128$
$\Rightarrow x^2 = 64$
$\Rightarrow x = 8$ metre
Capacity of tank $(8)^3 = 512\ m^3$
So, option (a) is correct.

158. (c) Volume of solid cubical gold = $(1)^3 = 1\ cm^3$
Let radius of spherical solid balls be r.

$$4\pi r^2 = \pi^{\frac{1}{3}} = r^2 = \frac{\pi^{\frac{1}{3}}}{4\pi} = r = \frac{\pi^{\frac{1}{3}}}{2}$$

$$\text{No of balls} = \frac{1}{\frac{4}{2}\pi\left(\frac{\pi^{-\frac{1}{3}}}{2}\right)^3}$$

$$= \frac{3}{4\pi} \times \frac{8}{\pi^{-1}} = 6$$

So, option (c) is correct

159. (c) At sometimes, the contoiner was full.
At 25 minutes, it was holy full.
At 20 minutes, it was the fourth full.
So, option (c) is correct.

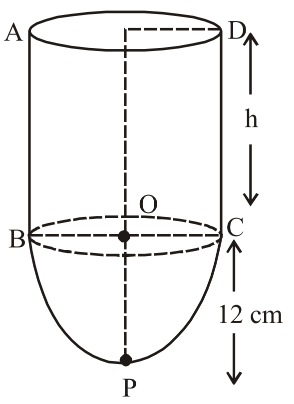
160. (c) Let the radius and height of each cylinder be 'r' and 'h' respectively.
Volume of 30 metallic cylinders
$= 30 \times \pi^2 h$
Let the no. of cones casted be 'N'

$$N \times \frac{1}{3}\pi r^2 h = 30\pi r^2 h$$

$$\boxed{N = 90}$$

The curved surface of cylinder in reactangle and curved surface of cone is semi-circle when they are flattened.
So, option (c) is correct.

161. (b)



Total capacity
= volume of cylinder + volume of hemisphere

$$\Rightarrow 3312\pi = \pi r^2 h + \frac{2}{3}\pi r^3$$

$$\Rightarrow 3312\pi = \pi\left[(12)^2 h + \frac{2}{3}(12)^3\right]$$

$$\Rightarrow 3312 = 144\left[h + \frac{2}{3} \times 12\right]$$

$\Rightarrow h + 8 = 23$
$\Rightarrow h = 15$ metre

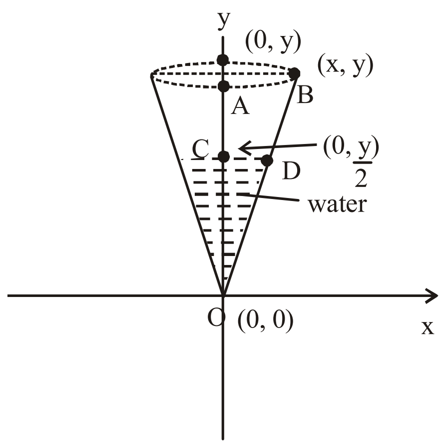
$$\frac{\text{Surface area of hemisphere}}{\text{Surface area of cylinder}} = \frac{2\pi r^2}{2\pi rh}$$

$$= \frac{r}{h} = \frac{12}{15} = \frac{4}{5}$$

So, option (b) is correct.

162. (b) Let sides of cuboid be a, b, c
$x = a^2$
$y = b^2$
$z = c^2$
volume of cuboid V = a. b. c
$V^2 = a^2 b^2 c^2$
$V^2 = x . y. z$
So, option (b) is correct

163. (a)



$$V = \frac{1}{3}\pi x^2 y \qquad \text{------------(i)}$$

Δ COD and Δ AOB are similar

$$\frac{CO}{AO} = \frac{CD}{AB}$$

$$\frac{\frac{y}{2}}{y} = \frac{CD}{x}$$

$$CD = \frac{x}{2}$$

$$\text{Volume of water} = \frac{1}{3}\pi(CD)^2 . \frac{y}{2} = \frac{1}{3}\pi.\left(\frac{x}{2}\right)^2 . \frac{y}{2}$$

$$= \frac{1}{8}\left[\frac{1}{3}\pi x^2 y\right] = \frac{1}{8}v$$

So, option (a) is correct.

164. (c) Let length of the cube be a
$6a^2 = 13254$
$a^2 = 2209$
$a = 47$
Length of diagonal = $47\sqrt{2}$ cm

165. (a) No. of bullets
$$= \frac{44\times44\times44}{\frac{4}{3}\times\frac{22}{7}\times\left(\frac{4}{2}\right)\times\left(\frac{4}{2}\right)\times\left(\frac{4}{2}\right)}$$
$$= \frac{11\times11\times11\times21\times8}{22\times4} = 2541$$
So, option (a) is correct.

166. (b) Water running per hour in the river
$= 2.5 \times 45 \times 3600\ m^3$.
Water running per minutes in the river
$$= \frac{2.5\times45\times3600}{60}\ m^3$$
$= 2.5 \times 45 \times 60$
$= 6750\ m^3$
So, option (a) is correct

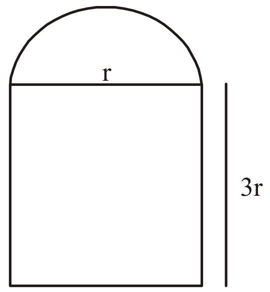
167. (b) area of 4 wall = 2 (l + b) h
$l = 2b \quad b = b \quad h = 4$
$2(2b + b) \times 4 = 120$
$3b = 15$
$b = 5$
$\ell = 2 \times 5 = 10$
area $= 5 \times 10 = 50\ m^2$

168. (c) C.S.A cylinder : T.S.A. cylinder = 1 : 2
$$\frac{2\pi rh}{2\pi r(r+h)} = \frac{1}{2} = \frac{h}{r+h}$$
$\Rightarrow 2h = h + r$
$h = r$
$\Rightarrow$ Gives T.S.A = 616 cm^2
$2\pi r[r + r] = 61.6\ cm^2$
$4\pi r^2 = 616$
$$r^2 = \frac{616\times7}{22\times4} = 49$$
$r = 7$
$h = 7$
Volume $\pi r^2 h = \frac{22}{7}\times7\times7\times7 = 1078\ cm^3$

169. (b) Let side of square be a
Volume = 1
$a^2 \times 9 = 1$
$$a = \frac{1}{3}$$
Volume of cube $= a^3 = \frac{1}{27}\ m^3$
Required weight $= 9000 \times \frac{1}{27} = \frac{1000}{3}$ kg

170 (c) R = 8.4m, h = 3.5 Space required for 1 bag of wheat = 1.96 m^3
Let number of bag = n
$$n(1.96) = \frac{1}{3}\pi r^2 h$$
$$n = \frac{1}{3}\,\frac{22}{7}\times\frac{8.4\times8.4}{1.96}\times3.5 = 132$$

171. (a) Radius of hemisphere = r



radius of cylinder = r
height of cylinder = 2r
Total volume = $V_{hemisphere} + V_{cylinder}$
$$= \frac{2}{3}\pi r^3 + \pi(r)^2(2r) = 67\frac{1}{21}$$
$$= 2\pi r^3\left[\frac{1}{3}+1\right] = \frac{1408}{21}$$
$$\frac{2\times22}{7}\times\frac{4}{3}r^3 = \frac{1408}{21}$$
$$r^3 = \frac{1408}{2\times22\times4}$$
$r^3 = 8 = 2^3$
$r = 2$
height of building = 3r = 6

172. (c) Let radius and height of cylinder be 2x and 3x Respectively
$\pi r^2 h = 1617$
$\pi(2x)^2 3x = 1617$
$\pi 12x^3 = 1617$
$$x^3 = \frac{1617\times7}{22\times12} = \left(\frac{7}{2}\right)^3$$
$x = \frac{7}{2}; r = 7 \qquad h = \frac{21}{2}$
Total surface area = $2\pi r(r + h)$
$$= \frac{2\times22}{7}\times7\left[7+\frac{21}{2}\right]$$
$$\Rightarrow 44\left[\frac{35}{2}\right] \Rightarrow 22\times35 \Rightarrow 770\ cm^2$$

173. (c) Radius of sphere $= \frac{14}{2}$ cm = 7 cm
Total surface area $= 4\pi r^2 = 4\times\frac{22}{7}\times7\times7$

Total cost of painting = ₹8008

Cost per square cm = $\frac{8008}{4\times22\times7}$ = ₹ 13 per cm^2

174. (a) Frustum

$r=\frac{4}{2}, R=\frac{18}{2}, h=24$

$\text{Volume}=\frac{\pi}{3}h\left[r_1^2+r_2^2+r_1r_2\right]$

$\pi x=\frac{22}{7\times3}\times24\ [(2)^2+(9)^2+2\times9]=824\text{ cm}^3$

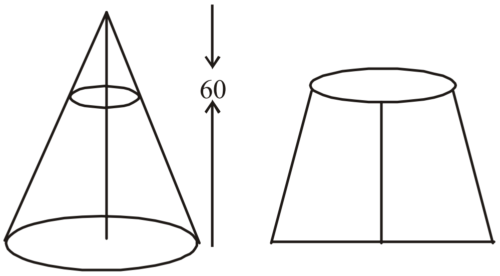
175. (c) Volume of roof = volume of cylindrical vessel

$22\text{m}\times20\text{m}\times x\text{m}=\pi r^2h$

$22\times20\times x=\frac{22}{7}\times1\times1\times3.5$

$x=\frac{22\times3.5}{722\times20}=\frac{.5}{20}=.025\text{ m}=2.5\text{ cm}$

176. (d)



Volume of cone be V_1 and V_2

$\frac{V_1}{V_2}=\frac{64}{1}=\frac{r^2h}{R^2H},\left[\frac{r}{R}=\frac{h}{H}\right]\therefore$ similar triangles

$\frac{h_1}{h_2}=\sqrt[3]{\left(\frac{4^3}{1}\right)}=\frac{4}{1}$

$=\frac{1}{4}\times60\text{ cm}=15\text{ cm}$

height of small cone = 15 cm

height from base to where cut = 60 – 15 = 45 cm

177. (a) Volume of sphere where R is 3 cm = $\frac{4}{3}\pi r^3$

$=\frac{4}{3}\pi3\times3\times3=36\pi\text{ cm}^3$

178. (b) Radius cone = $\frac{126}{2}=63\text{m}$

height of cone = 21 – 5 = 16 m

slant height = $\sqrt{63^2+16^2}=\sqrt{3969+256}$

$=\sqrt{4225}=65\text{ m}$

179. (a) Radius of cylinder = 63 m

height = 5 m

$\text{CSA}=2\pi rh=2\times\frac{22}{7}\times63\times5$

$\text{CSA}=1980\text{ m}^2$

180. (d) Canvas used = $\text{CSA}_{\text{cylinder}}+\text{CSA}_{\text{cone}}$

$=2\pi rh+\pi rl$

$=1980+\frac{22}{7}\times63\times65=14850\text{ m}^2$

181. (b) Let side of square = x

perimeter = 4x

Circumference of circle = 4x

$2\pi r=4x$

$r=\frac{4x}{2\pi}=\frac{2x}{\pi}$

$\frac{\text{Area Circle}}{\text{Area Square}}=\frac{x\left(\frac{2x}{\pi}\right)^2}{x^2}$

$=\frac{\pi4x^2}{\pi^2x}=\frac{4\times7}{22}=\frac{14}{11}$

182. (d) Volume of cone when R = r H = h = $\frac{1}{3}\pi r^2h$

Volume of cone when R = r + p%r H = n

$V=\frac{1}{3}\pi\left(r+\frac{pr}{100}\right)^2h$

$\text{increase}=\frac{1}{3}\pi h\left[r+\frac{pr}{100}\right]^2-\frac{1}{3}\pi r^2h$

$=\frac{1}{3}\pi h\left[r^2+\left(\frac{pr}{100}\right)^2+\frac{2rpr}{100}-r^2\right]$

$=\frac{1}{3}\pi h\frac{r^2p}{(100)}\left[R+\frac{P}{100}\right]$

$\%=\frac{\frac{1}{3}\pi hr^2p\left[R+\frac{P}{100}\right]}{\frac{1}{3}\pi r^2\pi100}\times100$

$=P\left[R+\frac{P}{100}\right]$

or

$\boxed{P+P+\frac{P\times P}{100}=P\left(2+\frac{P}{100}\right)}$

183. (b) When r = r Surface area = 9

When r = x Surface area = 1

$\frac{9}{1}=\frac{4\pi r^2}{4\pi x^2}$

$x^2=\frac{r^2}{9}=\left(\frac{r}{3}\right)^2$

$x=\frac{1}{3}^{\text{rd}}$

184. (c) Cylinder | Cone

Radius $= \frac{35}{2}/2\text{cm}$ | Radius $= \frac{4}{2}$ cm

height = 32 cm | height = 7 cm

volume $= \pi(\text{rs})^2\text{h}$ | Volume $= \frac{1}{3}\pi r^2 h$

Volume of cylinder = volume of n cone

$$\pi\left(\frac{35}{2}\right)^2 32 = \frac{n\pi}{3}(2)^2 7$$

n = 1050

185. (b) Since HCF of two polynomials is $x^2 + x - 2$, therefore splitting this polynomial by middle term, we get

$$x^2 + x - 2 = x^2 + 2x - x - 2 = x(x+2) - (x+2)$$
$$= (x-1)(x+2)$$

Being the HCF of the given polynomials, we conclude that $(x-1)(x+2)$ is a factor of $f(x)$ and $g(x)$. Therefore, dividing $f(x)$ by the HCF and dividing $g(x)$ by the HCF will give the values of a and b.

Now, $\frac{(x-1)(x^2+3x+a)}{(x-1)(x+2)} = \frac{(x^2+3x+a)}{(x+2)}$

and $\frac{(x+2)(x^2+2x+b)}{(x-1)(x+2)} = \frac{(x^2+2x+b)}{(x-1)}$

Since $(x+2)$ is a factor of (x^2+3x+a), therefore, $x = -2$will satisfy this polynomial. Thus,

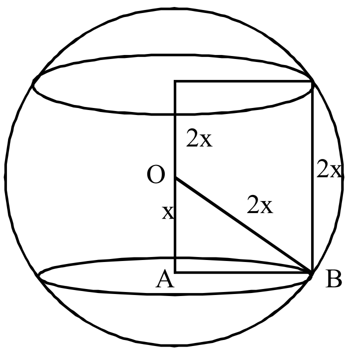
$$x^2 + 3x + a = 0 \Rightarrow (-2)^2 + 3.-2 + a = 0$$
$$\Rightarrow 4 - 6 + a = 0 \Rightarrow -2 + a = 0 \Rightarrow a = 2$$

Also, since $(x-1)$ is a factor of $x^2 + 2x + b$, therefore, $x = 1$ will satisfy this polynomial. Thus,

$$x^2 + 2x + b = 0 \Rightarrow 1 + 2 + b = 0 \Rightarrow b + 3 = 0 \Rightarrow b = -3$$

Hence, $a = 2, b = -3$

186. (a)



In ΔOAB, by Pythagoras theorem,
Radius of cylinder =

$$\sqrt{OB^2 - OA^2} = \sqrt{(2x)^2 - x^2} = \sqrt{4x^2 - x^2}$$
$$= \sqrt{3x^2} = \sqrt{3}x$$

CSA of the cylinder

$$= 2\pi rh = 2.\pi.\sqrt{3}x.2x = 4\sqrt{3}\pi x^2$$

Surface area of the sphere

$$= 4\pi r^2 = 4.\pi.4x^2 = 16\pi x^2$$

Ratio of CSA of the cylinder and surface area of the sphere $= \frac{4\sqrt{3}\pi x^2}{16\pi x^2} = \frac{\sqrt{3}}{4} = \sqrt{3} : 4$

187. (d) Since the sphere is dropped in the cylindrical vessel partially filled with water and is completely immersed, therefore the volumes of 3d-shapes will be equal.
Let r be the radius of cylinder and R be the radius of the sphere.
Thus, volume of cylinder = volume of sphere

$$\pi r^2 h = \frac{4}{3}\pi R^3 \Rightarrow 30 \times 30 \times h = \frac{4}{3} \times 15 \times 15 \times 15$$

$$\Rightarrow h = \frac{4 \times 15 \times 15 \times 15}{3 \times 30 \times 30} \Rightarrow h = 5 \text{ cm}$$

188. (c) Let the radius of the base of the cone be r cm.

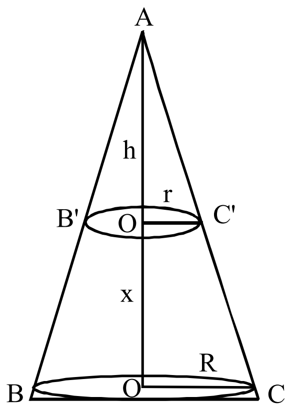
Slant height $= \sqrt{2}r$ cm (given)

Height of the cone =

$$\sqrt{l^2 - r^2} = \sqrt{(\sqrt{2}r)^2 - r^2} = \sqrt{2r^2 - r^2} = \sqrt{r^2} = r$$

Volume of the cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2.r = \frac{\pi r^3}{3}$

189. (d)



Let the radii of frustum of a cone be r_1 and r_2.

We are given that $\frac{R}{r} = \frac{2}{1}$ ...(1)

Now, let $\angle AC'O' = \angle ACO = \theta$

Therefore, in ΔAC'O', $\tan\theta = \frac{h}{r_2} = h$ ...(2) (by (1))

In ΔACO, $\tan\theta = \frac{h+x}{r_1} = \frac{h+x}{2}$ ...(3)

(2) and (3) $\Rightarrow h = \frac{h+x}{2} \Rightarrow h + x = 2h \Rightarrow x = h$.

Therefore, $H = 2h \Rightarrow \frac{H}{h} = \frac{2}{1}$

Now, volume of frustum of the cone

$$= \frac{\pi h}{3}(R^2 + Rr + r^2)$$

and volume of the cone $= \frac{1}{3}\pi r^2 h$

$$\text{Ratio} = \frac{\frac{\pi h}{3}(R^2 + Rr + r^2)}{\frac{1}{3}\pi R^2 H} = \frac{h(R^2 + Rr + r^2)}{R^2 H}$$

$$= \frac{h(4r^2 + r^2 + 2r^2)}{4r^2.2h} = \frac{7r^2}{8r^2} = \frac{7}{8} = 7:8$$

190. (d) Since the conical cavity in the cylinder is hollowed out, therefore inner surface area of the cavity is curved surface area of the cone.

Therefore, CSA of the cone $= \pi rl = \pi.6.\sqrt{8^2 + 6^2}$

$= \pi.6.\sqrt{64 + 36 = 100} = \pi.6.10 = 60\pi$

191. (b) Volume of cylinder $= \pi r^2 h = \pi.50.50.10$

$= 25000\pi$ cu. m

Volume of cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi.50.50.(15 - 10)$

$= \frac{1}{3}\pi.50.50.5 = \frac{12500\pi}{3}$ cu. m

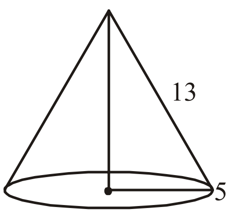
Total volume $= 25000\pi + \frac{12500\pi}{3} = \frac{87500\pi}{3}\text{cm}^3$

192. (b) The maximum possible volume of the circular cylinder that can be formed from a rectangular sheet will have the largest length and breadth i.e. we will consider the rectangular sheet with length 4π and breadth 2π.
The length of the rectangular sheet = circumference of the cylinder

$4\pi = 2\pi r \Rightarrow r = 2$

Volume of the cylinder $= \pi r^2 h = \pi.4.2\pi = 8\pi^2$

193. (a)



r = 5 cm $\ell = 13$ cm

$h = \sqrt{\ell^2 - r^2} = \sqrt{13^2 - 5^2} = \sqrt{144} = 12$

Hence, volume of cone $= \frac{1}{3} \times \pi \times r^2 \times h$

$= \frac{1}{3} \times \pi \times (5)^2 \times 12 = 100\pi$

194. (d) Let radius of cone be x and slant height of cone be 3x
Total surface area of cone $= \pi r(r + \ell)$
$= \pi \times x\,(x + 3x) = 4x^2\pi$
Curved surface area of cone $= \pi rl = \pi \times x \times 3x = 3x^2\pi$
Required ratio $= 4x^2\pi : 3x^2\pi = 4 : 3$

195. (b) Let volume of smaller cone be 64 unit and volume of frustum be 61 unit
Total volume of bigger cone = 64 + 61 = 125 units
Here smaller cone is cut from bigger cone then respective ratio of their radius, hight will be equal
∴ Respective ratio of area of bigger cone to that of smaller cone

$= \left(\sqrt[3]{125}\right)^2 : \left(\sqrt[3]{64}\right)^2 = 25:16$

Respective ratio of area of smaller cone to that of frustum
$= 16 : 25 - 16 = 16 : 9$

196. (c) Side of the room = 10m
Area of the room $= (10)^2 = 100\text{m}^2$
Side of table = 2m

Area of table $= \frac{\sqrt{3}}{4} \times (2)^2 = \sqrt{3} = 1.73\text{m}^2$

Area of 4 Book-shelves $= 4 \times (4 \times 1) = 16\text{m}^2$
Area of remaining room $= 100 - 1.73 - 16 = 82.27\,\text{m}^2$

Required cost $= \frac{82.27}{2} \times 100 = 4113$

197. (b)

198. (c) edge of cube = 2a

radius of cone $= \frac{2a}{2} = a$

hight of cone = 2a

Volume of cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times a^2 \times 2a$

$= \frac{2a^3\pi}{3} = \frac{2\pi a^3}{3}$

199. (b) Let radius of the sphere be r
According to the question

$4\pi r^2 = \frac{4}{3}\pi r^3$

$r^2 = \frac{r^3}{3}$

∴ r = 3 cm

200. (d) Radius of cone $= \frac{140}{2} = 70\text{ cm} = \frac{70}{100} = 0.7\text{ m}$

Curved surface are $= \pi r\,\ell = 1.76\,\text{m}^2$

$\frac{22}{7} \times 0.7 \times \ell = 1.76$

$\ell = \frac{1.76 \times 7}{22 \times 0.7} = 0.8\text{ m} = 80\text{ cm}$

height of cone $= \sqrt{\ell^2 - r^2} = \sqrt{80^2 - 70^2}$

$= \sqrt{1500} = 10\sqrt{15}$ cm

201. (c) According to the question

$\sqrt{3}\,a = 2r$ ∴ $a = \frac{2r}{\sqrt{3}}$

Required ratio $= \left(\frac{2r}{\sqrt{3}}\right)^3 : \frac{4}{3}\pi r^3$

$= \frac{8r^3}{3\sqrt{3}} : \frac{4}{3}\pi r^3 = 2 : \sqrt{3}\,\pi$

202. (d) According to the question
$2\pi r : h = 3 : 1$

$h = \frac{2\pi r}{3}$

$\ell = \sqrt{\left(\frac{2\pi r}{3}\right)^2 + r^2} = \sqrt{\frac{4\pi^2 r^2 + r^2}{9}}$

$= \sqrt{r^2 \frac{\left(4\pi^2 + 9\right)}{9}} = \frac{r\sqrt{4\pi^2 + 9}}{3}$

Curved surface area $= \pi r\ell = \pi \times r \times \frac{r\sqrt{4\pi^2 + 9}}{3}$

$= \frac{\pi r^2 \sqrt{4\pi^2 + 9}}{3}$

203. (c) length of wire $= 2\pi r = \frac{22}{7} \times 2 \times 98 = 616$
Perimeter of square $= 616$

Side of square $= \frac{616}{4} = 154$

204. (b) Side of square $= \frac{2}{\sqrt{\pi}}$

radius of circular disc $= \frac{2}{\sqrt{\pi}} \times \frac{1}{2} = \frac{1}{\sqrt{\pi}}$

area of disc $= \pi r^2 = \pi \times \left(\frac{1}{\sqrt{\pi}}\right)^2 = 1$ unit 2

205. (b) According to the question
$2\pi r\,(r + h) = 352$

$2 \times \frac{22}{7} \times r(r + 10) = 352$
$r^2 + 10r = 56$
$r^2 + 10r - 56 = 0$
$r^2 + 14r - 4r - 56 = 0$
$r(r + 4) - 4\,(r + 14) = 0$
$(r - 4)\,(r + 14) = 0$
$\therefore$ $r = 4, -14$
Hence radius of circle $= 4$
diameter $= 2 \times 4 = 8$

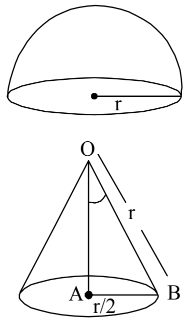
206. (b) Total area of fair walls of 5 rooms excluding 2 windows
$= 5 \times 2 \times h(\ell + b) - (2.5)^2 \times 2$
$= 5 \times 2 \times 2.5(6 + 4) - 12.5$
$= 237.5$

Number of cans required $= \frac{237.5}{20} \approx 12.$

207. (c) Lateral Surface area of the cone $= \pi.r.\ell$
so, $\pi.r. \times 35 = 462$

$r = \frac{462}{35 \times 22} \times 7 = 4.2$ cm

208. (c)



Since, the radius of semicircle = slant height of the cone = r
And the circumference of semicircle = circumference of base of cone.

$\pi r = 2\pi R \quad \Rightarrow \quad \frac{r}{2} = R.$

In ΔOAB

$\sin\theta = \frac{R}{r} = \frac{r/2}{r} = \frac{1}{2} = \sin 30°$
$\theta = 30°$

209. (b) lateral surface Area $= \frac{1}{2} \times$ total surface Area

$2\pi.r.h = \frac{1}{2}(2\pi r.h + 2\pi r^2).$

$2h = h + r \quad \Rightarrow \quad r = h = \pi$ cm.

210. (d) Volume of rectangular
$= 20 \times 15 \times 10 = 3000$
Now, Let number of equal dimensional small cube = n and dimension of cube is ℓ
then, $n.\ell^3 = 3000$
For, least possible integer value of n, ℓ must be maximum.
For $\ell = 6$, we are getting n in fraction and for $\ell = 5$
$n \times (5)^3 = 3000$

$n = \frac{3000}{125} = 24$

211. (d) Diagonal of a cube of side 'a'
$= \sqrt{3}.a = \ell$

$\therefore \quad a = \frac{\ell}{\sqrt{3}}$

total surface area of cube $= 6 \times a^2$

$= 6 \times \left(\frac{\ell}{\sqrt{3}}\right)^2 = 2\ell^2.$

212. (c)

Cone1		Cone2
Height	$\rightarrow$	1 : 3
Radius	$\rightarrow$	3 : 1

Volume of cone $= \frac{1}{3}\pi r^2 h$

Volume $\rightarrow$ 9 : 3
Required ratio = 3 : 1

213. (a) Let radius of the sphere is r then, volume $V = \frac{4}{3}\pi r^3$

When, volume increases by 700%

New volume $V' = V + \frac{700}{100} \times V = 8V$

Let changed radius is r'.

$\therefore \quad V' = \frac{4}{3}\pi(r')^3$

$8V = \frac{4}{3}\pi(r')^3$

$\therefore 8\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi(r')^3$

or $r' = (8r^3)^{\frac{1}{3}} = 2r$

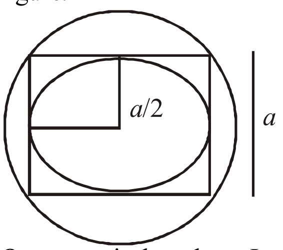
Percent change in surface area $= \left(\frac{S' - S}{S}\right) \times 100$

$= \frac{4\pi\{(r')^2 - r^2\}}{4\pi r^2} \times 100$

$= \left(\frac{(r')^2 - r^2}{r^2}\right) \times 100 = \left(\frac{(2r)^2 - r^2}{r^2}\right) \times 100 = 300\%$

Sol. (214-216):

The top view of the given assembly will look like the figure.



Outermost is the sphere. Inside that there is a cube and within that there is a cone and cylinder with same radius.

Here side of cube $= a$

Diameter of sphere = Diagonal of the cube $= \sqrt{3}\, a$

Radius of sphere $= \frac{\sqrt{3}\, a}{2} = r1$ (let)

Height of Cylinder = Height of cone

= side of cube $= a = h$

Radius of cylinder = Radius of cone

= side of cube/2 $= \frac{a}{2} = r_2$ (let).

214. (a) Required ratio

$= \frac{\text{Volume of sphere}}{\text{Volume of cone}} = \frac{\frac{4}{3}\pi(r_1)^3}{\frac{1}{3}\pi(r_2)^2 \times h} = \frac{6\sqrt{3}}{1}$

215. (c) Required ratio $= \frac{\text{Volume of cube}}{\text{Volume of cylinder}} = \frac{a^3}{\pi(r_2)^2 h}$

$= \frac{a^3}{\pi\left(\frac{a^2}{4}\right)a} = \frac{14}{11}$

216. (d) Surface area of the sphere $= 4\pi(r_1)^2$

$= 4\pi\left(\frac{\sqrt{3}a}{2}\right)^2 = 3\pi a^2.$

Curved surface area of cone

$= \pi r_2.l = \pi.r_2(h^2 + r_2^2)^{\frac{1}{2}}$

$= \sqrt{5}\pi\frac{a^2}{4}$

Surface area of cube $= 6a^2$

Curved surface area of the cylinder $= 2\pi r_2.h = \pi.a^2$

Hence, neither statement 1 nor 2 are true.

Sol. (217-218):

217. (a) Let lengths, breadth and height of cuboid be l, b and h respectively

According to question

$l + b + h = 22$ cm ...(i)

and $\sqrt{l^2 + b^2 + h^2} = 14$ ...(ii)

Surface area of cuboid $= 2(lb + bh + lh)$

Squaring eq. (i) gives

$l^2 + b^2 + h^2 + 2\,(lb + bh + lh) = 484$

Substituting $l^2 + b^2 + h^2$ from eq (i)

$2\,(lb + bh + lh) = 484 - 196 = 288$ cm^2

218. (c) $S = l^3 + b^3 + h^3$

and $V = lbh$

Now, $S - 3V = l^3 + b^3 + h^3 - 3lbh$

$= (l + b + h)\,(l^2 + b^2 + h^2 - (lb + bh + lh))$

$= 22\,(196 - 144) = 1144$ cm^2

219. (a) volume of cone $= \frac{1}{3}\pi r_c^2 h_c$

volume of hollow sphere $= \frac{4}{3}\pi(R_1^3 - R_2^3)$

Now, $\frac{1}{3}\pi r_c^2 h_c = \frac{4}{3}\pi(R_1^3 - R_2^3)$

$(4)^2\, H_c = 4(3^3 - 2^3)$

$16\, H_c = 19 \times 4$

$\therefore \quad H_c = \frac{19}{4} = 4.75$ cm

220. (b) Volume of solid metallic cylender $= \pi r^2 h$

$= \pi \times 36 \times 10 = 360\,\pi$

flat surface area of cylinder $= 2\pi r^2$

$= 2 \times \pi \times 6 \times 6 = 72\pi$

After melted to make two cones in the ratio of volume 1 : 2

volume of first cone $= \frac{1}{3}\pi r_1^2 h \quad = 120\,\pi$

$r_1^2 h = 360$

$\therefore \quad r_1^2 = 36$

volume of secound cone = $\frac{1}{3}\pi r_2^2 h = 240\pi$

$r_2^2 h = 720$

$\therefore \quad r_2^2 = 72$

Flat surface area of two cone = 72p + 36p = 108p

Change in surface = $108\pi - 72\pi = 36\pi$

% change = $\frac{36\pi}{72\pi} \times 100 = 50\%$

221. (d) Length of the rod = 24 feet.
Number of pieces of rod used to form a cube = 12 So,
side length of the cube $= \frac{24}{12} = 2$ feet
$\therefore$ area of one of the faces = 2 × 2 = 4 square feet

222. (d) Let R = radius of larger sphere
ATQ, $\frac{4\pi}{3}(3^3 + 4^3 + 5^3) = \frac{4\pi}{3}R^3$
$\Rightarrow R^3 = 3^3 + 4^3 + 5^3 = 216 = 6^3$
$\therefore \quad R = 6$

223. (b) Let r = radius of hemisphere
Volume of hemisphere = 155232
$\therefore \frac{2}{3} \times \frac{22}{7} r^3 = 155232$
$\Rightarrow r^3 = \frac{155232 \times 21}{44} = 3528 \times 21$
$= 2^3 \times 3^3 \times 7^3 = (42)^3$
$\Rightarrow \quad r = 42$

224. (b) From formula we know that Volume of bucket
$= \frac{1}{3}\pi H(R^2 + \gamma R + \gamma^2)$
$= \frac{1}{3} \times \frac{22}{7} \times 7(36 + 18 + 9)$
$= 21 \times 22 = 462\,cm^3$

225. (d) Total surface area of cone = $\pi R^2 + \pi LR$
$= \pi R\left[R + \sqrt{R^2 + H^2}\right] \because l = \sqrt{R^2 + H^2}$
$= \pi \times 6(6 + 10) = 96\pi\,cm^2$

226. (c) Resulting surface area of cuboid
$= 2(l \times b + b \times h + l \times h)$
$= 2(12 \times 72 + 12 \times 12 + 72 \times 12)$
$= 2((864 + 144 + 864)cm^2$
$= 2 \times 1872\,cm^2$
$= 3744\,cm^2$

227. (b) ℓ = length, b = breadth, h = height
ATQ, $x = \ell b$, $y = bh$, $z = \ell h$
$\Rightarrow \quad xyz = \ell^2 b^2 h^2 = (\ell bh)^2 = v^2$

228. (b) Volume = $\pi R_1^2 H_1 = \pi R_2^2 H_2$
$\Rightarrow \left(\frac{R_1}{R_2}\right)^2 = \frac{H_2}{H_1} \Rightarrow \frac{R_1}{R_2} = \sqrt{\frac{H_2}{H_1}} = \sqrt{\frac{3}{2}}$

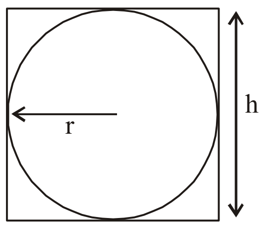
229. (c) Diameter = 20 cm
$\therefore$ Radius = 10 cm
Curved surface area = $2\pi rh$
$\therefore \quad 2\pi rh = 1000$
$\pi h = \frac{1000}{2 \times 10} = 50$
$\therefore$ Volume of the cylinder
$= \pi r^2 h = 50 \times 10 \times 10$
$= 5000\,cm^3$

230. (c) As per the given condition, the radius of sphere and cylinder will be same.
Volume of cylinder = $\pi R^2 H = \pi R^2.D = 2\pi R^3$
(where R = Radius)
Volume of sphere $V_s = \frac{4}{3}\pi R^3$
$\Rightarrow \frac{V_S}{V} = \frac{\frac{4}{3}\pi R^3}{2\pi R^3} = \frac{2}{3} \qquad \Rightarrow V_S = \frac{2}{3}V$

231. (c) $2\pi rh = 1$ litre
Paint Reqd. = $\frac{2\pi rh + 2\pi(r+1)h + 2\pi[(r+1) + r]}{2\pi rh}$
$= \frac{rh + (r+1)h + 2r + 1}{rh} \qquad = \frac{21 + 28 + 7}{3 \times 7} = \frac{8}{3}$

232. (d) $V = \pi r^2 h = \frac{22}{7} \times 1.4 \times 1.4 \times 5$
$= 30.8\,m^3$

233. (a) $p = 4\pi r^2$
$q = 2\pi r.h = 2\pi r.\,2r$
$= 4\pi r^2$
Hence, P = q.



CHAPTER 21

Lines & Angles

1. What is the angle which equals one-fifth of its supplement? **[2007-I]**

(a) 15º (b) 30º
(c) 36º (d) 75º

2. Consider the following statements

If two parallel lines are intersected by a transversal, then

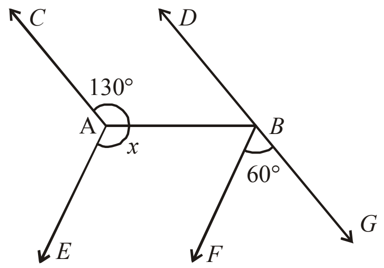
I. each pair of corresponding angles are equal.

II. each pair of alternate are unequal.

Which of the statements given above is/are correct? **[2007-I]**

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

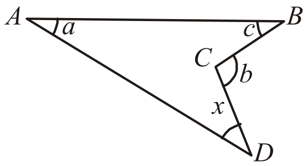
3.



In the figure given above, $AC \parallel BD$ and $AE \parallel BF$. What is $\angle x$? **[2007-II]**

(a) 130º (b) 110º
(c) 70º (d) 50º

4.



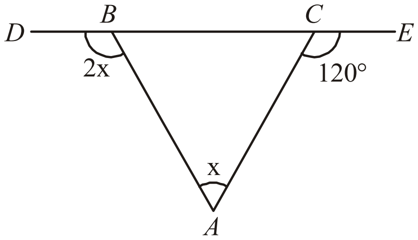
What is the value of x in the figure given above? **[2007-II]**

(a) $b - a - c$ (b) $b - a + c$
(c) $b + a - c$ (d) $\pi - (a + b + c)$

5. The angle between the legs of a compass is 60º and each leg is 10 cm long. How far apart are the points on which the legs of the compass rest? **[2008-I]**

(a) 5 cm (b) 10 cm
(c) $5\sqrt{3}$ cm (d) $10\sqrt{3}$ cm

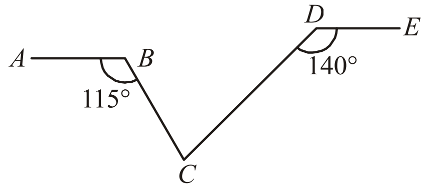
6.



In the figure given above, what is the value of x? **[2008-I]**

(a) 30º (b) 40º
(c) 45º (d) 60º

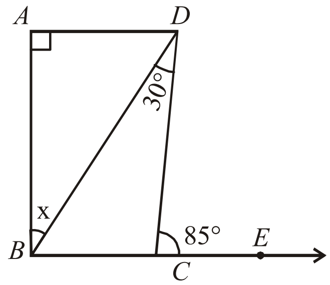
7.



If given that $AB \parallel DE$, $\angle ABC = 115º$, $\angle CDE = 140º$. Then, what is the value of $\angle BCD$? **[2008-I]**

(a) 45º (b) 55º
(c) 65º (d) 75º

8.



If $AD \parallel BE$, $\angle DCE = 85º$ and $\angle BDC = 30º$, then what is the value of x? **[2008-I]**

(a) 30º (b) 35º
(c) 45º (d) 55º

9. LM is a straight line and O is a point on LM. Line ON is drawn not coinciding with OL or OM. If $\angle MON$ is one-third of $\angle LON$, then what is $\angle MON$ equal to? **[2008-I]**

(a) 45º (b) 60º
(c) 75º (d) 80º

10. What is the least number of straight lines for a bounded plane figure? **[2008-I]**

(a) 1 (b) 2
(c) 3 (d) 4

11. Consider the following statements

Two lines intersected by a transversal are parallel, if

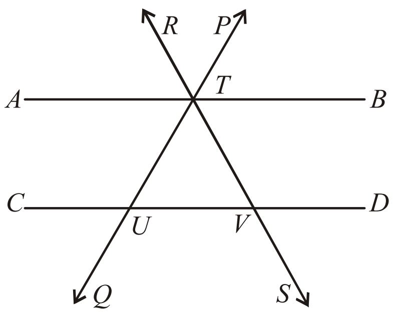
I. the pairs of cooresponding angles are equal.

II. the interior angles on the same side of the transversal are supplementary.

Which of the statements given above is/are correct? **[2008-I]**

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

12. **Assertion** (A) Two distinct lines cannot have more than one point in common.
Reason (R) Any number of lines can be drawn through one point. **[2008-II]**
(a) *A* and *R* are correct and *R* is correct explanation of *A*
(b) *A* and *R* are correct but *R* is not correct explanation of *A*
(c) *A* is correct but *R* is wrong
(d) *A* is wrong but *R* is correct

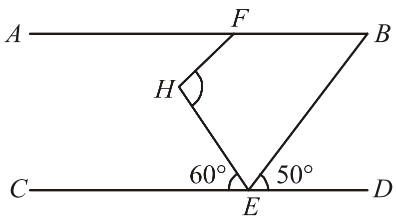
13.


In the given figure, If $AB \parallel CD$, $\angle PTB = 55^\circ$ and $\angle DVS = 45^\circ$, then what is the sum of the measures of $\angle CUQ$ and $\angle RTP$? **[2008-II]**
(a) 180º (b) 135º
(c) 110º (d) 100º

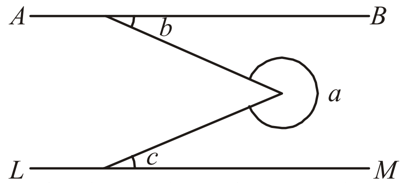
14. Let *AB* and *AC* be two rays intersecting at *A*. If *D*, *E* be the points lying on *AB*, *AC* respectively and *P* be the point such that *P* divides the line *DE* such that $PD : PE = AD : AE$. Then, what is the locus of the point *P*? **[2009-I]**
(a) The angle bisector of angle *A*
(b) The angle trisector of angle *A*
(c) The perpendicular bisector of angle *A*
(d) None of the above

15. The length of a line segment *AB* is 2 unit. It is divided into two parts at the point *C* such that $AC^2 = AB \times CB$. What is the length of *CB*? **[2009-II]**
(a) $3+\sqrt{5}$ units (b) $3-\sqrt{5}$ units
(c) $2-\sqrt{5}$ units (d) $\sqrt{3}$ units

16. If a point *P* moves such that its distance from two given points *A* and *B* are equal. Then, what is the locus of the point *P*? **[2009-II]**
(a) A straight line which is the right bisector of *AB*
(b) A circle with centre at *B*
(c) A straight line passing through *A* and *B*.
(d) A straight line passing through either *A* or *B*

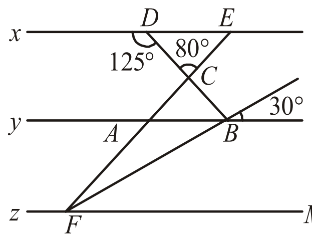
17.


In the figure given above, AB is parallel to CD and BE is parallel to FH. What is $\angle FHE$ equal to? **[2009-II]**
(a) 110º (b) 120º
(c) 125º (d) 130º

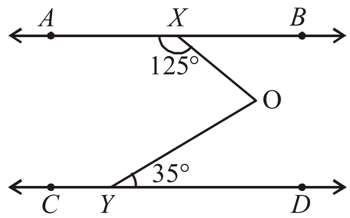
18.


In the figure given above, *AB* is parallel to *LM*, What is the angle *a* equal to? **[2010-I]**
(a) $\pi + b + c$ (b) $2\pi - b + c$
(c) $2\pi - b - c$ (d) $2\pi + b - c$

19. Three lines intersect each other in pairs. What is the number of angles so formed? **[2010-I]**
(a) 3 (b) 6
(c) 9 (d) 12

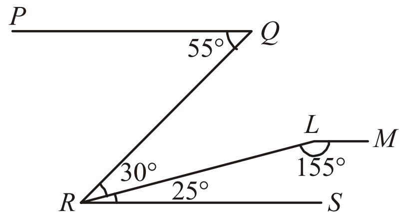
20.


Three straight lines *x*, *y*, and *z* are parallel and the angles are as shown in the figure above. What is $\angle AFB$ equal to? **[2010-I]**
(a) 20º (b) 15º
(c) 30º (d) 10º

21.


In the figure given above, *AB* is parallel to *CD*. What is the value $\angle XOY$? **[2010-II]**
(a) 80º (b) 90º
(c) 95º (d) 100º

22. The line segments *AB* and *CD* intersect at *O*, *OF* is the internal bisector of obtuse $\angle BOC$ and *OE* is the internal bisector of acute $\angle AOC$. If $\angle BOC = 130^\circ$, what is the measure of $\angle FOE$? **[2010-II]**
(a) 90º (b) 110º
(c) 115º (d) 120º

23.


In the figure given above, PQ is parallel to RS, What is the angle between the lines PQ and LM? **[2010-II]**

(a) 175º (b) 177º
(c) 179º (d) 180º

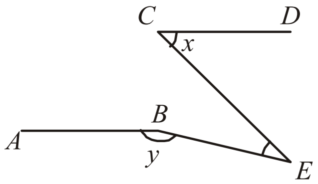
24. In a ΔABC, side AB is extended beyond B, side BC beyond C and side CA beyond A, What is the sum of the three exterior angles? **[2011-I]**

(a) 270º (b) 305º
(c) 360º (d) 540º

25. Let A and B be two points. What is the locus of the point P such that $\angle APB = 90º$? **[2011-II]**

(a) The line AB itself
(b) The point P itself
(c) The circumference of the circle with AB as diameter
(d) The line perpendicular to AB and bisecting AB

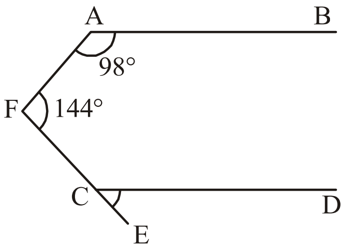
26.



In the figure given above, AB is parallel to CD. If $\angle DCE = x$ and $\angle ABE = y$, then what is $\angle CEB$ equal to? **[2011-II]**

(a) $y - x$ (b) $(x + y)/2$
(c) $x + y - (\pi/2)$ (d) $x + y - \pi$

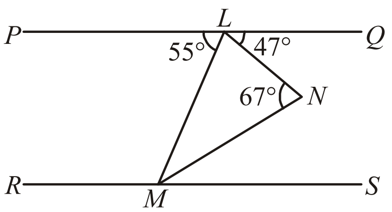
27.



In the figure given above, AB is parallel to CD. If $\angle BAF = 98º$ and $\angle AFC = 144º$, then what is $\angle ECD$ equal to? **[2011-II]**

(a) 62º (b) 64º
(c) 82º (d) 84º

28.



In the figure given above, PQ is parallel to RS. What is $\angle NMS$ equal to? **[2011-II]**

(a) 20º (b) 23º
(c) 27º (d) 47º

29. Consider the following statements
If two straight lines intersect, then
I. Vertically opposite angles are equal.
II. Vertically opposite angles are supplementary.
III. adjacent angles are complementary.
Which of the statements given above is/are correct? **[2012-I]**

(a) Only III (b) Only I
(c) I and III (d) II and III

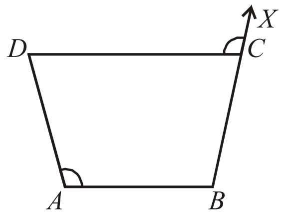
30. Two transversals S and T cut a set of distinct parallel lines. S cuts the parallel lines in points A, B, C, D, and T cuts the parallel lines in points E, F, G and H, respectively. If $AB = 4$, $CD = 3$ and $EF = 12$, then what is the length of GH? **[2012-I]**

(a) 4 (b) 6
(c) 8 (d) 9

31. The ratio of two complementary angle is 1 : 5. What is the difference between the two angles? **[2012-I]**

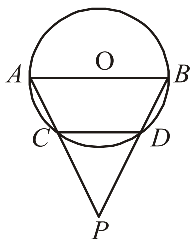
(a) 60º (b) 90º
(c) 120º
(d) Cannot be determined with the given data

32. In the quadrilateral $ABCD$ shown below $\angle DAB = \angle DCX = 120º$. If $\angle ABC = 105º$, what is the value of $\angle ADC$? **[2012-I]**



(a) 45º (b) 60º
(c) 75º (d) 95º

33. In the figure given below $AO = CD$, where O is the centre of the circle. What is the value of $\angle APB$? **[2012-I]**



(a) 60º (b) 50º
(c) 45º (d) 30º

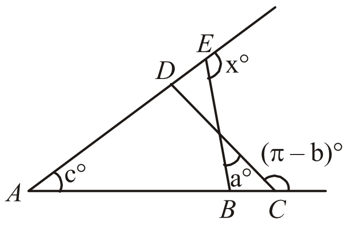
34. In a ΔABC, $\frac{1}{2}\angle A + \frac{1}{3}\angle C + \frac{1}{2}\angle B = 80º$, then what is the value of $\angle C$? **[2012-II]**

(a) 35º (b) 40º
(c) 60º (d) 70º

35. Consider the following statements
I. The locus of points which are equidistant from two parallel lines is a line parallel to both of them and drawn mid way between them.
II. The perpendicular distances of any point on this locus line from two original parallel lines are equal. Further, no point outside this locus line has this property.
Which of the above statements is/are correct? **[2012-II]**

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

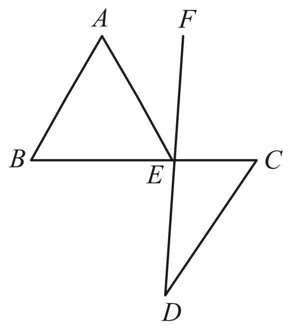
36. The angles xº, aº, cº and $(\pi - b)$º are indicated in the figure given below



Which one of the following is correct? **[2012-II]**

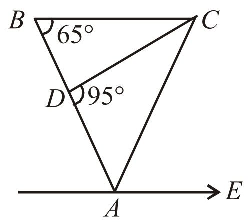
(a) $x = a + c - b$ (b) $x = b - a - c$
(c) $x = a + b + c$ (d) $x = a - b + c$

37. In the figure given below, *AB* is parallel to *CD*. $\angle ABC = 65°$, $\angle CDE = 15°$ and $AB = AE$, **[2012-II]**



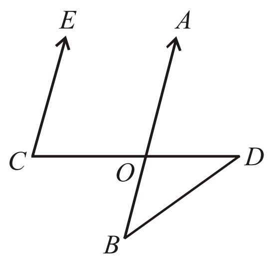
What is the value of $\angle AEF$?
(a) 30º (b) 35º
(c) 40º (d) 45º

38. In the figure given below, *ABC* is a triangle. *BC* is parallel to *AE*. If $BC = AC$, then what is the value of $\angle CAE$? **[2012-II]**



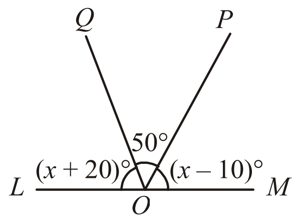
(a) 20º (b) 30º
(c) 40º (d) 50º

39. In the figure given below, *EC* is parallel to *AB*, $\angle ECD = 70°$ and $\angle BDO = 20°$. **[2012-II]**



What is the value of $\angle OBD$?
(a) 20º (b) 30º
(c) 40º (d) 50º

40. In the given figure below *LOM* is a straight line. **[2012-II]**



What is the value of $x°$?
(a) 45º (b) 60º
(c) 70º (d) 80º

41. If the arms of one angle are respectively parallel to the arms of another angle, then the two angles are **[2013-I]**
(a) neither equal nor supplementary
(b) not equal but supplementary
(c) equal but not supplementary
(d) either equal or supplementary

42. Let *OA*, *OB*, *OC* and *OD* are rays in the anticlockwise direction such that $\angle AOB = \angle COD = 100°$, $\angle BOC = 82°$ and $\angle AOD = 78°$. Consider the following statements : **[2015-I]**
1. *AOC* and *BOD* are lines.
2. $\angle BOC$ and $\angle AOD$ are supplementary.

Which of the above statements is /are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

43. At 8 : 30, the hour hand and the minute hand of a clock form an angle of **[2015-I]**
(a) 80° (b) 75°
(c) 70° (d) 60°

44. A clock is started at noon. By 10 minutes past 5, through what angle, the hour hand moves ? **[2015-II]**
(a) 160° (b) 145°
(c) 150° (d) 155°

45. Two poles are placed at *P* and *Q* on either side of a road such that the line joining *P* and *Q* is perpendicular to the length of the road. A person moves *x* metre away from *P* parallel to the road and places another pole at *R*. Then the person moves further *x* metre in the same direction and turns and moves a distance *y* metre away from the road perpendicularly, where he finds himself, *Q* and *R* on the same line. The distance between *P* and *Q* (*i.e.*, the width of the road in metre is) **[2016-I]**

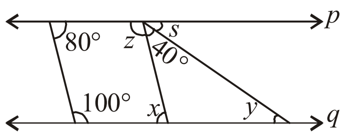
(a) x (b) $\frac{x}{2}$
(c) y (d) $2y$

46. There are five lines in a plane, no two of which are parallel. The maximum number of points in which they can intersect is **[2016-I]**
(a) 4 (b) 6
(c) 10 (d) None of the above

47. If a transversal intersects four parallel straight lines, then the number of distinct values of the angles formed will be **[2016-I]**

(a) 2 (b) 4
(c) 8 (d) 16

48.



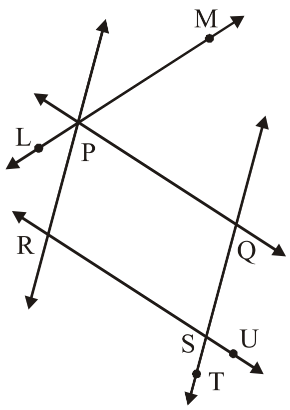
In the figure given above, P and q are parallel lines. What are the values of the angles x, y and z? **[2016-I]**

(a) $x = 80°, y = 40°, z = 100°$
(b) $x = 80°, y = 50°, z = 105°$
(c) $x = 70°, y = 40°, z = 110°$
(d) $x = 60°, y = 20°, z = 120°$

49. If D is the number of degrees and R is the number of radians in an angle θ, then which one of the following is correct ? **[2017-I]**

(a) $\pi D = 180R$ (b) $\pi D = 90R$
(c) $\pi R = 180D$ (d) $\pi R = 90D$

50. In the figure given below, PQ is parallel to RS and PR is parallel to QS. If ∠LPR = 35° and ∠UST = 70°, then what is ∠MPQ equal to ? **[2017-I]**



(a) 55° (b) 70°
(c) 75° (d) 80°

51. a, b, c, b are non-zero integers such that (ab) divides (cd). If a and c are coprime, then which one of the following is correct? **[2017-II]**

(a) a is a factor of c (b) a is a factor of b
(c) a is a factor of d (d) d is a factor of a

52. If θ measured in radians is the angle between the hour hand and the minute hand of a clock when the time is 4 : 36 pm, then which one of the following is correct? **[2017-II]**

(a) $\frac{3\pi}{5} < \theta < \frac{4\pi}{5}$ (b) $\frac{2\pi}{5} < \theta < \frac{3\pi}{5}$

(c) $\frac{\pi}{5} \le \theta \le \frac{2\pi}{5}$ (d) $\frac{7\pi}{15} \le \theta \le \frac{8\pi}{15}$

53. Consider the following statements in respect of three straight lines A, B and C on a plane : **[2017-II]**

1. If A and C are parallel and B and C are parallel; then A and B are parallel.
2. If A is perpendicular to C and B is perpendicular to C; then A and B are parallel.
3. If the acute angle between A and C is equal to the acute angle between B and C; then A and B are parallel.

Which of the above statements are correct?

(a) 1, 2 and 3 (b) 1 and 2 only
(c) 1 and 3 only (d) 2 and 3 only

54. What angle does the hour hand of a clock describe in 10 minutes of time? **[2018-I]**

(a) 1° (b) 5°
(c) 6° (d) 10°

55. The length of a line segment AB is 2 cm. It is divided into two parts at a point C such that $AC^2 = AB \times CB$. What is the length of CB? **[2018-I]**

(a) $3\sqrt{5}$ cm (b) $3 - \sqrt{5}$ cm

(c) $5\sqrt{3}$ cm (d) $\sqrt{5} - 1$ cm

56. The locus of a point equidistant from two intersecting lines is **[2018-I]**

(a) A straight line (b) A circle
(c) A pair of straight lines (d) None of the above

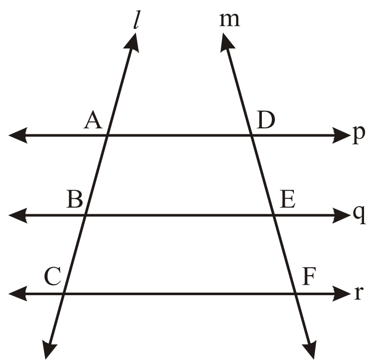
DIRECTIONS (Qs. 57-58): *In a triangle ABC, a, b and c are the lengths of the sides and p, q and r are the lengths of its medians.*

[2018-I]

57. Which one of the following is correct?

(a) $2(p + q + r) = (a + b + c)$
(b) $2(p + q + r) > 3(a + b + c)$
(c) $2(p + q + r) < 3(a + b + c)$
(d) $11(p + q + r) > 10(a + b + c)$

58. In the figure given below, p, q, r are parallel lines; l and m are two transversals.



Consider the following :

1. AB : AC = DE : DF
2. AB × EF = BC × DE

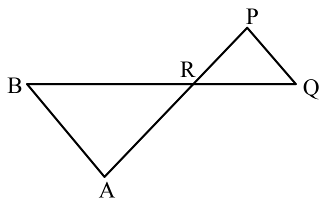
Which of the above is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

59. If two lines AB and CD intersect at O such that ∠AOC = 5 ∠AOD, then the four angles at O are **[2018-II]**
(a) 40°, 40°, 140°, 140° (b) 30°, 30°, 150°, 150°
(c) 30°, 45°, 75°, 210° (d) 60°, 60°, 120°, 120°

60. If a point P moves such that the sum of the squares of its distances from two fixed points A and B is a constant, then the locus of the point P is **[2018-II]**
(a) A straight line
(b) A circle
(c) Perpendicular bisector of AB
(d) An arbitrary curve

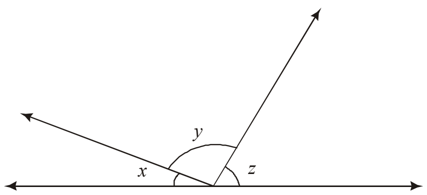
61. In the figure given below, ΔABR ~ ΔPQR. If PQ = 3 cm, AB = 6 cm, BR = 8.2 cm and PR = 5.2 cm, then QR and AR are respectively **[2018-II]**
(a) 8.2 m, 10.4 cm
(b) 4.1 cm, 6 cm
(c) 2.6 cm, 5.2 cm
(d) 4.1 cm, 10.4 cm



62. A piece of wire of length 33 cm is bent into an arc of a circle of radius 14 cm. What is the angle subtended by the arc at the centre of the circle ? **[2019-II]**
(a) 75° (b) 90° (c) 135° (d) 150°

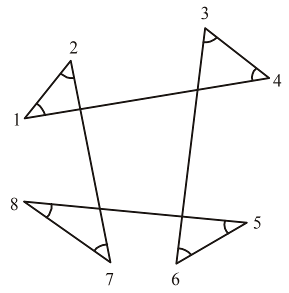
63. Three parallel lines *x, y* and *z* are cut by two transversals *m* and *n.* Transversal *m* cuts the lines *x*, *y*, *z* at *P*, *Q*, *R* respectively; and Transversal *n* cuts the lines *x*, *y*, *z* at *L*, *M*, *N* respectively. If *PQ* = 3 cm, *QR* = 9 cm and *MN* = 10.5 cm, then what is the length of *LM* ? **[2019-II]**
(a) 3 cm (b) 3.5 cm (c) 4 cm (d) 4.5 cm

64. In the given figure, if $\frac{y}{x} = 6$ and $\frac{z}{x} = 5$, then what is the value of *x* ? **[2020-I]**



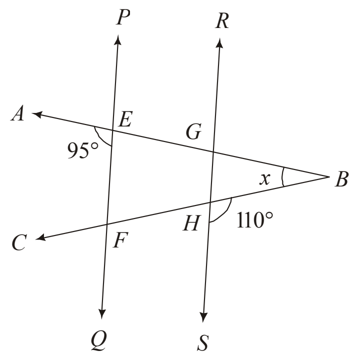
(a) 45° (b) 30° (c) 15° (d) 10°

65. Angles are shown in the given figure. What is value of ∠1 + ∠2 + ∠3 + ∠4 + ∠5 + ∠6 + ∠7 + ∠8 ? **[2020-I]**



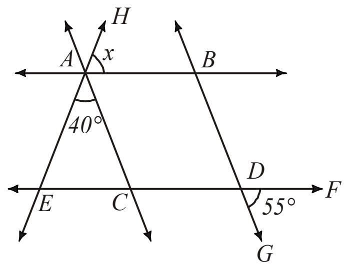
(a) 240° (b) 360° (c) 540° (d) 720°

66. In the given figure *PQ* is parallel to *RS*, ∠*AEF* = 95°, ∠*BHS* = 110° and ∠*ABC* = *x*°. Then what is the value of *x* ? **[2020-I]**



(a) 15 (b) 25 (c) 30 (d) 35

67. In the given figure *AB* is parallel to *CD* and *AC* is parallel to *BD*. If ∠*EAC* = 40°, ∠*FDG* = 55°, ∠*HAB* = *x*°, then what is the value of *x* ? **[2020-I]**



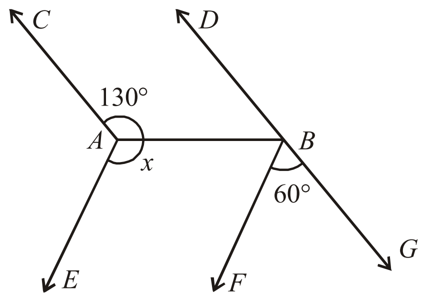
(a) 85 (b) 80 (c) 75 (d) 65

HINTS & SOLUTIONS

1. (b) Let the angle be x, then its supplement angle is $(180^\circ - x)$.
By given condition,
$$x = \frac{1}{5}(180^\circ - x) \Rightarrow 5x = 180^\circ - x$$
$$\Rightarrow \quad x = \frac{180^\circ}{6} = 30^\circ$$

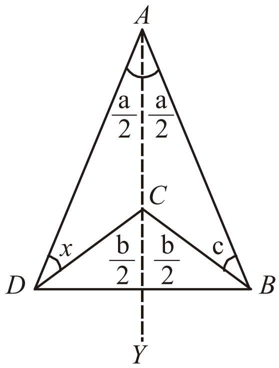
2. (a) If two parallel lines are intersected by a transversal, then each pair of corresponding angles and of alternate angles are equal.
Therefore, statement I is correct.

3. (b) Since, $AC \parallel BD$



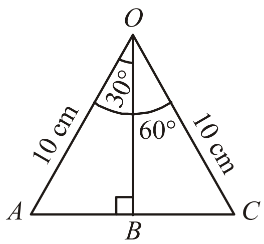
$\therefore \quad \angle DBA = 180^\circ - 130^\circ = 50^\circ$
Since, DBG is straight line,
$\therefore \quad \angle DBA + \angle ABF + \angle FBG = 180^\circ$
$\Rightarrow \quad 50^\circ + \angle ABF + 60^\circ = 180^\circ$
$\Rightarrow \quad \angle ABF = 70^\circ$
Since, $AE \parallel BF$
$\therefore \quad x = 180^\circ - \angle ABF = 180^\circ - 70^\circ = 110^\circ$

4. (a) Construction:

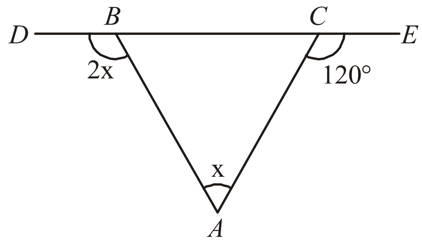


Draw a bisector AY of $\angle A$ and $\angle C$
In ΔADC,
$$x + \frac{a}{2} = \frac{b}{2} \quad \text{...(i)}$$
In ACB,
$$c + \frac{a}{2} = \frac{b}{2} \quad \text{...(ii)}$$
on adding eq. (i) and (ii)
$$x + \frac{a}{2} + \frac{a}{2} + c = \frac{b}{2} + \frac{b}{2}$$
$$x + a + c = b$$
$$\therefore \quad x = b - a - c$$

5. (b) In ΔAOB,
$$\sin 30^\circ = \frac{AB}{OA} \Rightarrow \frac{1}{2} = \frac{AB}{10}$$
$\Rightarrow \quad AB = 5$ cm
Now, $AC = 2\,AB = 2 \times 5 = 10$ cm

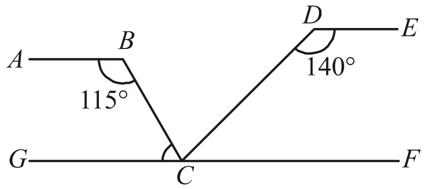


6. (d) $\therefore \ \angle ABC = 180^\circ - \angle DBA$



$= 180^\circ - 2x$
and $\angle ACB = 180^\circ - \angle ACE = 180^\circ - 120^\circ = 60^\circ$
We know that,
$\angle ABC + \angle ACB + \angle BAC = 180^\circ$
$\Rightarrow \quad 180^\circ - 2x + 60^\circ + x = 180^\circ$
$\Rightarrow \quad 240^\circ - 180^\circ = x$
$\therefore \quad x = 60^\circ$

7. (d) Draw a line GF through C parallel to AB and DE.
Now.



$\therefore \quad \angle BCG = 180^\circ - \angle ABC$
$= 180^\circ - 115^\circ = 65^\circ$ (Interior angles on the same side of the transversal)
and $\angle DCF = 180^\circ - \angle CDE$
$= 180^\circ - 140^\circ = 40^\circ$
(Interior angles on the same side of the transversal)
Now, $\angle BCG + \angle BCD + \angle DCF = 180^\circ$
$\Rightarrow \quad 65^\circ + \angle BCD + 40^\circ = 180^\circ$
$\therefore \quad \angle BCD = 180^\circ - 105^\circ = 75^\circ$

8. (b) $AD \parallel BE$

$\therefore \quad \angle ADC = \angle DCE$ (alternate angles)

$\Rightarrow \quad \angle ADB + 30^\circ = 85^\circ$

$\Rightarrow \quad \angle ADB = 55^\circ$

and $\angle BAD = 90^\circ$ (given)

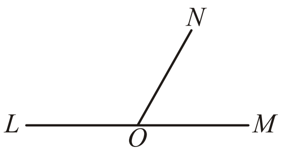
Now, in ΔABD,

$\angle ABD + \angle ADB + \angle BAD = 180^\circ$

$\Rightarrow \quad x + 55^\circ + 90^\circ = 180^\circ$

$\Rightarrow \quad x = 180^\circ - 145^\circ = 35^\circ$

9. (a) Given that,



$$\angle MON = \frac{1}{3}\angle LON$$

Let $\angle LON = x$

Then, $\angle MON = \frac{x}{3}$

We know that, $\angle LON + \angle MON = 180^\circ$ (linear pair)

$$\Rightarrow \quad x + \frac{x}{3} = 180^\circ \Rightarrow x = \frac{180^\circ \times 3}{4} = 135^\circ$$

Thus, $\angle MON = \frac{x}{3} = \frac{135^\circ}{3} = 45^\circ$

10. (c) The least number of straight lines for a bounded plane figure is 3.

11. (c) Hence, both Statements I and II are correct.

12. (b) A. It is true that two distinct lines intersect only one point.

R. It is true that, from a one point we can draw any number of lines but R is not a correct explanation of A.

13. (b)
$$\left.\begin{array}{l} AB \parallel CD \\ \angle PTB = 55^\circ \\ \angle DVS = 45^\circ \end{array}\right] \text{(Given)}$$

$\angle PTB = \angle ATU = 55^\circ$ (Vertically opposite angles)

Similarly,

$\angle DVS = \angle BTV = 45^\circ$ (Corresponding angles)

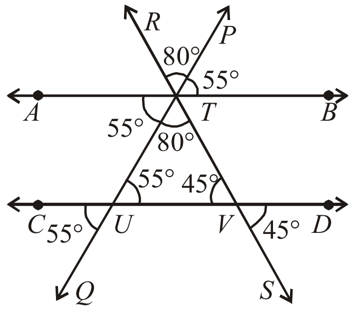
$\therefore \angle RTP = 180^\circ - (45^\circ + 55^\circ) = 80^\circ$

$\angle PTB = \angle TUV = 55^\circ$ (Corresponding angle)

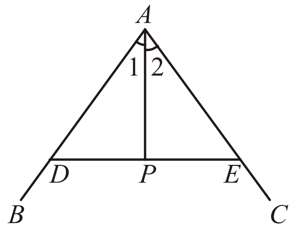
and $\angle CUQ = \angle TUV = 55^\circ$ (Vertically opposite angles)

$\therefore$ Sum of angles $= \angle RTP + \angle CUQ$

$= 80^\circ + 55^\circ = 135^\circ$



14. (a) Since, $\frac{PD}{PE} = \frac{AD}{AE} = \frac{AP}{AP}$



ΔDAP and ΔAPE are similar. So, $\angle 1 = \angle 2$

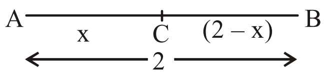
AP is bisector of $\angle A$.

Hence, the locus of P is the bisector of angle A.

15. (b) Given, $AC^2 = AB \times CB$

$\Rightarrow \quad x^2 = 2 \times (2 - x)$

$\Rightarrow \quad x^2 = 4 - 2x$



$\Rightarrow \quad x^2 + 2x - 4 = 0$

$$\Rightarrow \quad x = \frac{-2 \pm \sqrt{4+16}}{2 \times 1}$$

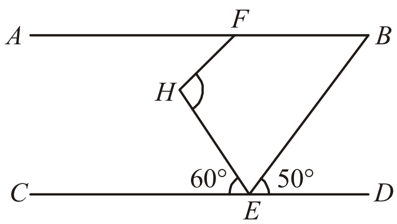
$\Rightarrow \quad x = -1 \pm \sqrt{5}$

Now, $BC = 2 - (-1 \pm \sqrt{5})$

$= 3 - \sqrt{5}$ (neglect $3 + \sqrt{5} \Rightarrow 3 + \sqrt{5} > 2$)

16. (a) The locus of P is a straight line which is the right bisector of AB.

17. (a) $\angle HEB = 180^\circ - 60^\circ - 50^\circ = 70^\circ$



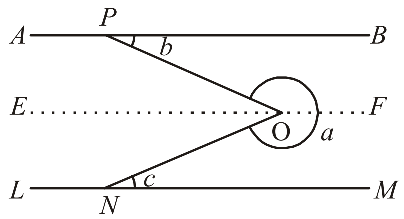
Since, $HF \parallel BE$ and HE is transversal line

$\therefore \angle FHE + \angle HEB = 180^\circ$(interior angle)

$\Rightarrow \angle FHE + 70^\circ = 180^\circ$

$\Rightarrow \angle FHE = 110^\circ$.

18. (c) Let we draw a line parallel to AB which is EF.



$\therefore \angle EOP = \angle OPB$ (alternate angle)

$\Rightarrow \angle EOP = b$

and $\angle EON = \angle ONM$ (alternate angle)

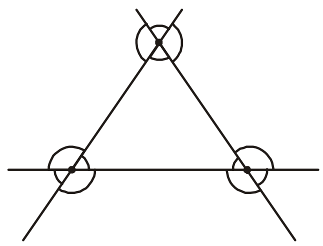
$\Rightarrow \angle EON = c$

$\therefore \angle PON = b + c$

$\because \angle PON + a = 2\pi \Rightarrow a = 2\pi - (b + c)$

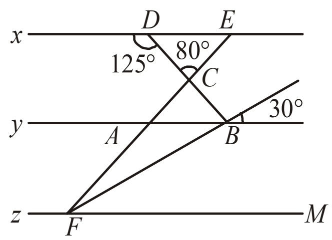
19. (d) We know that, when two lines intersect each other it makes 4 angles.

The total number of pairs = 3



$\therefore$ Total number of angles $= 3 \times 4 = 12$

20. (b) $\angle CDE = 180° - 125° = 55°$



In $\Delta DCE, \angle CED = 180° - 55° - 80° = 45°$

and $\angle ABF = 30°$ (vertically opposite angle)

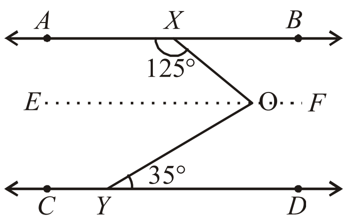
Also, $\angle ABF = \angle BFM = 30°$ (alternate angle)

and $\angle DEF = \angle EFM$ (alternate angle)

$\angle EFM = 45° \Rightarrow \angle EFB + \angle BFM = 45°$

$\Rightarrow \angle EFB = 45° - 30° \Rightarrow \angle AFB = 15°$

21. (b) Draw a line EF such that $EF \parallel AB \parallel CD$.



Now, $AB \parallel EF$

$\therefore \angle AXO + \angle XOE = 180°$

(Interior angle as the same side of line)

$\Rightarrow \angle XOE = 180° - 125° = 55°$

Also, $EF \parallel CD$

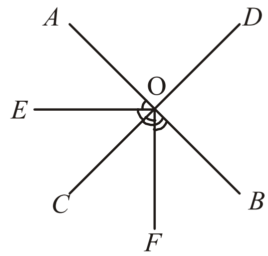
$\Rightarrow \angle EOY = \angle OYD = 35°$ (linear pair)

Hence, $\angle XOY = \angle XOE + \angle EOY$

$= 55° + 35° = 90°$

22. (a) Given, $\angle BOC = 130°$

$\therefore \angle BOC + \angle AOC = 180°$ (linear pair)



$\Rightarrow 130° + \angle AOC = 180° \Rightarrow \angle AOC = 50°$

Now $\angle BOC = 130° \Rightarrow \angle BOF + \angle FOC = 130°$

$\Rightarrow \angle FOC + \angle FOC = 130°$

($\because$ OF is bisector of $\angle BOC$)

$\Rightarrow \angle FOC = 65°$ now $\angle AOC = 50°$

$\Rightarrow \angle AOE + \angle EOC = 50°$

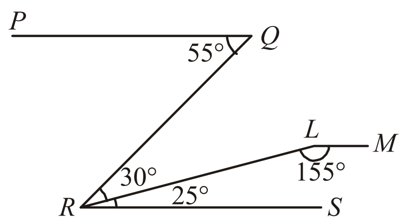
$\Rightarrow \angle EOC + \angle EOC = 50°$

($\because$ OE is bisector of $\angle AOC$)

$\Rightarrow \angle EOC = 25°$

Hence, $\angle EOF = \angle EOC + \angle FOC = 25° + 65° = 90°$

23. (d) Since, $PQ \parallel RS$



$\therefore \angle PQR = \angle QRS$ (alternate angle)

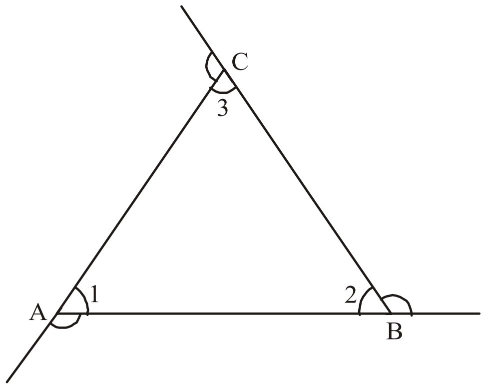
and $\angle SRL + \angle RLM = 180° \Rightarrow RS \parallel LM$ ...(ii)

From Eqs.(i) and (ii),

$PQ \parallel LM$

Therefore, angle between the lines PQ and LM is 180°

24. (c)



Sum of the three exterior angles

$= (\angle 1 + \angle 2) + (\angle 2 + \angle 3) + (\angle 3 + \angle 1)$

$= 2(\angle 1 + \angle 2 + \angle 3)$

(Sum of the interior angles are 180°)

$= 2 \times 180° = 360°$

Sum of exterior angles of any polygon = 360°

Alternate Method

Sum of the Exterior angles

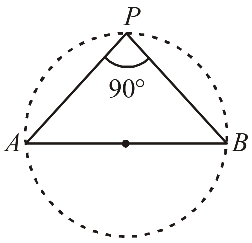
$= (180^\circ - \angle A) + (180^\circ - \angle B) + (180^\circ - \angle C)$

$= 540^\circ - (\angle A + \angle B + \angle C)$

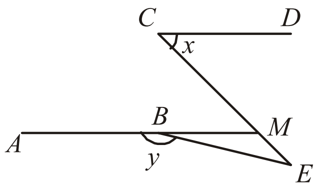
$= 540^\circ - 180^\circ$ (Sum of angles of a triangle is 180°)

$= 360^\circ$

25. (c) Hence, the locus of a point is the circumference of the circle with AB as diameter.



26. (a) Here, we produced AB line to M.



Since, AM is parallel to CD.

$\angle DCM = \angle BMC = x$ (alternate angle)

Also, ABM is a straight line.

$\angle EBM = \pi - y$

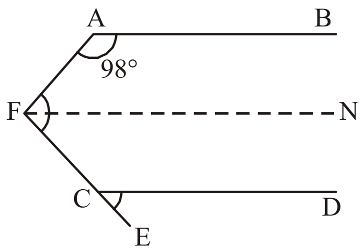
Now in ΔBEM.

$\angle B + \angle M + \angle E = \pi$

$\Rightarrow \pi - y + x + \angle E = \pi$

$\Rightarrow \angle E = y - x$

27. (a)



parallel to AB and CD.

$\therefore \angle AFN = 180^\circ - 98^\circ = 82^\circ$

$\Rightarrow \angle CFN = 144^\circ - 82^\circ = 62^\circ$

$\therefore \angle FCD = 180^\circ - 62^\circ = 118^\circ$

$\Rightarrow \angle ECD = 180^\circ - 118^\circ = 62^\circ$

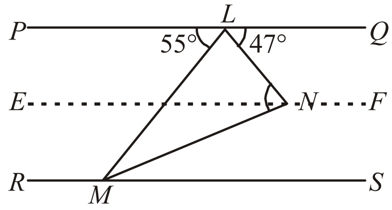
28. (a) $\angle PLM = \angle LMS$ (alternate angle)

$= 55^\circ$

Let draw EF line which is parallel to PQ and bisects by LN

Then, $\angle QLN = \angle LNE = 47^\circ$

$\because \angle ENL + \angle MNE = 67^\circ$



Then, $\angle LNE = 47^\circ$

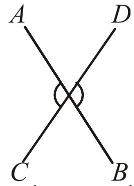
$\Rightarrow 47^\circ + \angle MNE = 67^\circ$

$\angle MNE = 67^\circ - 47^\circ$

$\angle MNE = 20^\circ$

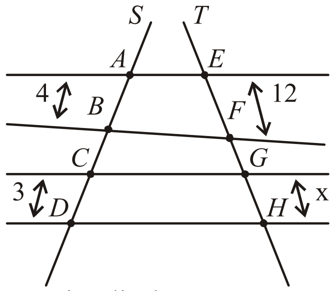
Similarly, $EF \parallel RS$, then $\angle ENM = \angle NMS = 20^\circ$

29. (b) AB and CD are two lines.



We know that when two lines intersect each other then opposite vertically angles are equal

30. (d) From figure.



By proportionality law,

$$\frac{AB}{CD} = \frac{EF}{GH} \Rightarrow \frac{4}{3} = \frac{12}{x}$$

$\therefore x = 3 \times 3 = 9$

31. (a) Given that, $\frac{\alpha}{\beta} = \frac{1}{5} \Rightarrow \alpha = k$ and $\beta = 5k$ (say)

or complementary angles,

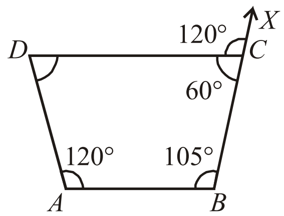
$\alpha = 90^\circ - \beta \Rightarrow k = 90^\circ - 5k$

$\Rightarrow k = 15^\circ$

$\therefore \alpha = 15^\circ$ and $\beta = 75^\circ$

$\therefore$ Difference between angles $= 75^\circ - 15^\circ = 60^\circ$

32. (c)



Given, $\angle ABC = 105^\circ$

$\angle DAB = 120^\circ$

$\angle DCX = 120^\circ$

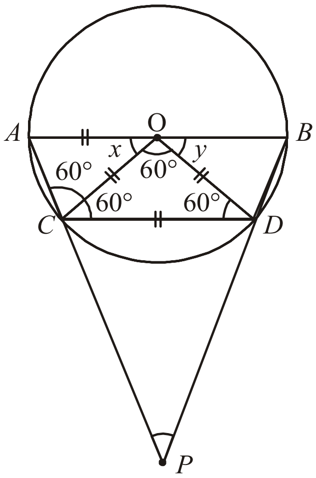
$\Rightarrow \angle DCB = 180^\circ - 120^\circ = 60^\circ$

Angles of a quadrilateral is equal to 360º

$\therefore \quad \angle ADC = 360^\circ - (120^\circ + 105^\circ + 60^\circ)$

$= 360^\circ - 285^\circ = 75^\circ$

33. (a)



$AO = CD$

$\Rightarrow \quad OC = OD = CD$

$(\because AO = OC = OD = \text{radii})$

So, ΔCOD is equilateral,

$\angle x + \angle y = 180 - 60$

and $\angle x = \angle y$

$\therefore \quad \angle 2x = 120°$

$\angle x = 60°$

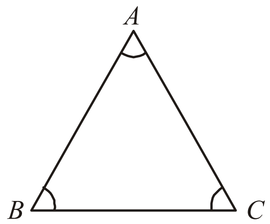
$\angle$AOC is also equilateral

$\angle DCP = 180^\circ - 120^\circ = 60^\circ$

and $\angle CDP = 60^\circ$

$\therefore \quad \angle APB = 360^\circ - (60^\circ + 120^\circ + 120^\circ) = 60^\circ$

34. (c) We know



$\angle A + \angle B + \angle C = 180^\circ$

$\angle A + \angle B = 180^\circ - \angle C \qquad$ ...(i)

$\frac{\angle A}{2} + \frac{\angle B}{2} + \frac{\angle C}{3} = 80^\circ$ (Given)

$\left(\frac{\angle A + \angle B}{2}\right) + \frac{\angle C}{3} = 80^\circ$

from equation (i)

$\Rightarrow \quad \frac{180^\circ - \angle C}{2} + \frac{\angle C}{3} = 80^\circ$

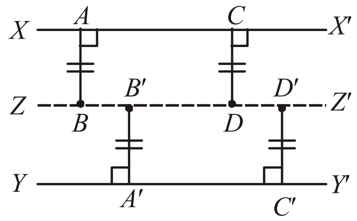
$\Rightarrow \quad \frac{180^\circ \times 3 - 3\angle C + 2\angle C}{6} = 80^\circ$

$\Rightarrow \quad \angle C = (540^\circ - 480^\circ)$

$\Rightarrow \quad \angle C = 60^\circ$

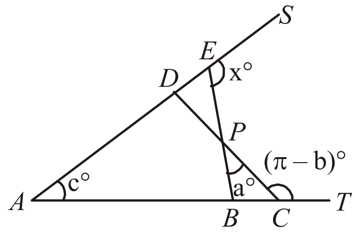
35. (c) Statements I and II are both true, because the locus of points which are equidistant from two parallel lines is a line parallel to both of them and draw mid way between them.

Also, it is true that the perpendicular distances of any point on this locus line from two original parallel lines are equal. Further, no point outside this locus line has this property.



36. (c) $\because \angle PCT + \angle PCB = \pi$

$\angle PCB = \pi - (\pi - b^\circ) = b^\circ$



In ΔBPC,

$\angle PCB + \angle BPC + \angle PBC = \pi$

$\Rightarrow \angle PBC = \pi - \angle PCB - \angle BPC$

$= \pi - b^\circ - a^\circ$

$\because \angle ABE + \angle EBC = \pi \qquad$ (linear pair)

$\Rightarrow \angle ABE = \pi - \angle PBC = \pi - (\pi - b^\circ - a^\circ)$

$= a^\circ + b^\circ \qquad (\because \angle PBC = \angle EBC)$

Now,

Sum of two interior angles = Exterior angle

$\therefore$ in $\Delta ABE, \angle EAB + \angle ABE = \angle BES \Rightarrow c^\circ + b^\circ + a^\circ = x^\circ$

$\therefore x^\circ = a^\circ + b^\circ + c^\circ$

37. (b) $AB = AE \qquad$ (Given)

$\therefore \quad \angle ABC = 65^\circ = \angle AEB$

$AB \parallel CD$, then

$\angle ABE = \angle DCE = 65^\circ$ (Alternate angles)

In ΔDCE,

$\angle D + \angle C + \angle E = 180^\circ$

$\angle E = 180^\circ - 65^\circ - 15^\circ = 100^\circ$

BC and FD intersect each other at E, then

$\angle BEF = \angle DEC = 100^\circ$

$\angle AEF = 100^\circ - 65^\circ = 35^\circ$

(Because $\angle BEF = \angle BEA + \angle AEF$)

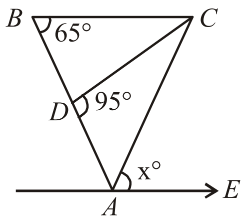
38. (d) Given that, $BC \parallel AE$

$\angle CBA + \angle EAB = 180^\circ$

$\Rightarrow \quad \angle EAB = 180^\circ - 65^\circ = 115^\circ$

$\because \quad BC = $ AC

Hence, ΔABC is an isosceles triangle



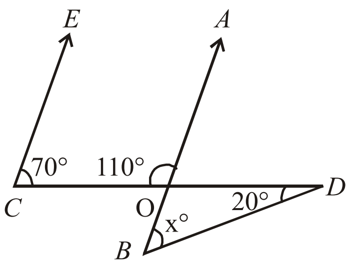
$\Rightarrow \angle CBA = \angle CAB = 65^\circ$

Now, $\angle EAB = \angle EAC + \angle CAB$

$\Rightarrow \quad 115^\circ = x + 65^\circ \Rightarrow x = 50^\circ$

39. (d) Given that, $EC \parallel AB$

$\angle ECO + \angle AOC = 180^\circ$



$\Rightarrow \angle AOC = 180^\circ - 70^\circ = 110^\circ$

$\therefore \angle BOD = \angle AOC = 110^\circ$ (alternate angle)

Now, in ΔOBD

$\angle BOD + \angle ODB + \angle DBO = 180^\circ$

$\therefore \quad 110^\circ + 20^\circ + x^\circ = 180^\circ \Rightarrow x^\circ = 50^\circ$

40. (b) From the given figure. We know that sum of all angles is 180º because of straight lines.

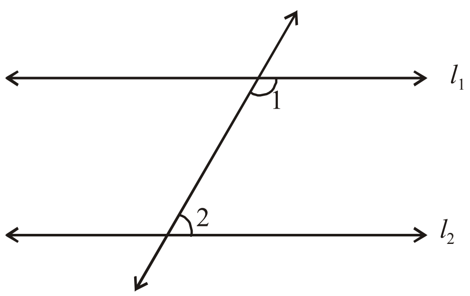
$\angle LOQ + \angle QOP + \angle POM = 180^\circ$

$\therefore \quad (x^\circ + 20^\circ) + 50^\circ + (x^\circ - 10^\circ) = 180^\circ$

$\Rightarrow \quad 2x^\circ + 60^\circ = 180^\circ \Rightarrow 2x^\circ = 120^\circ$

$x^\circ = 60^\circ$

41. (b) l_1 and l_2 are two parallel lines and $\angle 1$ and $\angle 2$ are interior angles on the same side of the transversal.

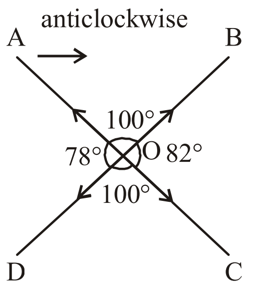


$\angle 1 \neq \angle 2$

$\angle 1 + \angle 2 = 180^\circ$.

Therefore, these are supplementary angles or consecutive interior angles.

42. (d)



Statement 1

AOC and BOD are not lines because $\angle AOD = 78^\circ$, So, it is not correct.

Statement 2

$\angle BOC$ and $\angle AOD$ are 82° and 78° respectively, so it is not supplementry angle.

So, neither statement 1 nor 2 are correct.

43. (b) Total hour = 8 : 30 = $8 + \dfrac{30}{60} = 8 + \dfrac{1}{2} = \dfrac{17}{2}h$

12 h angle made by hour hand = 360°

1 h angle made by hour hand $= \dfrac{360}{12}$

$\dfrac{17}{2}h$ angle made by hour hand = $\dfrac{360}{12} \times \dfrac{17}{2}$

$= 15 \times 17 = 255^\circ$

Now, 60 min angle made by min hand = 360°

1 min angle made by min hand = $\dfrac{360^\circ}{60}$

30 min angle made by min hand = $6 \times 30^\circ = 180^\circ$

Angle between hour and min hand

$= 255^\circ - 180^\circ$

$= 75^\circ$

44. (d) Hour hand moves by 30° in 1 hour.

In one hour movement of hour hand = 30°

In $\left(5 + \dfrac{10}{60}\right)$ hour movement is $= 30\left(5 + \dfrac{10}{60}\right)$

$$30 = \left(5 + \frac{1}{6}\right)$$

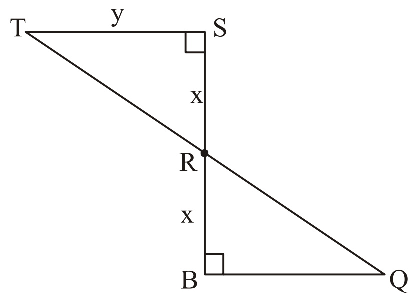
$$= 30 \times \frac{31}{6} = 155^\circ$$

So, option (d) is correct.

45. (c) In the figure,

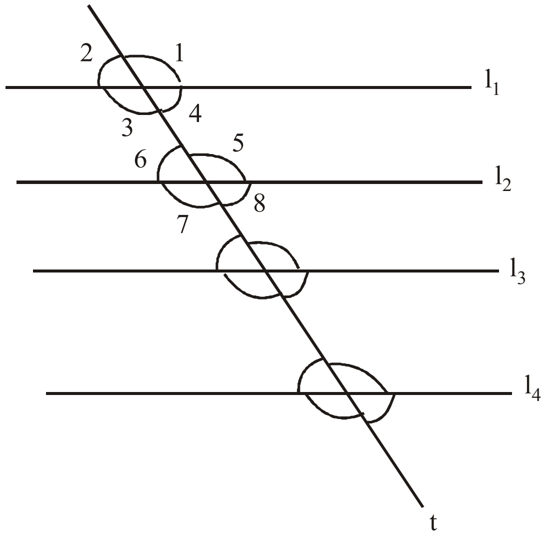
$\Delta PQR \cong \Delta STR$ (By ASA)

$\therefore PQ = TS = y$ metre



46. (d)

47. (a) Let l_1, l_2, l_3 and l_4 be your straight lines and t be a transversal.



$\angle 1 = \angle 3$

$\angle 2 = \angle 4$ [alternate angles]

Similarly angles formed by

l_2, l_3 and l_4 are also equal in same way as l_1

corresponding angles

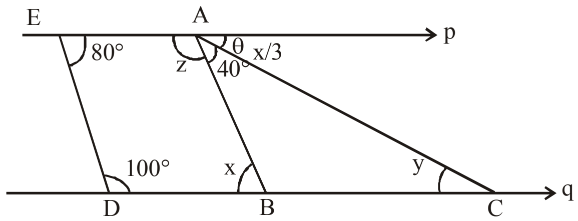
$\angle 1 = \angle 5$ & $\angle 5 = \angle 7$

$\angle 6 = \angle 2$ & $\angle 6 = \angle 8$ and so on

Only two distinct angles are formed

$\therefore$ Option (a) is correct.

48. (d)



Now ABDE is a quadrilateral

AB works as a transversal on parallel lines.

$\Rightarrow \quad \angle x = 40° + \frac{x}{3}$ (alternative angles)

$\Rightarrow \quad x - \frac{x}{3} = 40$

$\Rightarrow \quad x = \frac{40 \times 3}{2}$

$\Rightarrow \quad x = 60°$

$\Rightarrow \quad \frac{x}{3} = 20° = \angle y$ (alternate angles)

$\Rightarrow \quad x = 60°$

$y = 20°$

Since ABDE is a quadrilateral.

$\Rightarrow \quad \angle z + \angle x = 180°$

$\Rightarrow \quad \angle z = 180 - \angle x$

$= 180° - 60°$

$= 120°$

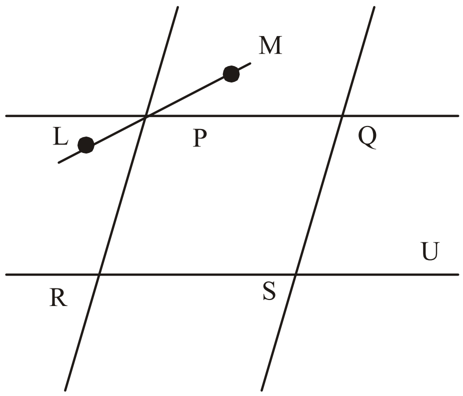
$\therefore$ option (d) is correct.

49. (d) $\pi^C = 180°$

$\pi°R = 180D$

$\pi R = 180D$

50. (c)



$PQ \parallel RS$

$PR \parallel QS$

$\therefore$ PQRS is a $\parallel$ gm

$\angle LPR = 35°$ and $\angle UST = 70°$

$\angle UST = \angle RSQ$ (Vertically opposite)

$\angle RSQ = \angle RPQ$ (opposite angle of 11 gm)

$\angle LPR + \angle RPQ + \angle MPQ = 180°$

$35° + 70° + \angle MPQ = 180°$

$\angle MPQ = 180 - 105$

$\angle MPQ = 75°$

51. (c) Since we are given that *a* and c are co-prime i.e. HCF of a and c is 1, therefore we can say that a definitely divides *d* exactly. So, *a* is a factor of *d*.

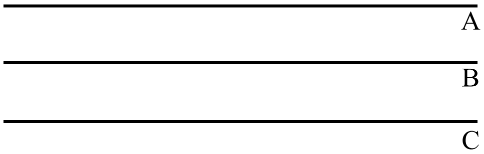
52. (b) To calculate the angle between the hour hand and the minute hand of a clock when the time is 4 : 36 pm, we can say that the angle will be approximately equal to the angle made from 4 : 20 pm to 4 : 36 pm.

Therefore, we need to calculate the angle made by the hands of a clock in 16 minutes.

In 60 minutes, the angle made by the hands of a clock is 360°. So, the angle made by the hands of a clock in 16 minutes will be $\frac{360}{60} \times 16 = 96°$.

Thus, the angle lies between 72° to 108° i.e. $\frac{2\pi}{5} < \theta < \frac{3\pi}{5}$.

53. (b)



The above two figures shows that (b) is the only correct answer.

54. (b) Angle described by hour hand of a clock in 12 hrs $= 360°$

Angle described by hour hand of clock in 1 hr

$= \frac{360}{12} = 30°$

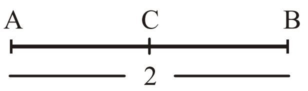
Angle described by hour hand of clock in 1 min

$= \frac{30}{60} = \frac{1°}{2}$

Angle described by hour hand of clock in 10 min

$\frac{1}{2} \times 10 = 5°$

55. (b)



$AB = 2$ cm

$AC^2 = AB \times CB$

$(2 - CB)^2 = 2CB$

$CB^2 - 4CB + 4 - 2CB = 0$

$CB^2 - 6CB + 4 = 0$

Hence, from $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, value of $CB = 3 - \sqrt{5}$

56. (c) 57. (c) 58. (c)

59. (b) Let two lines AB and CD intersect each other as shown in the fig. such that

$\angle AOD = x$

Then, $\angle AOC = 5x$

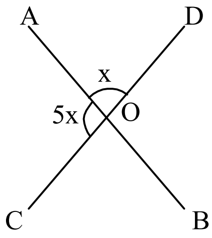
Now, $\angle AOD + \angle AOC = \angle COD = 180°$

$x + 5x = 180°$

$6x = 180° \Rightarrow x = 30°$

Now, $\angle AOD = \angle BOC = x = 30°$

and $\angle AOC = \angle BOD = 5x = 150°$



60. (b) A circle is the lows of any point that sum of square of its distance from any two fixed point is always constant.

61. (d) As $\Delta ABR \sim \Delta PQR$.

$\therefore \quad \frac{AB}{PQ} = \frac{BR}{QR} = \frac{AR}{PR}$

$\Rightarrow \quad \frac{6}{3} = \frac{8.2}{QR} = \frac{AR}{5.2}$

$\Rightarrow \quad QR = \frac{8.2}{2} = 4.1\text{cm}$

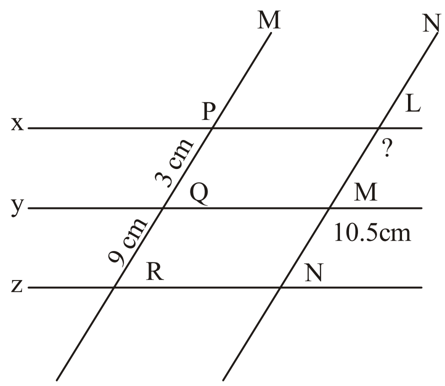
and $AR = 2 \times 5.2 = 10.4$ cm

62. (c) Angle subtended by the are at the centre of the circle $= \theta$

Then $\frac{\theta}{360}.2\pi R = 33$

$\Rightarrow \theta = \frac{33 \times 360 \times 7}{2 \times 22 \times 14} = 3 \times 45 = 135°$

63. (b)



Now, $\frac{PQ}{QR} = \frac{LM}{MN}$

$\Rightarrow \frac{3}{9} = \frac{x}{10.5} \quad \therefore \quad x = \frac{10.5 \times 3}{9} = 3.5\text{cm}$

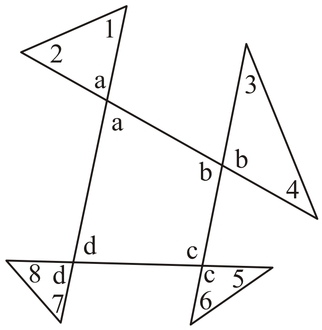
64. (c) From question $\frac{y}{x} = 6 \quad y = 6x \quad \frac{z}{x} = 5 \quad z = 5x$

Again $x + 6x + 5x = 12x = 180° \Rightarrow \quad x = 15°$

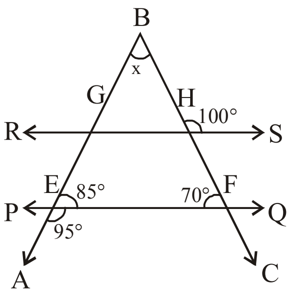
65. (b) $a + b + c + d = 360°$

$(1+2+a) + (3+4+b) + (5+6+c) + (7+8+d) = 180° \times 4$

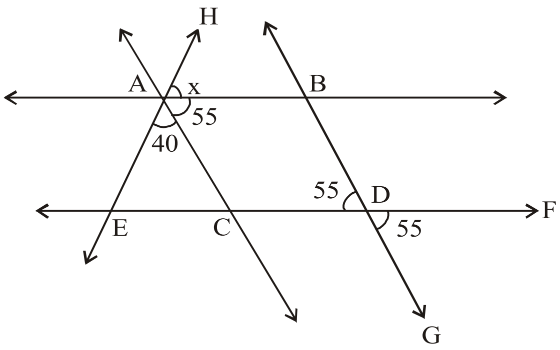
$\therefore \quad 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 = 360°$



66. (b) $x = 180 - 85 - 70$
$= 25°$



67. (a) $x = 180 - 40 - 55$
$x = 85°$



CHAPTER

22

Triangles and its Properties

1. In ΔABC and ΔDEF, it is given that $AB = 5\ cm$, $BC = 4$ cm and $CA = 4.2\ cm$, $DE = 10\ cm$, $EF = 8\ cm$ and $FD = 8.4$ cm. If AL is perpendicular to BC and DM is perpendicular to EF, then what is the ratio of AL to DM? **[2007-I]**

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{1}{4}$ (d) 1

2. A ΔABC is permitted to move around when its vertex A is fixed. What is the locus of the circumcentre? **[2007-II]**

(a) A straight line (b) A circle

(c) A point (d) A curve other than a circle

3. **Assertion** (A) ABC is a triangle and AD is its angular bisector. If $AB = 6\ cm$, $BC = 7\ cm$, $AC = 8\ cm$, then $BD = 3\ cm$ and $CD = 4\ cm$.

Reason (R) The angular bisector AD of a triangle cuts the base BC in the ratio $AB : AC$. **[2007-II]**

(a) Both A and R individually true and R is the correct explanation of A

(b) Both A and R are individually true but R is not the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

4. BAC is triangle with $\angle A = 90°$. From A, a perpendicular AD is drawn on BC. Which one of the following is correct? **[2007-II]**

(a) Only $\Delta ABC \sim \Delta DAC$

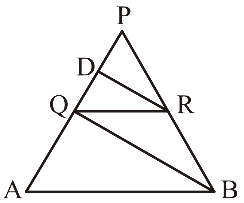
(b) Only $\Delta DAC \sim \Delta DBA$

(c) Only $\Delta ABC \sim \Delta DBA \sim \Delta DAC$

(d) Only $\Delta ABC \sim \Delta DAB$

where ~ stands for the notation of similarity.

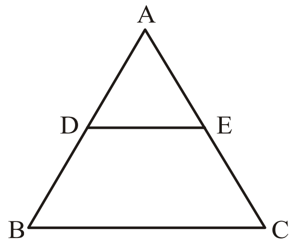
5. In the given figure, QR is parallel to AB and DR is parallel to QB. What is the number of distinct pairs of similar triangles? **[2007-II]**



(a) 1 (b) 2

(c) 3 (d) 4

6.



In the figure given above. BC is parallel to DE and $DE : BC = 3 : 5$. What is the ratio of area of the ΔABC to that of ΔDEA? **[2007-II]**

(a) 3 : 1 (b) 5 : 3

(c) 9 : 2 (d) 25 : 9

7. The vertical angle of an isosceles triangle is 15° more than each of its base angles. What is the vertical angle? **[2008-I]**

(a) 35° (b) 55°

(c) 65° (d) 70°

8. **Assertion** (A) If two triangles are congruent, then their corresponding angles are equal. **[2008-I]**

Reason (R) Two congruent triangles have same area.

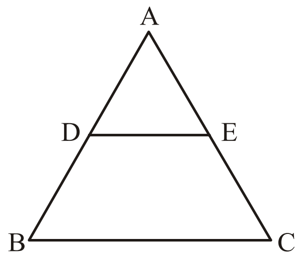
(a) Both A and R individually true and R is the correct explanation of A

(b) Both A and R are individually true but R is not the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

9.



In the figure shown above, DE is parallel to BC and the ratio of the areas of ΔADE and trapezium $BDEC$ is 4 : 5. What is $DE : BC$? **[2008-I]**

(a) 1 : 2 (b) 2 : 3

(c) 4 : 5 (d) None of these

10. Consider the following statements

A triangle can be constructed if its **[2008-I]**

I. two sides and the included angles are given.

II. three angles are given.

III. two angles and the included side are given.

Which of the statements given above are correct?

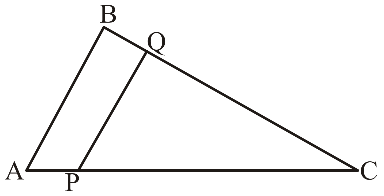
(a) I and II (b) I and III

(c) II and III (d) All of these

11. AB, EF and CD are parallel lines. If $EG = 5$ cm $GC = 10$ cm, $AB = 15$ cm and $DC = 18$ cm, then what is the value of AC? **[2008-I]**

(a) 20 cm (b) 24 cm
(c) 25 cm (d) 28 cm

12.



In the given triangle, AB is parallel to PQ. $AP = c$, $PC = b$, $PQ = a$, $AB = x$. What is the value of x? **[2008-I]**

(a) $a+\frac{ab}{c}$ (b) $a+\frac{bc}{a}$
(c) $b+\frac{ca}{b}$ (d) $a+\frac{ac}{b}$

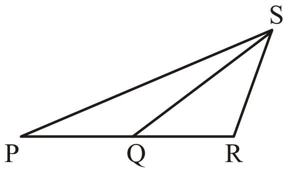
13. **Assertion** (A) Triangles on the same base and between the same parallel lines are equal in area.
Reason (R) The distance between two parallel lines is same everywhere. **[2008-II]**

(a) Both A and R individually true and R is the correct explanation of A
(b) Both A and R are individually true but R is not the correct explanation of A
(c) A is true but R is false
(d) A is false but R is true

14. The hypotenuse of a right triangle is $3\sqrt{10}$ unit. If the smaller side is tripled and the longer side is doubled, new hypotenuse becomes $9\sqrt{5}$ unit. What are the lengths of the smaller and longer sides of the right triangle, respectively? **[2008-II]**

(a) 5 and 9 units (b) 5 and 6 units
(c) 3 and 9 units (d) 3 and 6 units

15.



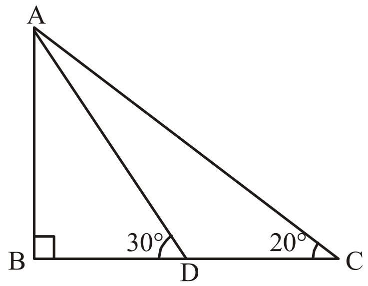
In the figure given above, $PQ = QS$ and $QR = RS$. If $\angle SRQ = 100°$, then how many degrees is $\angle QPS$? **[2008-II]**

(a) 40° (b) 30°
(c) 20° (d) 15°

16. ABC is a right angled triangle, right angled at C and p is the length of the perpendicular from C on AB. If a, b and c are the sides of the triangle, then which one of the following is correct? **[2008-II]**

(a) $(a^2+b^2)p^2 = a^2b^2$ (b) $a^2+b^2 = a^2b^2p^2$
(c) $p^2 = a^2+b^2$ (d) $p^2 = a^2-b^2$

17.



In the given figure, $\angle ABD = 90°$, $\angle BDA = 30°$ and $\angle BCA = 20°$. What is $\angle CAD$? **[2008-II]**

(a) 10° (b) 20°
(c) 30° (d) 15°

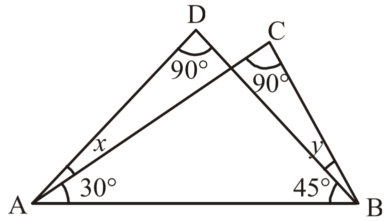
18. If the angles of a triangle are 90°, 60° and 30°, then what is the ratio of the sides opposite to these angles? **[2008-II]**

(a) $\sqrt{3}:\sqrt{2}:1$ (b) $1:\sqrt{2}:2$
(c) $2:\sqrt{3}:1$ (d) $3:2:1$

19. The sides of a triangle are 50 m, 40 m and 30 m. What is the length of the altitude of the vertex opposite to the side 50 m long? **[2008-II]**

(a) 22 m (b) 24 m
(c) 25 m (d) 26 m

20.



In the figure given above. What is the value of $(2x - y)$? **[2008-II]**

(a) 10° (b) 15° (c) 20° (d) 25°

21. If ABC is a triangle, right angled at B, M and N are mid-points of AB and BC respectively, then what is $4\,(AN^2 + CM^2)$ equal to? **[2008-II]**

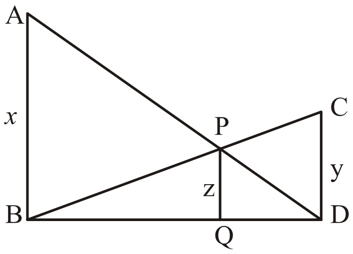
(a) $3AC^2$ (b) $4AC^2$
(c) $5AC^2$ (d) $6AC^2$

22. If A is the area of a right angled triangle and b is one of the sides containing the right angle, then what is the length of the altitude on the hypotenuse? **[2008-II]**

(a) $\frac{2Ab}{\sqrt{b^4+4A^2}}$ (b) $\frac{2A^2b}{\sqrt{b^4+4A^2}}$
(c) $\frac{2Ab^2}{\sqrt{b^4+4A^2}}$ (d) $\frac{2A^2b^2}{\sqrt{b^4+A^2}}$

23. ΔABC is right angled at A. $AB = 3$ units, $AC = 4$ units and AD is perpendicular to BC. What is the area of the ΔADB? **[2008-II]**

(a) $\frac{9}{25}$ sq units (b) $\frac{54}{25}$ sq units
(c) $\frac{72}{25}$ sq units (d) $\frac{96}{25}$ sq units

24. 

In the figure given above. $\angle ABD = \angle PQD = \angle CDQ = \frac{\pi}{2}$.

If $AB = x$, $PQ = z$ and $CD = y$, then which one of the following is correct? **[2009-I]**

(a) $\frac{1}{x}+\frac{1}{y}=\frac{1}{z}$ (b) $\frac{1}{x}+\frac{1}{z}=\frac{1}{y}$

(c) $\frac{1}{z}+\frac{1}{y}=\frac{1}{x}$ (d) $\frac{1}{x}+\frac{1}{y}=\frac{2}{z}$

25. ΔPQR is right angled at Q, $PR = 5\ cm$ and $QR = 4\ cm$. If the lengths of sides of another ΔABC are $3\ cm$, $4\ cm$ and $5\ cm$, then which one of the following is correct? **[2009-I]**

(a) Area of ΔPQR is double that of ΔABC

(b) Area of ΔABC is double that of ΔPQR

(c) $\angle B = \frac{\angle Q}{2}$

(d) Both triangles are congruent

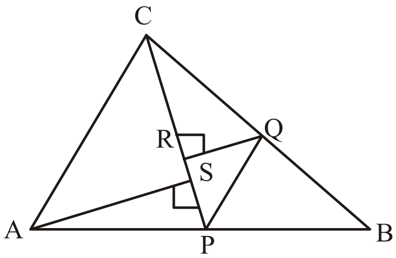
26. If C_1 and C_2 and r_1 and r_2 are respectively the centroids and radii of incircles of two congruent triangles, then which one of the following is correct? **[2009-I]**

(a) C_1 and C_2 are the same points and $r_1 = r_2$

(b) C_1 and C_2 are not necessarily the same point and $r_1 = r_2$

(c) C_1 and C_2 are same point and r_1 is not necessarily equal to r_2

(d) C_1 and C_2 are not necessarily the same point and r_1 is not necessarily equal to r_2

27. 

In the figure given above, P is a point on AB and PQ is parallel to AC. What is the number of pairs of distinct similar triangles in the figure. **[2009-I]**

(a) 1 (b) 2

(c) 3 (d) 4

28. If the medians of two equilateral triangles are in the ratio 3 : 2, then what is the ratio of their sides? **[2009-I]**

(a) $1:1$ (b) $2:3$

(c) $3:2$ (d) $\sqrt{3}:\sqrt{2}$

29. The centroid and the orthocentre are coincident for which one of the following triangles? **[2009-I]**

(a) Scalene triangle

(b) Isosceles triangle

(c) Equilateral triangle

(d) Right angled triangle

DIRECTIONS (Qs. 30-32) : *The following three questions consists of two statements, one labelled as the 'Assertion (A)' and the other as 'Reason (R)'. You are to examine these two statements carefully and select the answers to these items using the codes given below* :

(a) Both A and R are individually true and R is the correct explanation of A.

(b) Both A and R are individually true but R is not the correct explanation of a.

(c) A is true but R is false

(d) A is false but R is true.

30. **Assertion** (A) If two triangles have same perimeter, then they are congruent.

Reason (R) If under a given correspondence, the three sides of one triangle are equal to the three sides of the other triangle, then the two triangle, are congruent. **[2009-I]**

31. ABC is a triangle. Let D, E denote the mid-points of BC and CA, respectively. Let AD and BE intersect at G. Let O be a point on AD such $AO : OD = 2 : 7$. Where, AD is a median. **[2009-I]**

Assertion (A) $AO = \frac{(2GD)}{3}$

Reason (R) $OD = \frac{(2AG)}{3}$

32. ABC is a triangle. AD, BE and CF are altitudes of ΔABC.

Assertion (A)

$(AB^2 + BC^2 + CA^2) > (AD^2 + BE^2 + CF^2)$

Reason (R)

$(AE^2 - AF^2) + (BF^2 - BD^2) + (CD^2 - CE^2) = 0$ **[2009-I]**

33. ABC is a triangle right angled at C. If p is the length of the perpendicular from C to AB and a, b and c are the sides, then which one of the following is correct? **[2009-II]**

(a) $pa = bc$ (b) $pb = ca$

(c) $pc = ab$ (d) $p^2 = ab$

34. In the ΔABC, $AB = 2\ cm$, BC = $3\ cm$ and AC = $4\ cm$. D is the middle point of AC. If a square is constructed on the side BD, what is the area of the square? **[2009-II]**

(a) $4.5\ cm^2$ (b) $2.5\ cm^2$

(c) $6.35\ cm^2$ (d) None of these

35. ABC is a triangle. X is a point outside the ΔABC such that $CD = CX$, where D is the point of intersection of BC and AX and $\angle BAX = \angle XAC$. **[2009-II]**

Which one of the following is correct?

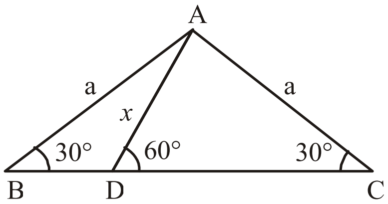
(a) ΔABD and ΔACX are similar
(b) $\angle ABD < \angle DXC$
(c) $AC = CX$
(d) $\angle ADB > \angle DCX$

36. Consider the following statements
I. Congruent triangles are similar.
II. Similar triangles are congruent.
III. If the hypotenuse and a side of one right triangle are equal to the hypotenuse and a side of another right triangle respectively, then the two right triangles are congruent.
Which of the statement given above is/are correct? **[2009-II]**
(a) Only I (b) Only I
(c) Both II and III (d) Both I and III

37. ABC is a triangle and the perpendicular drawn from A meets BC in D. If $AD^2 = BC.DC$, then
Which one of the following is correct? **[2009-II]**
(a) ABC must be an obtuse angled triangle
(b) ABC must be an acute angled triangle
(c) Either $\angle B \geq 45°$ or $\angle C \geq 45°$
(d) $BC^2 = AB^2 + AC^2$

38. The bisectors of the angles ABC and BCA of a Δ ABC meet in a point O. What is the angle at O facing the side BC? **[2009-II]**
(a) $90° - \left(\frac{A}{2}\right)$ (b) $90° + \left(\frac{A}{2}\right)$
(c) $90° - A$ (d) $90° + A$

39. What is the number of points in the plane of a ΔABC which are at equal distance from the vertices of the triangle? **[2009-II]**
(a) 0 (b) 1
(c) 2 (d) 3

40.


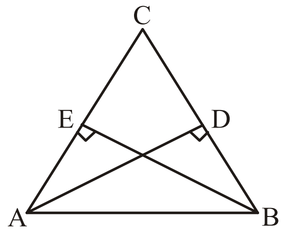
In the above figure, what is x equal to? **[2010-I]**
(a) $\frac{a}{3}$ (b) $\frac{a}{2}$
(c) $\frac{a}{\sqrt{3}}$ (d) $\frac{a}{\sqrt{2}}$

41. **Statement I** Let LMN be a triangle. Let P, Q be the mid-points of the sides LM and LN, respectively. If $PQ^2 = MP^2 + NQ^2$, then LMN is a right angled triangle at L.
Statement II If in a $\Delta ABC, AB^2 > BC^2 + CA^2$, then $\angle ACB$ is obtuse. **[2010-I]**
Which of the following is correct of the above statements?
(a) Both Statements I and II are individually true and Statements II is the correct explanation of Statement I
(b) Both Statements I and II are individually true but Statement II is not the correct explanation on Statement I
(c) Statement I is true and statement II is false
(d) Statement I is false and statement II is true

42. ABC is a triangle. The internal bisector of $\angle ABC$ and the external bisector of $\angle ACB$ meet at D.
Which one of the following is correct? **[2010-I]**
(a) $\angle BDC = \angle BAC$ (b) $\angle BDC = \frac{1}{2}\angle BAC$
(c) $\angle BDC = \angle DBC$ (d) $\angle BDC = \frac{1}{2}\angle ABC$

43. The following sets of conditions relate to two triangle ABC and DEF. Which set of conditions does not guarantee the congruence of ΔABC and ΔDEF? **[2010-I]**
(a) $a = d, b = e, c = f$
(b) $\angle B = \angle E, \angle C = \angle F, a = d$
(c) $a = f, b = e, \angle A = \angle D$
(d) $c = f, b = e, \angle B = \angle E$

44. Consider the following statements in respect of any triangle
I. The three medians of a triangle divide it into six triangles of equal area.
II. The perimeter of a triangle is greater than the sum of the lengths of its three medians.
Which of the statements given above is/are correct? **[2010-I]**
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

45. Consider the following in respect of the given figure


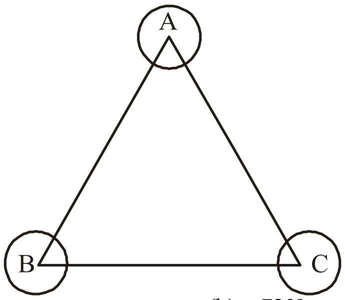
I. $\Delta DAC \sim \Delta EBC$ II. $CA/CB = CD/CE$
III. $AD/BE = CD/CE$
Which of the above are correct? **[2010-I]**
(a) I, II and III (b) I and II
(c) I and III (d) II and III

46. The median BD of the ΔABC meets AC at D. If $BD = \frac{1}{2}AC$, then which one of the following is correct? **[2010-I]**
(a) $\angle ACB = 1$ right angle
(b) $\angle BAC = 1$ right angle
(c) $\angle ABC = 1$ right angle
(d) None of the above

47. The three sides of a triangle are 10, 100 and x. Which one of the following is correct? **[2010-I]**

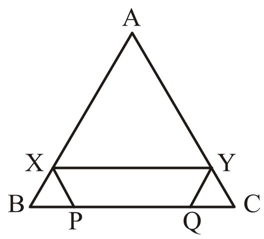
(a) $10 < x < 100$ (b) $90 < x < 110$
(c) $90 \le x \le 100$ (d) $90 \le x < 110$

48. In the figure given below, what is the sum of the angles formed around A, B, C except the angles of the ΔABC? **[2010-II]**



(a) 360º (b) 720º
(c) 900º (d) 1000º

49. In the given figure, ABC is an equilateral triangle of side length 30 *cm*. XY is parallel to BC, XP is parallel to AC and YQ is parallel to AB. If $(XY + XP + YQ)$ is 40 *cm*, then what is PQ equal to? **[2010-II]**



(a) 5 *cm* (b) 12 *cm*
(c) 15 *cm* (d) None of these

50. Consider the following statements
I. If two triangles are equiangular, then they are similar.
II. If two triangles have equal area, then they are similar.
Which of the statements given above is/are correct? **[2010-II]**

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

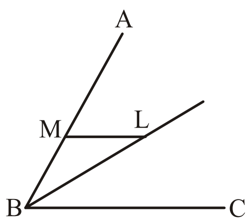
51. If two corresponding sides of two similar triangles are in the ratio 9 : 4, then what is the ratio of their areas? **[2010-II]**

(a) 9 : 4 (b) 3 : 2
(c) 81 : 16 (d) 27 : 8

52. In a triangle, if sum of two angles is equal to the third angle (considering the interior angles only), then the triangle is **[2010-II]**

(a) right angled (b) acute angled
(c) equilateral (d) obtuse angled

53. In the given figure, L is any point on the bisector of the acute angle ABC and the line ML is parallel to BC. Which one of the following is correct? **[2010-II]**



(a) The Δ BML is equilateral
(b) The Δ BML is isosceles but right angled
(c) The Δ BML is isosceles but not right angled
(d) The Δ BML is not isosceles

54. The side BC of the ΔABC is extended to D. If $\angle ACD =$ 120º and $\angle ABC = \frac{2}{3} \angle CAB$, then what is $\angle BAC$? **[2010-II]**

(a) 60º (b) 45º
(c) 30º (d) 72º

55. Let D, E be the points on sides AB and AC respectively of a ΔABC such that DE is parallel to BC. Let $AD = 2$ *cm*, $DB = 1$ *cm*, $AE = 3$ *cm* and area of $\Delta ADE = 3$ cm^2. What is EC equal to? **[2011-I]**

(a) 1.5 *cm* (b) 1.6 *cm*
(c) 1.8 *cm* (d) 2.1 *cm*

56. In ΔABC, a line PQ is drawn parallel to BC, points P, Q being on AB and AC, respectively. If $AB = 3\ AP$, then what is the ratio of the area of ΔAPQ to the area of ΔABC? **[2011-I]**

(a) 1 : 3 (b) 1 : 5
(c) 1 : 7 (d) 1 : 9

57. Consider the following statements

Statement I Let PQR be a triangle in which $PQ = 3$ *cm*, $QR = 4$ *cm* and $RP = 5$ *cm*. If D is a point in the plane of the ΔPQR such that D is either outside it or inside it, then $DP + DQ + DR > 6$ *cm*

Statement II PQR is a right–angled triangle.

Which one of the following is correct in respect of the above two statements? **[2011-I]**

(a) Both Statements I and II are individually true and Statements II is the correct explanation of StatementI

(b) Both Statements I and II are individually true but Statement II is not the correct explanation on Statement I

(c) Statement I is true and statement II is false

(d) Statement I is false and statement II is true

58. ABC is a triangle right angled at B and D is a point on BC produced (BD > BC), such that BD = 2DC.

Which one of the following is correct? **[2011-I]**

(a) $AC^2 = AD^2 - 3CD^2$ (b) $AC^2 = AD^2 - 2CD^2$

(c) $AC^2 = AD^2 - 4CD^2$ (d) $AC^2 = AD^2 - 5CD^2$

59. PQR is a triangle right angled at Q. If X and Y are the mid-points of the sides PQ and OR respectively then which one of the following is not correct? **[2011-I]**

(a) $RX^2 + PY^2 = 5XY^2$

(b) $RX^2 + PY^2 = XY^2 + PR^2$

(c) $4(RX^2 + PY^2) = 5PR^2$

(d) $RX^2 + PY^2 = 3(PQ^2 + QR^2)$

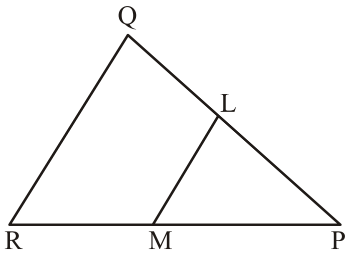
60. The centroid of a ΔABC is 8 *cm* from the vertex *A*. What is the length of the median of the trianlgle through *A*? **[2011-I]**

(a) 20 *cm* (b) 16 *cm*
(c) 12 *cm* (d) 10 *cm*

61. ΔABC is an isosceles triangle such that $AB = BC = 8$ *cm* and $\angle ABC = 90^\circ$. What is the length of the perpendicular drawn from *B* on *AC*? **[2011-II]**

(a) 4 *cm* (b) $4\sqrt{2}$ *cm*
(c) $2\sqrt{2}$ *cm* (d) 2 *cm*

62.



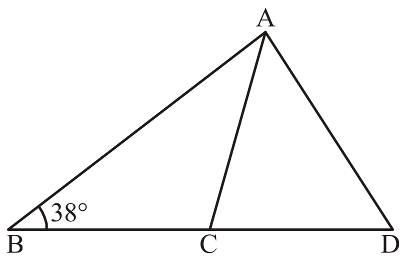
In the figure given above, *LM* is parallel to *QR*. If *LM* divides the ΔPQR such that area of trapezium *LMRQ* is two times the area of ΔPLM, then what is $\frac{PL}{PQ}$ equal to? **[2011-II]**

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{3}}$
(c) $\frac{1}{2}$ (d) $\frac{1}{3}$

63. The point of concurrence of the altitudes of a triangle is called **[2011-II]**

(a) circumcentre (b) orthocentre
(c) incentre (d) centroid

64.



In the figure given, $\angle B = 38^\circ$, $AC = BC$ and $AD = CD$. What is $\angle D$ equal to? **[2011-II]**

(a) 26° (b) 28°
(c) 38° (d) 52°

65. **Statement I** Let the side *DE* of a ΔDEF be divided at *S*. so that $\frac{DS}{DE} = \frac{1}{\sqrt{2}}$. If a line through *S* parallel to *EF* meets *DF* at *T*, then the area of ΔDEF is twice the area of the ΔDST.

Statement II The areas of the similar triangles are proportional to the squares on the corresponding sides.

Which one of the following is correct in respect of the above statements? **[2012-I]**

(a) Both Statements I and II are true and Statement II is the correct explanation of Statement I
(b) Both Statements I and II are true but Statement II is not the correct explanation of Statement I
(c) Statement I is true but Statement II is false
(d) Statement II is true but statement I is false

66. The base of a triangular wall is 7 times its height. If the cost of painting the wall at ₹ 350 per 100 sq *m* is ₹ 1225, then what is the base length? **[2012-I]**

(a) 50 *m* (b) 70 *m*
(c) 75 *m* (d) 100 *m*

67. Which one among the following is not correct? **[2012-I]**

(a) Two congruent triangles are necessarily similar
(b) All equiangular triangles are similar
(c) Two isosceles right triangles are similar
(d) All isosceles triangles are similar

68. Consider the following statement in respect of an equilateral ΔABC.

I. There is a point *P* inside the ΔABC such that each of its sides subtends an angle of 120° at *P*.
II. There is a point P inside the Δ *ABC* such that the Δ *PBC* is obtuse angled and A is the orthocentre of ΔPBC.

Which of the above statements is/are correct? **[2012-I]**

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

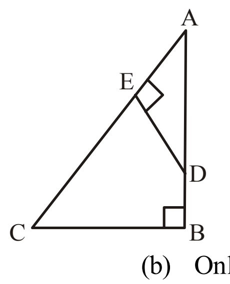
69. The medians of ΔABC intersect at *G*. Which one of the following is correct? **[2012-I]**

(a) Five times the area of ΔAGB is equal to four times the area of ΔABC
(b) Four times the area of ΔAGB is equal to three times the area of ΔABC
(c) Three times the area of ΔAGB is equal to the area of ΔABC
(d) None of the above

70. In the figure given below $\angle ABC = \angle AED = 90^\circ$. Consider the following statements **[2012-I]**

I. *ABC* and *ADE* are similar triangles.
II. The four points *B*, *C*, *E* and *D* may lie on a circle.

Which of the above statements is/are correct?



(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

71. The side *BC* of a ΔABC is produced to *D*, bisectors of the $\angle ABC$ and $\angle ACD$ meet at *P*. If $\angle BPC = x^\circ$ and $\angle BAC = y^\circ$, then which one of the following option is correct? **[2013-I]**

(a) $x^\circ = y^\circ$ (b) $x^\circ + y^\circ = 90^\circ$
(c) $x^\circ + y^\circ = 180^\circ$ (d) $2x^\circ = y^\circ$

72. ABC is a right angled triangle such that $AB = a - b$, $BC = a$ and $CA = a + b$. D is a point on BC such that $BD = AB$. The ratio of $BD : DC$ for any value of a and b is given by **[2013-I]**

(a) $3:2$ (b) $4:3$

(c) $5:4$ (d) $3:1$

73. Let ABC be an equilateral triangle. If the side BC is produced to the point D so that $BC = 2\ CD$, then AD^2 is equal to **[2013-I]**

(a) $3CD^2$ (b) $4CD^2$

(c) $5CD^2$ (d) $7CD^2$

74. ABC is a triangle, where $BC = 2AB$, $\angle C = 30°$ and $\angle A = 90°$. The magnitude of the side AC is **[2013-I]**

(a) $\frac{2BC}{3}$ (b) $\frac{3BC}{4}$

(3) $\frac{BC}{\sqrt{3}}$ ` (4) $\frac{\sqrt{3}BC}{2}$

75. The bisectors BI and CI of the $\angle B$ and $\angle C$ of a ΔABC meet in I. What is $\angle BIC$ equal to? **[2013-I]**

(a) $90° - \frac{A}{4}$ (b) $90° + \frac{A}{4}$

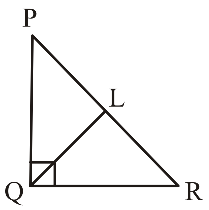
(c) $90° - \frac{A}{2}$ (d) $90° + \frac{A}{2}$

76. In a ΔABC, $\angle BCA = 90°$ and CD is perpendicular to AB. If $AD = 4$ *cm* and $BD = 9$ *cm*, then the value of DC will be **[2013-I]**

(a) $\sqrt{18}$ *cm* (b) $\sqrt{20}$ *cm*

(c) $\sqrt{65}$ *cm* (d) 6 *cm*

77.



In the figure given above, $\angle PQR = 90°$ and QL is a median, $PQ = 5$ *cm* and $QR = 12$ *cm*. Then, QL is equal to **[2013-I]**

(a) 5 *cm* (b) 5.5 *cm*

(c) 6 *cm* (d) 6.5 *cm*

78. In a right angled ΔABC, $\angle C = 90°$ and CD is perpendicular to AB. If $AB \times CD = CA \times CB$, then $\frac{1}{CD^2}$ is equal to **[2013-I]**

(a) $\frac{1}{AB^2} - \frac{1}{CA^2}$

(b) $\frac{1}{AB^2} - \frac{1}{CB^2}$

(c) $\frac{1}{BC^2} - \frac{1}{CA^2}$

(d) $\frac{1}{BC^2} - \frac{1}{CA^2}$, if $CA > CB$

79. Each side of the equilateral triangle is 6 *cm*. Its altitude is **[2013-I]**

(a) $6\sqrt{3}$ *cm* (b) $3\sqrt{3}$ *cm*

(c) $2\sqrt{3}$ *cm* (d) $\sqrt{3}$ *cm*

80. In a ΔABC, $\angle BCA = 60°$ and $AB^2 = BC^2 + CA^2 + X$, What is the value of X? **[2013-I]**

(a) $(BC)(CA)$ (b) $-(BC)(CA)$

(c) $(AB)(BC)$ (d) Zero

81. In ΔABC, XY is drawn parallel to BC, cutting sides at X and Y, where $AB = 4.8$ *cm*, $BC = 7.2$ *cm* and $BX = 2$ *cm*. What is the length of XY? **[2013-I]**

(a) 4 *cm* (b) 4.1 *cm*

(c) 4.2 *cm* (d) 4.3 *cm*

82. Let ABC be a triangle with $AB = 3$ *cm* and $AC = 5$ *cm*. If AD is a median drawn from the vertex A to the side BC, then which one of the following is correct? **[2013-I]**

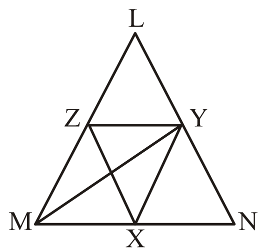
(a) AD is always greater than 4 *cm* but less than 5 *cm*

(b) AD is always greater than 5 *cm*

(c) AD is always less than 4 *cm*

(d) None of the above

83. In the figure given below, YZ is parallel to MN, XY is parallel to LM and XZ is parallel to LN. Then MY is **[2013-I]**



(a) The median of ΔLMN

(b) the angular bisector of $\angle LMN$

(c) perpendicular to LN

(d) perpendicular bisector of LN

84. The sum of the perpendiculars drawn from an interior point of an equilateral triangle is 20 *cm*. What is the length of side of the triangle? **[2013-I]**

(a) $\frac{40}{\sqrt{3}}$ *cm* (b) $40\sqrt{3}$ *cm*

(c) $20\sqrt{3}$ *cm* (d) $\frac{20}{\sqrt{3}}$ *cm*

85. The lengths of three line segments (in *cm*) are given in each of the four cases. Which one of the following cases is not suitable to be the three sides of a triangle? **[2013-I]**

(a) 2, 3, 4 (b) 2, 3, 5

(c) 2, 4, 5 (d) 3, 4, 5

86. Consider the following statements

I. If G is the centroid of ΔABC, then $GA = GB = GC$.

II. If H is the orthocentre of ΔABC, then $HA = HB = HC$.

Which of the statements given above is/are correct? [2013-II]

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

87. If the bisectors BI and CI of the angles B and C of a ΔABC meet at the point I, then what is $\angle BIC$ equal to? [2013-II]

(a) $2A$ (b) $90^\circ + \frac{A}{2}$
(c) $90^\circ - \frac{A}{2}$ (d) $90^\circ + A$

88. E is the mid-point of the median AD of a ΔABC, *If* BE produced meets the side AC at F, then CF is equal to [2013-II]

(a) $\frac{AC}{3}$ (b) $\frac{2AC}{3}$
(c) $\frac{AC}{2}$ (d) None of these

89. *A* ΔDEF is formed by joining the mid-points of the sides of ΔABC. Similarly, a ΔPQR is formed by joining the mid-points of the sides of the ΔDEF. If the sides of the ΔPQR are of lengths 1, 2 and 3 units, what is the perimeter of the ΔABC? [2013-II]

(a) 18 units (b) 24 units
(c) 48 units (d) Cannot be determined

90. PQR is an equilateral triangle. O is the point of intersection of altitudes PL, QM and RN. If $OP = 8$ *cm*, then what is the perimeter of the ΔPQR? [2013-II]

(a) $8\sqrt{3}$ *cm* (b) $12\sqrt{3}$ *cm*
(c) $16\sqrt{3}$ *cm* (d) $24\sqrt{3}$ *cm*

91. In a ΔABC, $\angle B = 90^\circ$ and $\angle C = 2\angle A$, then what is AB^2 equal to? [2013-II]

(a) $2BC^2$ (b) $3BC^2$
(c) $4BC^2$ (d) $5BC^2$

92. The side AC of a ΔABC is produced to D such that $BC = CD$. If ΔACB is 70°, then what is ΔADB equal to? [2013-II]

(a) 35° (b) 45°
(c) 70° (d) 110°

93. Consider the following statements

I. If the diagonals of a parallelogram $ABCD$ are perpendicular, then $ABCD$ may be a rhombus.
II. If the diagonals of a quadrilateral $ABCD$ are equal and perpendicular, then $ABCD$ is a square.

Which of the statements given above is/are correct? [2013-II]

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

94. ABC is triangle right angled at A and a perpendicular AD is drawn on the hypotenuse BC. What is $BC.AD$ equal to? [2013-II]

(a) $AB.AC$ (b) $AB.AD$
(c) $CA.CD$ (d) $AD.DB$

95. ABC and XYZ are two similar triangles with $\angle C = \angle Z$, whose areas are respectively 32 cm^2 and 60.5 cm^2. If XY = 7.7 *cm*, then what is AB equal to? [2013-II]

(a) 5.6 *cm* (b) 5.8 *cm*
(c) 6.0 *cm* (d) 6.2 *cm*

96. The diameter of circle with centre at C is 50 *cm*. CP is a radial segment of the circle. AB is a chord perpendicular to CP and passes through P. CP produced intersects the circle at D. If $DP = 18$ *cm* then what is the length of AB? [2013-II]

(a) 24 *cm* (b) 32 *cm*
(c) 40 *cm* (d) 48 *cm*

97. Consider the following statements

I. The perpendicular bisector of a chord of a circle does not pass through the centre of the circle.
II. The angle in a semi-circle is a right angle.

Which of the statements given above is/are correct? [2013-II]

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

98. The three sides of a triangle are 15, 25, x units. Which one of the following is correct? [2014-I]

(a) $10 < x < 40$ (b) $10 \le x \le 40$
(c) $10 \le x < 40$ (d) $10 < x \le 40$

99. Which one of the following is a Pythagorean triple in which one side differs from the hypotenuse by two units? [2014-I]

(a) $(2n+1, 4n, 2n^2+2n)$ (b) $(2n, 4n, n^2+1)$
(c) $(2n^2, 2n, 2n+1)$ (d) $(2n, n^2-1, n^2+1)$

Where, n is a positive real number.

100. The sides of a right angled triangle are equal to three consecutive numbers expressed in centimeters. What can be the area of such a triangle? [2014-I]

(a) 6 cm^2 (b) 8 cm^2
(c) 10 cm^2 (d) 12 cm^2

101. If triangles ABC and DEF are similar such that $2AB = DE$ and $BC = 8$ *cm*, then what is EF equal to? [2014-I]

(a) 16 *cm* (b) 12 *cm*
(c) 10 *cm* (d) 8 *cm*

102. The sides of a triangle are in geometric progression with common ratio r < 1. If the triangle is a right angled triangle, the square of common ratio is given by [2014-I]

(a) $\frac{\sqrt{5}+1}{2}$ (b) $\frac{\sqrt{5}-1}{2}$
(c) $\frac{\sqrt{3}+1}{2}$ (d) $\frac{\sqrt{3}-1}{2}$

103. In a ΔABC, AD is perpendicular of BC and BE is perpendicular to AC. Which of the following is correct? [2014-I]

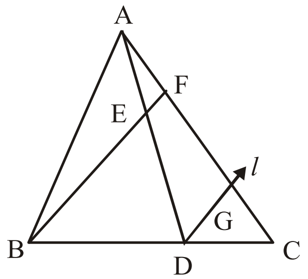
(a) $CE \times CB = CA \times CD$
(b) $CE \times CA = CD \times CB$
(c) $AD \times BD = AE \times BE$
(d) $AB \times AC = AD \times BE$

104. Let ABC is triangle right angled at B. If $AB = 6$ cm and $BC = 8$ cm, then what is the length of the circumradius of the ΔABC? **[2014-I]**
(a) 10 cm (b) 7 cm
(c) 6 cm (d) 5 cm

105. If AD is the internal angular bisector of ΔABC with $AB = 3$ cm and $AC = 1$ cm then what is $BD : BC$ equal to? **[2014-I]**
(a) 1 : 3 (b) 1 : 4
(c) 2 : 3 (d) 3 : 4

106. AB is a straight line, C and D are points the same side of AB such that AC is perpendicular to AB and DB is perpendicular to AB. Let AD and BC meet at E. What is $\frac{AE}{AD} + \frac{BE}{BC}$ equal to? **[2014-I]**
(a) 2 (b) 1.5
(c) 1 (d) None of these

107. In a ΔABC, AD is the median through A and E is the mid–point of AD and BE produced meets AC at F. Then, AF is equal to **[2014-II]**
(a) $AC/5$ (b) $AC/4$
(c) $AC/3$ (d) $AC/2$



108. Three straight lines are drawn through the three vertices of a ΔABC, the line through each vertex being parallel to the opposite side. The ΔDEF is bounded by these parallel lines. **[2014-II]**
Consider the following statements in respect of the ΔDEF.
1. Each side of ΔDEF is double the side of ΔABC to which it is parallel.
2. Area of ΔDEF is four times the area of ΔABC.

Which of the above statements is/are correct?
(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

109. In a ΔABC, if $\angle B = 2\angle C = 2\angle A$. Then, what is the ratio of AC to AB ? **[2014-II]**
(a) $\sqrt{2}:1$ (b) $\sqrt{3}:1$
(c) $1:1$ (d) $1:\sqrt{2}$

110. For a triangle, the radius of the circumcircle is double the radius of the inscribed circle, then which one of the following is correct ? **[2014-II]**
(a) The triangle is a right-angled
(b) The triangle is an isosceles
(c) The triangle is an equilateral
(d) None of the above

111. Consider the following statements in respect of an equilateral triangle : **[2014-II]**
1. The altitudes are congruent.
2. The three medians are congruent.
3. The centroid bisects the altitude.

Which of the above statements are correct ?
(a) 1 and 2 (b) 2 and 3
(c) 1 and 3 (d) 1,2 and 3

112. Consider the following : **[2014-II]**
ABC and DEF are triangles in a plane such that AB is parallel to DE, BC is parallel to EF and CA is parallel to FD.
Statement I If $\angle ABC$ is a right angle, then $\angle DEF$ is also a right angle.
Statement II Triangles of the type ABC and DEF are always congruent.
Which one of the following is correct in respect of the above statements ?
(a) Statements I and II are correct and Statement II is the correct explanation of Statement I
(b) Statements I and II are correct and Statement II is not the correct explanation of Statement I
(c) Statement I is correct and Statement II is incorrect
(d) Statement I is incorrect and Statement II is correct

113. Let the incircle to a ΔABC touch BC, AC and AB respectively at the points X, Y and Z. **[2014-II]**
Statement I If $AB > BC$, then $AB + AZ < BC + XC$
Statement II $AZ = AY$
Which one of the following is correct in respect of the above statements ?
(a) Statement I and II are correct and Statements II is the correct explanation of Statement I
(b) Statement I and II are correct and Statement II is not the correct explanation of Statement I
(c) Statement I is correct and Statement II is incorrect
(d) Statement I is incorrect and Statement II is correct

114. Let ABC be a triangle in which $\angle ACB = 60°$ and $AC = x < BC$. Let the circle with centre at C and radius x meet BC at D. Let CF be the perpendicular drawn from C meeting AD at F.
Statement I ΔACD is isosceles but not equilateral.
Statement II DF = $\frac{x}{2}$ **[2014-II]**
Which one of the following is correct in respect of the above statements ?
(a) Statements I and II are correct and Statement II is the correct explanation of Statement I
(b) Statementes I and II are correct and Statement II is not the correct explanation of Statement I
(c) Statement I is correct and Statement II is incorrect
(d) Statemente I is incorrect and Statement II is correct

115. If every side of an equilateral triangle is doubled, then the area of new triangle becomes k times the area of the old one. What is k equal to? **[2014-II]**
(a) $\sqrt{3}$ (b) 2
(c) 4 (d) 8

116. The angles of a triangle are in the ratio 4 : 1 : 1. Then the ratio of the largest side to the perimeter is **[2015-I]**

(a) $\frac{2}{3}$ (b) $\frac{1}{2+\sqrt{3}}$

(c) $\frac{\sqrt{3}}{2+\sqrt{3}}$ (d) $\frac{2}{1+\sqrt{3}}$

117. Let a, b, c be the sides of a right triangle, where c is the hypotenuse. The radius of the circle which touches the sides of the triangle is **[2015-I]**

(a) $(a+b-c)/2$ (b) $(a+b+c)/2$

(c) $(a+2b+2c)/2$ (d) $(2a+2b-c)/2$

118. The area of the largest triangle that can be inscribed in a semicircle of radius r is **[2015-I]**

(a) r^2 (b) $2r^2$

(c) $3r^2$ (d) $4r^2$

119. Consider the following statements :

1. Let D be a point on the side BC of a triangle ABC. If area of triangle ABD = area of triangle ACD, then for all points O on' AD, area of triangle ABO = area of triangle ACO.
2. If G is the point of concurrence of the medians of a triangle ABC, then area of triangle ABG = area of triangle BCG = area of triangle ACG.

Which of the above statements is /are correct? **[2015-I]**

(a) 1 only (b) 2 only

(c) Both 1 and 2 (d) Neither 1 nor 2

120. The point O is equidistant from the three sides of a triangle ABC. Consider the following statements : **[2015-II]**

1. $\angle OAC + \angle OCB + \angle OBA = 90°$
2. $\angle BOC = 2\angle BAC$
3. The perpendiculars drawn from any point on OA to AB and AC are always equal

Which of the above statements are correct ?

(a) 1 and 2 only (b) 2 and 3 only

(c) 1 and 3 only (d) 1, 2 and 3

121. How many right angled triangles can be formed by joining the vertices of a cuboid ? **[2015-II]**

(a) 24 (b) 28

(c) 32 (d) None of the above

122. Let ABC and $A'B'C'$ be two triangles in which $AB > AB'$, $BC > B'C'$ and $CA > C'A'$. Let D, E and F be the mid. Points of the sides BC, CA and AB respectively. Let D', E' and F' be the midpoints of the sides $B'C'$, $C'A'$ and $A'B'$ respectively. Consider the following statement: **[2016-I]**

Statement I.

$AD > A'D'$, $BE > B'E'$ and $CF > C'F'$ are always true.

Statement II.

$$\frac{AB^2+BC^2+CA^2}{AD^2+BE^2+CF^2} = \frac{A'B'^2+B'C'^2+C'A'^2}{A'D'^2+B'E'^2+C'F'^2}$$

Which one of the following is correct in respect of the above statements?

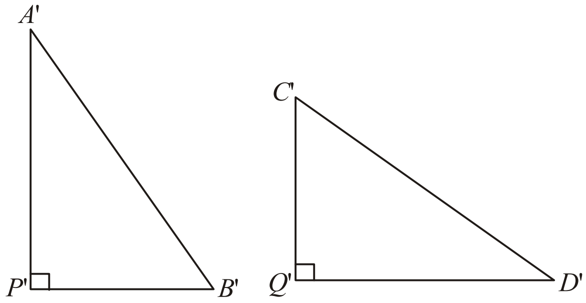
(a) Both Statement I and Statement II are true and Statement II are true and Statement II is the correct explanation of Statement I.

(b) Both Statement I and Statement II are true but Statement II is not the correct explanation of Statement I

(c) Statement I is true but Statement II is false

(d) Statement I is false but Statement II is true

123. Suppose chords AB and CD of a circle intersect at a point P inside the circle. Two right-angled triangles $A'P'B'$ and $C'Q'D'$ are formed as shown in the figures below such that $A'P' = AP$, $B'P' = BP$, $C'Q' = CP$, $D'Q' = DP$ and $\angle A'P'B' = 90° = \angle C'Q'D'$: **[2016-I]**



Which of the following statements are not correct?

1. $A'P'B'$ and $C'Q'D'$ are similar triangles, but need not be congruent.
2. $A'P'B'$ and $C'Q'D'$ are congruent triangles.
3. $A'P'B'$ and $C'Q'D'$ are triangles of same area.
4. $A'P'B'$ and $C'Q'D'$ are triangles of same perimeter.

Select the correct answer using the code given below.

(a) 2 and 3 only (b) 1 and 3 only

(c) 1, 2 and 4 only (d) 1, 2, 3 and 4

124. Suppose ABC is a triangle with AB of unit length D and E are the points lying on AB and AC respectively such that BC and DE are parallel. If the area of triangle ABC is twice the area of triangle ADE, then the length of AD is **[2016-I]**

(a) $\frac{1}{2}$ unit (b) $\frac{1}{3}$ unit

(c) $\frac{1}{\sqrt{2}}$ unit (d) $\frac{1}{\sqrt{3}}$ unit

125. Let the triangles ABC and DEF be such that $\angle ABC = \angle DEF$, $\angle ACB = \angle DFB$ and $\angle BAC = \angle EDF$. Let L be the midpoint of BC and M be the midpoint of EF. Consider the following statements: **[2016-I]**

Statement I.

Triangles ABL and DEM are similar.

Statement II.

Triangle ALC is congruent to triangle DMF even in $AC \neq DF$

Which one of the following is correct in respect of the above statements?

(a) Both Statement I and Statement II are true and Statement II is the correct explanation of Statement I.
(b) Both Statement I and Statement II are true but Statement II is not the correct explanation of Statement I
(c) Statement I is true but Statement II is false
(d) Statement I is false but Statement II is true

126. ABC and DEF are similar triangles. If the ratio of side AB to side DE is $(\sqrt{2}+1):\sqrt{3}$, then the ratio of area of triangle ABC to that of triangle DEF is **[2016-I]**

(a) $(3-2\sqrt{2}):3$ (b) $(9-6\sqrt{2}):2$
(c) $1:(9-6\sqrt{2})$ (d) $(3+2\sqrt{2}):3$

127. In a triangle ABC if $A-B=\frac{\pi}{2}$, then $C+2B$ is equal to **[2016-I]**

(a) $\frac{2\pi}{3}$ (b) $\frac{3\pi}{4}$
(c) π (d) $\frac{\pi}{2}$

128. Let ABC be a triangle in which $AB = AC$. Let L be the locus of points X inside or on the triangle such that $BX = CX$. Which of the following statements are correct? **[2016-I]**

1. L is a straight line passing through A and in-centre of triangle ABC is on L.
2. L is a straight line passing through A and orthocentre of triangle ABC is a point on L.
3. L is a straight line passing through A and centroid of triangle ABC is a point on L.

Select the correct answer using the code given below:
(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

129. In a triangle PQR, point X is on PQ and point Y is on PR such that $XP = 1\ 5$ units, $XQ = 6$ units, $PY = 2$ units and $YR = 8$ units. Which of the following are correct? **[2016-I]**

1. $QR = 5XY$
2. QR is parallel to XY.
3. Triangle PYX is similar to triangle PRQ.

Select the correct answer using the code given below.
(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

130. A person travels 7 km north and then turns right and travels 3 km and further turns right and travels 13 km. What is the shortest distance of the present position of the person from his starting point? **[2016-I]**

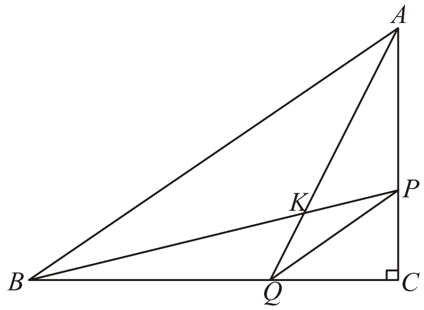
(a) 6 km (b) $3\sqrt{5}$ km
(c) 7 km (d) $4\sqrt{5}$ km

131. ABC is a triangle in which D is the midpoint of BC and E is the midpoint of AD. Which of the following statements is/are correct? **[2016-I]**

1. The area of triangle ABC is equal to four times the area of triangle BED.
2. The area of triangle ADC is twice the area of triangle BED.

Select the correct answer using the code given below.
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

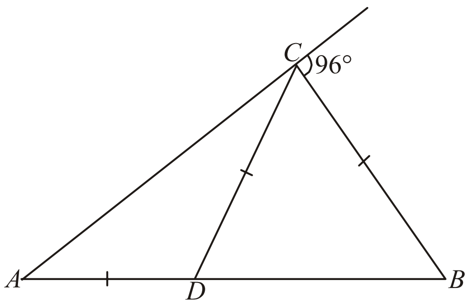
132.



ABC is a triangle right angled at C as shown in the figure above. Which one of the following is correct? **[2016-I]**

(a) $AQ^2 + AB^2 = BP^2 + PQ^2$
(b) $AQ^2 + PQ^2 = AB^2 + BP^2$
(c) $AQ^2 + BP^2 = AB^2 + PQ^2$
(d) $AQ^2 + AP^2 = BK^2 + KQ^2$

133.



In the figure given above, $AD = CD = BC$. What is the value of $\angle CDB$ **[2016-I]**

(a) 32°
(b) 64°
(c) 78°
(d) Cannot be determined due to insufficient data

134. ABC is an equilateral triangle and X, Y and Z are the points on BC, CA and AB respectively such that $BX = CY = AZ$. Which of the following is/are correct? **[2016-I]**

1. XYZ is an equilateral triangle.
2. Triangle XYZ is similar to triangle ABC.

Select the correct answer using the code given below.
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

135. Let ABC be a right angled triangle with BC = 5 cm and AC = 12 cm. Let D be a point on the hypotenuse AB such that $\angle BCD = 30°$. What is length of CD ? **[2016-II]**

(a) $\frac{60}{13}$ cm (b) $\frac{17}{2}$ cm

(c) $\frac{120}{5+12\sqrt{2}}$ cm (d) $\frac{120}{5+12\sqrt{3}}$ cm

136. In an equilateral triangle another equilateral triangle is drawn inside joining the mid–points of the sides of given equilateral triangle and the process is continued up to 7 times. What is the ratio of area of fourth triangle to that of seventh triangle ? **[2016-II]**

(a) 256 : 1 (b) 128 : 1
(c) 64 : 1 (d) 16 : 1

137. The sides of a triangle are given by $\sqrt{a^2+b^2}, \sqrt{c^2+a^2}$ and (b + c) where a, b, c are positive. What is the area of the triangle equal to ? **[2016-II]**

(a) $\frac{\sqrt{a^2+b^2+c^2}}{2}$

(b) $\frac{\sqrt{a^2b^2+b^2c^2+c^2a^2}}{2}$

(c) $\frac{a(b+c)}{2}$

(d) $\frac{\sqrt{3(a^2b^2+b^2c^2+c^2a^2)}}{2}$

138. What is area of largest triangle inscribed in a semi circle of radius r units ? **[2016-II]**

(a) r^2 square units (b) $2r^2$ square units
(c) $3r^2$ square units (d) $4r^2$ square units

139. Which one of the following triples does not represent the sides of a triangle ? **[2017-I]**

(a) (3, 4, 5) (b) (4, 7, 10)
(c) (3, 6, 8) (d) (2, 3, 6)

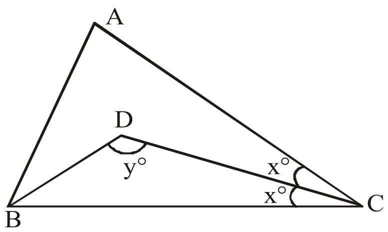
140. The angles of a triangle are in the ratio 2 : 4 : 3. The smallest angle of the triangle is. **[2017-I]**

(a) 20° (b) 40°
(c) 50° (d) 60°

141. ABC is a triangle and D is a point on the side BC. If BC = 12 cm, BD = 9 cm and ∠ADC = ∠BAC, then the length of AC is equal to **[2017-I]**

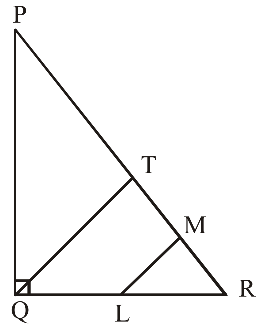
(a) 5 cm (b) 6 cm
(c) 8 cm (d) 9 cm

142. In the figure given below, ∠A = 80° and ∠ABC = 60°. BD and CD bisect angles B and C respectively. What are the values of x and y respectively ? **[2017-I]**



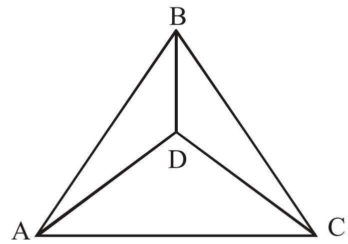
(a) 10 and 130 (b) 10 and 125
(c) 20 and 130 (d) 20 and 125

143. In the figure given below, PQR is a non-isosceles right-angled triangle, right angled at Q. If LM and QT are parallel and QT = PT, then what is ∠RLM equal to ? **[2017-I]**



(a) ∠PQT (b) ∠LRM
(c) ∠RML (d) ∠QPT

144. In the figure given below, ABC is a triangle with AB = BC and D is an interior point of the triangle ABC such that ∠DAC = ∠DCA. **[2017-I]**



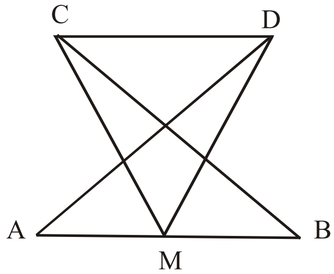
Consider the following statements :

1. Triangle ADC is an isosceles triangle.
2. D is the centroid of the triangle ABC.
3. Triangle ABD is congruent to the triangle CBD.

Which of the above statements are correct ?

(a) 1 and 2 only
(b) 2 and 3 Only
(c) 1 and 3 only
(d) 1, 2 and 3

145. In the figure given below, M is the mid-point of AB and ∠DAB = ∠CBA and ∠AMC = ∠BMD. Then the triangle ADM is congruent to the triangle BCM by **[2017-I]**



(a) SAS rule (b) SSS rule
(c) ASA rule (d) AAA rule

146. ABCD is a square. X is the mid-point of AB and Y is the mid-point of BC. **[2017-I]**

Consider the following statements :

1. Triangles ADX and BAY are congruent.
2. $\angle DXA = \angle AYB$.
3. DX is inclined at an angle 60° with AY.
4. DX is not perpendicular to AY.

Which of the above statements are correct ?

(a) 2, 3 and 4 only (b) 1, 2 and 4 only
(c) 1, 3 and 4 only (d) 1 and 2 only

147. In a triangle ABC, AD is perpendicular on BC. If $\angle BAC = 90°$, AB = c, BC = a, CA = b and AD = p, then which one of the following is correct? **[2017-II]**

(a) $p = abc$ (b) $p^2 = bc$
(c) $p = \frac{bc}{a}$ (d) $p = \frac{ab}{c}$

148. In an equilateral triangle ABC, BD is drawn perpendicular to AC. What is BD^2 equal to? **[2017-II]**

(a) AD^2 (b) $2AD^2$
(c) $3AD^2$ (d) $4AD^2$

149. If PL, QM and RN are the altitudes of triangle PQR whose orthocentre is O, then Q is the orthocentre of the triangle **[2017-II]**

(a) OPQ (b) OQR
(c) PLR (d) OPR

150. In triangle ABC, $\angle C = 90°$ and CD is the perpendicular from C to AB.

If $(CD)^{-2} = (BC)^{-2} + (CA)^{-2}$, then which one of the following is correct? **[2017-II]**

(a) BC . CD = AB . CA
(b) AB . BC = CD . CA
(c) $CA^2 + CB^2 = 2(AD^2 + CD^2)$
(d) AB . CD = BC . CA

151. In a triangle ABC, the medians AD and BE intersect at G. A line DF is drawn parallel to BE such that F is on AC. If AC = 9 cm, then what is CF equal to? **[2017-II]**

(a) 2.25 cm (b) 3 cm
(c) 4.5 cm (d) 6 cm

152. In a triangle PQR, X is a point on PR and Y is a point on QR such that PR = 10 cm, RX = 4 cm, YR = 2 cm, QR = 5 cm. Which one of the following is correct? **[2017-II]**

(a) XY is parallel to PQ (b) PQ = 2XY
(c) PX = QY (d) PQ = 3XY

153. One-fifth of the area of a triangle ABC is cut off by a line DE drawn parallel to BC such that D is on AB and E is on AC. If BC = 10 cm, then what is DE equal to? **[2017-II]**

(a) $\sqrt{5}$ cm (b) $2\sqrt{5}$ cm
(c) $3\sqrt{5}$ cm (d) $4\sqrt{5}$ cm

154. Consider the following statements : **[2017-II]**

1. The point of intersection of the perpendicular bisectors of the side of a triangle may lie outside the triangle.
2. The point of intersection of the perpendiculars drawn from the vertices to the opposite sides of a triangle may lie on two sides.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 not 2

155. ABC is a triangle right angled at C with BC = a and AC = b. If p is the length of the perpendicular from C on AB, then which one of the following is correct? **[2018-I]**

(a) $a^2 b^2 = p^2 (a^2 + b^2)$
(b) $a^2 b^2 = p^2 (b^2 - a^2)$
(c) $2a^2 b^2 = p^2 (a^2 + b^2)$
(d) $a^2 b^2 = 2p^2 (a^2 + b^2)$

156. Consider the following statements : **[2018-I]**

1. The orthocentre of a triangle always lies inside the triangle.
2. The centroid of a triangle always lies inside the triangle.
3. The orthocentre of a right angled triangle lies on the triangle.
4. The centroid of a right angled triangle lies on the triangle.

Which of the above statements are correct?

(a) 1 and 2 (b) 1 and 4
(c) 2 and 3 (d) 2 and 4

157. Consider the following statements : **[2018-I]**

Two triangles are said to be congruent, if

1. Three angles of one triangle are equal to the corresponding three angles of the other triangle.
2. Three sides of one triangle are equal to the corresponding three sides of the other triangle.
3. Two sides and the included angle of one triangle are equal to the corresponding two sides and the included angle of the other triangle.
4. Two angles and the included side of one triangle are equal to the corresponding two angles and the included side of the other triangle.

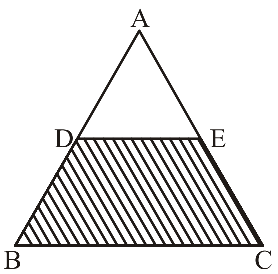
Which of the above statements are correct?

(a) 1, 2 and 3 (b) 1, 3 and 4
(c) 1, 2 and 4 (d) 2, 3 and 4

158. A square and an equilateral triangle have the same perimeter. If the diagonal of the square is $6\sqrt{2}$ cm, then what is the area of the triangle? **[2018-I]**

(a) $12\sqrt{2}$ cm^2 (b) $12\sqrt{3}$ cm^2
(c) $16\sqrt{2}$ cm^2 (d) $16\sqrt{3}$ cm^2

159. In the equilateral triangle ABC given below, AD = DB and AE = EC. If l is the length of a side of the triangle, then what is the area of the shaded region? **[2018-I]**



(a) $\frac{3\sqrt{3}\,l^2}{16}$ (b) $\frac{3\,l^2}{16}$ (c) $\frac{3\sqrt{3}\,l^2}{32}$ (d) $\frac{3\,l^2}{32}$

160. If base and hypotenuse of a right triangle are $(u^2 - v^2)$ and $(u^2 + v^2)$ respectively and the area of the triangle is 2016 square units, then the perimeter of the triangle may be **[2018-II]**

(a) 224 units (b) 288 units
(c) 448 units (d) 576 units

161. The sides of a triangle are 5 cm, 6 cm and 7 cm. The area of the triangle is approximately **[2018-II]**

(a) 14.9 cm^2 (b) 14.7 cm^2 (c) 14.5 cm^2 (d) 14.3 cm^2

162. The areas of two similar triangles are $(7-4\sqrt{3})\text{cm}^2$ and $(7+4\sqrt{3})\text{cm}^2$ respectively. The ratio of their corresponding sides is **[2018-II]**

(a) $7-4\sqrt{3}$ (b) $7-3\sqrt{3}$
(c) $5-\sqrt{3}$ (d) $5+\sqrt{3}$

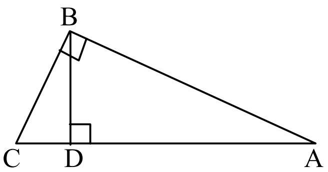
163. In a triangle ABC, the sides AB, AC are produced and the bisectors of exterior angles of ∠ABC and ∠ACB intersect at D. If ∠BAC = 50°, then ∠BDC is equal to **[2018-II]**

(a) 115° (b) 65° (c) 55° (d) 40°

164. If ABC is a right-angled triangle with AC as its hypotenuse, then which one of the following is correct? **2018-II]**

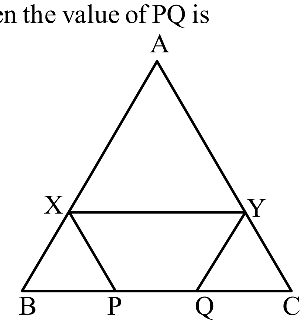
(a) $AC^3 < AB^3 + BC^3$ (b) $AC^3 > AB^3 + BC^3$
(c) $AC^3 \leq AB^3 + BC^3$ (d) $AC^3 \geq AB^3 + BC^3$

165. In the figure given below, ABC is a triangle with AB perpendicular to BC. Further BD is perpendicular to AC. If AD = 9 cm and DC = 4 cm, then what is the length of BD? **[2018-II]**



(a) 13/36 cm
(b) 36/13 cm
(c) 13/2 cm
(d) 6 cm

166. In the figure given below, ABC is an equilateral triangle with each side of length 30 cm. XY is parallel to BC, XP is parallel to AC ad YQ is parallel to AB. If XY + XP + YQ is 40 cm, then the value of PQ is **[2018-II]**



(a) 5 cm (b) 12 cm (c) 15 cm (d) 10 cm

167. What is the percentage decrease in the area of a triangle if its each side is halved ? **[2019-I]**

(a) 75% (b) 50% (c) 25% (d) No change

168. Three consecutive integers form the lengths of a right-angled triangle. How many sets of such three consecutive integers is/are possible ? **[2019-I]**

(a) Only one (b) Only two
(c) Only three (d) Infinitely many

169. The perimeter of a right-angled triangle is k times the shortest side. If the ratio of the other side to hypotenuse is 4 : 5, then what is the value of k ? **[2019-I]**

(a) 2 (b) 3 (c) 4 (d) 5

170. The angles of a triangle are in the ratio 1 : 1 : 4. If the perimeter of the triangle is k times its largest side, then what is the value of k ? **[2019-I]**

(a) $1+\frac{2}{\sqrt{3}}$ (b) $1-\frac{2}{\sqrt{3}}$
(c) $2+\frac{2}{\sqrt{3}}$ (d) 2

171. The hypotenuse of a right-angled triangle is 10 cm and its area is 24 cm^2. If the shorter side is halved and the longer side is doubled, the new hypotenuse becomes **[2019-I]**

(a) $\sqrt{245}$ cm (b) $\sqrt{255}$ cm (c) $\sqrt{265}$ cm (d) $\sqrt{275}$ cm

172. A ladder is resting against a vertical wall and its bottom is 2.5 m away from the wall. If it slips 0.8 m down the wall, then its bottom will move away from the wall by 1.4 m. What is the length of the ladder ? **[2019-I]**

(a) 6.2 m (b) 6.5 m (c) 6.8 m (d) 7.5 m

173. Consider the following inequalities in respect of any triangle ABC: **[2019-I]**

1. $AC - AB < BC$ 2. $BC - AC < AB$ 3. $AB - BC < AC$

Which of the above are correct ?

(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

174. Consider the following statements : **[2019-I]**

1. The perimeter of a triangle is greater than the sum of its three medians.
2. In any triangle ABC, if D is any point on BC, then $AB + BC + CA > 2AD$.

Which of the above statements is/are correct ?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

Consider the following for the next two (02) items:

An equilateral triangle ABC is inscribed in a circle of radius $20\sqrt{3}$ cm. **[2019-I]**

175. What is the length of the side of the triangle ?

(a) 30 cm (b) 40 cm (c) 50 cm (d) 60 cm

176. The centroid of the triangle ABC is at a distanced from the vertex A. What is d equal to ?

(a) 15 cm (b) 20 cm
(c) $20\sqrt{3}$ cm (d) $30\sqrt{3}$ cm

177. If l is the length of the median of an equilateral triangle, then what is its area ? **[2019-II]**

(a) $\frac{\sqrt{3}l^2}{3}$ (b) $\frac{\sqrt{3}l^2}{2}$ (c) $\sqrt{3}l^2$ (d) $2l^2$

178. Suppose P, Q and R are the mid-points of sides of a triangle of area 128 cm^2. If a triangle ABC is drawn by joining the mid-points of sides of triangle PQR, then what is the area of triangle ABC? **[2019-II]**

(a) 4 cm^2 (b) 8 cm^2 (c) 16 cm^2 (d) 32 cm^2

179. Let two lines p and q be parallel. Consider two points B and C on the line p and two points D and E on the line q. The line through B and E intersects the line through C and D at A in between the two lines p and q. If $AC : AD = 4 : 9$, then what is the ratio of area of triangle ABC to that of triangle ADE? **[2019-II]**

(a) 2 : 3 (b) 4 : 9 (c) 16 : 81 (d) 1 : 2

180. ABC is a triangle right angled at B. If $AB = 5$ cm and $BC = 10$ cm, then what is the length of the perpendicular drawn from the vertex B to the hypotenuse ? **[2019-II]**

(a) 4 cm (b) $2\sqrt{5}$ cm (c) $\frac{4}{\sqrt{5}}$ cm (d) 8 cm

181. If the length of the hypotenuse of a right angled triangle is 10 cm, then what is the maximum area of such a right angled triangle ? **[2019-II]**

(a) 100 cm^2 (b) 50 cm^2 (c) 25 cm^2 (d) 10 cm^2

182. Which one of the following is correct in respect of a right angled triangle ? **[2019-II]**

(a) Its orthocentre lies inside the triangle
(b) Its orthocentre lies outside the triangle
(c) Its orthocentre lies on the triangle
(d) It has no orthocentre

183. Let the bisector of the angle BAC of a triangle ABC meet BC in X. Which one of the following is correct ? **[2019-II]**

(a) $AB < BX$
(b) $AB > BX$
(c) $AX = CX$
(d) None of the above

184. The perimeter of a triangle is 22 cm. Through each vertex of the triangle, a straight line parallel to the opposite side is drawn. What is the perimeter of triangle formed by these lines? **[2019-II]**

(a) 33 cm (b) 44 cm
(c) 66 cm (d) 88 cm

185. The perimeters of two similar triangles ABC and PQR are 75 cm and 50 cm respectively. If the length of one side of the triangle PQR is 20 cm, then what is the length of corresponding side of the triangle ABC ? **[2019-II]**

(a) 25 cm (b) 30 cm (c) 40 cm (d) 45 cm

186. If the angles of a triangle are 30° and 45° and the included side is $\left(\sqrt{3}+1\right)$ cm, then what is the area of the triangle ? **[2020-I]**

(a) $\left(\sqrt{3}+1\right)$ cm^2 (b) $\left(\sqrt{3}+3\right)$ cm^2

(c) $\frac{1}{2}\left(\sqrt{3}+1\right)$ cm^2 (d) $2\left(\sqrt{3}+1\right)$ cm^2

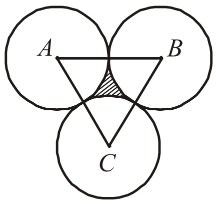
187. The lengths of sides of a triangle are $3x, 4\sqrt{y}, 5\sqrt[3]{z}$, where $3x < 4\sqrt{y} < 5\sqrt[3]{z}$. If one of the angles is 90°, then what are the minimum integral values of x, y, z respectively? **[2020-I]**

(a) 1, 2, 3 (b) 2, 3, 4
(c) 1, 1, 1 (d) 3, 4, 5

188. What is the maximum number of circum-circles that a triangle can have ? **[2020-I]**

(a) 1 (b) 2
(c) 3 (d) Infinite

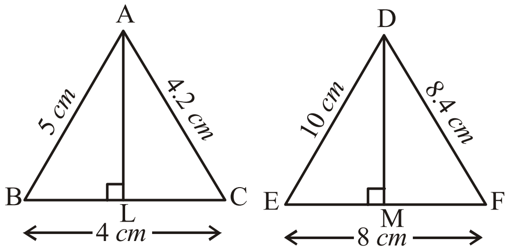
189. What is the area of the shaded region in the given figure, if the radius of each of the circles is 2 cm ? **[2020-I]**



(a) $4\sqrt{3}-2\pi$ cm^2
(b) $\sqrt{3}-\pi$ cm^2
(c) $\sqrt{3}-\frac{\pi}{2}$ cm^2
(d) $2\pi-2\sqrt{3}$ cm^2

HINTS & SOLUTIONS

1. (a) Given that, $AB = 5\ cm$, $BC = 4$ cm, $CA = 4.2\ cm$ and $DE = 10\ cm$, $EF = 8\ cm$ and $FD = 8.4\ cm$



Now, $\dfrac{AB}{DE} = \dfrac{5}{10} = \dfrac{1}{2}$

$\dfrac{BC}{EF} = \dfrac{4}{8} = \dfrac{1}{2}$

and $\dfrac{CA}{FD} = \dfrac{4.2}{8.4} = \dfrac{1}{2}$

$\therefore \quad \dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{CA}{FD}$

$\therefore \quad \Delta ABC \sim \Delta DEF$

We know that,

$\dfrac{AB}{DE} = \dfrac{AL}{DM}$

$\Rightarrow \quad \dfrac{AL}{DM} = \dfrac{1}{2}$

Alternate Method

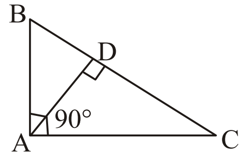
Every sides of second is double of first one.

Hence, $\dfrac{AL}{DM} = \dfrac{1}{2}$.

2. (b) When Δ ABC is moved around fixed ponit A then its locus is circle.

3. (a) $BD : CD = 3 : 4$ and $AB : AC = 6 : 8$ or $3 : 4$ in ΔABC, AD is a angular bisector.
Both A and R are true and R is the correct explanation of A.

4. (c) According to triangle property.



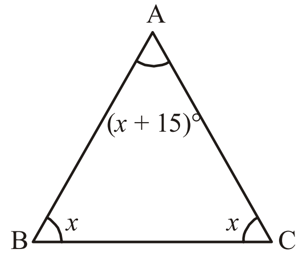
ΔABC is a right angled triangle at A.
$AD \perp BC$, then $\Delta ABC \sim \Delta ADC \sim \Delta ADB$ are similar is each other.
$\therefore$ Option (c) is correct.

5. (c) Also QR is parallel to AB.
$\therefore \ \Delta PQR \sim \Delta QPB$
DR is also parallel to QB.
$\Delta PQB \sim \Delta QDR$
Again, $DR \| QB$ and $QR \| AB$
$\therefore \ \Delta DQR \sim \Delta AQB$

6. (d) $DE : BE = 3 : 5$

$\therefore \quad \dfrac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DAE} = \left(\dfrac{BC}{DE}\right)^2 = \dfrac{25}{9}$ or $25 : 9$

7. (b): Let each base angle of isosceles triangle be x.



$\therefore$ Vertical angle of an isosceles triangle $= x + 15°$
We know that,
$\angle A + \angle B + C = 180°$
$\Rightarrow \ x + 15° + x + x = 180°$
$\Rightarrow \ 3x = 165°$
$\Rightarrow \ x = 55°$

8. (a) **Assertion** (A) is true, of two triangles are congruent then their corresponding angles and side are same.
Reason (R) is alsoe true, because two congruent triangle are same area.
So A, R are inddividually true R is correct explanation of A.

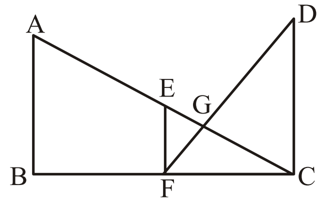
9. (b) Given, Area (ΔADE) : Area (trape $BDEC$) = 4 : 5
area (ΔADE) = $4x$
and area of trapezium $BDEC = 5x$
Area of $\Delta ABC = 4x + 5x = 9x$
Now, ΔADE and ΔABC

$\dfrac{\text{area } \Delta ADE}{\text{area } \Delta ABC} = \dfrac{DE^2}{BC^2}$

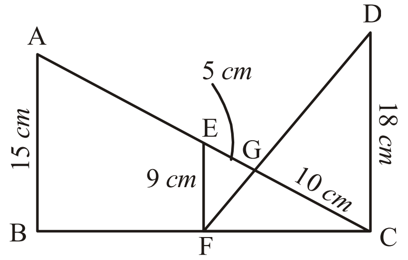
$\Rightarrow \quad \dfrac{4x}{9x} = \dfrac{DE^2}{BC^2} \Rightarrow \dfrac{DE}{BC} = \dfrac{2}{3}$

$\therefore \quad DE : BC = 2 : 3$

10. (d) A triangle can be constructed by given all three statements.



11. (c) : $\because AB \parallel EF \parallel CD$



In ΔEFG and ΔCDG, Here triangle EFG and ΔCDG similar

$$\frac{EG}{GC} = \frac{EF}{CD} \Rightarrow \frac{5}{10} = \frac{EF}{18}$$

$\Rightarrow$ $EF = 9\ cm$

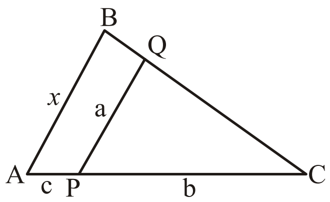
Also, ΔABC and ΔEFC are similar

In ΔABC and ΔEFC,

$$\frac{EC}{AC} = \frac{EF}{AB} \Rightarrow \frac{15}{AC} = \frac{9}{15}$$

$$\Rightarrow AC = \frac{15 \times 15}{9} = 25\ cm$$

12. (d) In ΔABC and ΔPQC,

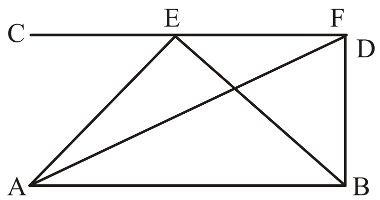


$$\therefore \frac{PC}{AC} = \frac{PQ}{AB}$$

$$\Rightarrow \frac{b}{c+a} = \frac{a}{x}$$

$$\therefore x = \frac{a(c+b)}{b} = \frac{ac}{b} + a$$

13. (a)



A. By the properties of triangle, it is true.

R. It is also true that the distance between two parallel lines is same everywhere.

Hence, A and R are true and R is the correct explanation of A.

14. (c) Suppose the smaller and larger sides of a right triangle be x and y, respectively.

By given condition,

$$x^2 + y^2 = (3\sqrt{10})^2$$

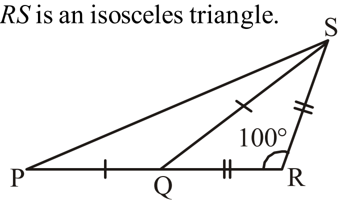
$$\Rightarrow x^2 + y^2 = 90 \quad \ldots (i)$$

and $9x^2 + 4y^2 = 405 \quad \ldots (ii)$

On solving Eqs. (i) and (ii), we get

$x = 3$ units and $y = 9$ units

15. (c) ΔQRS is an isosceles triangle.



$\therefore$ $\angle QSR = \angle SQR = 40°$

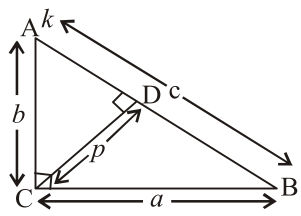
$\Rightarrow$ $\angle PQS = \angle 180° - \angle RQS = \angle 140°$

Again, $\Delta \angle PQS$ is an isosceles triangle.

$$\therefore \angle PQS = \angle QSR = \frac{180-140}{2} = \frac{40}{2} = 20°$$

16. (a) ΔACB is right angled at C.

$c^2 = a^2 + b^2$



$$\text{Area of } \Delta ACB = \frac{1}{2} \times AB \times DC$$

$$\frac{1}{2} \times a \times b = \frac{1}{2} \times c \times p$$

$$ab = p\left(\sqrt{a^2+b^2}\right) \ (\therefore C^2 = a^2 + b^2)$$

Squaring on both sides we get

$a^2b^2 = p^2(a^2+b^2)$

Alternate Method : ΔACB and ΔCDB are similar

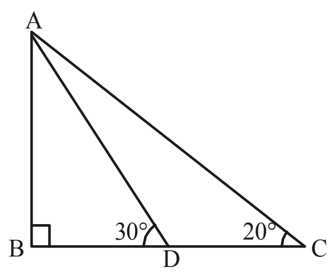
$$\therefore \frac{AC}{AB} = \frac{DC}{BC} \Rightarrow \frac{b}{c} = \frac{P}{a} \Rightarrow ab = cp$$

$$\Rightarrow ab = p\left(\sqrt{a^2+b^2}\right)$$

Squaring on both sides we get,

$\Rightarrow$ $a^2b^2 = p^2(a^2+b^2)$

17. (a) In ΔADC,



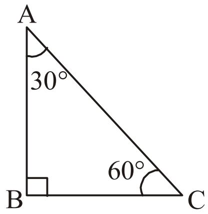
$\angle ADC = 180° - 30° = 150°$

$\therefore$ $\angle DAC = 180° - (150° + 20°) = 10°$

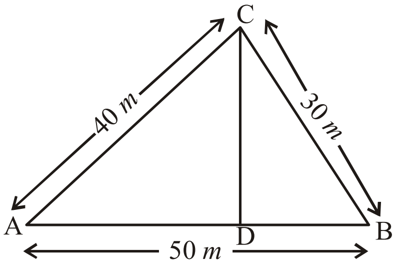
18. (c) Ratio of sides,
According to angles order.
$AC : AB : BC$

$= 1 : \frac{\sqrt{3}}{2} : \frac{1}{2}$

$= 2 : \sqrt{3} : 1$



19. (b) Let $a = 30\,m$, $b = 40\,m$ and $c = 50\,m$



Now, $2s = 30 + 40 + 50$

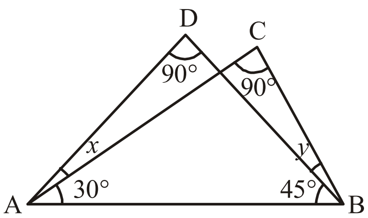
$\Rightarrow$ $s = 60\,m$

$\therefore$ $\Delta = \sqrt{60(60-30)(60-40)(60-50)}$

$= \sqrt{60 \times 30 \times 20 \times 10} = 600$

$\Rightarrow$ $\frac{1}{2} \times AB \times CD = 600 \Rightarrow CD = \frac{600 \times 2}{50} = 24\,m$

20. (b) In ΔABD,



$\angle DAB + \angle ABD + \angle BDA = 180°$

$\Rightarrow$ $(30° + x) + 45° + 90° = 180° \Rightarrow x = 15°$

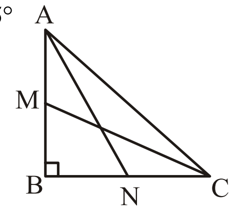
In ΔACB,

$\angle CAB + \angle ABC + \angle BCA = 180°$

$\Rightarrow$ $30° + (45° + y) + 90° = 180° \Rightarrow y = 15°$

$\therefore$ $2x - y = 2(15°) - 15° = 15°$

21. (c) In right angled ΔABN,
By Pythagoras theorem,



$$AN^2 = AB^2 + BN^2 = AB^2 + \frac{BC^2}{4}$$

$$\left(\because BN = \frac{BC}{2}\right) \quad \text{... }(i)$$

In ΔCBM,

$$CM^2 = BC^2 + BM^2 = BC^2 + \frac{AB^2}{4}$$

$$\left(\because BM = \frac{AB}{2}\right) \quad \text{... }(ii)$$

From Eqs. (i) and (ii)

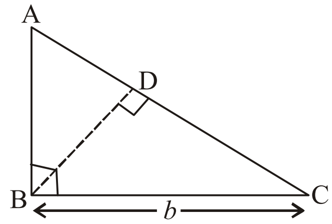
$$AN^2 + CM^2 = AB^2 + \frac{AB^2}{4} + BC^2 + \frac{BC^2}{4}$$

$$= \frac{5(AB^2 + BC^2)}{4}$$

$\Rightarrow$ $4(AN^2 + CM^2) = 5AC^2$

22. (a) Area of ΔABC, $A = \frac{1}{2} \times b \times AB$

$$AB = \frac{2A}{b} \quad \text{... }(i)$$



By Phthagoras theorem, $AC^2 = AB^2 + BC^2$

$$AC = \sqrt{\frac{4A^2}{b^2} + b^2}$$

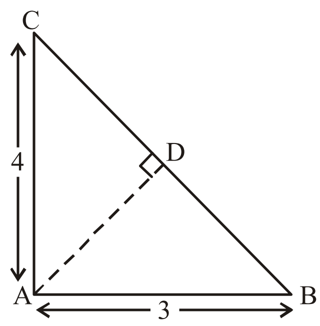
Again in ΔABC

$$A = \frac{1}{2} \times AC \times BD$$

$$BD = \frac{2A}{\sqrt{\frac{4A^2}{b^2} + \frac{b^2}{1}}} = \frac{2A}{\sqrt{\frac{4A^2 + b^4}{b^2}}}$$

$$= \frac{2Ab}{\sqrt{4A^2 + b^4}}$$

23. (b) By Pythagoras theorem,



$CB^2 = AC^2 + AB^2 = 16 + 9 = 25$

$\Rightarrow$ $CB = 5$

Area of $\Delta ABC = \frac{1}{2} \times 3 \times 4 = 6$

Again in ΔABC, Area $= \frac{1}{2} \times BC \times AD$

$6 = \frac{5}{2} AD \Rightarrow AD = \frac{12}{5}$

In right angled ΔADB,

$AB^2 = AD^2 + BD^2$

$\Rightarrow$ $(3)^2 = \left(\frac{12}{5}\right)^2 + BD^2$

$\Rightarrow$ $BD = \sqrt{9 - \frac{144}{25}} = \frac{9}{5}$

$\therefore$ Area of $\Delta ABD = \frac{1}{2} \times BD \times AD$

$= \frac{1}{2} \times \frac{9}{5} \times \frac{12}{5} = \frac{54}{25}$ sq units

24. (a) Since, $\angle ABD = \angle PQD = 90°$

So, $\Delta ABD \sim \Delta PQD$ (right angled triangle)

Then, $\frac{x}{z} = \frac{BD}{QD}$ (by Thales theorem) ...(i)

Since, $\angle CDB = \angle PQB = 90°$

So, $\Delta BCD \sim \Delta BPQ$

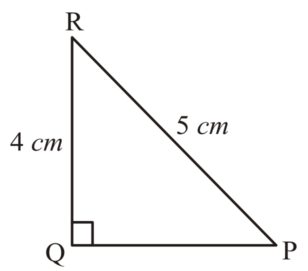
$\therefore$ $\frac{z}{y} = \frac{BQ}{BD}$ (by Thales theorem)

$\Rightarrow$ $\frac{z}{y} = \frac{BD - QD}{BD} \Rightarrow \frac{z}{y} = 1 - \frac{QD}{BD}$

$\Rightarrow$ $\frac{z}{y} = 1 - \frac{z}{x}$ [from Eq. (i)]

$\Rightarrow$ $\frac{z}{x} + \frac{z}{y} = 1 \Rightarrow \frac{1}{x} + \frac{1}{y} = \frac{1}{z}$

25. (d) In right angle ΔPQR



$QP^2 = (5)^2 - (4)^2 = 9 \Rightarrow QP = 3cm$

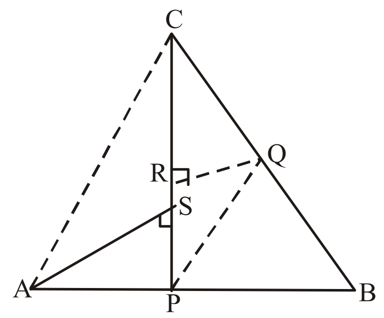
In second ΔABC whose sides are 3 cm, 4 cm and 5 cm, So, the sides of both triangle are same, hence they are congruent.

26. (a) When both triangles are congruent then both shape and size are equal.

So, C_1 and C_2 are the same points, and $r_1 = r_2$

27. (b) PQ is parallel to AC.

$\therefore$ $\Delta PQB \sim \Delta ABC$



In ΔPQR and ΔASC,

$\angle ASC = \angle RPQ$ (alternate angle)

$\angle R = \angle S = 90° \Rightarrow \angle A = \angle Q$ (rest angle)

$\therefore$ $\Delta PQR \sim \Delta ASC$

Two distinct similar triangles are exist.

28. (c) Median of an equilateral triangle $= \frac{\sqrt{3}}{2} a$

According to question, $\frac{\frac{\sqrt{3}}{2} a_1}{\frac{\sqrt{3}}{2} a_2} = \frac{3}{2}$

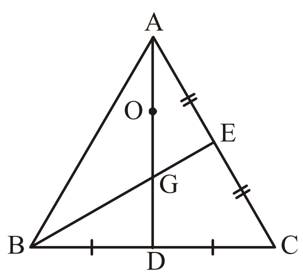
$\therefore$ $\frac{a_1}{a_2} = \frac{3}{2}$ or 3 : 2

29. (c) In an equilateral triangle, the centroid and the othocentre are coincident.

30. (d) (A) If two triangles have same perimeter, then it is not necessary that they have same area. So, they need not be congruent.

(R) This condition is true, because two triangles are congruent by (SSS) property.

31. (c) $\frac{AO}{OD} = 2 : 7$ (where, AD is a median)



$OA = \frac{2}{9} AD, OD = \frac{7}{9} AD$

We know that, centroid makes a ratio 2 : 1 on the median

So, $AG = \frac{2}{3}AD, GD = \frac{1}{3}AD$ ... (*i*)

A. $OA = \frac{2}{9}AD$

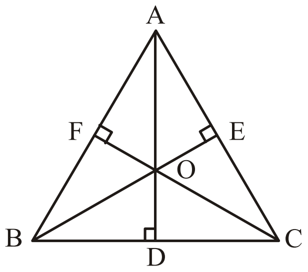
$$OA = \left(2.\frac{1}{3}AD\right)\frac{1}{3} = \frac{2GD}{3} \quad \text{[from Eq. } (i)\text{]}$$

R. $OD = \frac{7}{9}AD = \left(7.\frac{2}{3}AD\right)\frac{1}{3\times 2} = \frac{7AG}{6}$

Hence, A is true but R is false.

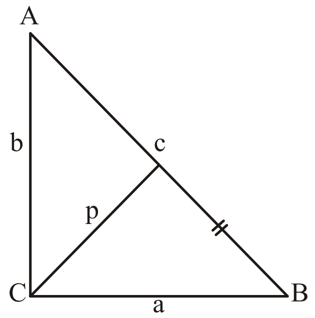
32. (b) We know that, in a right angled triangle, hypotenuse is a largest side.

In ΔABD, $AB^2 > AD^2$ ... (*i*)
In ΔBEC, $BC^2 > BE^2$ ... (*ii*)
In ΔACF, $AC^2 > CF^2$ ... (*iii*)



On adding Eqs. (*i*), (*ii*) and (*iii*), we get
$(AB^2 + BC^2 + AC^2) > (AD^2 + BE^2 + CF^2)$
R. Now,
$(AE^2 - AF^2) + (BF^2 - BD^2) + (CD^2 - CE^2)$
$= [(OA^2 - OE^2) - (OA^2 - OF^2)] + [(OB^2 - OF^2) - (OB^2 - OD^2)] + [(OC^2 - OD^2) - (OC^2 - OE^2)] = 0$
Hence, both A and R are individually true but R is not correct explanation of A.

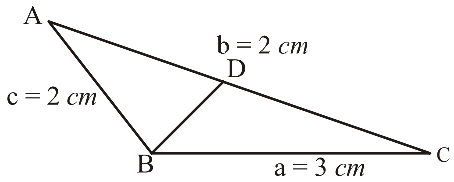
33. (c) Area of $\Delta ABC = \frac{1}{2}ab$



Again area of $\Delta ABC = \frac{1}{2}cp$

$\therefore \frac{1}{2}ab = \frac{1}{2}pc \Rightarrow pc = ab$

34. (b) In ΔABC,



$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{4^2 + 2^2 - 3^2}{2\times 4\times 2}$$

$$\Rightarrow \cos A = \frac{16 + 4 - 9}{16} = \frac{11}{16} \quad \text{...(i)}$$

In ΔABD, Let $BD = x$, $AB = c = 2$ cm, $AD = b = 2$ cm

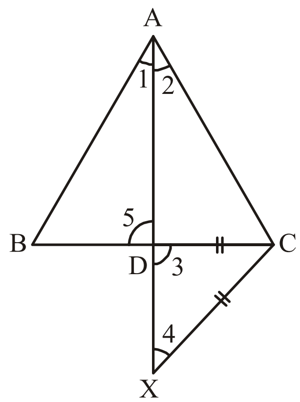
$$\therefore \cos A = \frac{b^2 + c^2 - a^2}{2bc} \Rightarrow \frac{11}{16} = \frac{2^2 + 2^2 - x^2}{2\times 2\times 2}$$

$\Rightarrow 11 = 16 - 2x^2$

$\Rightarrow x^2 = \frac{5}{2} \Rightarrow x^2 = 2.5$

$\therefore$ Area of square $= (BD)^2 = x^2 = 2.5\ cm^2$

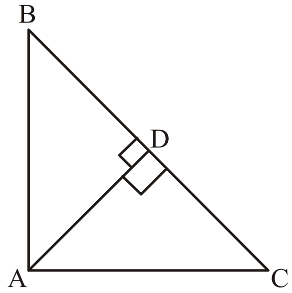
35. (a) In ΔDCX,
$CD = CX$ (given)
$\angle 3 = \angle 4$
(opposite angles of same sides are equal.)
But $\angle 3 = \angle 5$



So, $\angle 4 = \angle 5$ ...(i)
In ΔABD and ΔACX,
$\angle 1 = \angle 2$ (given)
$\angle 4 = \angle 5$ [from Eq. (i)]
$\angle ABD = \angle ACX$ (rest angle)
$\Rightarrow \Delta ABD \sim \Delta ACX$

36. (d) It is true that congruent triangles are similar but converse is not true. Also, Statement III is true.

37. (d) Here, $AD^2 = BD.DC$



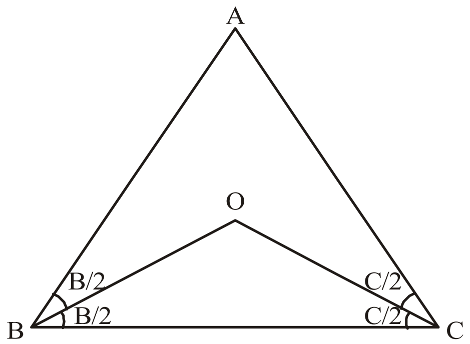
$\Rightarrow \quad \frac{AD}{BD} = \frac{DC}{AD}$

Hence, ΔABC must be right angled triangle.

So, $BC^2 = AB^2 + AC^2$

38. (b) $\because \angle A + \angle B + \angle C = 180^\circ$

According to triangle property



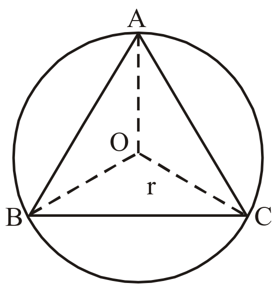
$\Rightarrow \quad \angle\frac{B}{2} + \angle\frac{C}{2} = 90^\circ - \angle\frac{A}{2}$

In ΔBOC, $\angle\frac{B}{2} + \angle\frac{C}{2} + \angle O = 180^\circ$

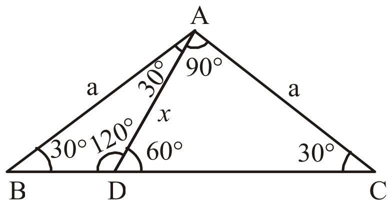
$\Rightarrow \quad 90^\circ - \angle\frac{A}{2} + \angle O = 180^\circ \Rightarrow \angle O = 90^\circ + \frac{A}{2}$

39. (b) Number of points is one, because circumcentre is the only point in the plane of a triangle, which is equidistant from the vertices of the triangle.

$OA = OB = OC = r$



40. (c)



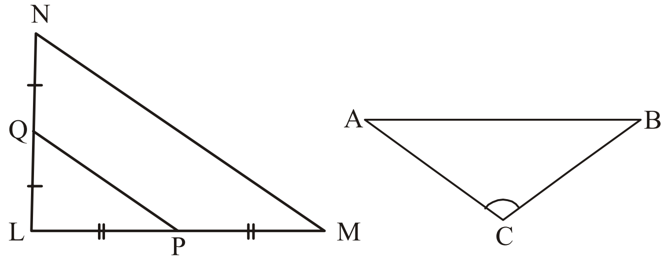
In ΔADC, $\angle ADB = 180° - \angle ADC = 180^\circ - 60^\circ = 120^\circ$

$\angle DAC = 180^\circ - 60^\circ - 30^\circ = 90^\circ$

Again, in right angled ΔDAC,

$\tan 60^\circ = \frac{AC}{DA} \Rightarrow \sqrt{3} = \frac{a}{x} \Rightarrow x = \frac{a}{\sqrt{3}}$

41. (b) Given, $PQ^2 = MP^2 + NQ^2$



Hence, both statements are individually true but statements II is not the correct explanation of statement I.

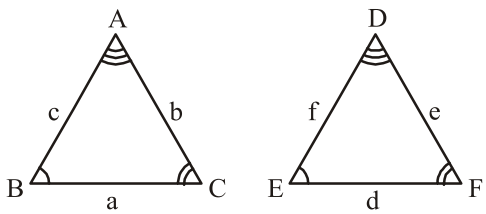
Hence, option (b) is correct.

42. (b) We know that angle subtended by the bisect or two internal angles, is the twice of the third angle and angle subtended the bisectors of one internal and one external angle, is the third angle.

According to this property

$\angle BDC = \frac{1}{2}\angle BAC.$

43. (d) Let, the set C = f, b = e, ∠B = ∠E.



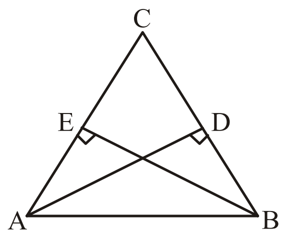
Does not guarantee the congruence of ΔABC and ΔDEF. (According to congruence property of triangle)

44. (c)

I. It is true that the three medians of a triangle divide it into six triangle of equal area.

II. It is also true that, the perimeter of a triangle is greater than the sum of its three medians.

45. (a) In ΔCAD and ΔCEB,



$\angle C = \angle C$ (common)
$\angle CEB = \angle ADC$ (each 90º)
$\angle CAD = \angle CBE$ (rest angle)
$\therefore$ $\Delta CAD \sim CEB$

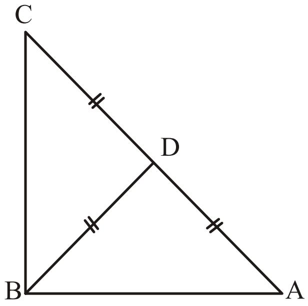
Since, the sides will be in same proportion,

$$\frac{CA}{CB} = \frac{CD}{CE}$$

and $\dfrac{AD}{BE} = \dfrac{CD}{CE}$

Hence, all three statements are correct.

46. (c) Here, we see that



$CD = BD = DA$

This is possible only when ABC is right angled triangle.

47. (b) We know that, the sum of two sides is always greater than third side.

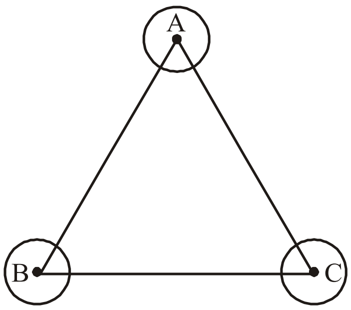
$\therefore$ $10 + 100 > x$ ...(i)
$x > 100 - 10$
and $x > 90$ ...(ii)

Difference of two sides of a triangle is always less than third side.

From Eqs. (i) and (ii), we get,

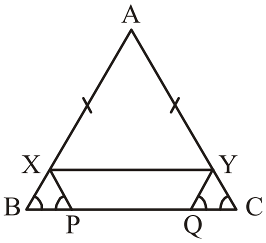
$$90 < x < 110$$

48. (c) $\angle A = 360º - \text{Ext } \angle A.$ ....(i)
$\angle B = 360 - \text{Ext } \angle B$ .... (ii)
$\angle C = 360 - \text{Ext } \angle C$ .... (iii)



Similarly,
$\angle A + \angle B + \angle C = 180º$
From Eqs. (i), (ii), (iii) and (iv),
$360º - \text{Ext } \angle A + 360º - \text{Ext } \angle B + 360º - \text{Ext } \angle C = 180º$
$\Rightarrow$ $\text{Ext } \angle A + \text{Ext } \angle B + \text{Ext } \angle C$
$= 1080º - 180º$
$= 900º$

49. (d) Since, $XP \parallel AC$, $YQ \parallel AB$
$\angle XBP = \angle YQC$ and $\angle XPB = \angle YCQ$



So, ΔXBP and ΔYCQ are equilateral triangles.
Now, $XY \parallel BC$

$\therefore$ $\dfrac{AX}{AB} = \dfrac{XY}{BC} \Rightarrow AX = XY$ $(\because AB = BC = 30\ cm)$

Also $XY + XP + YQ = 40$
$AX + XB + YQ = 40$ $(\because XY = AX, XP = XB)$
$\Rightarrow$ $AB + YQ = 40$
$\Rightarrow$ $YQ = 40 - 30 = 10\ cm$
$\therefore$ $YQ = XP = 10\ cm \Rightarrow BP = CQ = 10\ cm$
$PQ = 30 - BP - CQ = 30 - 10 - 10 = 10\ cm$

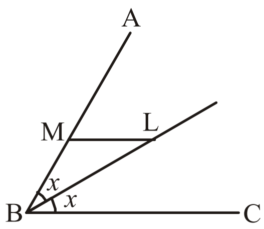
50. (a) We know that, if two triangles are equiangular, then they are similar (refer similarity conditions). Statement II is not true.

51. (c) Given, ratio of sides = 9 : 4
By properties of two similar triangle,

$$\frac{\text{Area of first triangle}}{\text{Area of second triangle}} = \frac{(9)^2}{(4)^2} = \frac{81}{16} \text{ or } 81 : 16$$

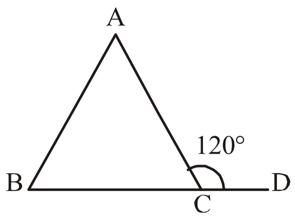
52. (a) In a triangle, if sum of two angles is equal to the third angle, then triangle is right angled.

53. (c) Since, BL is bisector of $\angle ABC$.



$\angle MBL = \angle LBC = x$ (say)
Also, $ML \parallel BC$
$\angle LBC = \angle MLB = x \Rightarrow \angle MLB = \angle MBL$
ΔBLM is an isosceles triangle and $\angle BML$ need not to be 90º.
ΔBML is isosceles but not right angled.

54. (d) Given, $\angle ACD = 120º$



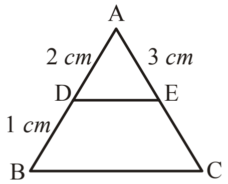
$\Rightarrow$ $\angle CAB + \angle ABC = 120º$

(since, exterior angle is equal to sum of two interior opposite angles)

$$\Rightarrow \quad \angle CAB + \frac{2}{3}\angle CAB = 120^\circ \Rightarrow \frac{5}{3}\angle CAB = 120^\circ$$

$$\Rightarrow \quad \angle CAB = \frac{120^\circ \times 3}{5} = 72^\circ$$

55. (a) In $\Delta\, ADE$ and $\Delta\, ABC$



$\angle A = \angle A$

$DE \parallel BC$

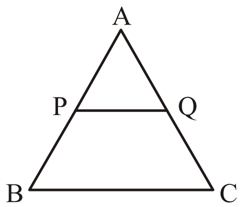
$\Delta ADE \sim \Delta ABC$

$$\frac{AD}{AB} = \frac{AE}{AC}$$

$$\Rightarrow \quad \frac{AD}{BD} = \frac{AE}{EC}$$

$$\frac{2}{1} = \frac{3}{EC} \Rightarrow EC = \frac{3}{2} = 1.5\ cm$$

56. (d)



$AB = 3AP$ .... (Given)

In ΔABC and ΔAPQ, $\angle A = \angle A$ and $PQ \parallel BC$

ΔAPQ and ΔABC are similar triangles,

$$\therefore \quad \frac{\text{Area of } \Delta APQ}{\text{Area of } \Delta ABC} = \frac{AP^2}{AB^2} = \left(\frac{1}{3}\right)^2 = \frac{1}{9} \text{ or } 1:9$$

57. (a) Given, PQ = 3 cm, QR = 4 cm and RP = 5 cm

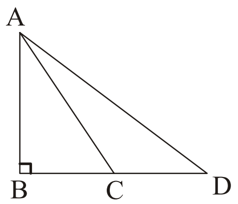
Here, $RP^2 = PQ^2 + QR^2$

So, PQR is a right angle triangle.

Both Statements I and II are individually true and Statement II is the correct explanation of Statement I.

Hence, option (a) is correct.

58. (a) Given, $BD = 2DC$



$\therefore \quad BC + CD = 2DC \Rightarrow BC = CD$ ...(i)

In ΔABC,

$AC^2 = AB^2 + BC^2$ ...(ii)

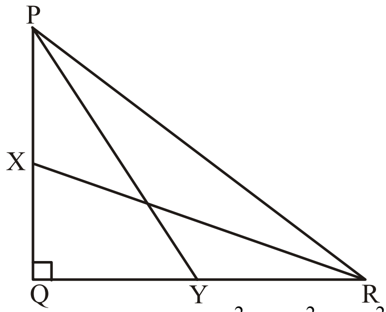
In ΔABD,

$AD^2 = AB^2 + BD^2$ ...(iii)

Subtracting Eq. (ii) from Eq (iii), we get

$$AD^2 - AC^2 = BD^2 - BC^2 = (BD - BC)(BD + BC)$$
$$= CD\,(2CD + CD) = 3CD^2$$
$$AC^2 = AD^2 - 3CD^2$$

59. (d) In $\Delta\, PQY$



According to Pythagoras theorem, $PY^2 = PQ^2 + QY^2$

$$\Rightarrow \quad PY^2 = PQ^2 + \left(\frac{QR}{2}\right)^2 \left(\therefore QY = YR = \frac{QR}{2}\right) \quad ...(i)$$

and in $\Delta\, XQR$,

$RX^2 = QX^2 + QR^2$

$$\Rightarrow \quad RX^2 = \left(\frac{PQ}{2}\right)^2 + QR^2 \left(\because PX = XQ = \frac{PQ}{2}\right) \quad ...(ii)$$

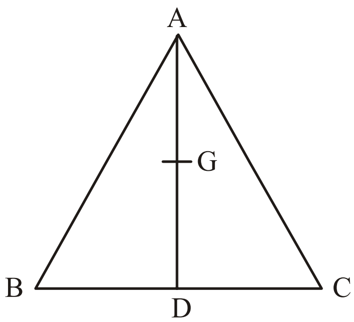
On adding Eqs. (i) and (ii), we get

$$PY^2 + RX^2 = \frac{5PQ^2}{4} + \frac{5QR^2}{4}$$

$\Rightarrow \quad 4(PY^2 + RX^2) = 5(PR^2)$

option (d) is not correct.

60. (c) Let G centroid of a triangle, then



In ΔABC,

$\because$ Then, AD is a median of ΔABC.

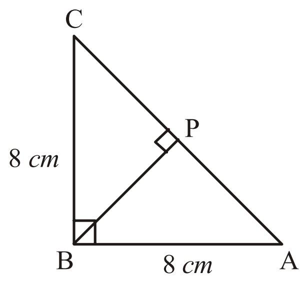
$AG : GD = 2 : 1$

$\Rightarrow \quad 8 : GD = 2 : 1$

$$\Rightarrow \quad GD = \frac{8}{2} = 4 \text{ cm}$$

$\therefore \quad AD = 8 + 4 = 12$ cm

61. (b) By using Pythagoras theorem in ΔABC.



$AC^2 = AB^2 + BC^2 = AC^2 = 64 + 64$

$\therefore AC^2 = 8\sqrt{2}$

ABC is isosceles right angle triangle, then

$AP = PC = PB = \dfrac{AC}{2} = 4\sqrt{2}$

62. (b) In the given figure.

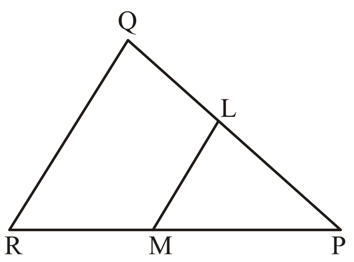
ar $\square MRQL = 2$ ar ΔPLM

Let area of Δ PLM be x, then

$\therefore$ the area of trapezium $= 2x$

$\therefore$ ar $\Delta PQR = 2x + x = 3x$

Here it is clear from the given figure that Δ $PQR \sim \Delta PLM$

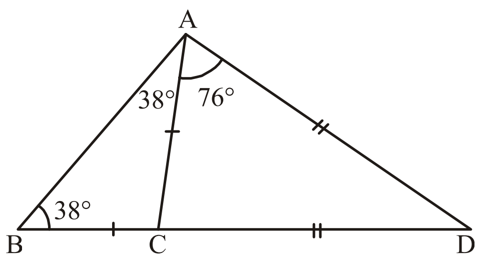


$\therefore \quad \dfrac{\text{ar}\,\Delta\, PQR}{\text{ar}\,\Delta\, PLM} = \dfrac{3x}{x}$

$\dfrac{PL^2}{PQ^2} = \dfrac{1}{3} \therefore \dfrac{PL}{PQ} = \dfrac{1}{\sqrt{3}}$

63. (b) The point of intersection of the altitudes of a triangle is called orthocentre.

64. (b) Given, $AC = BC$



In ΔABC, $\angle ABC = \angle CAB$

$\angle ABC = \angle CAB = 38°$ ($\therefore AC = BC$)

$\angle ACB = 180° - (\angle ABC + \angle CAB)$

$= 180° - (38° + 38°) = 180° - 76° = 104°$

In ΔACD, $\angle ACD = 180° - 104° = 76°$

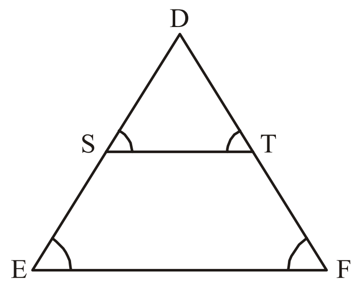
and $\angle ACD = \angle CAD = 76°$

($\because CD = AD$)

$\therefore \quad \angle ADC = 180° - (\angle ACD + \angle CAD)$

$= 180° - (76° + 76°) = 28°$

65. (a)



Given that, $\dfrac{DS}{DE} = \dfrac{1}{\sqrt{2}}$

$DE = \sqrt{2} \times DS$

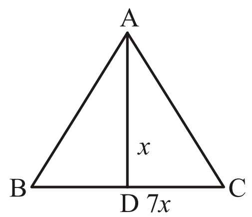
When two triangles ΔDTS and ΔDEF are similar then their ratio of area is equal to square of corresponding sides.

$\Rightarrow \quad \dfrac{\Delta\, DST}{\Delta\, DEF} = \left(\dfrac{1}{\sqrt{2}}\right)^2 = \dfrac{1}{2} \Rightarrow \Delta\, DEF = 2\,\Delta\, DST$

So, both Statements I and II are true and Statement II is the correct explanation of Statement I.

66. (b) Let the height of the triangle be x, then, $BC = 7x$

$\therefore$ Area of the Δ $ABC = \dfrac{1}{2} \times 7x \times x = \dfrac{7}{2}x^2$



Cost of painting the wall at ₹ 350 per 100 sq m = ₹ 1225

$\therefore$ Cost of painting 100 m^2 = ₹ 350

Cost of painting 1 m^2 = ₹ $\dfrac{350}{100}$

$\therefore$ Cost of painting $\dfrac{7}{2}x^2 = \dfrac{350}{100} \times \dfrac{7}{2}x^2$

$\dfrac{350}{100} \times \dfrac{7}{2} \times x^2 = 1225$ (Given)

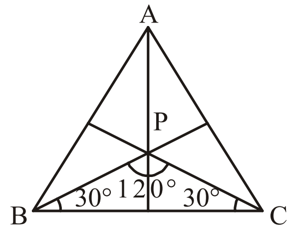
$\Rightarrow \quad x^2 = \dfrac{1225 \times 100 \times 2}{350 \times 7} = \dfrac{35 \times 10 \times 2}{7}$

$= 5 \times 10 \times 2 = 100 = 10\,m$

Base $= 7x = 70\,m$

67. (d) All isosceles triangles are similar.

68. (a) **Statement–I**



In equilateral triangle ABC, P is in incentre and $AP = BP = CP$ and each side of an equilateral triangle make 120° angle at P.

Statement–II

From the figure it is clear that A is not an orthocentre of the triangle PBC.

Therefore option (a) is correct

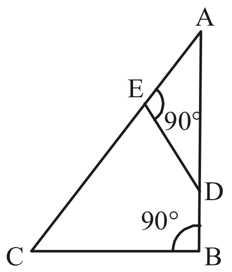
69. (c) Suppose ΔABC is an equilateral triangle.

A median divides an equilateral triangle into the three equal area of triangles.

$$\Delta AGB = \text{ar}\,\frac{(\Delta ABC)}{3} = \text{ar}\,BGC = ar\,\Delta AGC$$

$$\therefore\ \text{ar}\,\Delta AGB = \frac{1}{3}\Delta ABC.$$

70. (a)



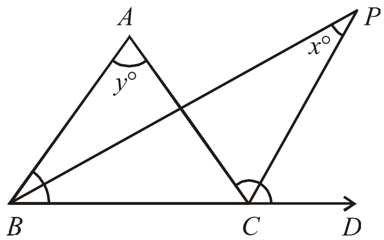
From fig. It is clear that $\angle ABC = \angle AED = 90^\circ$

So statement I is correct.

Statement II.

Clear from fig that B, C, E and D do not lie on a circle.

71. (d)



Given $\angle BAC = y^\circ$

and $\angle BPC = x^\circ$

According to triangle property, the angle subtended by the bisector of an internal angle and an external angle is half of the third angle.

$$\therefore\qquad y^\circ = 2x^\circ \text{ or } x = \frac{y}{2}$$

Alternate Method:—

In ΔABC, $y^\circ + \angle ABC + \angle ACB = 180^\circ$ ... (i)

In ΔBPC, $\angle BPC + \angle PBC + \angle PCB = 180^\circ$

$$x^\circ + \frac{\angle ABC}{2} + \left(\frac{\angle ACD}{2} + \angle ACB\right) = 180^\circ \quad \text{... (ii)}$$

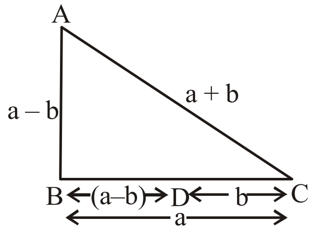
Comparing equation (i) and (ii)

$$x^\circ = \frac{y^\circ}{2}$$

72. (d) In ΔABC

Using Pythagoras theorem

$(a+b)^2 = (a-b)^2 + a^2$



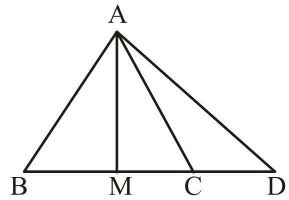
$\Rightarrow\ a^2 + b^2 + 2ab = a^2 + b^2 - 2ab + a^2$

$\Rightarrow\ 4ab = a^2 \Rightarrow 4b = a$

So, $\dfrac{BD}{DC} = \dfrac{a-b}{b} = \dfrac{4b-b}{b} = \dfrac{3b}{b} = \dfrac{3}{1}$

73. (d) Let $AB = BC = AC = x$ Because of equilateral triangle.

Now, $BM = MC = \dfrac{x}{2}$



Then AM is the median.

then, $CD = \dfrac{x}{2}$ (given $BC = 2CD$)

Again, in ΔAMC,

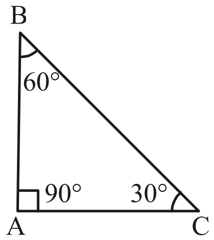
$AM^2 = AC^2 - MC^2 = x^2 - \dfrac{x^2}{4} = \dfrac{3x^2}{4}$ in ΔAMD,

$AD^2 = AM^2 + MD^2 = \dfrac{3x^2}{4} + x^2 = \dfrac{7x^2}{4}$

$= 7\left(\dfrac{x^2}{4}\right) = 7CD^2 \qquad \left(\because CD = \dfrac{x}{2}\right)$

74. (d) Given that, $\angle A = 90^\circ$ and $\angle C = 30^\circ$

Then $\angle B = 60^\circ$



In ΔABC, By Pythagoras theorem,

$BC^2 = AC^2 + AB^2 \Rightarrow (2AB)^2 = AC^2 + AB^2$

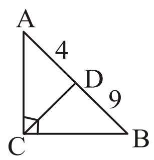
$\Rightarrow \quad AC^2 = 4AB^2 - AB^2 = 3AB^2 \Rightarrow AC = \sqrt{3}.AB$

$= \dfrac{\sqrt{3}}{2}.(2AB)$

$\therefore \quad AC = \dfrac{\sqrt{3}}{2}.BC \qquad$ [Given $\Delta ABC = 2AB$]

75. (d) If the bisectors of angles B and C of a triangle meet at point I, then BIC is equal to $90^\circ + \dfrac{A}{2}$

76. (d) $\Delta ADC \sim \Delta BDC$



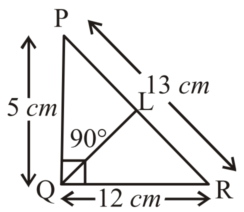
$\therefore \quad \dfrac{CD}{BD} = \dfrac{AD}{CD}$

$CD^2 = AD \times BD$

$CD^2 = 9 \times 4$

$CD = \sqrt{36} = 6\ cm$

77. (d)



$PQ = 5\,cm$

$QR = 12\,cm$

QL is a median

$\therefore \quad PL = LR = \dfrac{PR}{2} \qquad ...(1)$

In ΔPQR,

$PQ^2 + QR^2 = PR^2 \qquad$ (Pythagoras theorem)

$5^2 + 12^2 = PR^2$

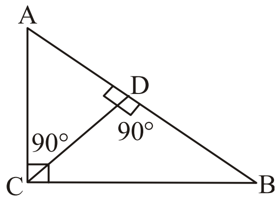
$25 + 144 = PR^2$

According to theorem, $PR = 13\ cm$

If L is the mid point of the hypotenuse PR of a right angled triangle PQR, then

$OL = \dfrac{1}{2}PR = \dfrac{1}{2} \times 13 = 6.5\ cm$

78. (c)



In ΔABC

$CD \perp AB$

and $AB \times CD = CA \times CB \qquad$ .... (i)

In ΔCDB

$BC^2 = BD^2 + CD^2 \qquad$ ...(ii)

$CD^2 = BC^2 - BD^2$

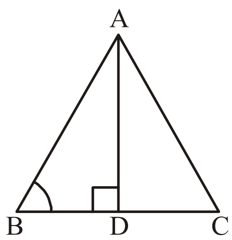
From eq. (i) and (ii)

$\dfrac{1}{CD^2} = \dfrac{1}{BC^2} - \dfrac{1}{CA^2}$

79. (b) Given that, ΔABC is an equilateral triangle.

$\therefore \quad AB = BC = CA = 6\,cm$

$AD =$ Altitude



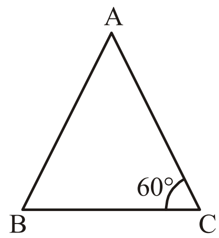
Area of equilateral triangle $= \dfrac{\sqrt{3}}{4}a^2$

$\Rightarrow \quad \dfrac{1}{2} \times b \times h = \dfrac{\sqrt{3}}{4}a^2 \qquad$ (here $a_2\ b$)

$\Rightarrow \quad \dfrac{1}{2} \times 6 \times h = \dfrac{\sqrt{3}}{4} \times 6 \times 6 = h = 3\sqrt{3}\ cm$

80. (b) According to cosine rule.

$\cos 60^\circ = \dfrac{AC^2 + BC^2 - AB^2}{2.AC.BC} = \dfrac{1}{2}$



$\Rightarrow \quad AC^2 + BC^2 - AB^2 = AC.BC$

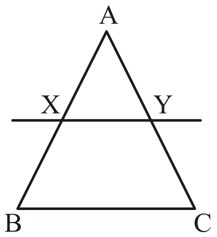
$\Rightarrow \quad AB^2 = AC^2 + BC^2 - AC.BC$

$\Rightarrow$ Required value of $X = -(BC)(CA)$

81. (c) Given

$AB = 4.8\ cm$

$BC = 7.2$ and $BX = 2\ cm$



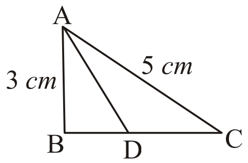
$\therefore \quad AX = AB - BX = 4.8 - 2 = 2.8$ cm

$\Delta AXY \approx \Delta ABC$

$$\therefore \quad \frac{XY}{BC} = \frac{AX}{AB} \Rightarrow XY = \frac{AX}{AB}.BC = \frac{2.8}{4.8} \times 7.2$$

$\therefore \quad XY = 4.2\ cm$

82. (c) According to theorem:– the sum of any two sides of a triangle is greater than twice the median drawn to the third side.



$(AB + AC) > 2\,AD$

$(3 + 5) > 2AD$

$AD < 4$

Thus, AD is always less than 4 cm.

83. (a) Given that, $YZ \parallel MN$ and $XZ \parallel LN$

$\therefore XNYZ$ is a parallelogram.

$\Rightarrow ZX = YN$ ...(i)

Also, $ZX \parallel YN$ and $XY \parallel ZL$

Hence, $XYLZ$ is a parallelogram.

$\therefore \quad XZ = LY$ ...(ii)

Now, From Eqs. (i) and (ii),

$\therefore \quad YN = LY$

So, MY is a median of ΔLMN.

84. (a) Area of equilateral triangle $= \dfrac{\sqrt{3}}{4}a^2$

$$\therefore \frac{1}{2}a \times h = \frac{\sqrt{3}}{4}a^2 \;\; \therefore h = \frac{\sqrt{3}}{2}a$$

or $a = \dfrac{2}{\sqrt{3}h}$

We know, the length of a side of an equilateral triangle

$= \dfrac{2}{\sqrt{3}}$ (sum of the perpendiculars drawn from an interior points)

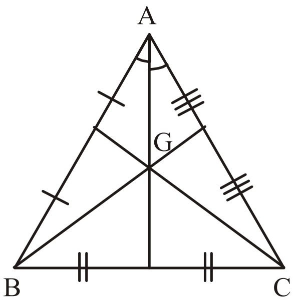
$$\frac{2}{\sqrt{3}} \times 20 = \frac{40}{\sqrt{3}} \text{ cm}$$

85. (b) We know that, in any triangle the sum of two sides is always greater than its third side and the difference of two sides is always less than its third side.

Only option (b) is not satisfy the above conditions

(i) $2 + 3 \ngtr 5$ (ii) $|5 - 2| \nless 3$

86. (d)

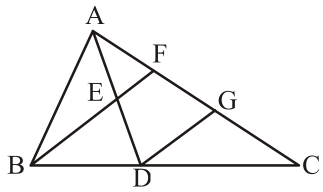


$GA = GB = GC$ is true. If ΔABC is an equilateral triangle and here it is not given that ABC is an equilateral triangle.

Statement II, If H is orthocantre, then HA = HB = HC is equal then ΔABD is an equilateral triangle so II is also not correct.

87. (b) If the bisectors of angles B and C of a triangle meet at point I, then $\angle BIC$ is equal to $90^\circ + \dfrac{A}{2}$

88. (b) We draw a line segment parallel to BF.



In ΔADG,

BF or $EF \parallel DG$ and $AE = ED$ (since, E is mid-point of AD)

$\therefore \quad AF = GC$ ... (i)

Similarly, in ΔBCF,

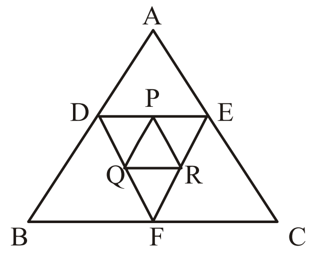
$DG \parallel BF$ and $BD = DC$

$FG = GC$ ... (ii)

From Eqs. (i) and (ii),

$$CF = \frac{2}{3}AC$$

89. (b) Perimeter of $\Delta PQR = 1 + 2 + 3 = 6$ units



Now, in ΔDEF,

$$\frac{DQ}{DF} = \frac{1}{2} = \frac{PQ}{FE}$$

So, $2PQ = FE$

Similarly, $DF = 2PR$ and $DE = 2QR$

$\therefore$ perimeter of $\Delta DEF = 2 \times 6 = 12$ units

Similarly, perimeter of ΔABC

$= 2 \times$ Perimeter of ΔDEF

$= 2 \times 12$

$= 24$ units

90. (d) PQR is an equilateral.

Then, PL is also the median of ΔPQR. Similarly, RN and QM are also the median and O is the centroid.

So, $\frac{PO}{OL} = \frac{2}{1}$

$OL = \frac{PO}{2} = \frac{8}{2} = 4\ cm$

Now, attitude of $\Delta PQR = \frac{\sqrt{3}a}{2}$

$PO + OL = \frac{\sqrt{3}a}{2}$

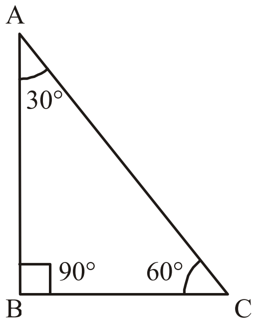
$8 + 4 = \frac{\sqrt{3}a}{2}$

a $= \frac{12 \times 2}{\sqrt{3}} = \frac{24}{\sqrt{3}} cm$

$\therefore$ perimeter of $\Delta PQR = 3a$

$= \frac{3 \times 24}{\sqrt{3}} = 24\sqrt{3}\ cm$

91. (b) Given, In ΔABC



$\angle C = 2\angle A$

$\angle B = 90^\circ$

$\angle A + 90^\circ + 2\angle A = 180^\circ$

$\angle 3A = 180^\circ - 90^\circ = 90^\circ$

$\angle A = \frac{90^\circ}{3} = 30^\circ$

$\therefore \angle C = 2 \times 30^\circ = 60^\circ$

$\sin 30^\circ = \frac{BC}{AC}$

$\frac{1}{2} = \frac{BC}{AC} \Rightarrow BC = \frac{1}{2} AC$

Now, $\sin 60^\circ = \frac{AB}{AC} \Rightarrow \frac{\sqrt{3}}{2} = \frac{AB}{AC}$

$AB = \frac{\sqrt{3}}{2} AC$ ... (ii)

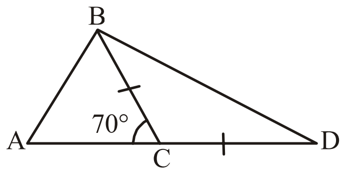
Squaring on both sides.

$AB^2 = \frac{3}{4} AC^2$, Now putting the value of AC.

$AB^2 = \frac{3}{4} \times 4\ BC^2 \Rightarrow AB^2 = 3BC^2$

92. (a) $\angle ACB + \angle BCD = 180^\circ$ (linear pair)

$\angle BCD = 180^\circ - 70^\circ = 110^\circ$



In ΔBCD, $BC = CD \Rightarrow \angle CBD = \angle CDB$ ...(i)

(angles opposite to equal side)

Now, $\angle BCD + \angle CBD + \angle CDB = 180^\circ$

$2\angle CDB = 180^\circ - \angle BCD = 180^\circ - 110^\circ = 70^\circ$

$\therefore \angle CDB = \angle ADB = \frac{70^\circ}{2} = 35^\circ$

93. (c) **Statement-1**

If the diagonal of a parallelogram ABCD are perpendicular then ABCD may Rectangle or Rhombus.

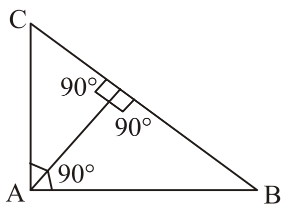
So it is true.

Statement-II

If the diagonal of quadrilateral $ABCD$ are equal and perpendicular then it is square.

So it is also true.

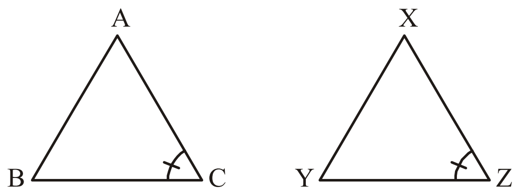
94. (a) In case of a right angled triangle, if we draw a perpendicular from the vertex containing right angle to the hypotenuse, we get three triangles, two smaller and one original and these three triangles are similar triangles.



So, $\Delta ABC \sim \Delta ABD \sim \Delta ADC$

$BC \cdot AD = AB \cdot AC$

95. (a) We know that when two triangles are similar then ratio of their areas is equal to square of corresponding sides.

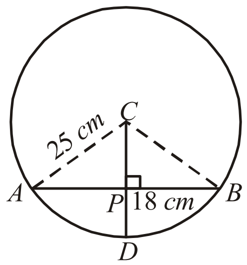


$$\frac{\text{area of } \Delta ABC}{\text{area of } \Delta XYZ} = \frac{AB^2}{XY^2} \Rightarrow \frac{32}{60.5} = \frac{AB^2}{(7.7)^2}$$

$$\Rightarrow \frac{32 \times 59.29}{60.5} = AB^2 \Rightarrow 31.36 = AB^2$$

$\therefore \quad AB = \sqrt{31.36} = 5.6\,cm$

96. (d)



In ΔACP, $CP = CD - PD = 25 - 18 = 7$

By Pythagoras theorem

$AC^2 = CP^2 + AP^2$

$AP = \sqrt{AC^2 - CP^2} = \sqrt{(25)^2 - (7)^2}$

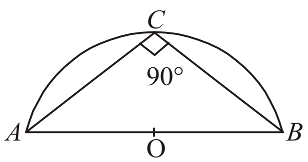
$= \sqrt{625 - 49} = \sqrt{576} = 24\,cm$

Similarly, $PB = 24\,cm$

$\therefore \quad AB = AP + PB$

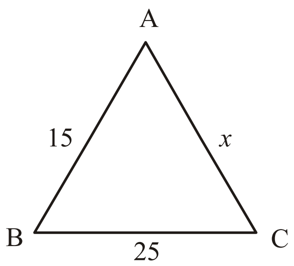
$= 24 + 24 = 48\,cm$

97. (b) The perpendicular bisector of the chord of a circle always pass through the centre. So, Statement I is wrong



The angle in a semi-circle is a right angle. So, Statement II is correct.

98. (a)



$AB = 15$

$BC = 25$

$AC = x$, then

We know that the sum of two sides of a triangle is always greater than third side.

$\Rightarrow \quad AB + BC > x$

$\Rightarrow \quad 15 + 25 > x \Rightarrow \quad 40 > x$ ... (i)

Also, the differences of two sides is always less than third side.

$BC - AB < AC$

$25 - 15 < x$

$10 < x$ ... (ii)

From eq. (i) and (ii)

$10 < x < 40$

99. (d) According to Pythagorean triplet.

The sum of square of base and perpendicular equal to square of hypotenuse.

By hit and trial method:—

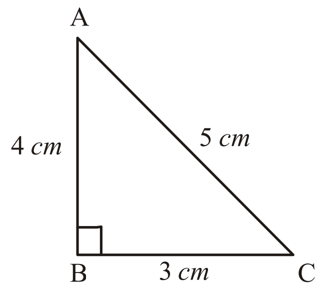
$(2n)^2 + (n^2 - 1)^2 = (n^2 + 1)^2$

$4n^2 + n^4 + 1 - 2n^2 = n^4 + 2n^2 + 1$

$n^4 + 2n^2 + 1 = n^4 + 2n^2 + 1$

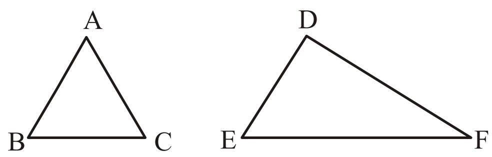
$LHS = RHS$

100. (a) In right angle triangle, the sides of triangle are three consecutive number then $AB = 4$ cm, $BC = 3$ cm, $AC = 5$ cm



$\therefore$ Area of triangle, $\frac{1}{2} \times 3 \times 4 = 6\ cm^2$

101. (a)



$\because$ Given that $\Delta ABC \sim \Delta DEF$

$$\therefore \quad \frac{AB}{DE} = \frac{BC}{EF} \Rightarrow \frac{1}{2} = \frac{8}{EF}$$

$EF = 16\,cm$

102. (b) The sides of a triangle in geometric progression are a, ar, ar^2

Triangle is right angled. Therefore, we use Pythagoras theorem.

$(a)^2 + (ar)^2 = (ar^2)^2$

$a^2 + a^2r^2 = a^2r^4$

$1 + r^2 = r^4$ or $r^4 - r^2 - 1 = 0$

$$\therefore \quad r^2 = \frac{-1 \pm \sqrt{1 - 4(-1)}}{2}$$

$$r^2 = \frac{-1 \pm \sqrt{5}}{2}$$

$$r = \frac{-1+\sqrt{5}}{2}$$

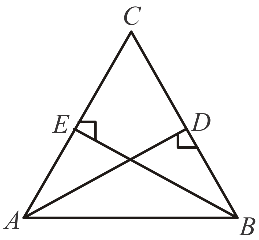
$$r \neq \frac{-1-\sqrt{5}}{2}$$ (Because Radius is not negative)

So, common ratio $= \frac{\sqrt{5}-1}{2}$.

103. (c) Area of ΔABD and ΔABE are same because both are on the same base.

Area of $\Delta ABD = \frac{1}{2} \times BD \times AD$ ... (i)

Area of $\Delta ABE = \frac{1}{2} \times AE \times BE$ ... (ii)

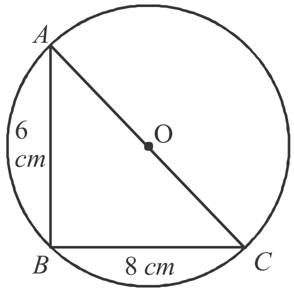


Now comparing both equations (i) and (ii)

$$\frac{1}{2} \times BD \times AD = \frac{1}{2} \times AE \times BE \Rightarrow BD \times AD = AE \times BE$$

104. (d) ΔABC is right angled at B. So that circum radius lies on its hypotenuse

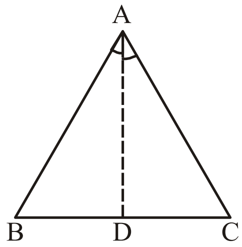
Here OA is circumradius $OA = \frac{AC}{2}$



By Pythagoras theorem,
$AC^2 = AB^2 + BC^2$
$AC^2 = (6)^2 + (8)^2$
$\Rightarrow AC^2 = 36 + 64 \therefore AC = 10$

$\therefore$ Radius of circumcircle $= \frac{10}{2} = 5$ cm $= OA = OC$

105. (d) In ΔABC,
AD is the internal angular bisector of angle A.



$$\frac{BD}{DC} = \frac{AB}{AC} \Rightarrow \frac{AC}{AB} = \frac{DC}{BD}$$

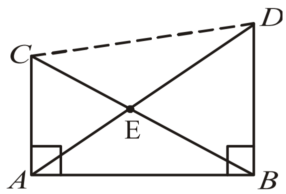
$$\Rightarrow \frac{DC+BD}{BD} = \frac{AC+AB}{AB}$$

$$\Rightarrow \frac{BC}{BD} = \frac{3+1}{3} = \frac{4}{3}$$

$$\frac{BD}{BC} = \frac{3}{4}$$

$BD : BC = 3 : 4$

106. (d) If AB is a straight line and C and D are points such that $AC \perp AB$ and $BD \perp AB$.



$\therefore$ $AC \parallel BD$
$ABCD$ forms trapezium.
Now, by property of trapezium diagonals intersect each other in the ratio of lengths of parallel side.

$$\frac{AE}{ED} = \frac{BE}{CE} \Rightarrow \frac{AE}{AD-AE} = \frac{BE}{BC-BE}$$

$$\frac{BC-BE}{BE} = \frac{AD-AE}{AE} \Rightarrow \frac{BC}{BE} - 1 = \frac{AD}{AE} - 1$$

$$\therefore \frac{BC}{BE} = \frac{AD}{AE}$$

$$\therefore \frac{AE}{AD} = \frac{BE}{BC}$$

But the value of $\frac{AE}{AD}$ or $\frac{BE}{BC}$ cannot be determined

Therefore, we cannot find the value of $\frac{AE}{AD} + \frac{BE}{BC}$.

107. (c) In ΔABC, we draw a line $l \parallel BF$ which intersect AC at G.
In ΔADG and ΔAEF;

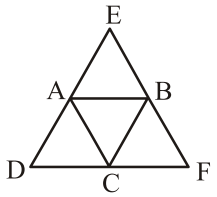
given that EA is the mid point of AD and $DL \parallel EF$.
So, concept of similar triangle.
F is also mid point of AG.
$AF = FG$ (i)
ΔADG and ΔAEF are similar.
Again
ΔFBC and ΔDCl
$BF \parallel DG$
given that AD is median so thad DB the mid point of BC.
G will be The mid point of CF
$CG = GF$ ...(ii)
From equations (i) and (ii), we get
$AF = FG = CG$ ...(iii)
From figure, $AC = AF = FG + CG$
$= AF + AF + AF + 3AF$

$\Rightarrow AF = \frac{1}{3} AC$

108. (c)
1. On drawing the three straight lines through the three vertices of ΔABC, we get the following figure.
Here, $AB \parallel DF$, $BC \parallel DE$ and $AC \parallel EF$.
Clearly, A, B and C are the mid–points of DE, EF and DF respectively.

By mid–point theorem, $BC = \frac{1}{2} DE$ or DE = $2BC$

Similarly, $DF = 2AB$ and $EF = 2AC$. Hence, Statement 1 is correct.

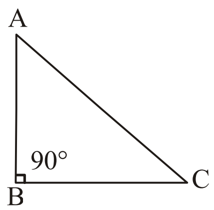


2. Also, area of ΔABC = $\frac{1}{4}$ area of ΔDEF or area of ΔDEF = 4 area of ΔABC. Hence, Statement 2 is also coreect.

109. (a) We know that, sum of angles of a triangle = 180°
$\Rightarrow \angle A + \angle B + \angle C = 180°$
$\Rightarrow \angle A + 2\angle A + \angle A = 180°$
$\Rightarrow 4\angle A = 180°$

$\Rightarrow \angle A = \frac{180°}{4} = 45°$

$\angle B = 90°$ and $\angle C = 45°$ Givan that $2\angle C + 2\angle A = \angle B$]
ΔABC is a right angled triangle,
$\angle B = 90°$, $\angle C = 45°$ and $\angle A = 45°$



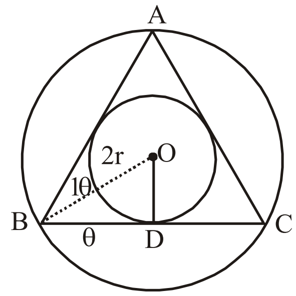
Pythagoras theorem,
$AB^2 + BC^2 = AC^2$
$\Rightarrow AB^2 + AB^2 = AC^2$ [$\because AB = BC$]
$\Rightarrow 2AB^2 = AC^2$

$\Rightarrow \frac{AC^2}{AB^2} = \frac{2}{1}$

$\Rightarrow \frac{AC}{AB} = \frac{\sqrt{2}}{1}$

$\therefore AC : AB = \sqrt{2} : 1$

110. (c)



Here, OD is the radius of incircle while OB is the radius of outer circle.
Given that
Radius of outer circle = 2 × radius of incircle.
$\Rightarrow$ OB = 2 × OD

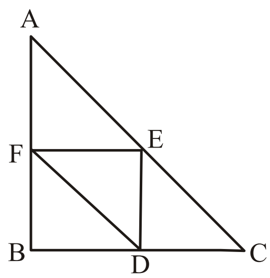
$\Rightarrow \frac{OB}{OD} = 2 \Rightarrow \frac{OD}{OB} = \frac{1}{2}$

$\Rightarrow \sin\angle BDO = \frac{1}{2} = \sin 30°$

$\angle$DBO = 30°, $\angle$ABC = 2 × $\angle$DBO = 2 × 30° = 60°
So, it is equilateral triangle.

111. (a) The altitude and medians of an equilateral triangle are congruent but centroid divide the altitude in 2 : 1. So, Statements 1 and 2 are correct.

112. (c) In ΔABC and ΔDEF,
$AB \parallel DE$, $BC \parallel EF$ and $CA \parallel FD$
If $\angle ABC$ is right angle, then $\angle DEF$ is also a right angle.



Both triangles are similar but not congruent.
Statement I is correct and Statement I is correct and Statement II is incorrect.

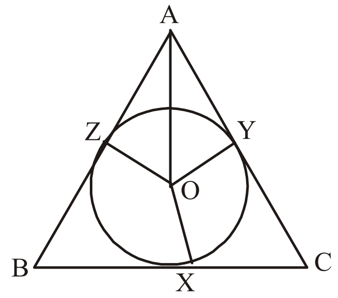
113. (d) In ΔAOZ and ΔAOY,

$AO = OA$ [common]

$\angle OAZ = \angle OAY$ [Since, OA bisect $\angle A$]

and $\angle AZO = \angle AYO$ [each 90°]

$\therefore$ $\Delta AZO \cong \Delta AYO$



So, $AZ = AY$ [by CPCT]

Similarly, $CX = CY$ and $BX = BZ$

Now, $AB > BC$

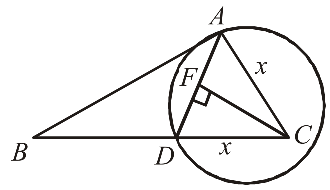
$\therefore$ $AZ + ZB > BX + XC$

$AZ > XC$ [$\because BX = ZB$]

If $AB > BC$, then $AB + AZ > BC + XC$

So, Statement I is incorrect and Statement II is correct.

114. (d) In ΔABC, $\angle ACB = 60°$ and $AC = x < BC$



The circle with centre at C and radius x meet BC at D.

$\therefore$ $CD = x =$ Radius

Again, in ΔACD, $AC = CD = x$

$$\therefore \angle CAD = \angle CDA = \frac{120°}{2} = 60°$$

ΔACD is an equilateral triangle.

Now, AC is a chord of circle, then perpendicular from C on chord AD bisect the chord.

$$\therefore DF = AF = \frac{AD}{2} = \frac{x}{2}$$

Statement I is incorrect and Statement II is correct.

115. (c) Let the sides of an old triangle be a, then area of an old equilateral triangle, $A_{old} = \frac{\sqrt{3}}{4}a^2$

Again, let the sides of a new triangle be 2a, then are of a new equilateral triangle ,

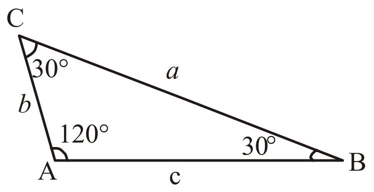
$$A_{new} = \frac{\sqrt{3}}{4}(2a)^2 = \frac{\sqrt{3}}{4} \times 4a^2$$

According to question, $A_{new} = KA_{old}$

$$\Rightarrow \frac{\sqrt{3}}{4} \times 4a^2 = k \times \frac{\sqrt{3}}{4}a^2$$

$\therefore$ $k = 4$

116. (c)



By Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\Rightarrow \frac{a}{\sin 120°} = \frac{b}{\sin 30°} = \frac{c}{\sin 30°}$$

$$\frac{b}{a} = \frac{\sin 30°}{\sin 120°} \text{ and } \frac{c}{a} = \frac{\sin 30°}{\sin 120°}$$

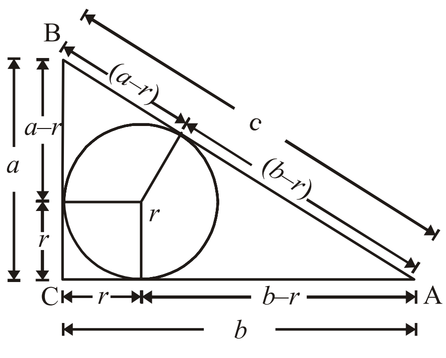
$$\frac{a}{a+b+c} = \frac{1}{1 + \frac{b}{a} + \frac{c}{a}}$$

$$= \frac{1}{1 + \frac{\sin 30°}{\sin 120°} + \frac{\sin 30°}{\sin 120°}}$$

$$= \frac{1}{1 + \frac{1/2}{\sqrt{3}/2} + \frac{1/2}{\sqrt{3}/2}}$$

$$= \frac{1}{1 + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}}} = \frac{\sqrt{3}}{2+\sqrt{3}}$$

117. (a)



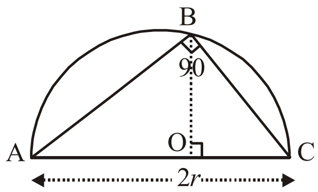
c $= b - r + a - r$

c $= a + b - 2r$

$2r = a + b -$ c

$$\therefore r = \frac{a+b-c}{2}$$

118. (a)



Let ABC is triangle, which have maximum area, while AC is $2r$

But OB = OC = r

By Pythagoras theorem

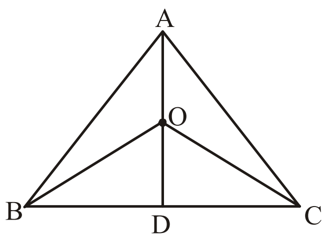
$$OB^2 + OC^2 = BC^2$$

$$BC = \sqrt{2}r = AC$$

Area of triangle = $\frac{1}{2} \times \sqrt{2}r \times \sqrt{2}r = r^2$

119. (c) **Statement 1**

AD divides Δ ABC in equal area of two parts. Then O is point on anywhere on AD



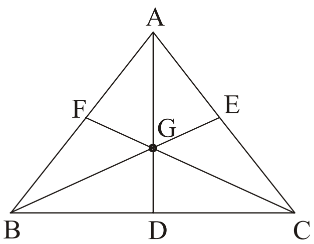
So area of triangle

$\Delta ABO = \Delta AOC$,

So statement 1 is true.

Statement 2

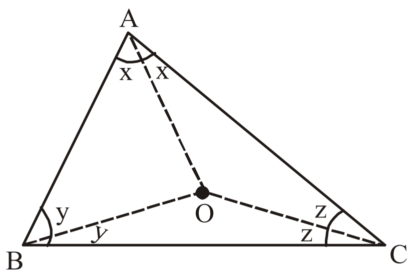
G is the point of concurrence of the medians then



area of Δ ABG = area of Δ BCG = Area of ΔACG

Both are true.

120. (a) If O is equidistant from all the sides, it will be circum-centre of the triangle so-



1. $\angle AOC = 2\angle ABC$; $\angle BOC = 2\angle BAC$; $\angle AOB = 2\angle ACB$ and $\angle OAC = \angle OCA$

   In ΔAOC–

   $\angle AOC + \angle OAC + \angle OCA = 180°$

   $\angle AOC + \angle OAC + \angle OAC = 180°$

   $2\angle OAC = 180 - 2\angle ABC$

   $\angle OAC = 90 - \angle ABC$ ------------(i)

   Similarly-

   $\angle OCB = 90 - \angle BAC$ -----------(ii)

   $\angle OBA = 90 - \angle ACB$ -----------(iii)

   Adding eq. (i), (ii) and (iii)-

   $\angle OAC + \angle OCB + \angle OBA$

   $= 90 + 90 + 90 - [\angle ABC + \angle BAC + \angle ACB]$

   $= 270° - 180°$

   $= 90°$

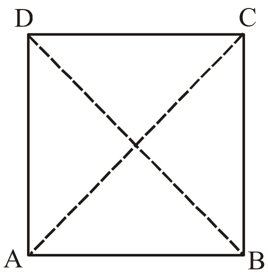
2. $\angle BOC = 2x + y + z = x + (x + y + z) = 90 + \angle A$

   It is not necessarily equal to $2\angle A$

3. This is a property of the angle bisector.

   So, option (a) is correct.

121. (a)



On single face of cube no. of right angled Triangles formed = 4 (i.e., ΔABD, ΔABC, ΔABD, ΔACD)

Total faces of a cube = 6

So, no. of right angle triangles = 4 × 6 = 24

So, option (a) is correct.

122. (a) Since $AB > A'B'$, $BC > B'C'$ and $CA > C'A'$

Also D, E, F are mid points of BC, CA and AB.

Also D′E′F′ are mid points of B′C′, C′A′ and A′B′

Now $AD = \frac{1}{2}BC$, $BE = \frac{1}{2}CA$, $CF = \frac{1}{2}AB$ ...(1)

Also $A'D' = \frac{1}{2}B'C'$, $B'E' = \frac{1}{2}C'A'$, $C'F' = \frac{1}{2}A'B'$ ...(2)

$\Rightarrow$ If $AB > A'B'$. $BC > B'C'$ and $CA > C'A'$

$\Rightarrow$ $A'D' > AD$, $BE > B'E'$ and $CF > C'F'$ $C'A'$

from (1) & (2)

Statement II

$$\frac{AB^2 + BC^2 + CA^2}{AD^2 + BF^2 + CF^2} = \frac{A'B'^2 + B'C'^2 + C'A'^2}{A'D'^2 + B'E'^2 + C'F'^2}$$

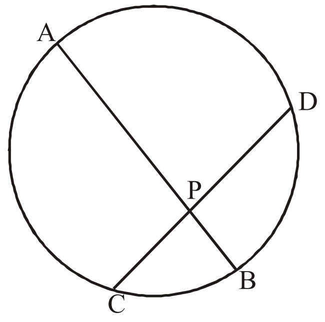
$$\Rightarrow \frac{(2CF)^2 + (2AD)^2 + (2BE)^2}{AD^2 + BE^2 + CF^2}$$

$$= \frac{(2C'F')^2 + (2A'D')^2 + (2B'F')^2}{A'D'^2 + B'E'^2 + C'F'^2}$$

$\Rightarrow$ $4 = 4 \Rightarrow$ Statement II is true.

$\therefore$ Option (a) is correct.

123. (b) Given two chords AB and CD intersects at a point P inside the circle.

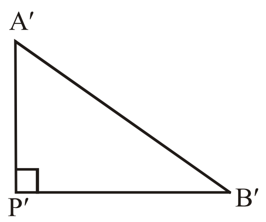


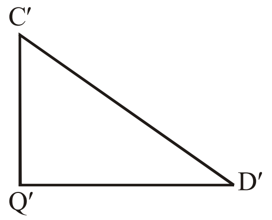
According to condition of circle if two chords intersects at P then

$AP \times PB = CP \times CD$

$$\frac{AP}{CP} = \frac{PD}{PB} \quad ...(1)$$

Given two right angled triangle $A'P'B'$ and $C'Q'D'$ then





$A'P' = AP$, $B'P' = BP$, $C'Q' = CP$, $D'Q' = DP$

From (1) we get

$$\frac{AP}{CP} = \frac{PD}{PB} \Rightarrow \frac{A'P'}{C'Q'} = \frac{D'Q'}{B'P'}$$

$\Rightarrow$ $\Delta A'P'B' \sim \Delta C'Q'D'$

$\because$ their corresponding ratios are equal.

Also we know that in similar triangles.

$$\frac{\text{Area of } \Delta A'P'B'}{\text{Area of } \Delta C'Q'D'} = \left(\frac{A'P'}{C'Q'}\right)^2 = \left(\frac{AP}{CP}\right)^2$$

$\Rightarrow$

$$\frac{\frac{1}{2} \times A'P' \times P'B'}{\frac{1}{2} \times C'Q' \times Q'D'} = \left(\frac{AP}{CP}\right)^2 \Rightarrow \frac{AP \times BP}{CP \times DP} = \left(\frac{AP}{CP}\right)^2$$

We know that

$$\frac{AP}{CP} = \frac{PD}{DB}$$

$$\Rightarrow \quad \left(\frac{AP}{CP}\right) = \left(\frac{BP}{DP}\right) = \left(\frac{AP}{CP}\right)^2$$

$$\Rightarrow \quad \left(\frac{AP}{CP}\right) = \left(\frac{AP}{CP}\right) = \left(\frac{AP}{CP}\right)^2$$

$$\Rightarrow \quad \left(\frac{AP}{CP}\right)^2 = \left(\frac{AP}{CP}\right)^2$$

$\Rightarrow$ Area of $\Delta A'P'B'$ = Area of $\Delta C'Q'D'$

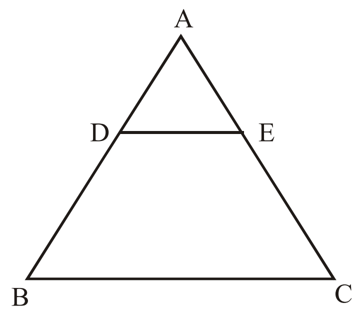
These triangles are not congruent because none of the criterion are satisfied by these triangles perimeter of triangle. Sum of all three sides. But in these triangles, all the sides are not equal, then their perimeter is not equal

$\Rightarrow$ Statement (1) and (3) is correct.

124. (c) We have $BE \parallel AC$ (Given)

So $\angle ADE = \angle B$ and $\angle AED = \angle C$

(corresponding angles)



Therefore $\Delta ABC \sim \Delta ADE$ by A similarity criterion.

Also given area of $\Delta ABC = 2$ area of ΔADE ...(1)

We know that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\Rightarrow \quad \frac{ar(ABC)}{ar(ADE)} = \left(\frac{AB}{AD}\right)^2 \quad ...(2)$$

From (1) we get

$$\frac{ar(ABC)}{ar(ADE)} = \frac{2}{1} \quad ...(3)$$

Therefore from (2) and (3)

$$\left(\frac{AB}{AD}\right)^2 = \frac{2}{1}$$

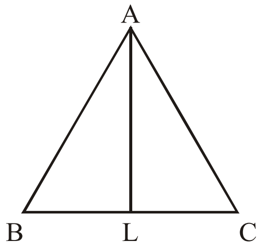
$\Rightarrow \quad \dfrac{AB}{AD} = \dfrac{\sqrt{2}}{1}$

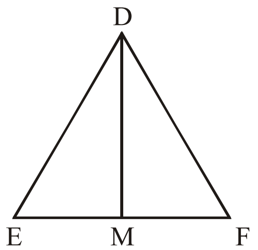
$\Rightarrow \quad \dfrac{1}{AD} = \sqrt{2}\text{unit} \quad (\because AB = 1 \text{ unit})$

$AD = \dfrac{1}{\sqrt{2}}\text{units}$

$\therefore$ Option (c) is correct.

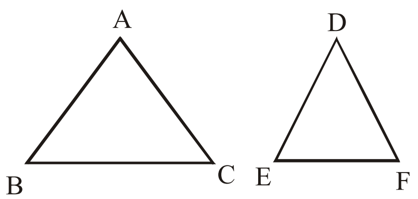
125. (b)





Here ABC and DEF be two triangles such that
$\angle A = \angle D, \angle B = \angle E$ and $\angle C = \angle F$ (Given)
Also $\angle L = \angle M = 90°$
$\Rightarrow \quad \angle ALB = \angle ALC = \angle DME = \angle DMF = 45°$
($\because$ M and L are mid points of EF and BC respectively)
$\Rightarrow \quad \Delta ABC \sim \Delta DEF$ by ΔAA similarity
Also $\Delta ABC \cong \Delta DEF$ by ΔAA Similarity
In ΔABL and ΔDEM
$\angle ALB = \angle DME = 45°$
$\angle A = \angle D$ and $\angle B = \angle E$ (Given)
$\Rightarrow \quad \Delta AB\angle \sim \Delta DEM$ by AAA similarity criterion.
$\therefore$ Statement I is true.
In ΔALC and ΔDMF
Given $AC \neq DF$
But $\angle ALC = \angle DMF = 45°$
Also $\angle A = \angle D$ and $\angle C = \angle F$ (Given)
$\Rightarrow \quad \Delta ALC \sim \Delta DMF$ by AAA similarity.
$\Rightarrow \quad \Delta ALC \cong \Delta DMF$ by AAA similarity.
$\therefore \quad$ Statement II is true.
But II is not the correct explanation of Statement-I
$\because$ these are different triangles.
$\therefore$ Option (b) is correct.

126. (d)



Here $\Delta ABC \sim \Delta DEF$
Then ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$\Rightarrow \quad \dfrac{\text{area of } \Delta ABC}{\text{area of } \Delta DEF} = \left(\dfrac{AB}{DE}\right)^2$

$= \left(\dfrac{\sqrt{2}+1}{\sqrt{3}}\right)^2$

$= \dfrac{2+1+2\sqrt{2}}{3} = \dfrac{3+2\sqrt{2}}{3}$

$= 3+2\sqrt{2} : 3$

127. (d) In a triangle ΔABC
$\angle A + \angle B + \angle C = \pi \qquad$ ...(1)

Given $A - B = \dfrac{\pi}{2}$

$\Rightarrow \quad A = B + \dfrac{\pi}{2}$

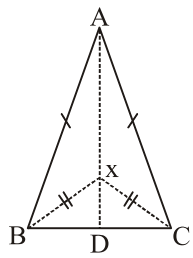
Putting this value of A in (1), we get

$B + \dfrac{\pi}{2} + B + C = \pi$

$C + 2B = \pi - \dfrac{\pi}{2} = \dfrac{\pi}{2}$

$\therefore$ Option (d) is correct.

128. (d) Locus of the point X is L.



Here L is line segment AD.
Now, $\Delta AXB \cong \Delta AXC$ (By SSS)
$\therefore \quad \angle BAX = \angle CAX$ (Corr. Angles)
$\Rightarrow$ AX and hence AD is the bisector of $\angle BAC$.
Hence incentre of the ΔABC lies on AD i.e., L (statemetn 1 correct)
Since AB = AC fand AD is the bisector of ΔABC

$\therefore \quad \dfrac{BD}{DC} = \dfrac{AB}{AC} = 1 \Rightarrow BD = DC$

Now $\Delta XBD \cong \Delta XCD$ (By SSS)
$\therefore \quad \angle XDB = \angle XDC = 90°$
Hence AD is the perpendicular bisector of BC.
Therefore, orthocentre of the ΔABC lies on AD i.e, L.

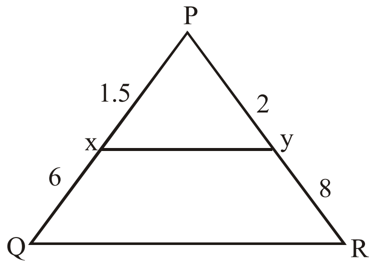
(Statement 2 correct)

Since D is the mid-point of BC, therefore AD is the mediam.

Hence centroid of the ΔABC lies ons AD i.e., L.

(Statement 3 correct)

129. (d) In ΔPYX and ΔPRQ



$$\frac{PX}{PQ} = \frac{PY}{PR}$$

$$\Rightarrow \quad \frac{1.5}{7.5} = \frac{2}{10}$$

$$\Rightarrow \quad \frac{1}{5} = \frac{1}{5}$$

Now corresponding ratios of two triangles are equal.

Also $\angle P = \angle P$ (Common)

$\Rightarrow \quad \Delta PXY \sim \Delta PQR$ by SAS similarity

$$\Rightarrow \quad \frac{PX}{PQ} = \frac{PY}{PR} = \frac{XY}{QR}$$

$$\Rightarrow \quad \frac{1}{5} = \frac{1}{5} = \frac{XY}{QR}$$

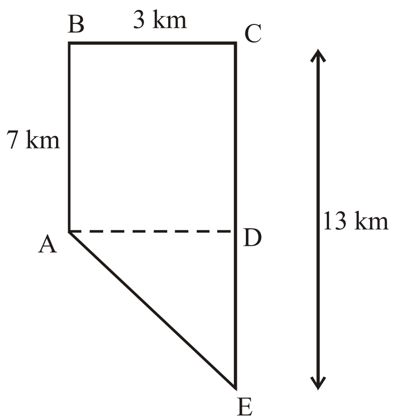
$\Rightarrow \quad QR = 5XY$

Also $QR \parallel XY$ (By B.P.T)

$\therefore$ Option (d) is correct.

130. (b) Let the position of person = A

We have to find out the distance between A and E.



AB = 7 km then CD = 7 km

BC = 3 m then AD = 3 km

DE = CE – CD = 13 – 7 = 6 km

Draw $AD \perp CE$

In right angled triangle AED, we get

$(AE)^2 = AD^2 = DE^2$

$= (3)^2 + (6)^2$

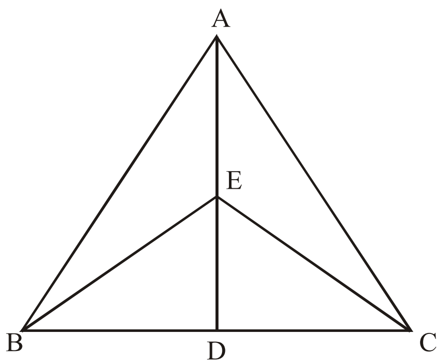
AE $= \sqrt{9+36}$

$= \sqrt{45} = \sqrt{5 \times 9}$

AE = $3\sqrt{5}$km

$\therefore$ Option (b) is correct.

131. (c) ABC is a triangle in which D and E are the mid points of BC and AD respectively.



$$\Rightarrow \quad AE = ED = \frac{AD}{2} \text{ and } BD = DC = \frac{BC}{2}$$

Statement (1) $\dfrac{\text{Area of triangle ABC}}{\text{Area of triangle BED}}$

$$= \frac{\frac{1}{2} \times BC \times AD}{\frac{1}{2} \times ED \times BD} = \frac{\frac{1}{2} \times BC \times AD}{\frac{1}{2} \times \frac{1}{2} AD \times \frac{1}{2} BC}$$

$$= \frac{1}{\frac{1}{4}} = 4$$

Area of ΔABC = 4 Area of ΔBED

(1) is true.

Statement (2) $\dfrac{\text{Area of } \Delta ADC}{\text{Area of } \Delta BED} = \dfrac{\frac{1}{2} \times AD \times DC}{\frac{1}{2} \times BD \times ED}$

$$= \frac{AD \times DC}{DC \times \frac{1}{2} AD} \quad \left(\because BD = DC \text{ and } ED = \frac{1}{2} AD\right)$$

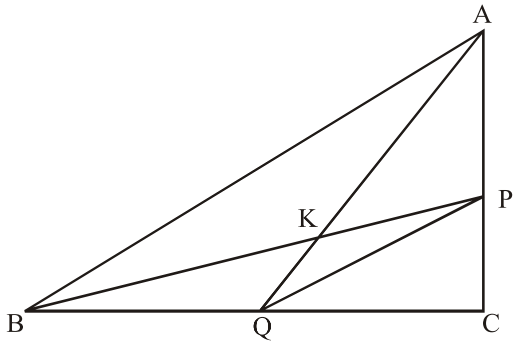
= 2

Area of ΔADC = 2 Area of ΔBED

2 is true.

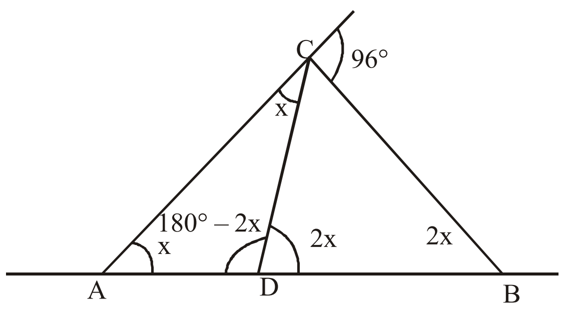
$\therefore$ Option (c) is correct.

132. (c) Since ABC is a right angled triangle QtC



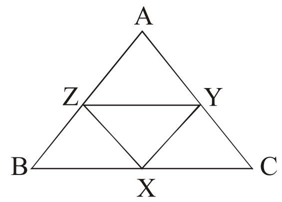
In ΔABC, we have by pythagorus theorem,
$AB^2 = AC^2 + BC^2$ ...(1)
Also In ΔBPC, we get
$BP^2 = BC^2 + CP^2$ ...(2)
In ΔAQC, we get
$AQ^2 = AC^2 = CQ^2$...(3)
In ΔPQC, we get
$PQ^2 = PC^2 + QC^2$ ...(4)
Adding (2) and (3) we get
$BP^2 + AQ^2 = BC^2 + CP^2 + AC^2 + CQ^2$
$= (BC^2 + AC^2) + (CP^2 + CQ^2)$
Using (1) and (4) we get
$BP^2 + AQ^2 = AB^2 + PQ^2$
$\therefore$ Option (c) is correct.

133. (b)



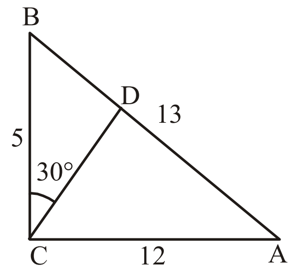
Let $\angle CAD = \angle ACD = x$
Since AD = CD = BC
$\because$ AD = CD
$\Rightarrow \quad \angle A = \angle C$
Also CD = BC
$\Rightarrow \quad \angle D = \angle B$
In ΔACD
$\angle A + \angle C + \angle D = 180°$
$\Rightarrow \quad \angle D = 180° - 2x$
$\angle CDB = 180° - \angle CDA$
$= \quad 180° - (180° - 2x)$
$= \quad 2x$
$\Rightarrow \quad \angle CDB = \angle CBD = 2x \Rightarrow \angle DCB = 180° - (2x + 2x)$
$= 180° - 4x$
At point c, we have
$x + (180° - 4x) + 96° = 180°$
$\Rightarrow \quad 180° - 3x + 96° = 180°$
$\Rightarrow \quad -3x = -96°$
$\Rightarrow \quad x = 32°$
Hence $\angle DBC = 2x = 2 \times 3^2 = 64°$
$\therefore$ Option b is correct.

134. (c) Let ABC be an equilateral triangle and x, y, z are points on BC, CA and AB.



Also given
BX = CY = AZ
Since $\angle A = \angle B = \angle C = 60°$ (Equilatual triangle)
$\Rightarrow \quad$ If BX = CY
$\Rightarrow \quad \angle X = \angle Y$
BX = AZ
$\Rightarrow \quad \angle X = \angle Z$
Also If $\angle Y$ = Az
$\Rightarrow \quad \angle Y = \angle Z$
$\Rightarrow \quad \angle X = \angle Y = \angle Z = 60°$
ΔXYZ is an equilateral triangle.
Consider triangle ΔABC and ΔXYZ
Since $\angle X = \angle Y = \angle Z = 60°$
and $\angle A = \angle B = \angle C = 60°$
$\Rightarrow \quad \Delta ABC \sim \Delta XYZ$ by AA similarity criterion
$\therefore$ Option (c) is correct.

135. (d)



in Δ BCD

$$\frac{\sin 30°}{BD} = \frac{\sin B}{CD}$$

$$\Rightarrow \frac{1}{2BD} = \frac{\sin B}{CD}$$

$$\Rightarrow \frac{1}{2BD} = \frac{12}{13CD} \quad \because \sin B = \frac{12}{13}$$

$$CD = \frac{24}{13} BD \quad ...(i)$$

in Δ ADC

$$\frac{\sin 60°}{AD} = \frac{\sin A}{CD} \quad \because \sin A = \frac{5}{13}$$

$$CD = \frac{5}{13} \times \frac{2}{\sqrt{3}} AD$$

$$\Rightarrow CD = \frac{10}{13\sqrt{3}}(13 - BD) \quad \text{...(ii)}$$

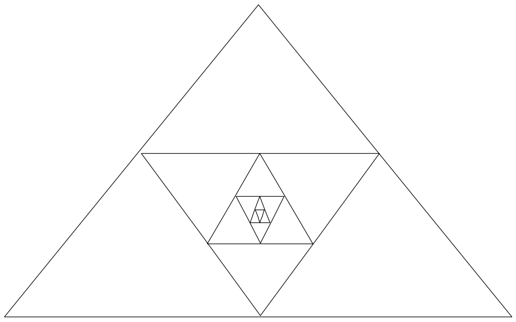
equating both eq (i) and (ii)

$$\frac{10}{13\sqrt{3}}(13 - BD) = \frac{24}{13} BD$$

$$BD = \frac{65}{5 + 12\sqrt{3}}$$

$$CD = \frac{24}{13} \quad \frac{65}{5 + 12\sqrt{3}} = \frac{120}{5 + 12\sqrt{3}}$$

136. (c)



Direction $\rightarrow$ make 2 more trangle
total equitateral Δ
$= \theta$

$$\text{Area} = \frac{\sqrt{3}}{4}(12\ \theta)^2 : \frac{\sqrt{3}}{4}(16)^2 = 64 : 1$$

137. (c) Let a = b = c then the sides an $\sqrt{2}a, \sqrt{2}a$ and 2a by heroni formula

$$S = \frac{2\sqrt{2}a + 2a}{2} = \sqrt{2}a + a$$

as $\Delta =$

$$\sqrt{\left(\sqrt{2}\,a + a\right)\left(\sqrt{2}\,a + a - \sqrt{2}\,a\right)\left(\sqrt{2}\,a + a - \sqrt{2}\,a\right)\left(\sqrt{2}\,a + a - 2a\right)}$$

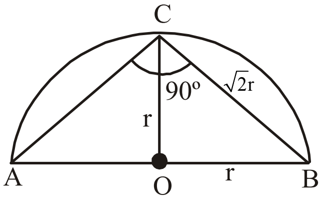
$$= \sqrt{\left(\sqrt{2}a + a\right)\left(\sqrt{2}a - a\right)a^2}$$

$$= \sqrt{(2a^2 - a^2)a^2} = a^2$$

Now by putting a = b = c in option
(c) satisfies the area a^2

$$\text{ie } \frac{a(b+c)}{2} = \frac{a(2\,a)}{2} = a^2$$

138. (a) Largest triangle in semicircle is



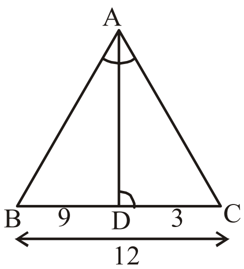
when height of triangle h = r

$$\text{then area} = \frac{1}{2} b \times h = \frac{1}{2} 2r \times r = r^2$$

139. (d) Remembering the fact that sum of any two small sides of triangle is larger than its third side
Option D is not a triangle
2 + 3 = 5 is not greater than 6

140. (b) let the angles be = 2x, 4x and 3x
$2x + 4x + 3x = 180°$
$9x = 180°$
$x = 20°$
Smallest angle = 2x
$= 2 \times 20 = 40°$

141. (b) in ΔABC and ΔDAC

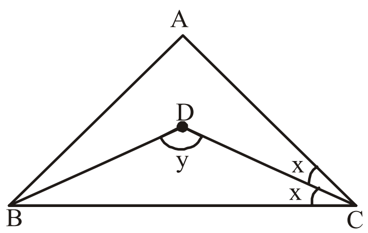


$\angle BAC = \angle ADC$
$\angle ACB = \angle DCA \quad \therefore$ common angle
$\therefore \Delta ABC \sim \Delta DAC$

$$\frac{BC}{AC} = \frac{AC}{DC}$$

$12 \times 3 = AC^2$
$AC = 6$

142. (c)



$\angle A = 80, \ \angle ABC = 60°$
$\angle ACB = 180 - (80 + 60) = 40°$

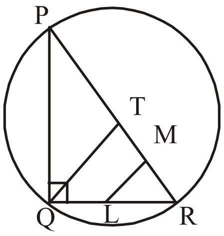
$$x = \frac{40}{2} = 20$$

$\angle DBC = \frac{60}{2} = 30$

$\angle y = 180 - (30 + 20) = 130$

$= x = 20, y = 130$

143. (b)



Assuming right angled triangle be in a circle where PR is diameter

PT = QT = TR (Radii of circle)

QT = TR

$\angle TQR = \angle TRQ$

$\angle TQR = \angle RLM$ Corresponding angles

$\angle RLM = \angle LRM$ i.e. ($\angle TRQ$)

144. (c) (1) $\because \angle DAC = \angle DCA \therefore DA = DC$ hence ΔADC is an isosceles triangle

(2) it cannot be said weather is the centroid

3. AB = CB

AD = DC

BD = BD

$\Delta ABD \cong CBD$

hence statement 1 and 3 are correct.

145. (c) $\angle AMC = \angle BMD$

$\angle CMD = \angle CMD$

$\angle AMC + \angle CMD = \angle BMD + \angle CMD$

{Adding both equation}

$\angle AMD = \angle BMC$

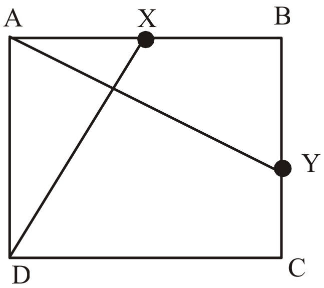
$\angle DAM = \angle CBM$

AM = BM

By ASA

$\Delta ADM \cong BCM$

146. (d)



1. in ΔADC and ΔBAY

$\angle A = \angle B = 90°$

AX = BY half of side of square

AD = AB

$\Delta ABY \cong \Delta DAX$ By SAS

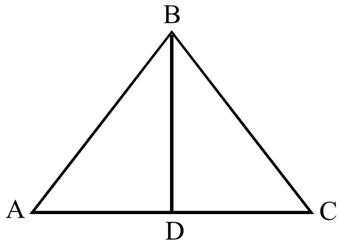
2. $\angle DXA = \angle AYB$ CPCT

3. DX is not included at 60° with AY

4. DX is not is perpendicular to AY

147. (c)

148. (c)



Since ΔABC is an equilateral triangle and BD is a perpendicular, therefore $AD = DC$.

In ΔBCD, by Pythagoras theorem, we have,

$$BC^2 = BD^2 + CD^2 \Rightarrow BD^2 = BC^2 - CD^2$$

$$\Rightarrow BD^2 = AC^2 - CD^2 \qquad (\because BC = AC)$$

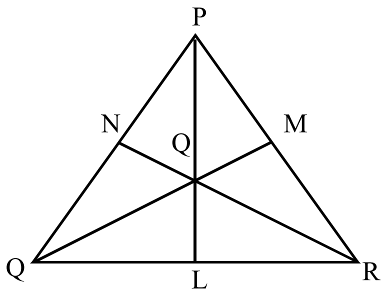
$$BD^2 = (AD + DC)^2 - CD^2 \Rightarrow BD^2$$

$$= AD^2 + DC^2 + 2ADCD - CD^2$$

$$BD^2 = AD^2 + 2DC^2 \Rightarrow BD^2 = AD^2 + 2AD^2$$

$$= 3AD^2 \qquad (\because CD = AD)$$

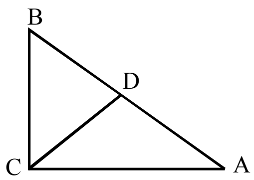
149. (d)



We can see that Q is outside triangle OPR and ΔOPR is obtuse-angled triangle.

Since orthocenter of an obtuse-angled triangle is always outside the triangle, therefore Q is the orthocenter of ΔOPR.

150. (d)



$$(CD)^{-2} = (BC)^{-2} + (CA)^{-2}$$

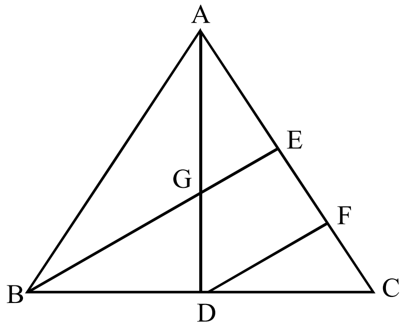
$$\Rightarrow \frac{1}{CD^2} = \frac{1}{BC^2} + \frac{1}{CA^2}$$

$$\Rightarrow \frac{1}{CD^2} = \frac{CA^2 + BC^2}{BC^2.CA^2} \Rightarrow \frac{1}{CD^2} = \frac{AB^2}{BC^2.CA^2}$$

$$CD^2 = \frac{BC^2.CA^2}{AB^2} \Rightarrow CD^2.AB^2 = BC^2.CA^2$$

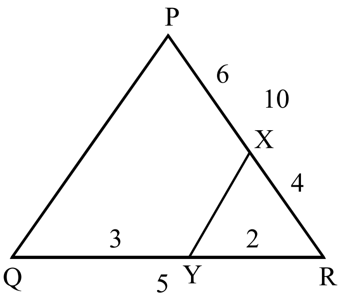
$$\Rightarrow CD.AB = BC.CA$$

151. (a)



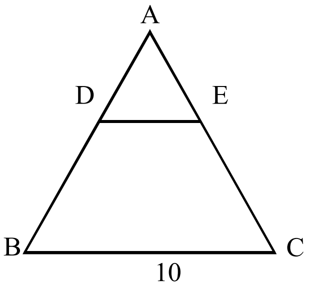
Since BE is the median of AC, therefore $AE = EC$.
Since $AC = 9$ cm, therefore $AE = EC = 3$ cm.
This means $FC < EC = 3$ cm. therefore, $FC = 2.25$ cm

152. (a)



Since $\frac{RY}{YQ} = \frac{RX}{XP} \Rightarrow \frac{2}{3} = \frac{4}{6} = \frac{2}{3}$ therefore, by converse of BPT theorem, XY 11 PQ.

153. (b)



According to the question and by BPT,

$\frac{AD}{AB} = \frac{AE}{AC}$ and $\angle A$ is common.

Therefore, $\Delta ADE \sim ABC$ (by SAS similarity)

Since we know that the ratio of areas of two similar triangles is equal to the square of its proportional sides, therefore, $\frac{ar.(\Delta ADE)}{ar.(\Delta ABC)} = \frac{(DE)^2}{(BC)^2}$ ...(1)

Also, we are given that

$$ar.(\Delta ADE) = \frac{1}{5} ar.(\Delta ABC)$$

Thus, (1)

$$\Rightarrow \frac{ar.(\Delta ABC)}{5ar.(\Delta ABC)} = \frac{(DE)^2}{(10)^2} = \frac{(DE)^2}{100}$$

$$\Rightarrow \frac{1}{5} = \frac{(DE)^2}{100} \Rightarrow DE = \sqrt{\frac{100}{5}} = 2\sqrt{5}$$

154. (a) Since the point of intersection of the perpendicular bisectors of the sides of a triangle is called circumcentre and the circumcentre for an obtuse triangle lie outside the triangle, therefore this statement is true.
Also, since the point of intersection of the perpendiculars drawn from the vertices to the opposite sides of a triangle is called orthocentre and orthocentre cannot lie on two sides, therefore this statement is false.
Consider the given table:

Subjects/ girls and boys	Mathematics	Physics	Statistic	Chemistry
Number of girls	240 – 150 = 90	300 – 180 = 120	250	$\frac{3}{5} \times 340$ = 68 × 3 = 204
Number of boys	150	60% of 300 = 180	320 – 250	136 = 70
Total	20% of 1200 = 240	$\frac{1200}{4}$ = 300	320	1200 – (240 + 300 + 320) = 340

155. (a)
156. (c)
157. (d)
158. (d) diagonal of square = $6\sqrt{2}$

$$\text{side of square} = \frac{6\sqrt{2}}{\sqrt{2}} = 6$$

3 × side of triangle = 6 × 4
side of triangle = 8

$$\text{area of triangle} = \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4} \times 8^2 = 16\sqrt{3}\ \text{cm}^2$$

159. (a) Area of $\Delta ABC = \frac{\sqrt{3}}{4}\ell^2$

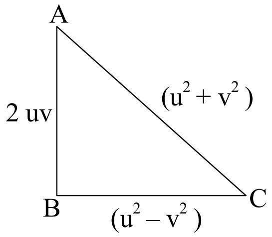
from mid point theoram

$$\text{Area of } \Delta ADE = \frac{\sqrt{3}}{4} \times \left(\frac{\ell}{2}\right)^2 = \frac{\sqrt{3}}{16}\ell^2$$

$$\text{Area of shaded region} = \frac{\sqrt{3}}{4}\ell^2 - \frac{\sqrt{3}}{16}\ell^2$$

$$= \frac{3\sqrt{3}\,\ell^2}{16}$$

160. (b)



Let ΔABC is a right angled triangle, with base $BC = (u^2 - v^2)$ and hypotenuse $AC = (u^2 + v^2)$

Now, $AB = \sqrt{(AC)^2 - (BC)^2}$

$$= \sqrt{\left(u^2 + v^2\right)^2 - \left(u^2 - v^2\right)^2}$$

$$= 2\,u.v$$

Area of triangle ABC $= \frac{1}{2} \times AB \times BC$

$$\Rightarrow \frac{1}{2} \times 2uv \times \left(u^2 - v^2\right) = 2016.$$

$$\Rightarrow uv\,(u^2 - v^2) = 2016.$$

Now, $2016 = 2^5 \times 3^2 \times 7.$

$= 9 \times 7\,(9 + 7)\,(9 - 7).$

$= 3^2 \times 7 \times 2^4 \times 2.$

$= 2^5 \times 3^2 \times 7$

$2016 = 9 \times 7\,(9^2 - 7^2) = uv\,(u^2 - v^2)$

On comparing the Values,

$u = 9, v = 7.$

Perimeter of the triangle

$= 2\,uv + u^2 - v^2 + u^2 + v^2$

$= 2\,u\,(u + v)$

$= 2 \times 9\,(9 + 7)$

$= 288$ units

161. (b) Here $a = 5$ cm, $b = 6$ cm, $c = 7$ cm.

Now, Area of ΔABC of sides a, b, c

$= \sqrt{S(S-a)(s-b)(s-c)}$

Where $s = \frac{a+b+c}{2} = \frac{5+6+7}{2} = 9\,\text{cm}$

$\therefore$ Area $= \sqrt{9(9-5)(9-6)(9-7)}$

$= \sqrt{9 \times 4 \times 3 \times 2} = 6\sqrt{6}$

$= 14.7\ \text{cm}^2$

162. (a) Area of triangle $(A_1) = \left(7 - 4\sqrt{3}\right)$

$$= \left(2^2 + \left(\sqrt{3}\right)^2 - 2.2.\sqrt{3}\right)$$

$$= \left(2 - \sqrt{3}\right)^2$$

Area of triangle $(A_2) = \left(7 + 4\sqrt{3}\right)$

$$= \left(2^2 + \left(\sqrt{3}\right)^2 + 2.2.\sqrt{3}\right) \qquad = \left(2 + \sqrt{3}\right)^2$$

Ratio of corresponding sides $= \sqrt{\left(\frac{A^1}{A_2}\right)}$

$$= \sqrt{\frac{\left(2-\sqrt{3}\right)^2}{\left(2+\sqrt{3}\right)^2}} = \frac{2-\sqrt{3}}{2+\sqrt{3}} \qquad = 7 - 4\sqrt{3}$$

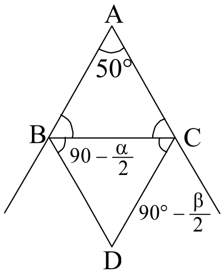
163. (b) Let In $\Delta ABC = \alpha$ and $\angle ACB = \beta$

and $\angle A = 50°$ (given)

so, in ΔABC, $\alpha + \beta + 50 = 180°$

$\alpha + \beta = 130°$

As BD and CD bisect exterior angle $\angle B$ and $\angle C$ respectively.



then, $\angle DBC = 90° - \frac{\alpha}{2}$

and $\angle DCB = 90° - \frac{\beta}{2}$

Again, In ΔBDC, sum of angles $= 180°$

$$\therefore\ \angle D + 90° - \frac{\alpha}{2} + 90° - \frac{\beta}{2} = 180°$$

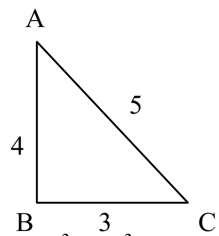
$$\angle D - \left(\frac{\alpha + \beta}{2}\right) = 0 \Rightarrow \angle D = \left(\frac{\propto + \beta}{2}\right)$$

From ΔABC, $\propto + \beta + 50° = 180°$

$\therefore \propto + \beta = 130°$

$$\angle D = \left(\frac{\alpha + \beta}{2}\right) = \frac{130°}{2} = 65°$$

164. (b) Let ΔABC is a right triangle with side AB, BC and AC are 4, 3 and 5 units.



$(AB)^3 = (4)^3 = 64$

$(BC)^3 = (3)^3 = 27$

$(AC)^3 = (5)^3 = 125.$

Here, we see that $125 > 64 + 27$

$\Rightarrow (AC)^3 > (AB)^3 + (BC)^3$

165. (d) Let BD = h cm.
In ΔABC,

$BC = \sqrt{h^2 + 16}$

In ΔBDA

$AB = \sqrt{h^2 + 81}$ ...(i)

In ΔABC

$AB = \sqrt{169 - (h^2 + 16)}$

$= \sqrt{169 - h^2 - 16}$

$= \sqrt{153 - h^2}$ ...(ii)

From (i) and (ii)

$\sqrt{h^2 + 81} = \sqrt{153 - h^2}$

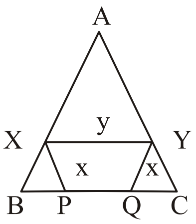
$h^2 + h^2 = 153 - 81$

$2h^2 = 72$

$h^2 = 36$

h = 6 cm

166. (d)



From question = $\begin{bmatrix} \because XY \parallel BC \\ PX \parallel AC \\ YQ \parallel AB \end{bmatrix}$

Let XP = x and XY = y

$\Rightarrow \quad 2x + y = 40$ ...(i)

$\because \quad XP \parallel AC.$

In ΔXBP

$\Rightarrow \quad \angle X = \angle P = \angle B = 60°$

$\left(\begin{matrix} \because \angle BXP = \angle BAC \\ \text{and } \angle BPX = \angle BCA \end{matrix}\right)$

⇒ ΔXBP is an equilateral triangle ⇒ BX = XP = x

Similarly, ΔXAY is an equilateral triangle.

⇒ AX = XY = y

Given = AB = AX + XB = 30

x + y = 30 ...(ii)

Solving (i) & (ii), we get x = 10

$\because \quad BP = QC = x = 10cm$

Given BC = BP + PQ + QC = 30

$\Rightarrow \quad PQ = 10cm$

167. (a) Let ΔABC is a equilateral triangle with side l.

Then, Area (A) $= \frac{\sqrt{3}}{4}(l)^2$

When, $l' = \frac{l}{2}$

Then area $(A') = \frac{\sqrt{3}}{4}(l')^2 = \frac{\sqrt{3}}{4}\left(\frac{l}{2}\right)^2$

Percent decrease in area $= \left(\frac{A - A'}{A}\right) \times 100$

$= \left\{\frac{\frac{\sqrt{3}}{4}l^2 - \frac{\sqrt{3}}{4}\left(\frac{l}{2}\right)^2}{\frac{\sqrt{3}}{4}l^2}\right\} \times 100 = \left(1 - \frac{1}{4}\right) \times 100 = 75\%$

168. (a) Let three consecutive integers are n, $(n + 1)$ and $(n + 2)$.
Then, from pythagorus rule,

$(n + 2)^2 = (n)^2 + (n + 1)^2$

$n^2 + 4n + 4 = 2n^2 + 2n + 1$

$(n^2 - 2n + 1) = 4$

$(n - 1)^2 = 4$

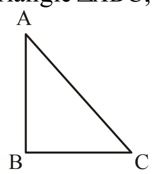
$(n - 1) = \pm 2$

$\therefore n = 3$ or -1. (But negative value is not valid)

$\therefore n = 3, (n + 1) = 4, (n + 2) = 5.$

Hence, set of integer = (3, 4, 5)

169. (c) Let BC is shortest side in right angle triangle ΔABC, such that $AB : AC = 4 : 5$.



$\therefore \ BC = \sqrt{(AC)^2 - (AB)^2}$

$= \sqrt{(5x)^2 - (4x)^2} = 3x$

Perimeter $= AB + BC + AC = 4x + 3x + 5x = 12x$

According to the question

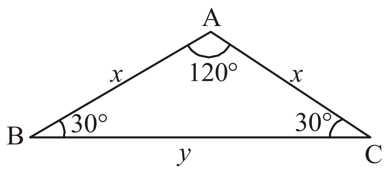
$12x = k(3x)$

$\therefore k = \frac{12x}{3x} = 4$

170. (a) We know that, sum of angles of any triangle = 180°

$x + x + 4x = 180°$

$x = \frac{180°}{6} = 30°.$



So, angle of triangles are 30°, 30°, 120°.

Let side of the triangles ABC are x, x and y

From cosine formula,

$\cos(120°) = \frac{AB^2 + AC^2 - BC^2}{2.AB.AC} = \frac{x^2 + x^2 - y^2}{2.x.x}$

$\frac{-1}{2} = \frac{2x^2 - y^2}{2x^2} \Rightarrow y^2 = 3x^2 \quad y = \sqrt{3}x$

Perimeter of the triangle $= (x + x + y) = k(y)$ (given)

$\left(\frac{2}{\sqrt{3}}y + y\right) = K.y. \quad \therefore K = 1 + \frac{2}{\sqrt{3}}$

171. (c) Let shorter and longer side of right angle triangle are x and y cm respectively.

Then, $x^2 + y^2 = (10)^2 \Rightarrow x^2 + y^2 = 100$ ...(i)

and Area $= \frac{1}{2}xy = 24 \Rightarrow x = \frac{48}{y}$

Plug. in $x = \frac{48}{y}$ into equation (i), we get

$\left(\frac{48}{y}\right)^2 + y^2 = 100$

$(48)^2 + y^4 = 100\,y^2$

$y^4 - 100y^2 + (48)^2 = 0$

On solving, we get $y = 6$ or 8

$\therefore x = 6, y = 8$

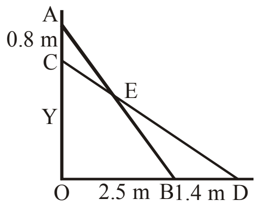
when, x becomes half and y becomes double then,

$x' = 3, y' = 16$

Hypotenuse $= \sqrt{(x)^2 + (y')^2}$

$= \sqrt{3^2 + (16)^2} = \sqrt{265}$ cm

172. (b) Let the length of the Ladder is x cm then AB = CD = x cm.



And, $OC = y$ m

From ΔOCD,

$y^2 + 3.9^2 = x^2$ ...(i)

From ΔAOB

$(y + 0.8)^2 + 2.5^2 = x^2$ ...(ii)

From (i) and (ii)

$y^2 + 3.9^2 = (y + 0.8)^2 + 2.5^2$

$\therefore y = 5.2$ m

$x = \sqrt{(5.2^2 + 3.9^2)}$

$x = 6.5$ m

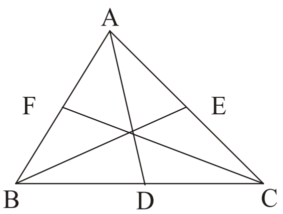
173. (d) In any triangle, sum of two sides is always greater than third side.

I. $AC - AB < BC \Rightarrow AC < BC + AB$ {True}

II. $BC - AC < AB \Rightarrow BC < AB + AC$ {True}

III. $AB - BC < AC \Rightarrow AB < AC + BC$ {True}

174. (c) 1. Perimeter of triangle is greater than the sum of 3 medians



Let ABC be the triangle and D, E and F are midpoints of BC, CA and AB respectively.

Recall that the sum of two sides of a traingle is greater than twice the median bisecting the third side, (Theorem)

Hence in ΔABD, AD is a median

$\Rightarrow AB + AC > 2(AD)$

Similarly, we get

$BC + AC > 2CF$

$BC + AB > 2BE$

On adding the above inequations, we get

$(AB + AC) + (BC + AC) + (BC + AB) > 2AD + 2CD + BE$

$2(AB + BC + AC) > 2(AD + BE + CF)$

$\therefore AB + BC + AC > AD + BE + CF$

2. In triangle ABD, $AB + BD > AD$ [because, the sum of any two sides of a triangle is always greater than the third side] ...(1)

Similarly,

In triangle ADC, $AC + DC > AD$ ...(2)

Adding 1 and 2 we get,

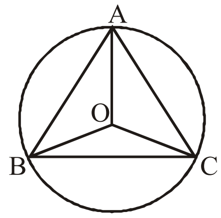
$AB + BD + AC + DC > AD + AD$

$\Rightarrow AB + (BD + DC) + AC > 2AD$

$\Rightarrow AB + BC + AC > 2AD$

Hence, both statements 1 and 2 are true.

175. (d)



Here, radius $OA = OB = OC = r = 20\sqrt{3}$ cm

We know that, side of equilateral triangle

$= \sqrt{3} \times$ radius of circumcircle

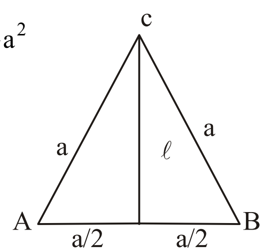
$= \sqrt{3} \times 20\sqrt{3} = 60$ cm

176. (c) For equilateral triangle circumcenter and centroid are the same points.

So distance from vertex = radius of circumcircle

$= 20\sqrt{3}$

177. (a) Area of eq. triangles $= \frac{\sqrt{3}}{4}a^2$



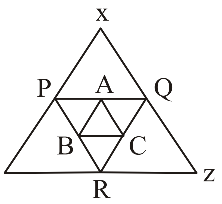
$\ell = \sqrt{a^2 - \left(a/2\right)^2} = \frac{\sqrt{3}}{2}a$

$\therefore\ a = \frac{2}{\sqrt{3}}\ell$

$= \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}\left(\frac{2}{\sqrt{3}}\ell\right)^2$

$= \frac{\sqrt{3}}{4} \times \frac{4}{3}\ell^2 = \frac{\sqrt{3}\ell^2}{3}$

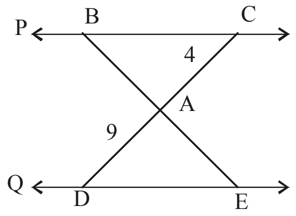
178. (b) Area of $\Delta XYZ = 128\ cm^2$



Area of $\Delta PQR = \frac{128}{4} = 32 cm^2$

Area of $\Delta ABC = \frac{32}{4} = 8 cm^2$

179. (c)



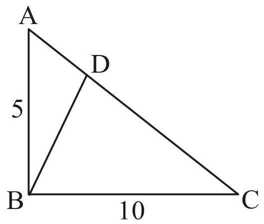
Ratio of area = (Ratio of sides)^2

$$\therefore \quad \frac{\Delta ABC}{\Delta ADE} = \left(\frac{AC}{DA}\right)^2 = \left(\frac{AB}{AE}\right)^2 = \left(\frac{BC}{DE}\right)^2$$

$$\frac{\Delta ABC}{\Delta ADE} = \left(\frac{AC}{DA}\right)^2$$

$$\therefore \quad \frac{\Delta ABC}{\Delta ADE} = \left(\frac{4}{9}\right)^2 = \frac{16}{81}$$

180. (b) From properties of perpendicular in a right angled triangle



$$\frac{1}{BD^2} = \frac{1}{BC^2} + \frac{1}{AB^2}$$

$$= \frac{1}{25} + \frac{1}{100} = \frac{1}{20}$$

$BD = \sqrt{20} = 2\sqrt{5}$ cm

181. (c) Area will be maximum when P and B will be same

So $P^2 + P^2 = H^2 \Rightarrow P^2 = \frac{H^2}{2}$

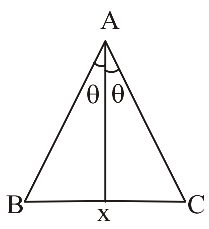
$$\Rightarrow \quad P = \frac{H}{\sqrt{2}}$$

$$\text{Area} = \frac{1}{2}BP = \frac{1}{2}P^2 = \frac{1}{2}.\frac{H^2}{2} = \frac{H^2}{4}$$

$$= \frac{100}{4} = 25\ cm^2$$

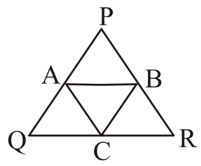
182. (c) In a right angle triangle. Its orthocentre lies on the triangle

183. (b)

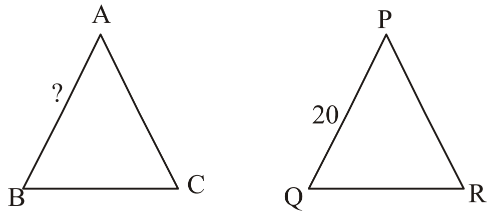


Here, AB > BX

184. (b) Perimeter of $\Delta ABC = 22$
Perimeter of ΔPQR
$= 2(AB + BC + AC)$
$= 2 \times 22 = 44$



185. (b)



Perimeter of $\Delta ABC = 75$ cm
Perimeter of $\Delta PQR = 50$ cm

So, $\frac{\Delta ABC}{\Delta PQR} = \frac{AB}{PQ} \quad \Rightarrow \quad \frac{75}{50} = \frac{?}{20}$

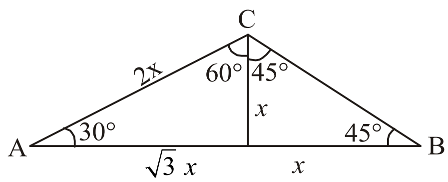
$$\therefore \quad ? = \frac{75 \times 20}{50} = 30\ cm$$

186. (c) Let the triangle is as like it shown in the fig. then;

$\sqrt{3} + x = \sqrt{3} + 1 \Rightarrow x = 1$

$$\text{Area} = \frac{1}{2} \times x \times \left(\sqrt{3} + 1\right)x$$

$$= \frac{\left(\sqrt{3}+1\right)}{2}\ cm^2$$



187. (c) $(3x)^2 + (4\sqrt{y})^2 = (5\sqrt[3]{z})^2$
$9x^2 + 16y = 25z^{2/3}$
if putting value x = 1, y = 1, z = 1
then, 9 + 16 = 25 (satisfy the equation)
$\therefore$ x = y = z = 1

188. (a) Option (a) by definition

189. (a) Area of all three sectors $= 3 \times \frac{\pi}{3} \times \frac{(2)^2}{2} = 2\pi$

Area of equilateral triangle $= \frac{\sqrt{3}}{4} \times 4^2 = 4\sqrt{3}$

Area of shaded portion $= 4\sqrt{3} - 2\pi$

CHAPTER 23

Quadrilateral and Polygon

1. An obtuse angle made by a side of a parallelogram $PQRS$ with other pair of parallel sides is 150°. If the perpendicular distance between these parallel sides (PQ and SR) is 20 cm, what is the length of the side RQ? **[2007-I]**

(a) 40 cm (b) 50 cm
(c) 60 cm (d) 70 cm

2. **Assertion (A)** If the side of a rhombus is 10 cm. Its diagonals should have values 16 cm and 12 cm.

Reason (R) The diagonals of a rhombus cut at right angles. **[2007-II]**

(a) A and R are correct and R is correct explanation of A
(b) A and R are correct but R is not correct explanation of A
(c) A is correct but R is wrong
(d) A is wrong but R is correct

3. An equilateral triangle and a regular hexagon are inscribed in a given circle. If a and b are the lengths of their sides respectively, then which one of the following is correct. **[2007-II]**

(a) $a^2 = 2b^2$ (b) $b^2 = 3a^3$
(c) $b^2 = 2a^2$ (d) $a^2 = 3b^2$

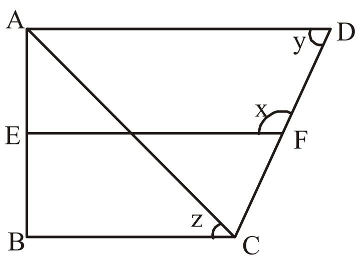
4. In a cricket match, the first 5 bastmen of a team scored runs : 30, 40, 50, 30 and 40. If these data represent a four sided figure with 50 as its one of the diagonals, then what does second diagonal represent? **[2007-II]**

(a) 30 runs (b) 40 runs
(c) 50 runs (d) 70 runs

5. The incircle of a quadrilateral of perimeter $2p$ has radius r. What is the area of the quadrilateral? **[2007-II]**

(a) $p(r + 1)$ (b) 2 pr
(c) pr (d) None of these

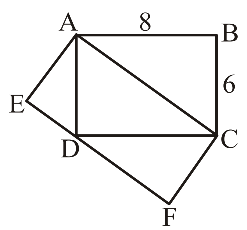
6.



$ABCD$ is a trapezium in which EF is parallel to BC. $\angle x = 120°$ and $\angle x = 50°$, then what is $\angle y$? **[2007-II]**

(a) 50° (b) 60°
(c) 70° (d) 80°

7.



$ABCD$ is a rectangle of dimensions 8 units and 6 units. AEFC is a rectangle drawn in such a way that diagonal AC of the first rectangle is one side and side opposite to it is touching the first rectangle at D as shown in the figure given above. What is the ratio of the area of rectangle $ABCD$ to that of $AEFC$? **[2008-II]**

(a) 2 (b) 3/2
(c) 1 (d) 8/9

8. $ABCD$ is a square. The diagonals AC and BD meet at O. Let K, L be the points on AB such that AO = AK and $BO = BL$. If $\theta = \angle LOK$, then what is the value of $\tan\theta$? **[2008-II]**

(a) $1/\sqrt{3}$ (b) $\sqrt{3}$
(c) 1 (d) 1/2

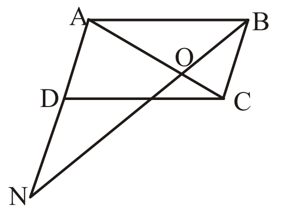
9. Two sides of a parallelogram are 10 cm and 15 cm. If the altitude corresponding to the side of length 15 cm is 5 cm, then what is the altitude to the side of length 10 cm? **[2009-I]**

(a) 5 cm (b) 7.5 cm
(c) 10 cm (d) 15 cm

10. Which one of the following figures has only one line of symmetry? **[2009-I]**

(a) Rhombus (b) Rectangle
(c) Isosceles trapezium (d) Parallelogram

11.



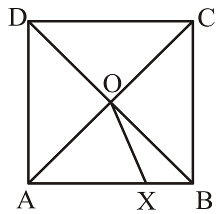
In the figure given above, M is the mid-point of the side CD of the parallelogram $ABCD$. What is ON : OB ? **[2009-I]**

(a) 3 : 2 (b) 2 : 1
(c) 3 : 1 (d) 5 : 2

12. ABC is a triangle in which $AB = AC$. Let BC be produced to D. From a point E on the line AC let EF be a straight line such that EF is parallel to AB. Consider the quadrilateral ECDF thus formed. If $\angle ABC = 65°$ and $\angle EFD = 80°$, then what is $\angle D$ equal to? **[2009-II]**

(a) 43° (b) 41°
(c) 37° (d) 35°

13.



In the figure given above, $ABCD$ is a square in which $AO = AX$ What is $\angle XOB$? **[2009-II]**

(a) 22.5° (b) 25°

(c) 30° (d) 45°

14. The quadrilateral formed by joining the mid-points of the sides AB, BC, CD and DA of a quadrilateral $ABCD$ is **[2009-II]**

(a) a trapezium but not a parallelogram

(b) a quadrilateral but not a trapezium

(c) a parallelogram only

(d) a rhombus

15. $ABCD$ is a square, P, Q, R and S are points on the sides AB, BC, CD and DA respectively such that $AP = BQ = CR = DS$. What is $\angle SPQ$ equal to? **[2010-I]**

(a) 30° (b) 45°

(c) 60° (d) 90°

16. The middle points of the parallel sides AB and CD of a parallelogram $ABCD$ are P and Q, respectively. If AQ and CP divide the diagonal BD into three parts BX, XY and YD, then which one of the following is correct? **[2010-I]**

(a) $BX \neq XY \neq YD$ (b) $BX = YD \neq XY$

(c) $BX = XY = YD$ (d) $XY = 2BX$

17. A parallelogram and a rectangle stand on the same base and on the same side of the base with the same height. If I_1, I_2 be the perimeters of the parallelogram and the rectangle respectively, then which one of the following is correct? **[2010-I]**

(a) $I_1 < I_2$ (b) $I_1 = I_2$

(c) $I_1 > I_2$ but $I_1 \neq 2I_2$ (d) $I_1 = 2l_2$

18. Two similar parallelograms have corresponding sides in the ratio $1 : k$. What is the ratio of their areas? **[2010-I]**

(a) $1 : 3k^2$ (b) $1 : 4k^2$

(c) $1 : k^2$ (d) $1 : 2k^2$

19. Consider the following statements in respect of a quadrilateral.

I. The line segments joining the mid-points of the two pairs of opposite sides bisect each other at the point of intersection.

II. The area of the quadrilateral formed by joining the mid-points of the four adjacent sides is half of the total area of the quadrilateral.

Which of the statements given above is/are correct? **[2010-I]**

(a) Only I (b) Only II

(c) Both I and II (d) Neither I nor II

20. Let WXYZ be a square. If P, Q and R be the mid-points of WX, XY and ZW, respectively and K, L be the mid-point of PQ and PR, respectively. Then, what is the value of $\dfrac{\text{area of } \Delta PKL}{\text{area of square } WXYZ}$? **[2010-I]**

(a) $\frac{1}{32}$ (b) $\frac{1}{16}$ (c) $\frac{1}{8}$ (d) $\frac{1}{64}$

DIRECTIONS (Qs. 21-23) *Read the following information carefully to answer the questions that follow.*

Let $ABCD$ be a quadrilateral. Let the diagonals AC and BD meet at O. Let the perpendicular drawn from A to CD, meet CD at E. Further, $AO : OC = BO : OD$, $AB = 30$ cm, $CD = 40$ cm and the area of the quadrilateral $ABCD$ is 1050 sq cm. **[2010-II]**

21. What is BE equal to?

(a) 30 cm (b) $30\sqrt{2}$ cm

(c) $30\sqrt{3}$ cm (d) None of these

22. What is the area of the ΔADC equal to?

(a) 300 cm^2 (b) 450 cm^2

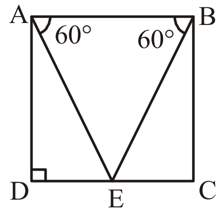
(c) 600 cm^2 (d) None of these

23. What is $\angle AEB$ equal to?

(a) 30° (b) 45°

(c) 60° (d) None of these

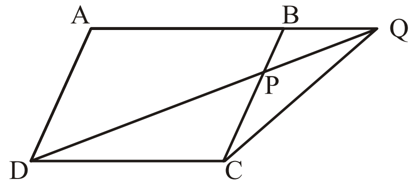
24. In the given figure, $ABCD$ is a quadrilateral with AB parallel to DC and AD parallel to BC, ADC is a right angle. If the perimeter of the ΔABE is 6 units. What is the area of the quadrilateral ? **[2010-II]**



(a) $2\sqrt{3}$ sq units (b) 4 sq units

(c) 3 sq units (d) $4\sqrt{3}$ sq units

25. In the figure given below, $ABCD$ is a parallelogram. P is a point in BC such that $PB : PC = 1 : 2$. DP produced meets AB produced at Q. If the area of the ΔBPQ is 20 sq units, what is the area of the ΔDCP? **[2010-II]**



(a) 20 sq units (b) 30 sq units

(c) 40 sq units (d) None of these

26. The sides of a parallelogram are 12 cm and 8 cm long and one of the diagonals is 10 cm long. If d is the length of other diagonal, then which one of the following is correct? **[2012-I]**

(a) $d < 8$ cm (b) 8 cm $< d <$ 10 cm

(c) 10 cm $< d <$ 12 cm (d) $d > 12$ cm

27. $ABCD$ is a rhombus with diagonals AC and BD. Then, which one among the following is correct? **[2012-I]**

(a) AC and BD bisect each other but not necessarily perpendicular to each other

(b) AC and BD are perpendicular to each other but not necessarily bisect each other

(c) AC and BD bisect each other and perpendicular to each other

(d) AC and BD neither bisect each other nor perpendicular to each other.

28. Let $ABCD$ be a parallelogram. Let m and n be positive integers such that $n < m < 2n$. Let $AC = 2$ mn, $BD = m^2 - n^2$ and $AB = (m^2 + n^2)/2$.

Statement I $AC > BD$

Statement II $ABCD$ is rhombus

Which one of the following is correct in respect of the above statements? **[2012-I]**

(a) Both Statements I and II are true and Statement II is the correct explanation of Statement I
(b) Both Statement I and II are true but Statement II is not the correct explanation of Statement I
(c) Statement I is true but Statement II is false
(d) Statement II is true but Statement I is false

29. $ABCD$ is a rectangle. Let E be a point on AB and F be a point on CD, such that DE is parallel to BF. If AE = 3 cm and if the area of ΔBFC = 6 sq cm.

Consider the following statements

I. Area of rectangle ABCD can be of the form pq^2 sq cm, where p and q are distinct primes.
II. Area of the figure $EBFD$ is of the form r^2 sq cm, where r is rational but not an integer.

Which of the above statements is/are correct? **[2012-I]**

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

30. Let $LMNP$ be a parallelogram and NR be perpendicular to LP. If the area of the parallelogram is six times the area of Δ RNP and RP = 6 cm what is LR equal to? **[2012-I]**

(a) 15 cm (b) 12 cm
(c) 9 cm (d) 8 cm

31. If the diagonals of a quadrilateral are equal and bisect each other at right angles, then the quadrilateral is a **[2012-II]**

(a) rectangle (b) square
(c) rhombus (d) trapezium

32. If two parallel lines are cut by two distinct transversals, then the quadrilateral formed by the four lines is always a **[2012-II]**

(a) square (b) parallelogram
(c) rhombus (d) trapezium

33. $ABCD$ is a parallelogram. If the bisectors of the $\angle A$ and $\angle C$ meet the diagonal BD at point P and Q respectively, then which one of the following is correct? **[2012-II]**

(a) $PCQA$ is a straight line
(b) ΔAPQ is similar to ΔPCQ
(c) $AP = CP$
(d) $AP = AQ$

34. Let X be any point within a square $ABCD$. On AX a square $AXYZ$ is described such that D is within it. Which one of the following is correct ? **[2012-II]**

(a) $AX = DZ$ (b) $\angle ADZ = \angle BAX$
(c) $AD = DZ$ (d) $BX = DZ$

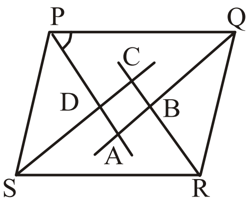
35. The locus of a point in rhombus $ABCD$ which is equidistant from A and C is **[2012-II]**

(a) a fixed point on diagonal BD
(b) diagonal BD
(c) diagonal AC
(d) None of the above

36. $ABCD$ is a trapezium with parallel sides AB = 2 cm and DC = 3 cm. E and F are the mid-points of the non-parallel sides. The ratio of area of $ABFE$ to area of $EFCD$ is **[2013-I]**

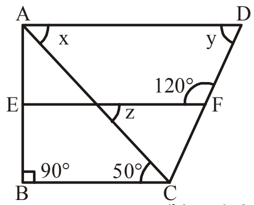
(a) 9 : 10 (b) 8 : 9
(c) 9 : 11 (d) 11 : 9

37. In the figure given below, $PQRS$ is a parallelogram. If AP, AQ, CR and CS are bisectors of $\angle P$, $\angle Q$, $\angle R$ and $\angle S$ respectively, then $ABCD$ is a **[2013-I]**



(a) square (b) rhombus
(c) rectangle (d) None of these

38. In the figure given above, $ABCD$ is a trapezium. EF is parallel to AD and BG. $\angle y$ is equal to **[2013-I]**



(a) 30° (b) 45°
(c) 60° (d) 65°

39. A quadrilateral $ABCD$ is inscribed in a circle. If AB is parallel to CD and $AC = BD$, then the quadrilateral must be a **[2013-II]**

(a) parallelogram (b) rhombus
(c) trapezium (d) None of these

40. $ABCD$ is a quadrilateral such that $BC = BA$ and $CD > AD$. Which one of the following is correct? **[2013-II]**

(a) $\angle BAD = \angle BCD$ (b) $\angle BAD < \angle BCD$
(c) $\angle BAD > \angle BCD$ (d) None of these

41. Two light rods $AB = a + b$, $CD = a - b$ symmetrically lying on a horizontal AB. There are kept intact by two strings AC and BD. The perpendicular distance between rods is a. The length of AC is given by **[2014-I]**

(a) a (b) b
(c) $\sqrt{a^2 - b^2}$ (d) $\sqrt{a^2 + b^2}$

42. If $PQRS$ be a rectangle such $PQ = \sqrt{3}\,QR$. Then, what is $\angle PRS$ equal to? **[2014-I]**

(a) 60° (b) 45°
(c) 30° (d) 15°

43. In a trapezium, the two non-parallel sides are equal in length, each being of 5 cm. The parallel sides are at a distance of 3 cm apart. If the smaller side of the parallel sides is of length 2 cm, then the sum of the diagonals of the trapezium is **[2014-I]**

(a) $10\sqrt{5}$ cm (b) $6\sqrt{5}$ cm
(c) $5\sqrt{5}$ cm (d) $3\sqrt{5}$ cm

44. The area of a rectangle lies between 40 cm^2 and 45 cm^2. If one of the sides is 5 cm, then its diagonal lies between **[2014-I]**
(a) 8 cm and 10 cm (b) 9 cm and 11 cm
(c) 10 cm and 12 cm (d) 11 cm and 13 cm

45. Let $ABCD$ be a parallelogram. Let P, Q, R and S be the mid-points of sides AB, BC, CD and, DA, respectively. Consider the following statements.
I. Area of triangle APS < Area of triangle DSR, if $BD < AC$.
II. Area of triangle ABC = 4 [Area of triangle BPQ].
Select the correct answer using the codes given below. **[2014-I]**
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

46. Consider the following statements
I. Let $ABCD$ be a parallelogram which is not a rectangle.
then, $2(AB^2 + BC^2) \neq AC^2 + BD^2$
II. If $ABCD$ is a rhombus with AB = 4cm, then $AC^2 + BD^2 = n^3$ for some positive integer n.
Which of the above statements is/are correct? **[2014-I]**
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

47. $ABCD$ is a parallelogram, E is a point on BC such that $BE : EC = m : n$. If AE and DB intersect in F, then what is the ratio of the area of ΔFEB to the area of ΔAFD? **[2014-I]**
(a) m/n (b) $(m/n)^2$
(c) (n/m^2) (d) $[m/(m+n)]^2$

48. Let $ABCD$ be a parallelogram. Let X and Y be the mid–points of the sides BC and AD, respectively. Let M and N be the mid–points of the sides AB and CD, respectively.
Consider the following statements: **[2014-II]**
1. The straight line MX cannot be parallel to YN.
2. The straight lines AC, BD, XY and MN meet at a point.
Which of the above statements is/are correct ?
(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

49. What is the maximum distance between two points of a cube of side 2 cm ? **[2014-II]**
(a) $\sqrt{3}$ cm (b) $2\sqrt{3}$ cm
(c) $4\sqrt{3}$ cm (d) $2\sqrt{2}$ cm

DIRECTIONS (Qs. 50–52) *Read the following information carefully and answer the given questions that follow.*

A piece of land is in the form of a parallelogram and the perimeter of the land is 86m. The length of one side exeeds the other by 13 m and one of the diagonals is 41m. **[2014-II]**

50. What is the area of the parallelogram ?
(a) 63 m^2 (b) 96 m^2
(c) 126 m^2 (d) 252 m^2

51. What is the shorter height of the parallelogram ?
(a) 9.0 m (b) 7.5 m
(c) 5.5 m (d) 4.5 m

52. Consider the following statements :
1. The difference between the diagonals of the parallelogram is more than 20 m.
2. The difference between the heights of the parallelogram is more than 10 m.
Which of the above statements is / are correct ?
(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

53. What is the number of pairs of perpendicular planes in a cuboid ? **[2014-II]**
(a) 4 (b) 8
(c) 12 (d) None of these

54. How many equilateral triangles can be formed by joining any three vertices of a cube ? **[2014-II]**
(a) 0 (b) 4
(c) 8 (d) None of these

DIRECTIONS (Qs. 55-56) *Read the following information carefully and answer the given questions that follow.*

ABCD is a trapezium, in which AB is parallel to CD. Let M be the mid-point of BC. **[2014-II]**

55. Consider the following statements :
1. 'Area of ΔADM + Area of ΔDCM' is equal to three-fourth of the area of trapezium $ABCD$, if $AB = CD$.
2. 'Area of ΔDCM + Area of ΔABM' is always greater than half of the area of trapezium $ABCD$.
Which of the above statements is/are correct?
(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

56. Consider the following statements :
1. 'Area of ΔADM – Area of ΔABM' is always equal to area of ΔDCM, if $AB = CD$.
2. Half of area of ΔABM is equal to one-eight of area of trapezium $ABCD$, if $AB = CD$.
Which of the above statements is /are correct?
(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

57. $ABCD$ is a parallelogram. P and R are the mid-points of DC and BC, respectively. The line PR intersects the diagonal AC at Q. The distance CQ will be **[2014-II]**
(a) $AC/4$ (b) $BD/3$
(c) $BD/4$ (d) $AC/3$

58. Bisectors of two adjacent angles A and B of a quadrilateral $ABCD$ intersect each other at a point P. Which one of the following is correct ? **[2014-II]**
(a) $2\angle APB = \angle C + \angle D$
(b) $\angle APB = \angle C + \angle D$
(c) $\angle APB = 180° - (\angle C + \angle B)$
(d) $\angle APB = 180° - (\angle C + \angle D)$

59. If each interior angle of a regular polygon is 135°, then the number of diagonals of the polygon is equal to **[2015-I]**
(a) 54 (b) 48
(c) 20 (d) 18

60. AD is the diameter of a circle and AB is a chord. If AD = 34 cm, AB = 30 cm, the distance of AB from the centre of the circle is **[2015-I]**
(a) 17 cm (b) 15 cm
(c) 13 cm (d) 8 cm

61. If a star figure is formed by elongating the sides of a regular pentagon, then the measure of each angle at the angular points of the star figure is **[2015-I]**
(a) 36° (b) 35°
(c) 32° (d) 30°

62. The area of a rhombus with side 13 cm and one diagonal 10 cm will be **[2015-I]**
(a) 140 square cm (b) 130 square cm
(c) 120 square cm (d) 110 square cm

63. The diagonals of a trapezium are at right angles, and the slant sides, if produced, form an equilateral triangle with the greater of the two parallel sides. If the area of the trapezium is 16 square cm, then the distance between the parallel sides is **[2015-I]**
(a) 2 cm
(b) 4 cm
(c) 8 cm
(d) Cannot be determined due to insufficient data

64. ABCD is a parallelogram with AB and AD as adjacent sides. If $\angle A = 60°$ and AB = 2AD, then the diagonal BD will be equal to **[2015-II]**
(a) $\sqrt{2}$AD (b) $\sqrt{3}$AD
(c) 2AD (d) 3AD

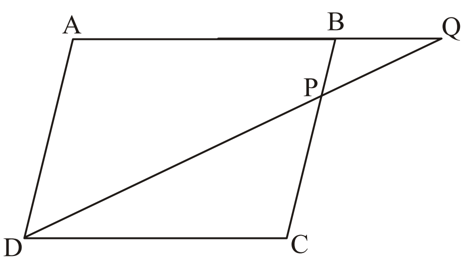
65. If X is any point within a square ABCD and on AX a square AXYZ is described, which of the following is/are correct? **[2015-II]**
1. BX = DZ or BZ = DX
2. $\angle ABX = \angle ADZ$ or $\angle ADX = \angle ABZ$

Select the correct answer using the code given below:
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

66. ABCD is a square. If the sides AB and CD are increased by 30%, sides BC and AD are increased by 20%, then the area of the resulting rectangle exceeds the area of the square by **[2015-II]**
(a) 50% (b) 52%
(c) 54% (d) 56%

67. Three rectangles R_1, R_2 and R_3 have the same area. Their lengths x_1, x_2 and x_3 respectively are such that $x_1 < x_2 < x_3$. If V_1, V_2 and V_3 are the volumes of the cylinders formed from the rectangles R_1, R_2 and R_3 respectively by joining the parallel sides along the breadth, then **[2015-II]** which one of the following is correct?
(a) $v_3 < v_2 < v_1$ (b) $v_1 < v_3 < v2$
(c) $v_1 < v_2 < v_3$ (d) $v_3 < v_1 < v_2$

68.



In the above figure, ABCD is a parallelogram. P is a point on BC such that PB : PC = 1 : 2. DP and AB when both produced meet at Q. If area of triangle BPQ is 20 square unit, the area of triangle DCP is **[2015-II]**
(a) 20 square unit (b) 30 square unit
(c) 40 square unit (d) None of the above

69. A circle of radius r is inscribed in a regular polygon with n sides (the circle touches all sides of the polygon). If the perimeter of the polygon is p, then the area of the polygon is **[2015-II]**
(a) $(p + n)\,r$ (b) $(2p - n)\,r$
(c) $\frac{pr}{2}$ (d) None of the above

70. A rhombus is formed by joining midpoints of the sides of a rectangle in the suitable order. If the area of the rhombus is 2 square units, then the area of the rectangle is **[2016-I]**
(a) $2\sqrt{2}$ square units (b) 4 square units
(c) $4\sqrt{2}$ square units (d) 8 square units

71. If each interior angle of a regular polygon is 140°, then the number of vertices of the polygon is equal to **[2016-I]**
(a) 10 (b) 9 (c) 8 (d) 7

72. A square is inscribed in a right-angled triangle with legs p and q, and has a common right angle with the triangle. The diagonal of the square is given by **[2016-I]**
(a) $\frac{pq}{p+2q}$ (b) $\frac{pq}{2p+q}$
(c) $\frac{\sqrt{2}pq}{p+q}$ (d) $\frac{2pq}{p+q}$

73. Consider the following statements: **[2016-I]**
1. If $n \geq 3$ and $m \geq 3$ are distinct positive integers, then the sum of the exterior angles of a regular polygon of m sides is different from the sum of the exterior angles of a regular polygon of *n* sides.
2. Let m, n be integers such that $m > n \geq 3$. Then the sum of the interior angles of a regular polygon of m sides is greater than the sum of the interior angles of a regular polygon of n sides, and their sum is $(m+n)\frac{\pi}{2}$.

Which of the above statements is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

74. Consider the following statements: **[2016-I]**
1. There exists a regular polygon whose exterior angle is 70°.
2. Let $n \geq 5$. Then the exterior angle of any regular polygon of *n* sides is acute.

Which of the above statements is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

75. If a quadrilateral has an inscribed circle, then the sum of a pair of opposite sides equals **[2016-II]**
(a) Half the sum of the diagonals
(b) Sum of the other pair of opposite sides
(c) Sum of two adjacent sides
(d) None of the above

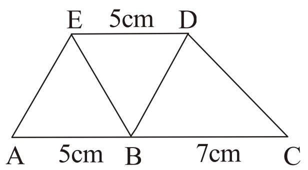
76. Let ABCD be a rectangle. Let P, Q, R, S be the mid-points of sides AB, BC, CD, DA respectively. Then the quadrilateral PQRS is a **[2017-I]**
(a) Square
(b) Rectangle, but need not be a square
(c) Rhombus, but need not be a square
(d) Parallelogram, but need not be a rhombus

77. ABCDEF is a regular polygon. Two poles at C and D are standing vertically and subtend angles of elevation 30° and 60° at A respectively. What is the ratio of the height of the pole at C to that of the pole at D ? **[2017-I]**
(a) 1 : 1 (b) $1:2\sqrt{3}$
(c) $2\sqrt{3}:1$ (d) $2:\sqrt{3}$

78. In a trapezium ABCD, AB is parallel to CD and the diagonals intersect each other at O. What is the ratio of OA to OC equal to ? **[2017-I]**
(a) Ratio of OB to OD (b) Ratio of BC to CD.
(c) Ratio of AD to AB (d) Ratio of AC to BD

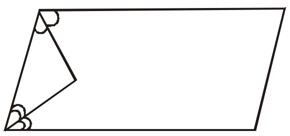
79. ABCD is a rectangle. The diagonals AC and BD intersect at O If AB = 32 cm and AD = 24 cm, then what is OD equal to ? **[2017-I]**
(a) 22 cm (b) 20 cm
(c) 18 cm (d) 16 cm

80. In the figure given below, AC is parallel to ED and AB = DE = 5 cm and BC = 7 cm. What is the area ABDE : area BDE : area BCD equal to ? **[2017-I]**



(a) 10 : 5 : 7 (b) 8 : 4 : 7
(c) 2 : 1 : 2 (d) 8 : 4 : 5

81. In the figure given below, PQRS is a parallelogram. PA bisects angle P and SA bisects angle S. What is angle PAS equal to? **[2017-I]**



(a) 60° (b) 75°
(c) 90° (d) 100°

82. If a point O in the interior of a rectangle ABCD is joined with each of the vertices A, B, C and D, then $OB^2 + OD^2$ will be equal to **[2017-II]**
(a) $2OC^2 + OA^2$ (b) $OC^2 - OA^2$
(c) $OC^2 + OA^2$ (d) $OC^2 + 2OA^2$

83. The diagonals of a rhombus are of length 20 cm and 48 cm. What is the length of a side of the rhombus? **[2017-II]**
(a) 13 cm (b) 26 cm
(c) 36 cm (d) 39 cm

84. A closed polygon has six sides and one of its angles is 30° greater than each of the other five equal angles. What is the value of one of the equal angles? **[2017-II]**
(a) 55° (b) 115°
(c) 150° (d) 175°

85. Given that the angles of a polygon are all equal and each angle is a right angle. **[2018-1]**
Statement-1 : The polygon has exactly four sides.
Statement-2 : The sum of the angles of a polygon having n sides is (3n – 8) right angles.
Which one of the following is correct in respect of the above statements?
(a) Both Statement-1 and Statement-2 are true and Statement-2 is the correct explanation of Statement-1
(b) Both Statement-1 and Statement-2 are true but Statement-2 is not the correct explanation of Statement-1
(c) Statement-1 is true but Statement-2 is false
(d) Statement-1 is false but Statement-2 is true

86. The number of sides of two regular polygons are in the ratio 5 : 4. The difference between their interior angles is 9º. Consider the following statements : **[2018-II]**
1. One of them is a pentagon and the other is a rectangle.
2. One of them is a decagon and the other is an octagon.
3. The sum of their exterior angle is 720º.
Which of the above statements is/are correct?
(a) 1 only (b) 2 only
(c) 1 and 3 (d) 2 and 3

87. Let S be the parallelogram obtained by joining the mid-points of the parallelogram T. Consider the following statements :
1. The ratio of area of T to that of S is 2 : 1.
2. The perimeter of S is half of the sum of diagonals of T.
Which of the above statements is/are correct? **[2018-II]**
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

88. The corners of a square of side 'a' are cut away so as to form a regular octagon. What is the side of the octagon ? **[2019-I]**
(a) $a\left(\sqrt{2}-1\right)$ (b) $a\left(\sqrt{3}-1\right)$
(c) $\dfrac{a}{\sqrt{2}+2}$ (d) $\dfrac{a}{3}$

89. Consider the following statements : **[2019-I]**
1. An isosceles trapezium is always cyclic,
2. Any cyclic parallelogram is a rectangle.
Which of the above statements is/are correct ?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

Consider the following for the next three (03) items :
$ABCD$ is a quadrilateral with $AB = 9$ cm, $BC = 40$ cm, $CD = 28$ cm, $DA = 15$ cm and angle ABC is a right-angle. **[2019-I]**

90. What is the area of triangle ADC ?
(a) 126 cm^2 (b) 124 cm^2
(c) 122 cm^2 (d) 120 cm^2

91. What is the area of quadrilateral $ABCD$?
(a) 300 cm^2 (b) 306 cm^2
(c) 312 cm^2 (d) 316 cm^2

92. What is the difference between perimeter of triangle ABC and perimeter of triangle ADC ?
(a) 4 cm (b) 5 cm
(c) 6 cm (d) 7 cm

93. Consider a trapezium *ABCD,* in which *AB* is parallel to *CD* and *AD* is perpendicular to *AB*. If the trapezium has an incircle which touches *AB* at *E* and *CD* at *F*, where *EB* = 25 cm and *FC* = 16 cm, then what is the diameter of the circle? **[2019-II]**

(a) 16 cm (b) 25 cm
(c) 36 cm (d) 40 cm

94. An equilateral triangle and a square are constructed using metallic wires of equal length. What is the ratio of area of triangle to that of square ? **[2019-II]**

(a) 3 : 4 (b) 2 : 3
(c) $4\sqrt{3}:9$ (d) $2\sqrt{3}:9$

95. All the four sides of a parallelogram are of equal length. The diagonals are in the ratio 1 : 2. If the sum of the lengths of the diagonals is 12 cm, then what is the area of the parallelogram ? **[2019-II]**

(a) 9 cm^2 (b) 12 cm^2
(c) 16 cm^2 (d) 25 cm^2

96. The sides *AD, BC* of a trapezium *ABCD* are parallel and the diagonals *AC* and *BD* meet at *O.* If the area of triangle *AOB* is 3cm^2 and the area of triangle *BDC* is 8cm^2, then what is the area of triangle *AOD* ? **[2019-II]**

(a) 8 cm^2 (b) 5 cm^2
(c) 3.6 cm^2 (d) 1.8 cm^2

97. Let *PQRS* be a parallelogram whose diagonals *PR* and *QS* intersect at *O.* If triangle *QRS* is an equilateral triangle having a side of length 10 cm, then what is the length of the diagonal *PR* ? **[2019-II]**

(a) $5\sqrt{3}$ cm (b) $10\sqrt{3}$ cm
(c) $15\sqrt{3}$ cm (d) $20\sqrt{3}$ cm

98. *ABCD* is a plate in the shape of a parallelogram. *EF* is the line parallel to *DA* and passing through the point of intersection *O* of the diagonals *AC* and *BD.* Further, *E* lies on *DC* and *F* lies on *AB*. The triangular portion *DOE* is cut out from the plate *ABCD.* What is the ratio of area of remaining portion of the plate to the whole? **[2020-I]**

(a) $\frac{5}{8}$ (b) $\frac{5}{7}$
(c) $\frac{3}{4}$ (d) $\frac{7}{8}$

99. *ABCD* is a parallelogram where *AC* and *BD* are the diagonals. If $\angle BAD = 60°$, $\angle ADB = 90°$, then what is BD^2 equal to ? **[2020-I]**

(a) $\frac{3}{5}AB^2$ (b) $\frac{3}{4}AB^2$
(c) $\frac{1}{2}AB^2$ (d) $\frac{2}{3}AB^2$

100. A line through the vertex *A* of a parallelogram *ABCD* meets *DC* in *P* and *BC* produced in *Q.* If *P* is the mid-point of *DC,* then which of the following is/are correct ? **[2020-I]**

1. Area of ΔPDA is equal to that of ΔPCQ
2. Area of ΔQAB is equal to twice that of ΔPCQ Select the correct answer using the code given below :

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

101. If the diagonals of a rhombus are *x* and *y,* then what is its area? **[2020-I]**

(a) $\frac{xy}{2}$ (b) $\frac{xy}{4}$
(c) xy (d) $x^2 - y^2$

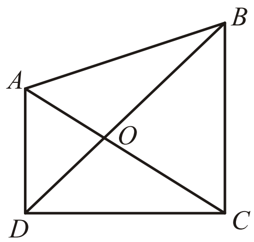
102. *ABCD* is a quadrilateral such that *AD* = *DC* = *CA* = 20 units, *BC* = 12 units and $\angle ABC = 90°$. What is the approximate area of the quadrilateral *ABCD* ? **[2020-I]**

(a) 269 sq. units (b) 300 sq. units
(c) 325 sq. units (d) 349 sq. units

103. *ABCD* is a trapezium, where *AB* is parallel to *DC.* If *AB* = 4 cm, *BC* = 3 cm, *CD* = 7 cm and *DA* = 2 cm, then what is the area of the trapezium ? **[2020-I]**

(a) $22\sqrt{\frac{2}{3}}$ cm^2 (b) $22\sqrt{\frac{3}{2}}$ cm^2
(c) $22\sqrt{3}$ cm^2 (d) $\frac{22\sqrt{2}}{3}$ cm^2

104. Consider the following statements with reference to the given figure : **[2020-I]**



1. The sum of the areas of ΔAOD and ΔBOC is equal to the sum of the areas of ΔAOB and ΔDOC.
2. $\angle AOD = \angle BOC$
3. $AB + BC + CD + DA > AC + BD$

Which of the above statements are correct ?

(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

HINTS & SOLUTIONS

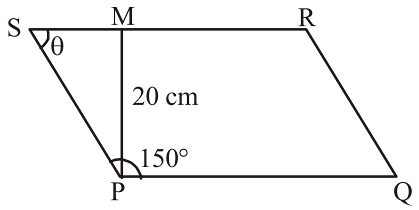
1. (a) Given that, $\angle SPQ = 150°$ and $PM = 20$ cm

In parallelogram $PQRS$,

$\angle RSP + SPQ = 180°$ (interior angles)

$\angle RSP = 180° - 150° = 30°$

$\Rightarrow \angle RSP = \theta = 30°$



In ΔPSM,

$$\sin\theta = \sin 30° = \frac{PM}{SP}$$

$$\Rightarrow \frac{1}{2} = \frac{20}{SP} \Rightarrow SP = 40 \text{ cm}$$

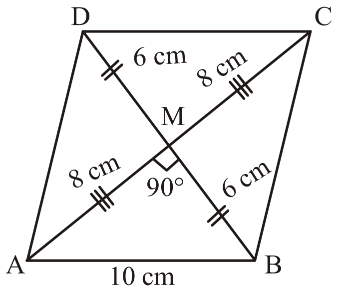
$\therefore RQ = SP = 40$ cm.

2. (a) $H^2 = P^2 + B^2$

$\therefore AB^2 + BC^2 + CD^2 + AD^2 = AC^2 + BD^2$

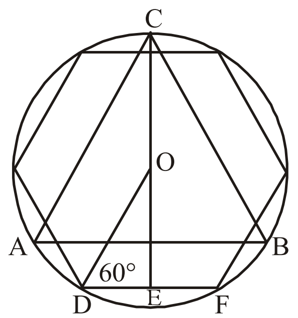
$\Rightarrow (10)^2 + (10)^2 + (10)^2 + (10)^2 = (16)^2 + (12)^2$

$\Rightarrow 400 = 400$



Hence, both A and R are true and R is the correct explanation of A.

3. (d) We know altitude of equilateral ΔABC is $\frac{\sqrt{3}}{2}a$.



$\therefore$ Length of $OC = \frac{\sqrt{3}}{2}a \times \frac{2}{3} = \frac{a}{\sqrt{3}}$ = radius

Also, $DF = b \Rightarrow DE = \frac{b}{2}$

In ΔODE, $\cos 60° = \frac{DE}{OD} = \frac{b/2}{a/\sqrt{3}}$

$$\Rightarrow \frac{1}{2} = \frac{\sqrt{3}b}{2a} \Rightarrow a = \sqrt{3}b$$

$\therefore a^2 = 3b^2$

4. (c) Here we see $(50)^2 = (30)^2 + (40)^2$

$\Rightarrow 2500 = 900 + 1600 \Rightarrow 2500 = 2500$

It means given scores are the sides of a rectangle. So, other diagonal should be 50 runs.

5. (c) We know that, if r be the radius of incircle and $2p$ be the perimeter of a quadrilateral, then

Area of quadrilateral = pr

6. (b) $ABCD$ is a trapezium.

$\therefore AD \| BC$ and $EF \| BC$ (given)

Hence, $EF \| AD$

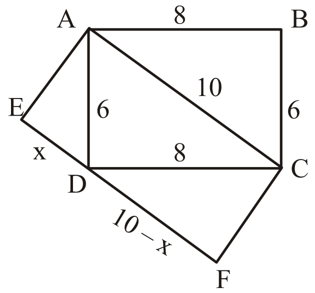
$\therefore \angle x + \angle y = 180°$(interior angles)

$\therefore \angle y = 180° - 120° = 60°$

7. (c) Let $ED = x$

and area of rectangle $ABCD$

$= AB \times BC = 8 \times 6 = 48$ units



Now, $AC = \sqrt{8^2 + 6^2} = 10$

In ΔAED, $AE^2 + ED^2 = AD^2$

$AE^2 = AD^2 - x^2 = 36 - x^2$ ...(*i*)

and in ΔCFD,

$CF^2 + DF^2 = CD^2$

$\Rightarrow CF^2 = (8)^2 - (10 - x)^2$ ... (*ii*)

From eqs. (*i*) and (*ii*), we get

$36 - x^2 = 64 - (10 - x)^2$ ($\because AE = FC$)

$\Rightarrow 36 - x^2 = 64 - (100 + x^2 - 20x)$

(because $AECE$ is rectangle)

$$\Rightarrow 20x = 72 \Rightarrow x = \frac{18}{5}$$

From eq. (i) $AE^2 = 36 - \left(\frac{18}{5}\right)^2$

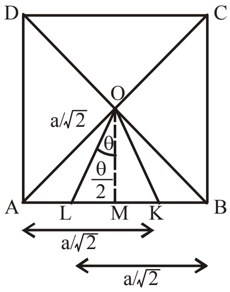
$$AE^2 = 36 - \frac{324}{25} = \frac{900-324}{25}$$

$$\Rightarrow \quad AE^2 = \frac{576}{25}$$

$$\Rightarrow \quad AE = \frac{24}{5}$$

$$\therefore \quad \frac{\text{Area of rectangle } ABCD}{\text{Area of rectangle } AEFC} = \frac{8\times 6}{10\times\frac{24}{5}} = 1$$

8. (c) Let sides of the square be a.



Then, $AC = a\sqrt{2}$ and $AO = OC = \frac{a}{\sqrt{2}}$

Here, $AM = \frac{a}{2}$

$\therefore \quad LM = \frac{a}{\sqrt{2}} - \frac{a}{2}$ and $OM = \frac{a}{2}$

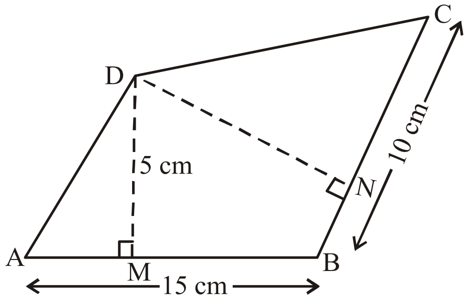
In ΔOML, $\tan\frac{\theta}{2} = \frac{\frac{a}{\sqrt{2}}-\frac{a}{2}}{\frac{a}{2}} = \frac{\frac{\sqrt{2}-1}{2}}{\frac{1}{2}} = \sqrt{2}-1$

$$\therefore \quad \tan\theta = \frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}} = \frac{2(\sqrt{2}-1)}{1-(2+1-2\sqrt{2})}$$

$$= \frac{2(\sqrt{2}-1)}{1-3+2\sqrt{2}} = \frac{2(\sqrt{2}-1)}{2\sqrt{2}-2}$$

$\Rightarrow \quad \tan\theta = 1$

9. (b) Area of parallelogram = Base × Height
= 15 × 5 = 75 sq cm

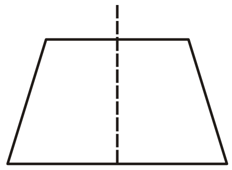


Area of parallelogram = Base × Height = 10 × DN

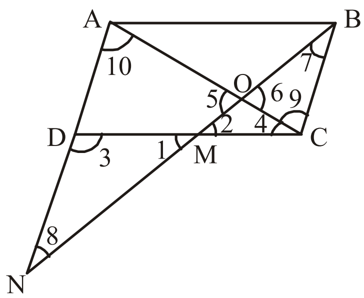
$\Rightarrow \quad 10 \times DN = 75$

$\therefore \quad DN = \frac{75}{10} = 7.5$ cm

10. (c) Isosceles trapezium has only line of symmetry.



11. (b) In ΔDMN and ΔBMC,
$DM = MC$ (mid-point) (given)



$\angle 1 = \angle 2$ (vertically opposite angle)
$\angle 3 = \angle 4 + \angle 9$ (alternate interior angle)
Since, $BC \| AD$ and intersects by CD.
$\Delta DMN \cong \Delta BMC$
$DN = BC = AD$ (ASA)

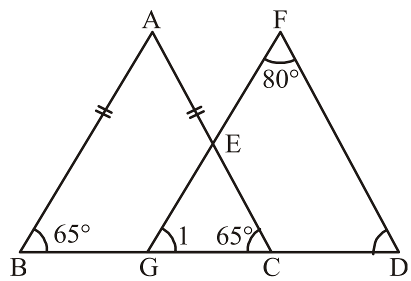
So, $AN = 2\,BC \Rightarrow \frac{AN}{BC} = \frac{2}{1}$ ... (i)

In ΔOAN an ΔOBC,
$\angle 5 = \angle 6$ (vertically opposite angle)
$\angle 7 = \angle 8$ (alternate interior angle)
$\angle 9 = \angle 10$ (rest angle)
$\therefore \quad \Delta OAN \sim \Delta OBC$
So, the sides will be in same ratio

$$\frac{AN}{BC} = \frac{ON}{OB}$$

$\Rightarrow \quad \frac{2}{1} = \frac{ON}{OB}$ [from eq. (i)]

12. (d) Here, $\angle B = \angle C = 65°$
Here, $GF \| AB$, which is intersects.



Now, $BG \quad \angle 1 = \angle B = 65°$

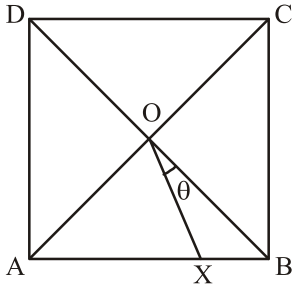
(corresponding angles)

In ΔFGD,

$\angle 1 + \angle F + \angle D = 180°$

$\Rightarrow \quad 65° + 80° + \angle D = 180° \Rightarrow \angle D = 35°$

13. (a) Let $\angle XOB = \theta$



In ΔOXB,

$\angle XOB + \angle OBX + \angle OXB = 180°$

$\Rightarrow \quad \theta + 45° + \angle OXB = 180°$

$\Rightarrow \quad \angle OXB = 180° - 45° - \theta$

$\Rightarrow \quad \angle OXB = 135° - \theta$

Here, $\angle OXA + \angle OXB = 180°$

$\Rightarrow \quad \angle OXA + 135° - \theta = 180°$

$\Rightarrow \quad \angle OXA = 45° + \theta$

In ΔOAX,

$AO = AX$

$\therefore \quad \angle OXA = \angle AOX = 45° + \theta$

Since, $\angle AOX + \angle XOB = 90°$

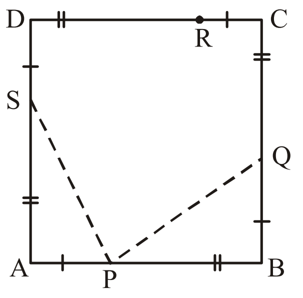
$\Rightarrow \quad 45 + \theta + \theta = 90°$

$\Rightarrow \quad 2\theta = 45°$

$\therefore \quad \theta = 22.5°$

14. (c) The quadrilateral formed by joining the mid-points of the sides is a parallelogram.

15. (d) In ΔAPS and ΔPBQ,



$AS = PB$ (given)

$AP = BQ$ (given)

and $\angle A = \angle B = 90°$

(since, $ABCD$ is square)

Therefore, $\Delta APS \cong \Delta BQP$ are congruent.

$\therefore \quad SP = PQ$

$\angle SPA = \angle BQP$ and $\angle ASP = \angle BPQ$

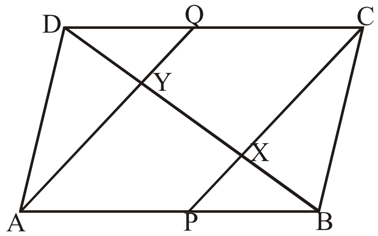
$\therefore \quad \angle SPQ = 90°$ (by RHS rule)

16. (c) Since ABCD is a parallelogram and P, Q are the mid-points of AB, CD respectively.

$\therefore$ AP = QC and AP || QC

$\Rightarrow$ APCQ is a parallelogram.

Now in ΔDXC, Q is the mid point of CD and QY || CX, therefore Y will be the mid point of DX.



$\therefore$ DY = YX ...(i)

Similarly

YX = XB ...(ii)

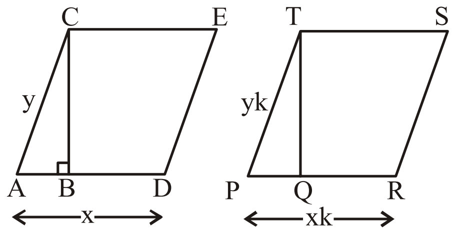
From equations (i) and (ii)

DY = YX = XB

17. (c) If a parallelogram and a rectangle stand on the same base and on the same side of the base with the same height, then perimeter of parallelogram is greater than perimeter of rectangle.

$\therefore \quad I_1 > I_2$

18. (c) Let the sides of a parallelograms are x, y and xk, yk.



Since, sides of two parallelogram are in 1 : k.

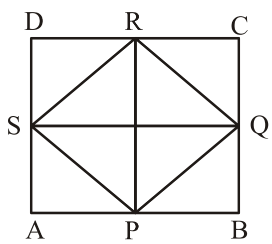
$\therefore \quad \Delta ABC \sim \Delta PQT \Rightarrow \dfrac{AC}{PT} = \dfrac{BC}{QT} \Rightarrow \dfrac{BC}{QT} = \dfrac{y}{yk} = \dfrac{1}{k}$

Let $BC = z$ and $QT = zk$

$\therefore$ Ratio of areas of two similar parallelograms

$$= \frac{x \times z}{xk \times zk} = \frac{1}{k^2}$$

19. (c)



$AP = PB$

$CQ = QB$

$RC = DR$

$SD = AS$

According to above ABCD is a parallelogram.

Then, the diagonals PR and SQ bisect each other.

Now, ar $(RSQ) = \dfrac{1}{2}$ ar $(SQCD)$ ... (i)

and ar $(PQS) = \dfrac{1}{2}$ ar$(ABQS)$ ... (ii)

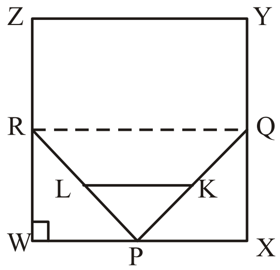
From addition of both (*i*) and (*ii*), we get

$$\text{ar}(PQRS) = \frac{1}{2}\text{ ar}(ABCD)$$

Thus, both statements are correct.

20. (b) $\text{ar}(PRQ) = \frac{1}{2}\text{ar}(WXQR)$

$$= \frac{1}{2}\left(\frac{1}{2}\text{ar}(WXYZ)\right) = \frac{1}{4}\text{ar}(WXYZ) \qquad \ldots (i)$$



$$\frac{\text{ar}(PRQ)}{\text{ar}(PLK)} = \frac{RP^2}{LP^2}$$

(by properties of similar triangle)

$$\Rightarrow \frac{\text{ar}(PRQ)}{\text{ar}(PLK)} = \frac{(2LP)^2}{LP^2}$$

$\Rightarrow$ ar(PRQ)= 4 ar(PLK)

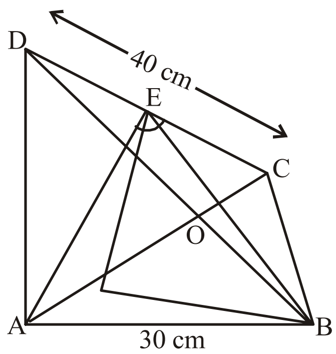
$\Rightarrow$ $\frac{1}{4}$ar($WXYZ$)= 4 ar(PLK) [from eq. (*i*)]

$\Rightarrow$ $\frac{1}{16}$ ar($WXYZ$)= ar(PLK) $\Rightarrow \frac{\text{ar}(PLK)}{\text{ar}(WXYZ)} = \frac{1}{16}$

Explanations (Q. Nos. 21-23) :

Given, $AO : OC = BO : OD$

and AB= 30 cm and CD = 40 cm



$\therefore$ $\Delta AOB \sim$ ACOD $\Rightarrow \frac{OA}{OC} = \frac{AB}{CD} = \frac{3}{4}$

$\therefore$ $\angle OAB = \angle OCD$ and $\angle OBA = \angle ODC$

It means $DC \parallel AB$. So, it is a trapezium.

Area of quadrilateral $ABCD = \frac{1}{2}(AB + CD) \times AE$

$\Rightarrow$ $1050 = \frac{1}{2}(30 + 40) \times AE$

$\Rightarrow$ AE = 30 cm

Also, $\angle BAE = 90°$

21. (b) In right ΔEAB,

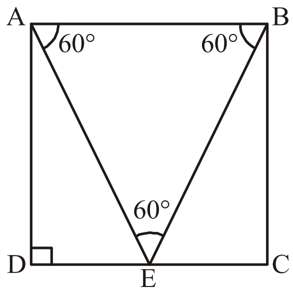
$EB = \sqrt{AE^2 + AB^2} \Rightarrow EB = \sqrt{30^2 + 30^2} = 30\sqrt{2}$ cm

22. (c) Area of ΔADC = $\frac{1}{2} \times CD \times AE = \frac{1}{2} \times 40 \times 30 = 600$ cm^2

23. (b) Also $\angle BAE = 90°$, $AE = AB = 30$ cm

$\therefore$ $\angle AEB = \angle ABE = 45°$

24. (a) $AB \| DC$ and $AD \| BC$ ... (Given)



In ΔABE,

$\angle EAB = \angle ABE = 60°$

$\Rightarrow$ $\angle AEB = 60°$

$\Rightarrow$ $\Delta\ ABE$ is an equilateral triangle.

Now, $AB = BE = EA$ perimeter of $\Delta ABE = 6$

$\Rightarrow$ $AB + BE + EA = 6$

$\Rightarrow$ AB = 2 units

$AB = BE = EA = 2$

and in ΔADE, $AE^2 = AD^2 + ED^2$

$\Rightarrow$ $4 = AD^2 + 1$

(since, E is mid-point of CD)

$\Rightarrow$ $AD = \sqrt{3}$ units

Therefore, area of quadrilateral $ABCD = AB \times AD$

$= 2 \times \sqrt{3} = 2\sqrt{3}$ sq units.

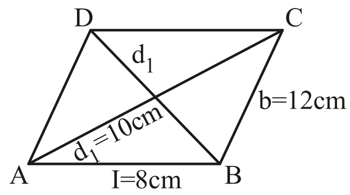
25. (d) We know that, ratio of the areas of two similar triangles are equal to the ratios of squares of their corresponding sides.

$$\therefore \frac{\text{ar}(\Delta BPQ)}{\text{ar}(\Delta DPC)} = \frac{PB^2}{PC^2}$$

$$\Rightarrow \frac{20}{\text{ar}(\Delta DPC)} = \frac{1}{4}$$

$\Rightarrow$ ar (Δ DPC)= 80 sq units

26. (d) In parallelogram, $d^2 + d_2^2 = 2\ (l^2 + b^2)$



$\therefore$ $d^2 + (10)^2 = 2\ (64 + 144)$

$\Rightarrow \quad d^2 = 2 \times 208 - 100$

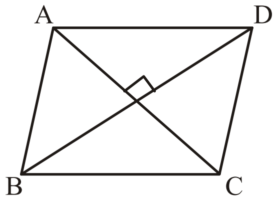
$\Rightarrow \quad d^2 = 416 - 100 = 316 \Rightarrow d = \sqrt{316}$

$\Rightarrow \quad d = 17.76$ cm

$d > 12$

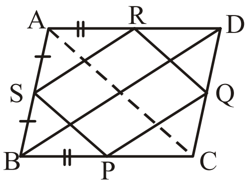
27. (c) $ABCD$ is a rhombus.

$\therefore \quad AB = BC = CD = DA$



and diagonals bisect each other at right angles

28. (b) In parallelogram $ABCD$.



$AC = 2\ mn$, $BD = m^2 - n^2$ and $AB = \dfrac{m^2 + n^2}{2}$

We know that,

$(AC^2 + BD^2) = 2\ (AB^2 + BC^2)$

$\Rightarrow \quad (m^2 + n^2 + m^4 + n^4 - 2m^2n^2)$

$$= 2\left\{\frac{1}{4}(m^2+n^2)^2 + BC^2\right\}$$

$\Rightarrow \quad (m^2 + n^2)^2 = \dfrac{1}{2}(m^2+n^2)^2 + 2BC^2$

$\Rightarrow \quad 2BC^2 = \dfrac{1}{2}(m^2+n^2)^2$

$\Rightarrow \quad BC^2 = \dfrac{(m^2+n^2)^2}{4}$

$\Rightarrow \quad BC = \dfrac{m^2+n^2}{2}$

Therefore, $ABCD$ is a rhombus.

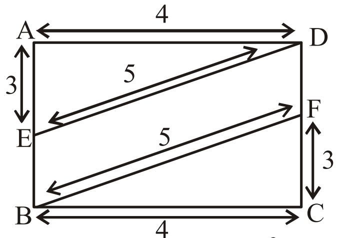
Let $AC > BD \Rightarrow 2mn > m^2 - n^2$

$\Rightarrow \quad (m + n)^2 > 2m^2$

which is always true for every positive integers m and n, where $n < m < 2n$.

29. (a) **Statement I**

Let side $BC = x$ cm.



Given Area of $\Delta BFC = 6$ cm^2

$\dfrac{1}{2} \times 3 \times x = 6 \Rightarrow x = 4$ cm

In ΔBFC, $\quad BF^2 = x^2 + 9 = 16 + 9 \Rightarrow BF^2 = 25$

$\therefore \quad BF = 5$

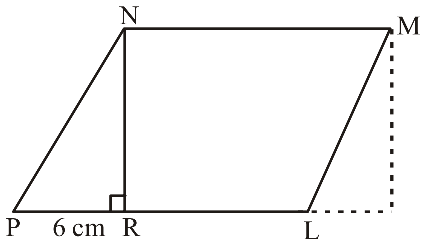
$\therefore \quad$ area of rectangle $ABCD$, $pq^2 = p\ (2)^2$ cm^2

Statement II

which is of the form pq^2.

While the area of $EBFD$ cannot be the form of r^2 cm^2.

30. (b)



Area of parallelogram = 6 × Area of Δ NPR

$\therefore \quad NR \times PL = 6 \times \dfrac{1}{2} \times NR \times PR$

$\Rightarrow \quad PL = 3PR$ (here, $PL = PR + RL$)

$\Rightarrow \quad PR + RL = 3PR$

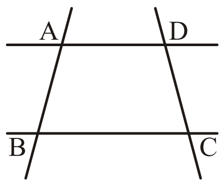
$\Rightarrow \quad RL = 2PL = 2 \times 6 = 12$ cm

31. (b) We know that in a square diagonals are equal and bisect each other at 90°.

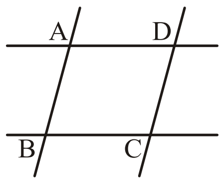
Hence, the required quadrilateral is a square.

32. (d) If two parallel lines are cut by two distinct transversals, the quadrillateral formed by the four lines is always a 'Trapezium'.

Case I If two distinct transversals (are not parallel), then always → (Trapezium)



Case II If two distinct transversals are parallel, then always (Trapezium + Parallelogram)



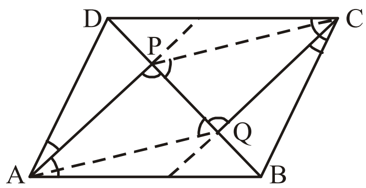
33. (b) Since, line segment

AP and CQ bisects the $\angle A$ and $\angle C$, respectively.

Then, $AP \| CQ$

Now in ΔAPQ and ΔCQP

$\because \quad AP \parallel QC$



$\because \quad \angle APQ = \angle PQC$ (alternate angle)

$PQ = \angle PQ$ (common)

Also, $PC \parallel AQ$

$\therefore \angle CPQ = \angle PQA$ (alternate angle)

$\therefore \Delta APQ \sim \Delta CQP$ (by ASA)

Thus, ΔAPQ is similar to ΔPCQ.

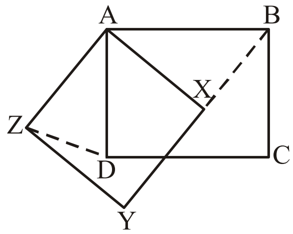
34. (d) In Δ ABX and ΔADZ.

$AB = AD$ (side of a square $ABCD$)

and $AX = AZ$ (side of square $AXYZ$)

Let $\angle BAX = \theta$

$\therefore \angle XAD = 90° - \theta$



Also, $AXYZ$ is a square,

$\therefore \angle ZAX = 90° \Rightarrow \angle ZAD + \angle XAD = 90°$

$\Rightarrow \angle ZAD = 90° - (90° - \theta) = \theta$

i.e., $\angle BAX = \angle ZAD$

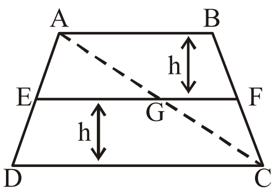
$\therefore \Delta ABX \cong \Delta ADZ$

$\therefore BX = DZ$ (by CPCT)

35. (b) Diagonals of a rhombus are perpendicular bisector of each other. Hence each point on the diagonal BD is equidistant from vertices A and C. Therefore required locus is the diagonal BD.

36. (c) Join AC.

In ΔACD, $EG \parallel DC$ and E and G are mid-points of AD and AC, respectively.



$$\therefore \quad EG = \frac{1}{2} DC = \frac{3}{2}$$

Similarly, in ΔABC

$$GF = \frac{1}{2} AB = 1$$

$$EF = EG + GF = 1 + \frac{3}{2} = \frac{5}{2}$$

$\therefore$ Area of trapezium $= \frac{1}{2}$ (Sum of parallel side × Height)

Now, the ratio $= \dfrac{\text{Area of } ABFE}{\text{Area of } EFCD}$

$$= \frac{\frac{1}{2}\left(2 + \frac{5}{2}\right) \times h}{\frac{1}{2}\left(3 + \frac{5}{2}\right) \times h} = \frac{9}{11}$$

37. (c) In parallelogram $PQRS$, AP, AQ, CR and CS are bisector of $\angle P$, $\angle Q$, $\angle R$ and $\angle S$.

In Δ RBQ $\angle B = 90°$ (Because, $\angle Q + \angle R = 180°$ and

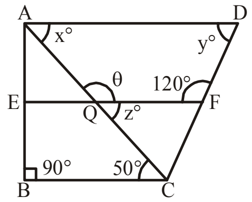
$$\frac{\angle Q}{2} + \frac{\angle R}{2} = \frac{180°}{2}$$

In Δ PDS, $\angle D = 90°$ (Because $\angle P + \angle S = 180°$ and

$$\frac{\angle P}{2} + \frac{\angle S}{2} = \frac{180}{2} = 90°$$

$\therefore$ ABCD is a rectangle.

38. (c) From figure.



$BC \parallel EF \parallel AD$

$\therefore x° = z° = 50°$ (corresponding interior angle

$\therefore \theta + z° = 180°$ (linear pair)

$\therefore \theta = 180° - 50° = 130°$

In quadrilateral

$AQFD$, $x° + y° + 120° + \theta = 360°$

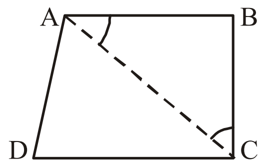
$50° + y° + 120° + 130° = 360°$

$y = 360° - 300° = 60°$

39. (c) The quadrilateral must be a trapezium because a quadrilateral where only one pair of opposite sides are parallel (in this case $AB \parallel CD$) is a trapezium.

40. (c) Constraction : In quadrilateral $ABCD$, form A to C.

Now, in ΔABC



$\because AB = BC$ ...(Given)

$\therefore \angle BAC = \angle BCA$

(angles opposite to equal side)

In ΔADC,

$\because CD > AD$

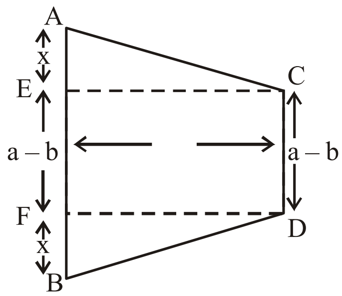
$\therefore \angle DAC > \angle DCA$

(since in a triangle, angle opposite to greater side is bigger than the angle opposite to smaller side)

On adding eqs. (*i*) and (*ii*), we get

$\angle BAD > \angle BCD$

41. (d) Since, they are symmetrically lying on horizontal plane.



$\therefore \quad AC = BD$

$\therefore \quad AE = BF = x$

Now, $AB = (a - b) + 2x$

i.e., $a + b = a - b + 2x \Rightarrow 2b = 2x$

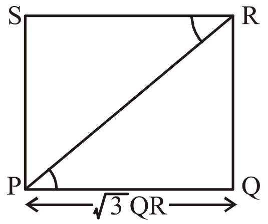
$\therefore \quad x = b$

Now in ΔACE,

$x^2 + a^2 = AC^2$

$\Rightarrow \quad AC^2 = b^2 + a^2 \Rightarrow AC = \sqrt{b^2 + a^2}$

42. (c) In rectangle *PQRS*,



$PQ \parallel RS$

$\therefore \quad \angle RPQ = \angle PRS$

(Alternate interior angles) ... (*i*)

Now in ΔPQR,

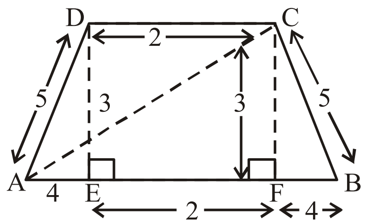
$$\tan \angle QPR = \frac{RQ}{PQ}$$

$$\tan \angle QPR = \frac{QR}{\sqrt{3}QR} = \frac{1}{\sqrt{3}}$$

$\Rightarrow \quad \angle QPR = 30°$

$\therefore \quad \angle PRS = 30°$ [from eq. (*i*)]

43. (b) In ΔBCF,



Pythagoras theorem,

$(5)^2 = (3)^2 + (BF)^2 \Rightarrow BF = 4\text{cm}$

$AB = 2 + 4 + 4 = 10$ cm

Now in ΔACF, $AC^2 = CF^2 + FA^2 \Rightarrow AC^2 = 3^2 + 6^2$

$AC = \sqrt{45}$ cm

Similarly, $BD = \sqrt{45}$ cm

$\therefore$ Sum of diagonal $= 2 \times \sqrt{45} = 2 \times 3\sqrt{5}$

$= 6\sqrt{5}$ cm

44. (b) Here, Area of Rectangle lies between 40cm^2 and 45cm^2

Given that one sides = 5cm.

Area of Rectangle = 5 × second sides

Now, If Area = 40cm^2

then, 40 = 5 × second sides

$\because$ second sides = 8cm.

Again, If Area = 45cm^2

45 = 5 × second sides.

$\because$ Second sides = 9cm.

it means that second sides varies. between 8 cm to 9 cm.

Let diagonal = d

$\Rightarrow \sqrt{8^2 + 5^2} < d < \sqrt{5^2 + 9^2}$

$\Rightarrow \sqrt{64 + 25} < d < \sqrt{25 + 81}$

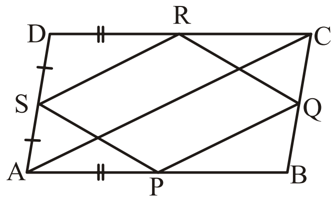
$\Rightarrow \sqrt{87} < d < \sqrt{106}$

$\Rightarrow \sqrt{81} < \sqrt{89} < d < \sqrt{106} < \sqrt{121}$

$\Rightarrow \sqrt{81} < d < \sqrt{121}$

$\Rightarrow$ 9cm < d < 11cm.

45. (b) Area of ΔAPS = Area of ΔDSR

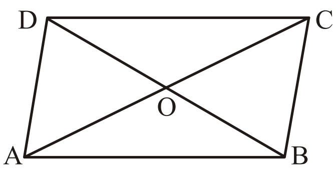


$\because \quad AS = SD$ and $AP = DR$

$\therefore \quad \text{ar}(\Delta ABC) = 4\text{ar}(\Delta BPQ)$

46. (b) I. *ABCD* is a parallelogram, then

$AC^2 + BD^2 = 2(AB^2 + BC^2)$



So it is not true.

II. *ABCD* is a rhombus and diagonals *AC* and *BD* bisect each other.

$\therefore \quad AO = OC$

and $OB = OD$

In ΔAOB, $AB^2 = AO^2 + OB^2$

$$(4)^2 = \left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2$$

$\therefore \quad AC^2 + BD^2 = 64$

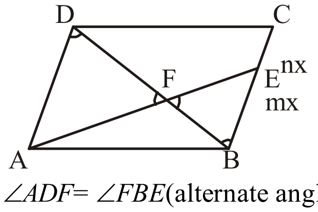
$= (4)^3$ *i.e.,* n^3

So only II is true.

47. (d) In ΔAFD and ΔBFE,

$\angle AFD = \angle BFE$

(vertically opposite angles)



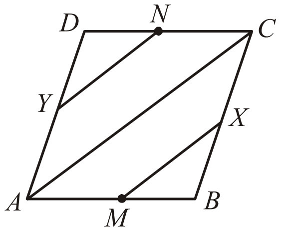
and $\angle ADF = \angle FBE$(alternate angles)

$\therefore \quad \Delta AFD \sim \Delta EFB$ (By AA)

So, $$\frac{\text{ar}(\Delta EFB)}{\text{ar}(\Delta AFD)} = \frac{EB^2}{AD^2}$$

$$= \frac{(mx)^2}{(mx + nx)^2} = \frac{m^2}{(m+n)^2} = \left[\frac{m}{(m+n)}\right]^2$$

48. (b) From Statement 1. Given, $ABCD$ is a parallelogram. X and Y are mid–points of BC and AD, respectively. M and N are the mid–points of AB and CD, respectively.



From statement 2. Here join point A and C.
In ΔABC, M and X are mid–points of AB and BC.

$\therefore$ $MX \parallel AC$ and $MX = \frac{1}{2}AC$ ...(i)

In ΔADC, Y and N are mid–points of AD and CD.

$\therefore$ $YN \parallel AC$ and $YN = \frac{1}{2}AC$ ...(ii)

From equations (i) and (ii), we get $MX \parallel YN$
From statement 2
So, Statement 1 is not correct.
Clearly, straight lines AC, BD, XY and MN meet at a point, So Statements 2 is correct.

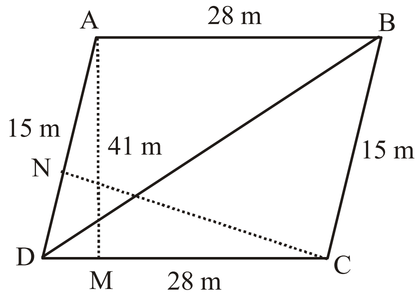
49. (b) Side of cube = 2 cm
$\therefore$ Maximum distance between two points of a cube = Length of diagonal
$= \sqrt{3} \times \text{side} = 2\sqrt{3}$ cm

50. (d) Perimeter of parallelogram land = 86 m and diagonal = 41m
Suppose one side of parallelogram be x m
other side = $(x + 13)$ m
$\therefore$ Perimeter $= 2(x + x + 13) = 86$

$\Rightarrow$ $2x + 13 = \frac{86}{2} = 43 \Rightarrow 2x = 43 - 13 = 30$

$\therefore$ $x = \frac{30}{2} = 15$

one side of parallelogram = 15 m
other side = 15 + 13 = 28 m



area of $\Delta ABD = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{42(42-15)(42-28)(42-41)}$

$\left[\because s = \frac{15+41+28}{2} = \frac{84}{2} = 42\right]$

$= \sqrt{42 \times 27 \times 14 \times 1} = 126 \text{ m}^2$

Required area of parallelogram = 2 × Area of ΔABD
$= 2 \times 126 = 252 \text{ m}^2$

51. (a) From the above figure, shorter height of parallelogrm $= AM$
From question 30
Area of parallelogram = Base Height = 252m^2

$28 \times AM = 252 \Rightarrow AM = \frac{252}{28} = 9$ m

Therefore, shorter height of the parallelogram is 9 m.

52. (a)
1. Let second diagonal be x.
Now, $x^2 + 41^2 = 2(15^2 + 28^2)$
$\Rightarrow$ $x^2 = 337 = 18.36$ (approx)
$\therefore$ Difference between the diagonals
$= 41 - 18.36 = 22.64$
Which is more than 20
Therefore, Statement 1 is correct.
2. $\because$ Second height of parallelogram $ABCD = CN$
$\therefore$ Base × Height = Area

$\Rightarrow$ $15 \times CN = 252 \Rightarrow CN = \frac{252}{15} = 16.8$ m

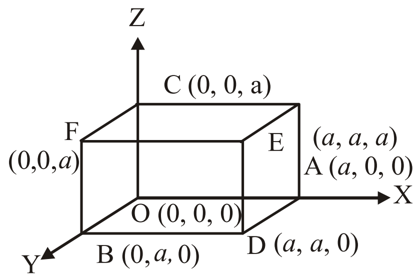
$\therefore$ Difference between the heights = 16.8 – 9 = 7.8 which is not more than 10.
Statement 2 is not correct.

53. (c) In a cuboid, 4 perpendicular face pairs in bottom surface, 4 perpendicular face pairs in top surface and 4 perpendicular face pairs in vertical surface. Total perpendicular pairs are 12.

54. (c) In a cube, there are six faces.
Let the sides of a cube be a.

$\therefore$ Diagonal of face $= \sqrt{a^2 + a^2} = a\sqrt{2}$

Hence, there is no equilateral triangle will be formed in faces.



In ΔABC,

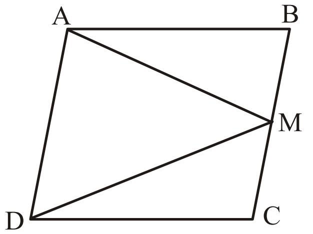
$AB = \sqrt{a^2 + a^2} = a\sqrt{2}, \quad BC = \sqrt{a^2 + a^2} = a\sqrt{2}$

and $AC = \sqrt{a^2 + a^2} = a\sqrt{2}$

ΔABC, is an equilateral triangle.
Similarly, In ΔABE, ΔODG, ΔODE, ΔCEB, ΔCEA, ΔFGO and ΔFGD
Eight equilateral triangles are possible.

55. (c) 1. Given, $ABCD$ is a trapezium.
If $AB = CD$, then it becomes a parallelogram.
M is the mid-point of BC.

Area of $\Delta AMD = \dfrac{1}{2}$ area of $ABCD$ ...(i)



Because, both are between same parallels and in same base.

By according to question

$\because$ Area of ΔABM + Area of ΔDCM

$= \dfrac{1}{2}$ area of $ABCD$

$\because$ Area of ΔABM = Area of ΔDCM

$\Rightarrow$ 2 Area of $\Delta DCM = \dfrac{1}{2}$ Area of $ABCD$

$\Rightarrow$ Area of $\Delta DCM = \dfrac{1}{4}$ Area of $ABCD$ ...(ii)

On adding equations (i) and (ii), we get

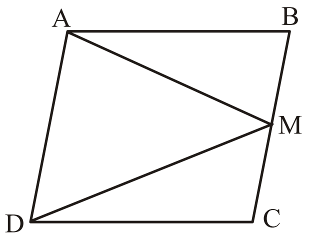
Area of ΔAMD + Area of ΔDCM

$= \dfrac{1}{2}$ Area of $ABCD + \dfrac{1}{4}$ Area of $ABCD$

$= \dfrac{3}{4}$ Area of $ABCD$

Therefore, Statement 1 is correct.

2. $ABCD$ is a trapezium.



$\therefore$ Area of trapezium = Area of ΔDCM + Area of ΔABM + Area of ΔAMD

$\Rightarrow$ Area of ΔDCM + Area of ΔABM

= Area of trapezium $ABCD$ – Area of ΔAMD

If $AD = BC$, then using eq. (i), we get

Area of ΔDCM + Area of ΔABM

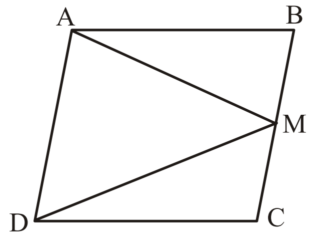
$= \dfrac{1}{2}$ Area of trapezium ABCD

If $AD = BD$, then it is true otherwise are of ΔDCM and area of ΔABM is greater than half of the area of trapezium $ABCD$.

Therefore Statement 2 is also correct.

56. (a) 1. If $AB = CD$, then $ABCD$ is a parallelogram

Area of $\Delta ADM = \dfrac{1}{2}$ Area of $ABCD$



[since, both are in same base and between same parallels]

$\Rightarrow$ Area of ΔADM – Area of ΔABM

$= \dfrac{1}{2}$ Area of $ABCD$

$-\dfrac{1}{4}$ Area of ABM = Area of ΔDMC

Statement 1 is correct.

2. Area of ΔABM + area of $\Delta DCM = \dfrac{1}{2}$ Area of $ABCD$

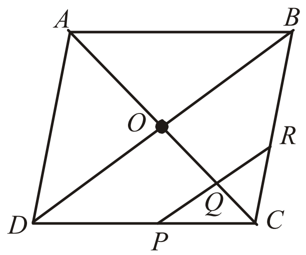
$\Rightarrow$ 2 Area of ΔABM $= \dfrac{1}{2}$ Area of $ABCD$

[$\because$ area Δ ABM = area of Δ DCM]

$\Rightarrow$ Area of Δ ABM $= \dfrac{1}{4}$ Area of $ABCD$

Statement 2 is not correct.

57. (a) Given, $ABCD$ is a parallelogram. Join AC and BD which intersect each other at O.



$\therefore$ $OC = \dfrac{1}{2}AC$

In ΔCBD, P and R are mid-points of DC and BC.

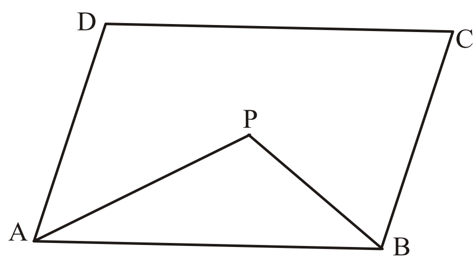
$\therefore$ $PR \parallel BD$ or $PQ \parallel DO$ and $RQ \parallel BO$

Again in ΔOCD, $PQ \parallel OD$

So, Q is mid-point of OC.

$\therefore$ $CQ = \dfrac{1}{2}OC = \dfrac{1}{2}\left(\dfrac{1}{2}AC\right) = \dfrac{1}{4}AC$

58. (a) A quadrilateral $ABCD$, AP and BP are bisectors of $\angle A$ and $\angle B$, respectively.



$\therefore \quad \angle APB = 180° - \left(\frac{1}{2}\angle A + \frac{1}{2}\angle B\right)$

We know that sum of all angles of a quadrilateral $= 360°$

$\Rightarrow \quad \angle A + \angle B + \angle C + \angle D = 360°$

$\therefore \quad \frac{1}{2}\angle A + \frac{1}{2}\angle B + \frac{1}{2}\angle C + \frac{1}{2}\angle D = \frac{360°}{2}$

$\Rightarrow \quad \frac{1}{2}\angle C + \frac{1}{2}\angle D = 180° - \left(\frac{1}{2}\angle A + \frac{1}{2}B\right)$

$\Rightarrow \quad \frac{1}{2}(\angle C + \angle D) = \angle APB$ [from eq. (i)]

$\Rightarrow \quad \angle C + \angle D = 2\angle APB$

59. (c) Sum of angle of regular polygon $= \frac{(n-2)180°}{n}$

$\Rightarrow \quad 135\,n = 180n - 360$

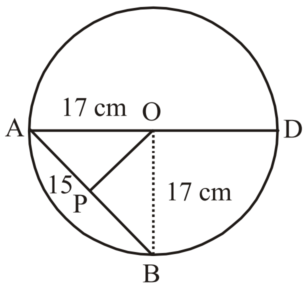
$\Rightarrow \quad 45\,n = 360$

$n = \frac{360}{45} = 8$

Number of diagonals = ${}^{8}C_2 - 8$

$= \frac{8 \times 7}{2} - 8 = 20$

60. (d) Let AD is diameter of circle of centre O. Find OP = q

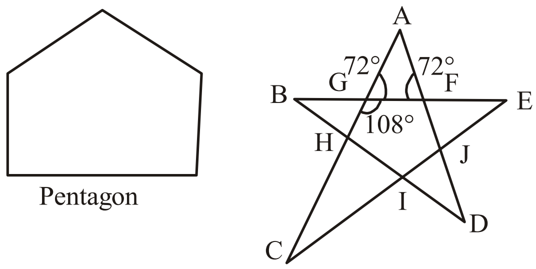


AD = 34 cm, AO = OD = 17 cm

AB = 30 cm, AP = $\frac{30}{2}$ = 15 cm

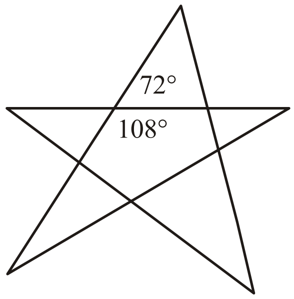
OP = $\sqrt{(17)^2 - (15)^2} = \sqrt{64}$ = 8cm.

61. (a)



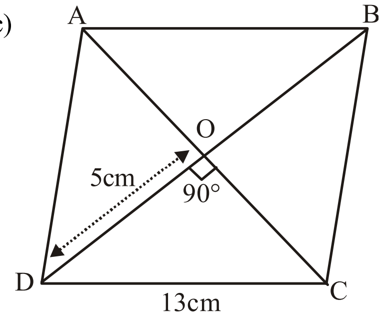
Sum of Interior angles
= (number of sides –2) × 180°
= (5 – 2) × 180° = 540°

Interior angle of regular pentagon = $\frac{540}{5} = 108°$



Now suppliment angle = 180° – 108° = 72°
Now, we can find angle at the top point of the star by adding the two equal base angles.
= 180° – (72° + 72°)
= 180° – 144° = 36°
So each point of the stars = 36°

62.(c)



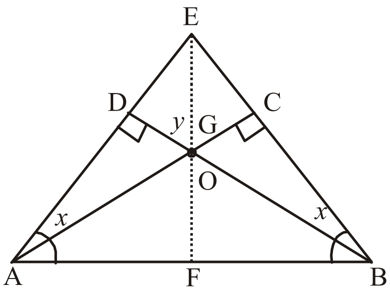
OC = $\sqrt{169 - 25} = \sqrt{144}$ = 12 cm

$\Rightarrow$ AC = 2OC = 24 cm

Area of Rhombus = $\frac{1}{2} \times d_1 \times d_2$

$= \frac{1}{2} \times 10 \times 24$ = 120 cm^2

63. (b)



Δ EAB is equilateral
Δ EDC is also equilateral
Area of trapezium ABCD

$= \left(\frac{1}{2} \times DB \times OA\right) + \frac{1}{2}(DB \times OC)$

$= \frac{1}{2} \times DB \times AC$

Let AO = OB = x and DO = OC = y

Area (ABCD) $= \frac{1}{2}(x+y)^2 = 16$ (given)

$\Rightarrow$ $x + y = 4\sqrt{2}$ ... (i)

Δ AOB is a right angled isosceles triangle.

So, $AB = \sqrt{x^2 + x^2} = \sqrt{2}x$

Similarly, DC = $\sqrt{2}y$

Now, FG = EF – EG

$\Rightarrow$ FG = AB sin 60° – DC sin 60°

$= \frac{\sqrt{3}}{2}(AB - DC) = \frac{\sqrt{6}}{2}(x - y)$ ... (ii)

Area of trapezium

= Area ΔEAB – Area ΔEDC

$= \frac{\sqrt{3}}{4}(AB^2 - DC^2)$

$= \frac{\sqrt{3}}{4}\left[(x\sqrt{2})^2 - (y\sqrt{2})^2\right]$

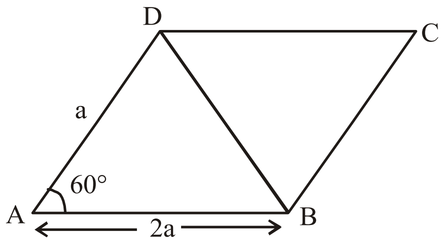
$\Rightarrow$ Area $= \frac{\sqrt{3}}{2}(x+y)(x-y)$

Now, $\frac{\sqrt{3}}{2}(x+y)(x-y) = 16$

$\Rightarrow$ $x - y = \frac{32}{\sqrt{3}(x+y)} \Rightarrow x - y = \frac{8}{\sqrt{6}}\ \left(\because x + y = 4\sqrt{2}\right)$

Height $= \frac{\sqrt{6}}{2}(x-y) = \frac{\sqrt{6}}{2} \times \frac{8}{\sqrt{6}} = 4$ cm

64. (b)



Using cosine law

$\cos A = \frac{AD^2 + AB^2 - BD^2}{2.AD.AB}$

$\Rightarrow \cos 60° = \frac{AD^2 + (2AD)^2 - BD^2}{2.AD.2AD}$

$\Rightarrow \frac{1}{2} = \frac{AD^2 + 4AD^2 - BD^2}{4AD^2}$

$\Rightarrow 2AD^2 = 5AD^2 - BD^2$

$\Rightarrow BD^2 = 5AD^2 - 2AD^2$

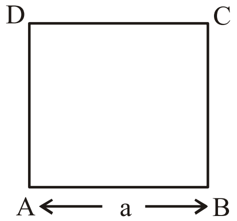
$\Rightarrow BD = 3AD^2$

$\Rightarrow BD = \sqrt{3}$ AD

So, option (b) is correct.

65. (c)

66. (d)



Let initially the side of square be ‘a’.

Area of square = a^2

Area of reactangle = 1.3 a × 1.2 a

Percentage increase = 1.56 a^2 in Area

$= \frac{(1.56a^2 - a^2)}{a^2} \times 100$

$= (1.56 - 1)\,100 = 56\%$

So, option (d) is correct.

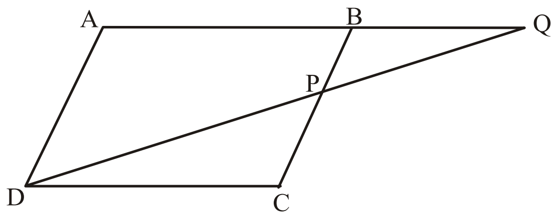
67. (c) The dimensions of the 3 rectangles are x_1y_1; x_2y_2; x_3y_3. $x_1 < x_2 < x_3$ and $x_1y_1 = x_2y_2 = x_3y_3$ ($\therefore$ $y_1 > y_2 > y_3$). By joining the parallel sides along the breadth to form a cylender, the length becames the circumference of the base (i.e.x = 2πr) and the breadth becomes the height. The quantities are fabulated below:

	R_1	R_2	R_3
Length	x_1	x_2	x_3
Breadth	y_1	y_2	y_3
Height of cylinder	y_1	y_2	y_3
Base radus	$\frac{x_1}{2\pi}$	$\frac{x_2}{2\pi}$	$\frac{x_3}{2\pi}$
Volume	$\frac{x_1y_1}{4\pi}$	$\frac{x_2y_2}{4\pi}$	$\frac{x_3y_3}{4\pi}$

As $x_1y_1 = x_2\,y_2 = x_3\,y_3$

and x_1, $x_2 < x_3$ of follows $v_1 < v_2 < v_3$.

68. (d)



ΔBPQ and ΔCPD are similar then,

$\Rightarrow \frac{\text{area of } \Delta BPQ}{\text{area of } \Delta CPD} = \left(\frac{BP}{PC}\right)^2$

$\Rightarrow \frac{20}{\text{area of } \Delta CPD} = \left(\frac{1}{2}\right)^2$

Area of Δ CPD = 4 × 20 = 80 square unit

So, option (d) is correct.

69. (c) The n-sided polygon can be dinded into ‘n’ triangle with O, the Centre of the circule as one veotex for each triange. The altitude of each triangle is r. Let the

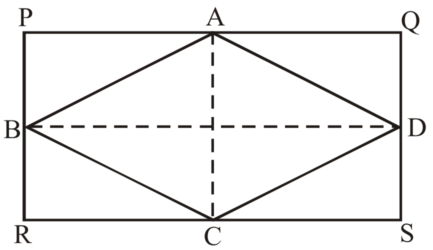
sides of the polygon be 'a_1', a_2 ____ a_n. (Given $a_1 = a_2 =$ ______ a_n)

$\therefore$ The area of polygon is $\frac{nr}{2} = \frac{pr}{2}$

Area of polygon $= \frac{a_1 r}{2} + \frac{a2r}{2} + - - - \frac{a_n r}{2} = \frac{pr}{2}$

So, option (c) is correct.

70. (b) Let PQRS be a rectangle and ABCD be a rhombus which is formed by joining the mid points of a rectangle.



Given Area of rhombus = 2 unit.

But Area $= \frac{1}{2} \times d_1 \times d_2 = 2$

$d_1 d_2 = 2 \times 2 = 4$ units

where d_1 = diagonal AC

d_2 = diagonal BD

But AC = Breadth of rectangle

BD = length of rectangle

$\Rightarrow$ Area of rectangle = AC × BD

$= d_1 \times d_2$

$= 4$ units

$\therefore$ option (b) is correct.

71. (b) Since interior angle of the regular polygon = 140°

Hence exterior angle = 180° – 140° = 40°

$\therefore$ No. of sides $= \frac{360°}{\text{Exterior angle}} = \frac{360}{40} = 9$

$\therefore$ No. of vertices or sides = 9

$\therefore$ Option (b) is correct.

72. (c) Let length of the side of the square = x units

$\therefore$ Length of the diagonal $= \sqrt{2}\,x$ units

$\Delta ADE \sim \Delta EFC$ (By AAA)

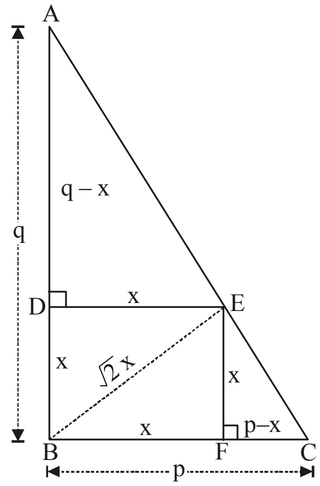
$\therefore \frac{AD}{EF} = \frac{DE}{FC}$

$\Rightarrow \frac{q-x}{x} = \frac{x}{p-x}$

$\Rightarrow x^2 = pq - px - qx + x^2$

$\Rightarrow x = \frac{pq}{p+q}$

$BE = \frac{\sqrt{2}\,pq}{p+q}$ = Length of the diagonal



73. (a) Statement (1) Since $m \geq 3$ and $n \geq 3$

Both of them can take equal values or different values

Let m = 4 and n = 3

$\Rightarrow$ Sum of Exterior angles $= \frac{360°}{\text{No. of sides}} = \frac{360}{m}$

$= \frac{360}{4} = 90°$

Sum of Exterior angles $= \frac{360}{n} = \frac{360}{3} = 120°$

It takes different values at different values of m and n

$\therefore$ Statement 1 is true.

Statement (2) take m = 4 and n = 3

Sum of interior angles = $(m-2) \times 180$

$= (4-2) \times 180 = 360°$

Sum of interior angles = $(n-2) \times 180°$

$= (3-2) \times 180 = 180°$

$\Rightarrow (m+n)\frac{\pi}{2} = (4+3) \times 90° = 630°$

But sum = 360 + 180 = 540

$\Rightarrow$ Statement 2 is not true.

$\therefore$ Option (a) is correct.

74. (b) (1) If Exterior angle = 70°

then no. of sides $= \frac{360°}{70} = 5.14$

which is not possible

Because side of polygon does not exist in decimals

(1) is not true.

Statement (2) Let $n \geq 5$

Let n = 5

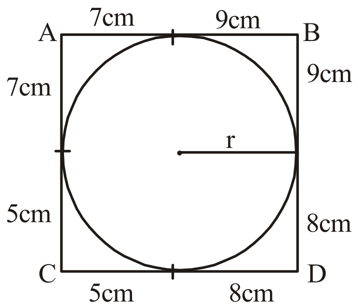
Exterior angle $= \frac{360°}{5} = 72°$

An exterior angle of any regular polygon of n sides is acute.

(2) is true.

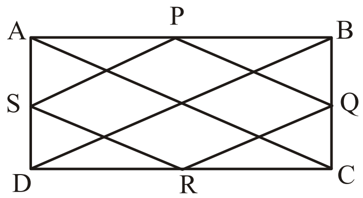
$\therefore$ Option (b) is correct.

75. (b)



BD = 9 + 8 = 17
AC = 7 + 5 = 12
AC + BD = 29
Same AB + CD = 29
So, the sum of a pair of opposite sides equals to sum of the other pair of opposite sides.

76. (c)



$PQ = \frac{1}{2}AC; SR = \frac{1}{2}AC$ {Mid point theorem}

Similarly $PS = \frac{1}{2}BD; QR = \frac{1}{2}BD$

$\therefore$ BD = AC(Diagonal of rectangle)

$\therefore$ PQ = QR = RS = SP

So PQRS is a Rhombus but need not be a square.

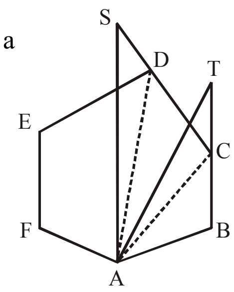
77. (b) $\angle ABC = 120°$(Angle of regular hexagon)

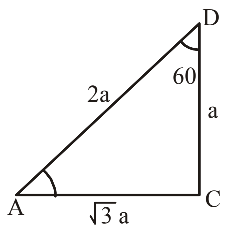
$\angle BAC = \angle BCA = \frac{180-120}{2} = 30°$

$\angle DCA = 120 - 30° = 90°$

i.e. ΔDCA is a right Δ

when side DC = a





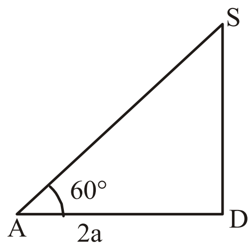
$\frac{AC}{a} = \cot 30°$

$AC = \sqrt{3}a$

$\frac{AD}{a} = \operatorname{cosec} 30°$

AD = 2a

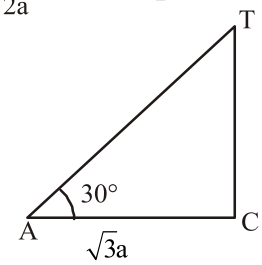
Now taking Δ ASD, Let 'S' is the vertex of pole.



$\frac{DS}{AD} = \tan 60°$

$DS = 2\sqrt{3}$

Taking ΔTCA



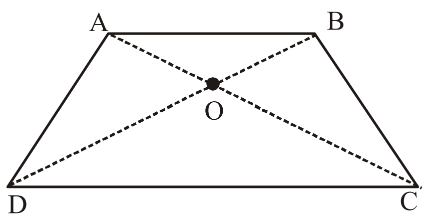
$\frac{TC}{AC} = \tan 30°$

$TC = \frac{\sqrt{3}a}{\sqrt{3}} = a$

Ratio ϕ

$\frac{CT}{DS} = \frac{a}{2\sqrt{3}\,a} = \frac{1}{2\sqrt{3}}$

78. (a)



in ΔADB and ΔCOD

$\angle D = \angle B$ {Alternate interior}

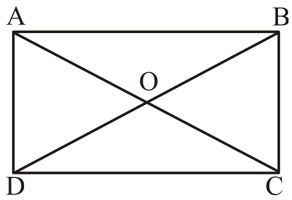
$\angle C = \angle A$

By ΔA

$\Delta AOB \sim \Delta COD$

$\frac{AO}{OC} = \frac{OB}{OD}$

79. (b)



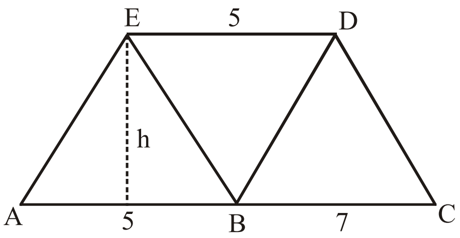
AB = 32, AD = 24 OD = ?

$DB^2 = 32^2 + 24^2$
$= 8^2\,[4^2 + 3^2]$
$DB^2 = 8\,[5^2] = (40)^2$
$DB = 40$
$OD = 20$

80. (a) Let height of triangle be = h
as $\Delta ABDE = 5 \times h$

ar $\Delta BDE = \frac{1}{2} \times 5 \times h$

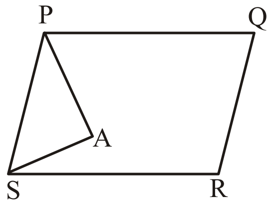
ar $\Delta BCD = \frac{1}{2} \times 7 \times h$



$\text{Ratio} = 5h : \frac{5h}{2} : \frac{7h}{2}$

$= 10 : 5 : 7$

81. (c)



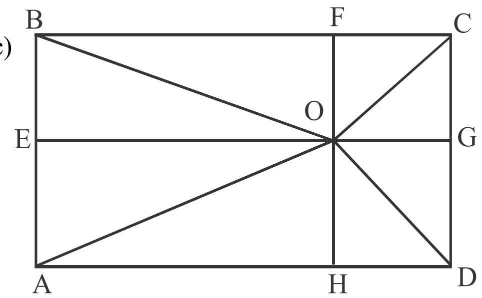
$\angle P + \angle S = 180$
{Sum of adjacent angles of 11 gm}

$\frac{\angle P}{2} + \frac{\angle S}{2} = 90$

$\frac{\angle P}{2} + \frac{\angle S}{2} + \angle A = 180$

$\angle A = 180 - 90 = 90$

82.(c)



Draw rectangle *ABCD* with arbitrary point *O* within it, and then draw lines *OA*, *OB*, *OC*, *OD*. Then draw lines from point *O* perpendicular to the sides: *OE*, *OF*, *OG*, *OH*. Using Pythagorean theorem we have from the above diagram:

$$OA^2 = AH^2 + OH^2 = AH^2 + AE^2$$
$$OC^2 = CG^2 + OG^2 = EB^2 + HD^2$$

$$OB^2 = EO^2 + BE^2 = AH^2 + BE^2$$
$$OD^2 = HD^2 + OH^2 = HD^2 + AE^2$$

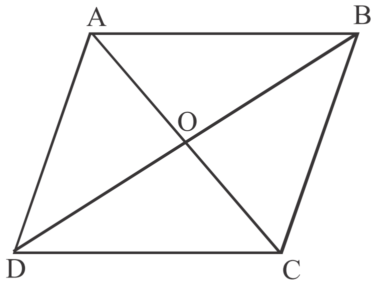
Adding these equalities, we get

$$OA^2 + OC^2 = AH^2 + HD^2 + AE^2 + EB^2$$
$$OB^2 + OD^2 = AH^2 + HD^2 + AE^2 + EB^2$$

From these equalities, we get

$$OA^2 + OC^2 = OB^2 + OD^2$$

83. (b)



Since the diagonals of rhombus bisect each other, therefore $AO = OC = 10$ cm and $BO = OD = 24$ cm.

In ΔAOB, by Pythagoras theorem, we get

$$AB = \sqrt{AO^2 + OB^2} \Rightarrow AB = \sqrt{10^2 + 24^2} \Rightarrow AB$$
$$= \sqrt{100 + 576} \Rightarrow AB = \sqrt{676} \Rightarrow AB = 26 \text{ cm.}$$

Hence, the side of rhombus is 26 cm.

84. (b) For finding the sum of the interior angles of a polygon is the same, whether the polygon is regular or irregular. So, we would use the formula $(n-2) \times 180°$, where *n* is the number of sides in the polygon.

Let one angle be of the polygon be *x* and other 5 equal angles be *y*.

So, according to the question,

$$x = y + 30 \quad (n-2) \times 180° = x + 5y$$
$$\Rightarrow (6-2) \times 180° = y + 30 + 5y \Rightarrow 180.4 = 6y + 30$$
$$\Rightarrow 720 = 6y + 30 \Rightarrow 6y = 690 \Rightarrow y = 115°$$

85. (c)

86. (d) **Statement 1**
No. of sides of pentagon = 5

Each exterior angle $= \dfrac{360°}{5} = 72°$

Each interior angle $= 180° - 72° = 108°$

Similarly,

Each interior angle of rectangle $= 90°$

So, difference of interior angles $= 108° - 90° = 18°$

It is not correct

Statement 2

Interior angle of decagon $= \dfrac{360°}{10} = 36°$

Also, interior angle of octagon $= \dfrac{360°}{8} = 45°$

So, difference of interior angle $= 45° - 36° = 9°$

It is correct

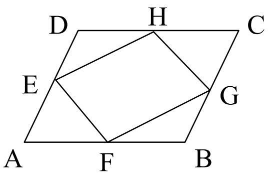
Statement 3

Sum of all exterior angles of a polygon $= 360°$

For 2 polygons, it is $2 \times 360° = 720°$

It is also correct

87. (a)



By symmetry area of

$\Delta EHD = \Delta BFG = \Delta AEF = \Delta CHG$

$\dfrac{1}{8} =$ of area of Parallelogran ABCD.

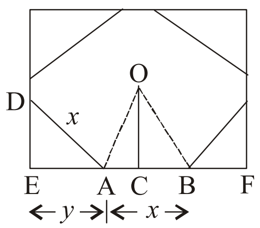
$\therefore$ Area of EFGH $= \dfrac{1}{2}$ of area of ABCD.

Ratio $= \dfrac{\text{Area of ABCD}}{\text{Area of EFGH}} = \dfrac{2}{1}$

But, Perimeter of EFGH $= \dfrac{1}{2} \times$ (Perimeter of ABCD)

88. (a) Let side of octagon $= x$ cm.

For regular octagon, each internal angle



$= \dfrac{2(8-2)\times 90°}{8} = \dfrac{6 \times 90°}{4} = 135°$

$\therefore \angle BAD = 135°$

then, $\angle DAE = 45°$

In ΔAED, $AE = DE$

$\therefore x^2 = (AE)^2 + (DE)^2 \Rightarrow AE = \dfrac{x}{\sqrt{2}}$

$\Rightarrow y = \dfrac{x}{\sqrt{2}}$

By symmetry $AE = BF = y$

Now, $EA + AB + BF = a$ (Side of square)

$y + x + y = a$

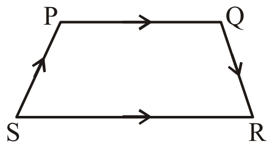
$2y + x = a$

$2.\left(\dfrac{x}{\sqrt{2}}\right) + x = a$

$(\sqrt{2}+1)x = a$

$x = \dfrac{a}{(\sqrt{2}+1)} = a(\sqrt{2}-1)$

89. (c) I. Let $PQRS$ is a trapezium,



So $\angle R = \angle S = x$ (let)

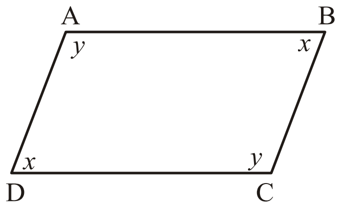
$\therefore \angle P = 180° - \angle S = 180° - x$ $\{\because PQ \| RS\}$

$\angle Q = 180° - x$

$\therefore \angle P + \angle R = 180° - x + x = 180°$

Thus, $PQRS$ is cyclic.

II. $ABCD$ is cyclic parallelogram with $AB \| CD$ and $AD \| BC$.



Considering angles

$A = C = y$ (Property of parallelogram) and

$B = D = x$

Also since it is cyclic

$A + C = B + D = 180$ degrees

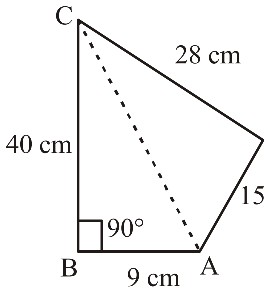
So $x = y = 90$ degrees

And also opposite sides are equal in a parallelogram

Thus $ABCD$ is a rectangle.

Hence, both statement I and II are correct.

Sol. (90-92):



In right triangle ABC,

$$AC = \sqrt{(AB)^2 + (BC)^2} = \sqrt{(40)^2 + (9)^2} = 41 \text{ cm}$$

90. (a) In ΔACD, $AC = 41$ cm, $AD = 15$ cm, $CD = 28$ cm

Area of $\Delta ACD = \sqrt{S(S-a)(S-b)(S-c)}$

Where $S = \dfrac{a+b+c}{2} = \dfrac{15+28+41}{2} = 42$ cm

$\therefore$ Area of $\Delta ACD = \sqrt{42(42-41)(42-28)(42-15)}$

$= \sqrt{42 \times 1 \times 14 \times 27} = 2 \times 3 \times 3 \times 7 = 126 \text{ cm}^2$.

91. (b) Area of $\Delta ABC = \dfrac{1}{2} \times 9 \times 40 = 180 \text{ cm}^2$

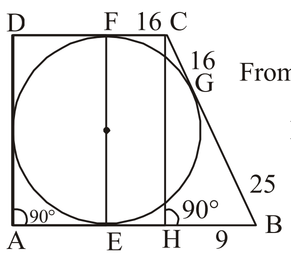
$\because$ Area of quadrilateral $ABCD$

$= 126 + 180 = 306 \text{ cm}^2$

92. (c) Difference = Perimeter of ΔABC – Perimeter of ΔADC

$= (40 + 9 + 41) - (28 + 15 + 41) = 6$ cm

93. (d)



From tangent properties

CF = CG = 16

BE = BG = 25

$\therefore$ BC = 16 + 25 = 41 cm

From ΔBCH,

$$CH = \text{diameter} = \sqrt{(41)^2 - (9)^2}$$

$$= \sqrt{1681 - 81}$$

$$= \sqrt{1600} = 40 \text{ cm}$$

94. (c) Let the length of sides of equilateral triangle and square are a_r and a_s respectiveiy

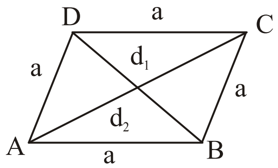
$\therefore$ Length $= 3a_r = 4a_s$

$\Rightarrow \quad a_s = \dfrac{3}{4} a_r$

Area of square $(a_s)^2 \left(\dfrac{3}{4} a_r\right)^2 = \dfrac{1}{16} a_r^2$

$$\frac{\text{Area of } \Delta}{\text{Area of } \square} = \frac{\frac{\sqrt{3}}{4} a_r^2}{a_s^2} = \frac{\sqrt{3}}{4} \times \frac{16}{9} = \frac{4\sqrt{3}}{9}$$

95. (c)



All sides are same for the parallogram

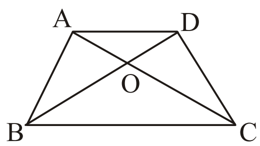
$\therefore$ It is a rhombus

Now $d_1 + d_2 = 12$ cm and $d_2 = 2d_1$

$\Rightarrow \quad d_1 = 4$ cm and $d_2 = 8$ cm

$$\text{Area} = \frac{1}{2} d_1 d_2 = \frac{1}{2} \times 4 \times 8 = 16 \text{ cm}^2$$

96. (d)



Area (ΔABC) = Area (ΔDBC)

Area (ΔAOB + ΔBOC) = Area (ΔDOC + ΔBOC)

Area ΔBOC = 8 – 3 = 5 cm² and Area (ΔDOC) = 3

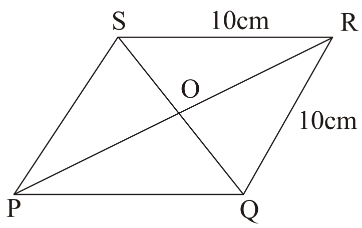
Now, $\dfrac{\text{Area (AOB)}}{\text{Area (BOC)}} = \dfrac{3}{5} = \dfrac{\frac{1}{2} \times AO \times \text{Length}}{\frac{1}{2} \times CO \times \text{Length}}$

$\Rightarrow \quad \dfrac{AO}{CO} = \dfrac{3}{5}$

$$\frac{\text{Area of } (\Delta AOD)}{\text{Area of } (\Delta BOC)} = \left(\frac{AO}{CO}\right)^2 = \frac{9}{25}$$

$$\text{Area } (\Delta AOD) = \frac{9 \times 5}{25} = 1.8 \text{ cm}^2$$

97. (b)



As ΔQRS is equilateral. So, QR = RS = QS = 10cm

$\therefore$ Given parallelogram is a rhombus.

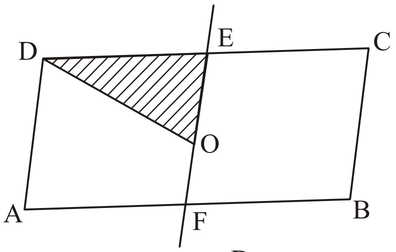
So area = 2 × area of ΔQRS

$$\frac{1}{2} d_1 d_2 = \frac{\sqrt{3}}{4} \times 2 \times (d_1)^2$$

$\Rightarrow \quad d_2 = d_1 \sqrt{3} = 10\sqrt{3}$ cm

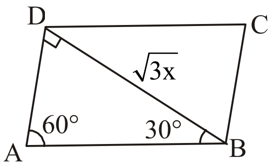
98. (d) $\text{Area (DOE)} = \dfrac{\text{Area (ABCD)}}{8}$

Reqd. Ratio $= \dfrac{7}{8}$



99. (b) $\dfrac{BD}{AB} = \dfrac{\sqrt{3}x}{2x} = \dfrac{\sqrt{3}}{2}$

$\Rightarrow \quad BD^2 = \dfrac{3}{4} AB^2$

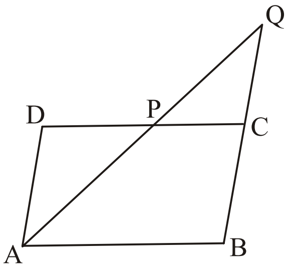


100. (d) $\Delta APD \cong \Delta QPC$

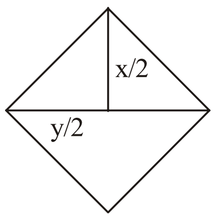
$\Delta QPC \sim \Delta QAB$

$\dfrac{PC}{AB} = \dfrac{1}{2}$

$\therefore \quad \dfrac{\text{Area}(\Delta QPC)}{\text{Area}(\Delta QAB)} = \left(\dfrac{1}{2}\right)^2 = \dfrac{1}{4}$

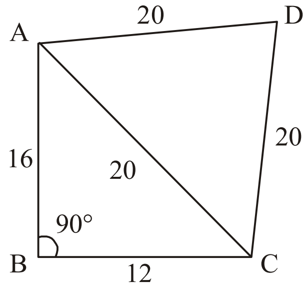


101. (a) $\text{Area} = 4 \times \dfrac{1}{2} \times \dfrac{x}{2} \times \dfrac{y}{2} = \dfrac{xy}{2}$



102. (a) $\text{Area} = \dfrac{\sqrt{3}}{4} \times 20^2 + \dfrac{1}{2} \times 16 \times 12$

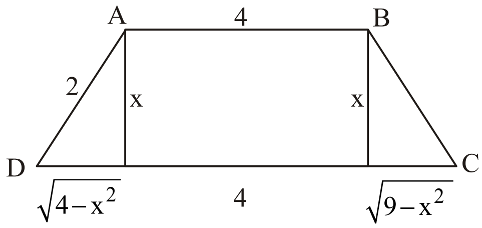
$= 269.2$



103. (d) $\sqrt{4 - x^2} + \sqrt{9 - x^2} = 3 \Rightarrow \sqrt{9 - x^2} = 3 - \sqrt{4 - x^2}$

on squaring both sides, we have

$9 - x^2 = 9 + 4 - x^2 - 6\sqrt{4 - x^2}$



$\Rightarrow \quad \sqrt{4 - x^2} = \dfrac{2}{3}$

$\Rightarrow \quad 4 - x^2 = \dfrac{4}{9} \Rightarrow x^2 = \dfrac{32}{9} \Rightarrow x = \dfrac{4\sqrt{2}}{3}$

$\Rightarrow \quad \text{Area} = \dfrac{1}{2}(4 + 7)\dfrac{4\sqrt{2}}{3} = \dfrac{22\sqrt{2}}{3} \text{ cm}^2$

104. (b) Statement 2 & 3 are true by basic properties

CHAPTER 24

Circle

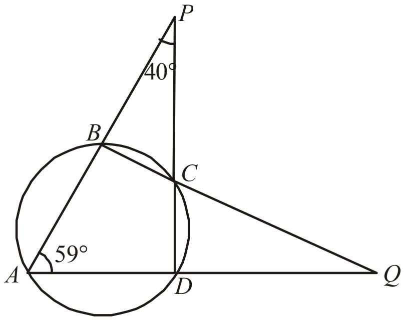
1. C is a point on the minor arc AB of a circle with centre O. If $\angle AOB = 100°$, then what is $\angle ACB$? **[2007-I]**

(a) 80° (b) 90°
(c) 100° (d) 130°

2. In a ΔABC, $AB = AC$. A circle through B touches AC at D and intersects AB at P. If D is the mid-point of AC, then which one of the following is correct? **[2007-I]**

(a) $AB = 2AP$ (b) $AB = 3AP$
(c) $AB = 4AP$ (d) $2\ AB = 5AP$

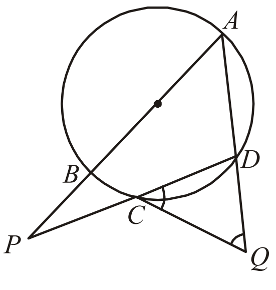
3.



In the given figure, if $\angle PAQ = 59°$, $\angle APD = 40°$, then what is $\angle AQB$? **[2007-I]**

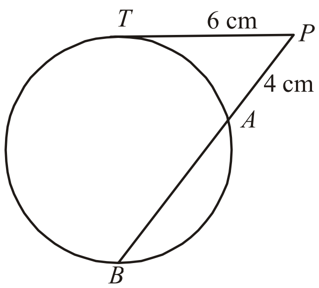
(a) 19° (b) 20°
(c) 22° (d) 27°

4. In the given figure, if $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$, where $\angle DCQ = x$, $\angle BPC = y$ and $\angle DQC = z$, then what are the values of x, y and z, respectively? **[2007-II]**



(a) 33°, 44° and 55° (b) 36°, 48° and 60°
(c) 39°, 52° and 65° (d) 42°, 56° and 70°

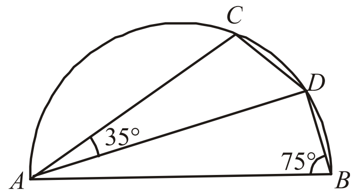
5.



In the figure given above, $PT = 6$ cm and $PA = 4$ cm. What is the length of AB? **[2007-II]**

(a) 9 cm (b) 5 cm
(c) 4 cm (d) 6 cm

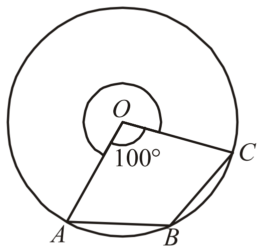
6.



In the figure given above, C and D are points on the semi-circle described on AB as diameter. If $\angle ABD = 75°$ and $\angle DAC = 35°$, then what is the $\angle BDC$? **[2007-II]**

(a) 130° (b) 110°
(c) 90° (d) 100°

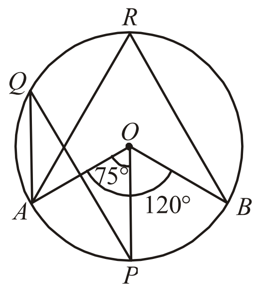
7.



In the figure given above, the $\angle AOC$ is 100°, where O is the centre of the circle. What is the $\angle ABC$? **[2007-II]**

(a) 100° (b) 80°
(c) 120° (d) 130°

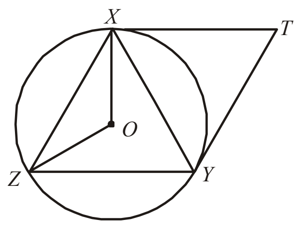
8.



In the figure given above, if $\angle AOP = 75°$ and $\angle AOB = 120°$, then what is the value of $\angle AQP$? **[2007-II]**

(a) 45° (b) 37.5°
(c) 30° (d) 22.5°

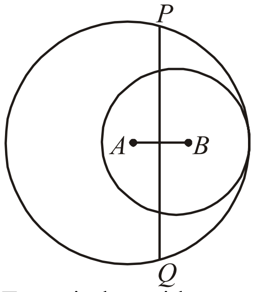
9.



In the figure given above, O is the centre of the circumcircle of the ΔXYZ. Tangents at X and Y intersect at T. If $\angle XTY = 80°$, then what is the value of $\angle ZXY$? **[2007-II]**

(a) 20° (b) 40°
(c) 60° (d) 80°

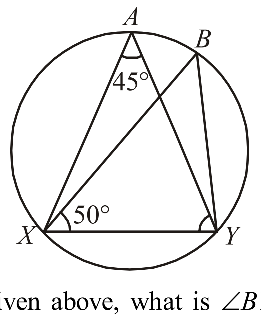
10.



Two circles with centres A and B touch each other internally, as shown in the figure given above. Their radii are 5 and 3 units, respectively. Perpendicular bisector of AB meets the bigger circle in P and Q. What is the length of PQ? **[2007-II]**

(a) $2\sqrt{6}$ (b) $\sqrt{34}$
(c) $4\sqrt{6}$ (d) $6\sqrt{2}$

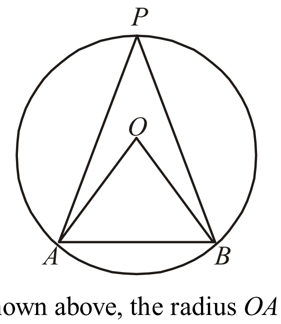
11.



In the figure given above, what is $\angle BYX$ equal to? **[2008-I]**

(a) 85° (b) 50°
(c) 45° (d) 90°

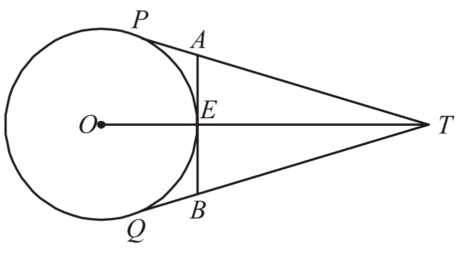
12.



In the figure shown above, the radius OA is equal to the chord AB. Then, what is $\angle APB$? **[2008-I]**

(a) 30° (b) 60°
(c) 15° (d) 45°

13.



In the figure given above, from a point T, 13 cm away from the centre O of a circle of radius 5 cm, tangents PT and QT are drawn. What is the length of AB? **[2008-I]**

(a) $\frac{19}{3}$ cm (b) $\frac{20}{3}$ cm
(c) $\frac{40}{13}$ cm (d) $\frac{22}{3}$ cm

14. With A, B and C as centres, three circles are drawn such that they touch each other externally. If the sides of the ΔABC are 4 cm, 6 cm and 8 cm, then what is the sum of the radii of the circles? **[2008-I]**

(a) 9 cm (b) 10 cm
(c) 12 cm (d) 14 cm

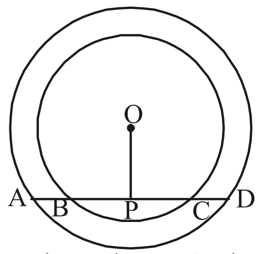
15. If two circles C_1 and C_2 have three points in common, then which of the following is correct? **[2008-I]**

(a) C_1 and C_2 are concentric
(b) C_1 anc C_2 are the same circle
(c) C_1 and C_2 have different centres
(d) None of the above

16. What is the number of tangents that can be drawn to a circle from a point on the circle? **[2008-I]**

(a) 0 (b) 1
(c) 2 (d) 3

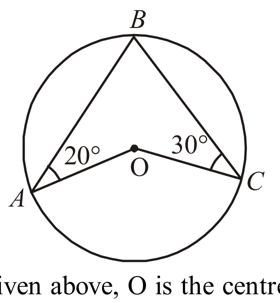
17.



In the figure given above, AD is a straight line, OP perpendicular to AD and O is the centre of both circles. If $OA = 20$ cm, $OB = 15$ cm and $OP = 12$ cm, then what is AB equal to ? **[2008-I]**

(a) 7 cm (b) 8 cm
(c) 10 cm (d) 12 cm

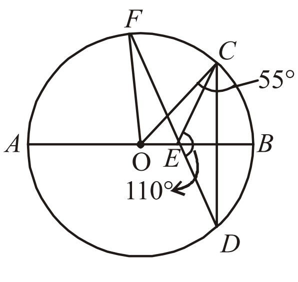
18.



In the figure given above, O is the centre of the circle. What is $\angle AOC$? **[2008-II]**

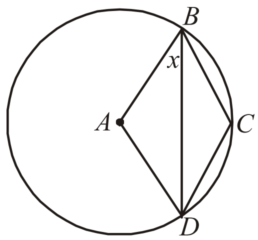
(a) 160° (b) 150°
(c) 120° (d) 100°

19.



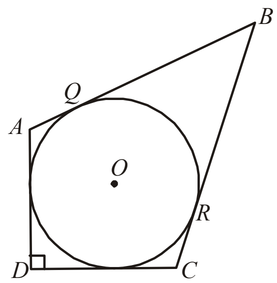
In the figure given above, AB is a diameter of the circle with centre O and $EC = ED$. What is $\angle EFO$? **[2008-II]**

(a) 15° (b) 20°
(c) 25° (d) 30°

20. 

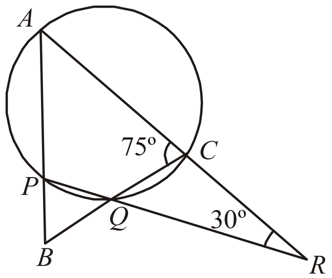
In the figure given above, A is the centre of the circle and $AB = BC = CD$. What is the value of x? **[2008-II]**

(a) 20° (b) $22\frac{1}{2}°$
(c) 25° (d) None of these

21. 

In the figure given above, a circle is inscribed in a quadrilateral $ABCD$. Given that, $BC = 38$ cm, $QB = 27$ cm, $DC = 25$ cm and AD is perpendicular to DC. What is the radius of the circle? **[2009-I]**

(a) 11 cm (b) 14 cm
(c) 15 cm (d) 16 cm

22. 

In the figure given above, what is $\angle CBA$? **[2009-I]**

(a) 30° (b) 45°
(c) 50° (d) 60°

23. A, B, C and D are four distinct points on a circle whose centre is at O.
If $\angle OBD - \angle CDB = \angle CBD - \angle ODB$, then what is $\angle A$ equal to? **[2009-I]**

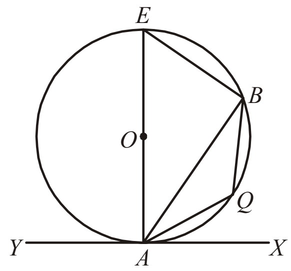
(a) 45° (b) 60°
(c) 120° (d) 135°

24. PQ is a common chord of two circles. APB is a secant line joining points A and B on the two circles. Two tangents AC and BC are drawn. If $\angle ACB = 45°$, then what is $\angle AQB$ equal to? **[2009-I]**

(a) 75° (b) 90°
(c) 120° (d) 135°

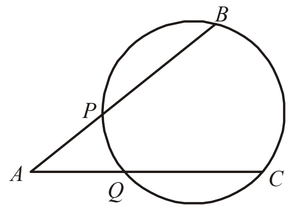
25. $ABCD$ is concyclic quadrilateral. The tangents at A and C intersect each other at P. If $\angle ABC = 100°$, then what is $\angle APC$ equal to? **[2009-I]**

(a) 10° (b) 20°
(c) 30° (d) 40°

26. 

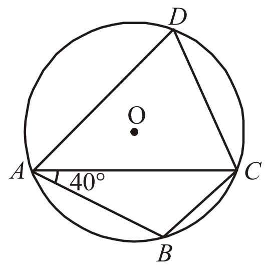
In the figure given above, YAX is a tangent to the circle with centre O. If $\angle BAX = 70°$ and $\angle BAQ = 40°$, then what is $\angle ABQ$ equal to? **[2009-I]**

(a) 20° (b) 30°
(c) 35° (d) 40°

27. 

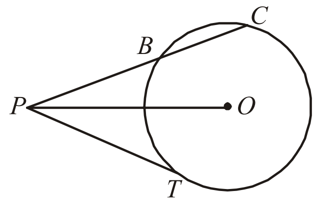
In the figure given above, If $AP = 3$ cm, $PB = 5$ cm, $AQ = 2$ cm and $QC = x$, then what is the value of x? **[2009-I]**

(a) 6 cm (b) 8 cm
(c) 10 cm (d) 12 cm

28. 

In the figure given above, O is the centre of a circle circumscribing a quadrilateral $ABCD$. If $AB = BC$ and $\angle BAC = 40°$, then what is $\angle ADC$ equal to? **[2009-I]**

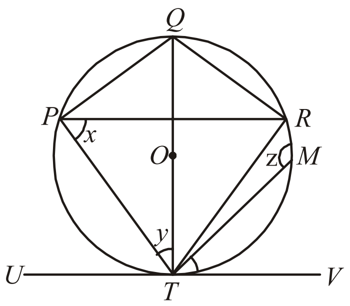
(a) 50° (b) 60°
(c) 70° (d) 80°

29. 

In the figure given above, PT is a tangent to a circle of radius 6 cm. If P is at a distance of 10 cm from the centre O and $PB = 5$ cm, then what is the length of the chord BC? **[2009-II]**

(a) 7.8 cm (b) 8 cm
(c) 8.4 cm (d) 9 cm

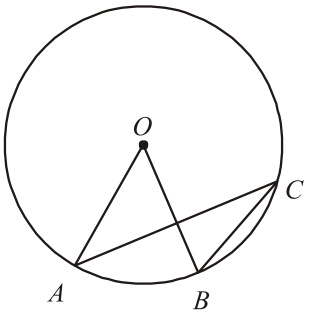
30.



In the figure given above, O is the centre of the circle. The line UTV is a tangent to the circle at T, $\angle VTR = 52°$ and ΔPTR is an isosceles triangle such that $TP = TR$. What is $\angle x + \angle y + \angle z$ equal to? **[2009-II]**

(a) 175° (b) 208°
(c) 218° (d) 250°

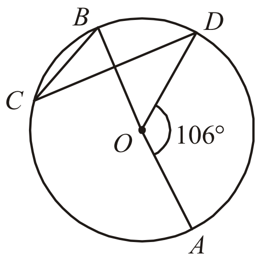
31.



In the figure given above, $\angle AOB = 46°$, AC and OB intersect each other at right angles. What is the measure of $\angle OBC$ (where, O is the centre of the circle)? **[2009-II]**

(a) 44° (b) 46°
(c) 67° (d) 78.5°

32.



In the figure given above. If O is the centre of the circle $\angle AOD = 106°$, then what is $\angle BCD$ equal to? **[2009-II]**

(a) 53° (b) 43°
(c) 40° (d) 37°

33. What is the number of circles passing through a given pair of points? **[2010-I]**

(a) One
(b) Two
(c) More than two, but finite
(d) Infinitely many

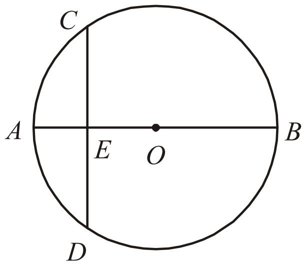
34. Two circles touch each other externally at P. Two secants APB and CPD are drawn through P to meet the circle at A, C and B, D respectively. Then, which one of the following is correct? **[2010-I]**

(a) AC is perpendicular to BD
(b) AC intersects BD
(c) AC is parallel to BD
(d) None of the above

35. What is the locus of centres of circles which touch a given line at a given point? **[2010-I]**

(a) A line perpendicular to the given line, passing through the given point
(b) A line parallel to the given line
(c) A circle tangent to the given line at the given point
(d) A closed curve other than a circle

36. In the given figure, AB is a diameter of a circle and CD is perpendicular to AB, if $AB = 10$ cm and $AE = 2$ cm, then what is the length of ED? **[2010-I]**



(a) 5 cm (b) 4 cm
(c) $\sqrt{10}$ cm (d) $\sqrt{20}$ cm

37. $ABCD$ is a quadrilateral, the sides of which touch a circle. Which one of the following is correct? **[2010-II]**

(a) $AB + AD = CB + CD$
(b) $AB : CD = AD : BC$
(c) $AB + CD = AD + BC$
(d) $AB : AD = CB : CD$

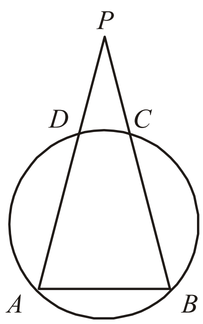
38. The diameter of two circles are 18 cm and 8 cm. The distance between their centres is 13 cm. What is the number of common tangents? **[2010-II]**

(a) 1 (b) 2
(c) 3 (d) None of these

39. Let PAB be a secant to a circle intersecting at points A and B and PC is a tangent. Which one of the following is correct? **[2010-II]**

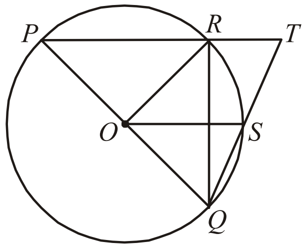
(a) The area of rectangle with PA, PB as sides is equal to the area of square with PC as sides
(b) The area of rectangle with PA, PC as sides is equal to the area of square with PB as sides
(c) The area of rectangular with PC, PB as sides is equal to the area of square with PA as side
(d) The perimeter of rectangle with PA, PB as sides is equal to the perimeter of square with PC as side

40. In the figure given below, if $\angle BAD = 60°$, $\angle ADC = 105°$, then what is $\angle DPC$ equal to? **[2010-II]**



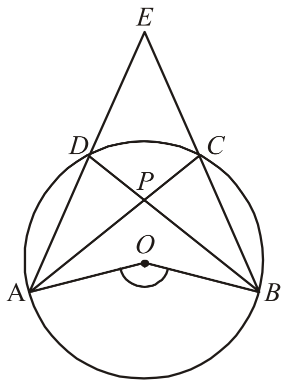
(a) 40° (b) 45°
(c) 50° (d) 60°

41. In the figure given below, PQ is a diameter of the circle whose centre is at O. If $\angle ROS = 44°$ and OR is a bisector of $\angle PRQ$, then what is the value of $\angle RTS$? **[2010-II]**



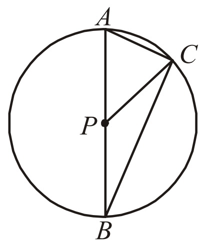
(a) 46° (b) 64°
(c) 69° (d) None of these

42. In the figure given below. O is the centre of the circle. AC and BD intersect at P. If $\angle AOB = 100°$ and $\angle DAP = 30°$, then what is $\angle APB$? **[2010-II]**



(a) 77° (b) 80°
(c) 85° (d) 90°

43. In the given figure, A and B are end points of diameter of a circle with centres at P and C is a point on the circumference of the circle such that $\angle ABC = 35°$, then what is $\angle PCA$? **[2010-II]**



(a) 25° (b) 30°
(c) 35° (d) 55°

44. What is the number of circles passing through all the vertices of a given triangle? **[2011-I]**
(a) One (b) Two
(c) Three (d) Infinite

45. Consider the following statements
I. Let P be a point on a straight line L. Let Q, R and S be the points on the same plane containing the line L such that PQ, PR and PS are perpendicular to L. Then, there exists no triangle with vertices Q, R and S.
II. Let C be a circle passing through three distinct points D, E and F such that the tangent at D to the circle C is parallel to EF. Then, DEF is an isosceles triangle.

Which of the statement (s) given above is/are correct? **[2011-I]**
(a) Only I (b) Only II
(c) Both I and II (d) Neither I and II

46. Two circles touch each other internally. Their radii are 4 cm and 6 cm. What is the length of the longest chord of the outer circle which is outside the inner circle? **[2011-I]**
(a) $4\sqrt{2}$ cm (b) $4\sqrt{3}$ cm
(c) $6\sqrt{3}$ cm (d) $8\sqrt{2}$ cm

47. The distance between the centres of two circles having radii 4.5 cm and 3.5 cm respectively is 10 cm. What is the length of the transverse common tangent of these circles? **[2011-I]**
(a) 8 cm (b) 7 cm
(c) 6 cm (d) None of these

48. ABC is an equilateral triangle inscribed in a circle with $AB = 5$ cm. Let the bisector of the angle A meet BC in X and the circle in Y. What is the value of $AX \cdot AY$? **[2011-I]**
(a) 16 cm^2 (b) 20 cm^2
(c) 25 cm^2 (d) 30 cm^2

49. Two unequal circle are touching each other externally at P, APB and CPD are two secants cutting the circles at A, B, C and D. Which one of the following is correct? **[2011-I]**
(a) $ACBD$ is parallelogram (b) $ACBD$ is a trapezium
(c) $ACBD$ is a rhombus (d) None of the above

50. Let C be a circle. A point P moves such that the tangents from P to C include an angle of 60°. What is the locus of P? **[2011-I]**
(a) Straight line
(b) A circle concentric with C
(c) A circle touching C
(d) A circle intersecting C at two points

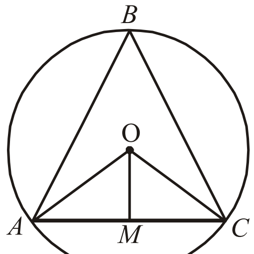
51. A bicycle is running straight towards North. What is the locus of the centre of the front wheel of the bicycle whose diameter is d? **[2011-I]**
(a) A line parallel to the path of the wheel of the bicycle at a height d cm
(b) A line parallel to the path of the wheel of the bicycle at a height $d/2$ cm
(c) A circle of radius $d/2$ cm
(d) A circle of radius d cm

52. What is the length of the perpendicular drawn from the centre of circle of radius r on the chord of length $\sqrt{3}r$? **[2011-II]**
(a) r (b) $\sqrt{2}r$
(c) $r/2$ (d) $r/4$

53. Consider the following statements
I. The opposite angles of a cyclic quadrilateral are supplementary.
II. Angle subtended by an arc at the centre is double

the angle subtended by it at any point on the remaining part of the circle.
Which one of the following is correct in respect of the above statements? **[2011-II]**
(a) Statement I $\Rightarrow$ Statement II
(b) Statement II $\Rightarrow$ Statement I
(c) Statement I $\Leftrightarrow$ Statement II
(d) Neither Statement I $\Rightarrow$ Statement II nor Statement II $\Rightarrow$ Statement I

54.



In the figure given above, O is the centre of the circle, If OA = 3 cm, AC = 3 cm and OM is perpendicular to AC, then what is $\angle ABC$ equal to? **[2011-II]**
(a) 60° (b) 45°
(c) 30° (d) None of these

55. AC is the diameter of the circumcircle of the cyclic quadrilateral $ABCD$. If $\angle BDC = 42°$, then what is $\angle ACB$ equal to? **[2011-II]**
(a) 42° (b) 45°
(c) 48° (d) 58°

56. What is the number of circles which pass through three given points not in a straight line? **[2011-II]**
(a) 0 (b) 1
(c) 2 (d) 3

57. Let AB be a line segment such that a point P on the locus satisfies $AB^2 - AP^2 - BP^2 = 0$. What is the locus? **[2012-I]**
(a) The straight line AB itself
(b) The perpendicular bisector of AB
(c) A straight line parallel to AB
(d) A circle

58. A circular ring with centre O is kept in the vertical position by two weightless thin strings TP and TQ attached to the ring at P and Q. The line OT meets the ring at E whereas a tangential string at E meets TP and TQ at A and B, respectively. If the radius of the ring is 5 cm and OT = 13 cm, then what is the length of AB? **[2012-I]**
(a) 10/3 cm (b) 20/3 cm
(c) 10 cm (d) 40/3 cm

59. The locus of the mid-points of all equal chords in a circle is **[2012-I]**
(a) The circumference of the circle concentric with the given circle and having radius equal to the length of the chords.
(b) The circumference of the circle concentric with the given circle and having radius equal to the distance of the chords from the centre.
(c) The circumference of the circle concentric with the given circle and having radius equal to half of the radius of the given circle.
(d) The circumference of the circle concentric with the given circle and having radius equal to half of the distance of the chords from the centre.

60. Consider a circle with centre at O and radius r. Points A and B lie on its circumference and a point M lies outside of it such that M, A and O lie on the same straight line. Then, the ratio of MA to MB is **[2013-I]**
(a) equal to 1 (b) equal to r
(c) greater than 1 (d) less than 1

61. Consider the following statements
I. The tangent of a circle is a line that meets the circle in one and only one point.
II. The tangent of a circle at the end point of the diameter is perpendicular to the diameter.
Which of the above statements is/are correct? **[2013-I]**
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

62. Consider the following statements in respect of two chords XY and ZT of a circle intersecting at P.
I. $PX \cdot PY = PZ \cdot PT$
II. PXZ and PTY are similar triangles.
Which of the statements given above is/are correct? **[2013-II]**
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

63. The diameter of a circle with centre at C is 50 cm. CP is a radial segment of the circle. AB is a chord perpendicular to CP and passes through P. CP produced intersects the circle at D. If DP = 18 cm, then what is the length of AB? **[2013-II]**
(a) 24 cm (b) 32 cm
(c) 40 cm (d) 48 cm

64. A regular hexagon is inscribed in a circle of radius 5 cm. If x is the area inside the circle but outside the regular hexagon, then which one of the following is correct? **[2013-II]**
(a) $13\text{ cm}^2 < x < 15\text{ cm}^2$ (b) $15\text{ cm}^2 < x < 17\text{ cm}^2$
(c) $17\text{ cm}^2 < x < 19\text{ cm}^2$ (d) $19\text{ cm}^2 < x < 21\text{ cm}^2$

65. Consider the following statements
I. The perpendicular bisector of a chord of a circle does not pass through the centre of the circle.
II. The angle in a semi-circle is a right angle.
Which of the statements given above is/are correct?
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

66. ABC is an equilateral triangle inscribed in a circle. D is any point on the arc BC. What is $\angle ADB$ equal to? **[2013-II]**
(a) 90° (b) 60°
(c) 45° (d) None of the above

67. A circle of radius 10 cm has an equilateral triangle inscribed in it. The length of the perpendicular drawn from the centre to any side of the triangle is **[2014-I]**
(a) $2.5\sqrt{3}$ cm (b) $5\sqrt{3}$ cm
(c) $10\sqrt{3}$ cm (d) None of these

68. AB and CD are two chords of a circle meeting externally at P. Then, which of the following is/are correct?

I. $PA \times PD = PC \times PB$

II. ΔPAC and ΔPDB are similar.

Select the correct answer using the codes given below. **[2014-I]**

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

69. In a ΔABC, $AB = BC = CA$. The ratio of the radius of the circumcircle to that of the incircle is **[2014-I]**

(a) 2 : 1 (b) 3 : 1
(c) 3 : 2 (d) None of these

70. If the chord of an arc of a circle is of length x, the height of the arc is y and the radius of the circle is z. Then, which one of the following is correct ? **[2014-II]**

(a) $y(2z - y) = x^2$ (b) $y(2z - y) = 4x^2$
(c) $2y(2z - y) = x^2$ (d) $4y(2z - y) = x^2$

71. A railroad curve is to be laid on a circle. What radius (approximate) should be used, if the track is to change direction by 25° in a distance of 120 m? **[2014-II]**

(a) 300 m (b) 280 m
(c) 275 m (d) 264 m

72. If the radius of a circle is increased by 6%, then its area will increase by **[2014-II]**

(a) 6% (b) 9%
(c) 12.36% (d) 16.64%

73. Two circles, each of radius r, with centres P and Q, are such that each circle passes through the centre of the other circle. Then the area common to the circles is less than one-third of the sum of the areas of the two circles by **[2015-I]**

(a) $\frac{\sqrt{3}r^2}{4}$ (b) $\frac{\sqrt{3}r^2}{3}$
(c) $\frac{\sqrt{3}r^2}{2}$ (d) $\sqrt{3}r^2$

74. Three equal circles each of diameter d are drawn on a plane in such a way that each circle touches the other two circles. A big circle is drawn in such a manner that it touches each of the small circles internally. The area of the big circle is **[2015-I]**

(a) πd^2 (b) $\pi d^2(2-\sqrt{3})^2$
(c) $\frac{\pi d^2(\sqrt{3}+1)^2}{2}$ (d) $\frac{\pi d^2(\sqrt{3}+2)^2}{12}$

75. If the angle between the radii of a circle is 130°, then the angle between the tangents at the ends of the radii is **[2015-I]**

(a) 90° (b) 70°
(c) 50° (d) 40°

76. Out of two concentric circles, the diameter of the outer circle is 26 cm and the chord MN of length 24cm is tangent to the inner circle. The radius of the inner circle is **[2015-I]**

(a) 5 cm (b) 6 cm
(c) 8 cm (d) 10 cm

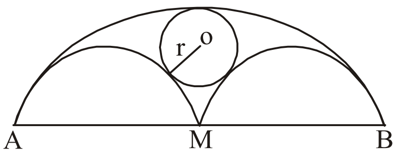
77. The two adjacent sides of a cyclic quadrilateral are 2 cm and 5 cm and the angle between them is 60°. If the third side is 3 cm, then the fourth side is of length **[2015-II]**

(a) 2 cm (b) 3 cm
(c) 4 cm (d) 5 cm

78. From a rectangular sheet of sides 18 cm and 14 cm, a semicircular portion with smaller side as diameter is taken out. Then the area of the remaining sheet will be **[2015-II]**

(a) 98 cm^2 (b) 100 cm^2
(c) 108 cm^2 (d) 175 cm^2

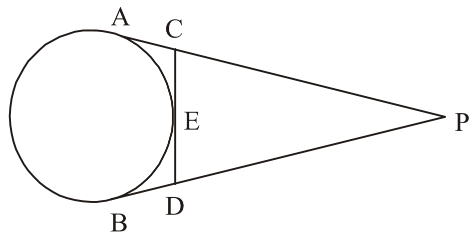
79.



AB is a line segment of length 2a, with M as mid–point. Semicircles are drawn on one side with AM, MB and AB as diameter as shown in the above figure. A circle with centre O and radius r is drawn such that this circle touches all the three semicircles. The value of r is **[2015-II]**

(a) $\frac{2a}{3}$ (b) $\frac{a}{2}$
(c) $\frac{a}{3}$ (d) $\frac{a}{4}$

80.



From an external point P tangents PA and PB are drawn to the circle as shown in the above figure. CD is the tangent to the circle at E. If AP = 16 cm, then the perimeter of the triangle PCD is equal to **[2015-II]**

(a) 24 cm (b) 28cm
(c) 30 cm (d) 32 cm

81. Chord CD intersects the diameter AB of a circle at right angle at a point P in the ratio 1 : 2. If diameter of circle is D, then CD is equal to **[2015-II]**

(a) $\frac{\sqrt{2}d}{3}$ (b) $\frac{2d}{3}$
(c) $\frac{2\sqrt{2}d}{3}$ (d) $\frac{2\sqrt{3}d}{3}$

82. In a circle of radius 2 units, a diameter AB intersects a chord of length 2 units perpendicularly at P. If $AP > BP$, then AP is equal to **[2016-I]**

(a) $(2+\sqrt{5})$ units (b) $(2+\sqrt{3})$ units
(c) $(2+\sqrt{2})$ units (d) 3 units

83. A truck moves along a circular path and describes 100 m when it has traced out 36° at the centre. The radius of the circle is equal to **[2016-I]**

(a) $\frac{100}{\pi}m$ (b) $\frac{250}{\pi}m$

(c) $\frac{500}{\pi}m$ (d) $\frac{600}{\pi}m$

84. A tangent is drawn from an external point D to a circle of radius 3 units at P such that DP = 4 units. If O is the centre of the circle, the the sine of the angle ODP is **[2016-I]**

(a) 4/5 (b) 3/4
(c) 3/5 (d) 1/2

85. Consider a circle with centre at O and radius 7 cm. Let OR be a chord of length 2 cm and let P be the midpoint of QR. Let CD be another chord of the circle passing through P such that $\angle CPQ$ is acute. If M is the midpoint of CD and $MP = \sqrt{24}$ cm, then which of the following statements are correct?

1. If $CP = 135°$
2. If CP = m cm and $PD = n$ cm, then m and n are the roots of the quadratic equation $x^2 - 10x + 1 = 0$
3. The ratio of the area of triangle OPR to the area of triangle OMP is $1 : 2\sqrt{2}$.

Select the correct answer using the code given below.

(a) 1 and 2 only (b) 2 and 2 only
(c) 1 and 3 only (d) 1, 2 and 3

86. Consider a circle with centre at C Let OP, OQ denote respectively the tangents to the circle drawn from a point O outside the circle. Let R be a point on OP and S be a point on OQ such that $OR \times SQ = OS \times RP$. Which of the following statement is/are correct? **[2016-I]**

1. If X is the circle with centre at O and radius OR, and Y is the circle with centre at O and radius OS, then $X = Y$.
2. $\angle POC + \angle QCO = 90°$

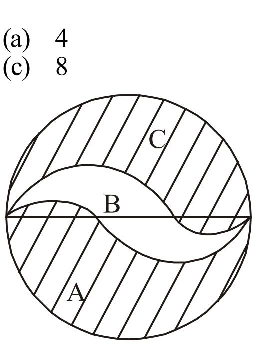
Select the correct answer using the code given below.

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

87. A circular path is made from two concentric circular rings in such a way that the smaller ring when allowed to roll over the circumference of the bigger ring, it takes three full revolutions. If the area of the pathway is equal to n times the area of the smaller ring, then n is equal to **[2016-I]**

(a) 4 (b) 6
(c) 8 (d) 10

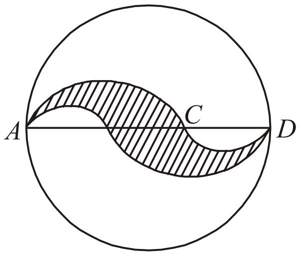
88.



A circle of 3 m radius is divided into three areas by semicircles of radii 1m and 2m as shown in the figure above. The ratio of the three areas A, B and C will be **[2016-I]**

(a) 2 : 3 : 2 (b) 1 : 1 : 1
(c) 4 : 3 : 4 (d) 1 : 2 : 1

89.



AD is the diameter of a circle with area 707 m^2 and AB = BC = CD as shown in the figure above. All curves inside the circle are semicircles with their diameters on AD. What is the cost of levelling the shaded region at the rate of 63 per square metre? **[2016-I]**

(a) ₹ 29,700 (b) ₹ 22,400
(c) ₹ 14,847 (d) None of the above

90. Two circles touch externally and sum of their areas is 130 π cm^2 and the distance between their centres is 14 cm. What is the difference in the radii of the circles ? **[2016-II]**

(a) 5 cm (b) 6 cm
(c) 7 cm (d) 8 cm

91. In a circle of radius 3 units, a diameter AB, intersects a chord of length 2 units perpendicularly at P. If AP > BP, then what'is the ratio of AP to BP ? **[2016-II]**

(a) $3+\sqrt{10} : 33-\sqrt{10}$
(b) $3+\sqrt{8} : 3-\sqrt{8}$
(c) $3+\sqrt{3} : 3-\sqrt{3}$
(d) $3 : \sqrt{3}$

92. What is the number of rounds that a wheel of diameter $\frac{5}{11}$m will make in travelling of 7 km ? **[2016-II]**

(a) 3300 (b) 3500
(c) 4400 (d) 4900

93. If two tangents inclined at an angle 60° are drawn to a circle of radius 3 cm, then what is the length of each tangent ? **[2016-II]**

(a) $3\sqrt{3}$cm (b) $\sqrt{3}$cm
(c) 6 cm (d) $2\sqrt{2}$cm

94. The diameter of the front wheel of an engine is 2x cm and that of rear wheel is 2y cm. To cover the same distance, what is the number of times the rear wheel revolves when the front wheel revolves n times ? **[2016-II]**

(a) $\frac{n}{xy}$ (b) $\frac{ny}{x}$

(c) $\frac{nx}{y}$ (d) $\frac{xy}{n}$

95. Three circles each of radius 3.5 cm touch one another. The area subtended between them is **[2017-I]**

(a) $6(\sqrt{3}\pi - 2)$ square units

(b) $6(2\pi - \sqrt{3}\pi)$ square units

(c) $\frac{49}{8}(2\sqrt{3} - \pi)$ square units

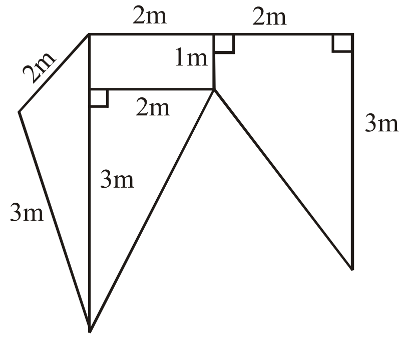
(d) $\frac{49}{8}(\sqrt{3} - \pi)$ square units

96. Two parallel chords of a circle whose diameter is 13 cm are respectively 5 cm and 12 cm in length. If both the chords are on the same side of the diameter, then the distance between these chords is **[2017-I]**
(a) 5.5 cm (b) 5 cm
(c) 3.5 cm (d) 3 cm

97. A copper wire when bent in the form of a square encloses an area of 121 cm^2. If the same wire is bent in the form of a circle, it encloses an area equal to **[2017-I]**
(a) 121 cm^2 (b) 144 cm^2
(c) 154 cm^2 (d) 168 cm^2

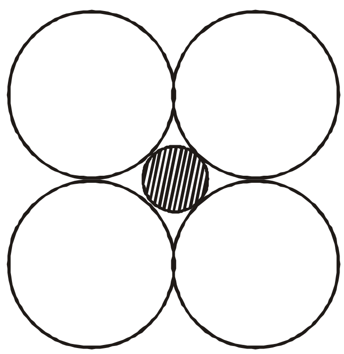
98. The radius of a circle is increased so that it's circumference increases by 15%. The area of the circle will increase by **[2017-I]**
(a) 31.25% (b) 32.25%
(c) 33.25% (d) 34.25%

99. A field is divided into four regions as shown in the given figure. What is the area of the field in square metres ? **[2017-I]**



(a) $6+\frac{3}{4}\sqrt{5}$ (b) $5+\frac{3}{2}\sqrt{3}$
(c) $9+\frac{3}{4}\sqrt{15}$ (d) $7+2\sqrt{2}$

100. In the figure given below, D is the diameter of each circle. What is the diameter of the shaded circle ? **[2017-I]**



(a) $D(\sqrt{2}-1)$ (b) $D(\sqrt{2}+1)$
(c) $D(\sqrt{2}+2)$ (d) $D(2-\sqrt{2})$

101. A hollow cube is formed by joining six identical squares. A rectangular cello tape of length 4 cm and breadth 0.5 cm is used for joining each pair of edges. What is the total area of cello tape used? **[2017-II]**
(a) 12 square cm (b) 24 square cm
(c) 36 square cm (d) 48 square cm

102. Two straight lines AB and AC include an angle. A circle is drawn in this angle which touches both these lines. One more circle is drawn which touches both these lines as well as the previous circle. If the area of the bigger circle is 9 times the area of the smaller circle, then what must be the angle A? **[2017-II]**
(a) 45° (b) 60°
(c) 75° (d) 90°

103. AB and CD are parallel chords of a circle 3 cm apart. If AB = 4 cm, CD = 10 cm, then what is the radius of the circle? **[2017-II]**
(a) 7 cm (b) $\sqrt{19}$ cm
(c) $\sqrt{29}$ cm (d) 14 cm

104. The diagonals of a cyclic quadrilateral ABCD intersect at P and the area of the triangle APB is 24 square cm. If AB = 8 cm and CD = 5 cm, then what is the area of the triangle CPD? **[2017-II]**
(a) 24 square cm (b) 15 square cm
(c) 12.5 square cm (d) 9.375 square cm

105. The distance between the centres of two circles having radii 9 cm and 4 cm is 13 cm. What is the length of the direct common tangent of these circles? **[2017-II]**
(a) 12 cm (b) 11 cm
(c) 10 cm (d) 9.5 cm

106. An arc of a circle subtends an angle π at the centre. If the length of the arc is 22 cm, then what is the radius of the circle? **[2017-II]**

(Take $\pi = \frac{22}{7}$)
(a) 5 cm (b) 7 cm
(c) 9 cm (d) 11 cm

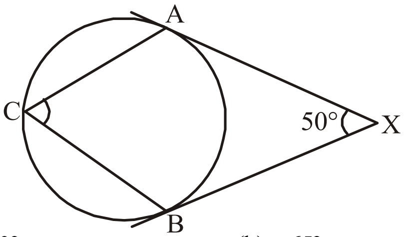
107. Two equal circular regions of greatest possible area are cut off from a given circular sheet of area A. What is the remaining area of the sheet? **[2018-1]**
(a) A/2 (b) A/3
(c) 3A/5 (d) 2A/5

108. The radii of two circles are 4.5 cm and 3.5 cm respectively. The distance between the centres of the circles is 10 cm. What is the length of the transverse common tangent? **[2018-1]**
(a) 4 cm (b) 5 cm
(c) 6 cm (d) 7 cm

109. The locus of the mid-points of the radii of length 16 cm of a circle is **[2018-1]**
(a) A concentric circle of radius 8 cm
(b) A concentric circle of radius 16 cm
(c) The diameter of the circle
(d) A straight line passing through the centre of the circle

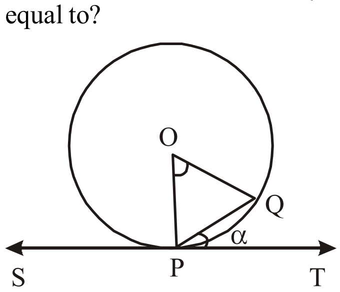
110. What is the area of the region bounded externally by a square of side of length 'a' and internally by a circle passing through the four corners of the square? **[2018-1]**
(a) $(\pi - 1)\,a^2$ square units
(b) $\frac{(\pi-1)a^2}{2}$ square units
(c) $(\pi - 2)\,a^2$ square units
(d) $\frac{(\pi-2)a^2}{2}$ square units

111. In the figure given below, XA and XB are two tangents to a circle. If $\angle AXB = 50°$ and AC is parallel to XB, then what is $\angle ACB$ equal to? **[2018-1]**



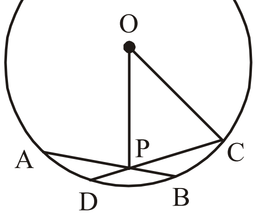
(a) 70° (b) 65°
(c) 60° (d) 55°

112. In the figure given below, SPT is a tangent to the circle at P and O is the centre of the circle. If $\angle QPT = \alpha$, then what is $\angle POQ$ equal to? **[2018-1]**



(a) α (b) 2α
(c) $90° - \alpha$ (d) $180° - 2\alpha$

113. In the figure given below, two equal chords cut at point P. If AB = CD = 10 cm, OC = 13 cm (O is the centre of the circle) and PB = 3 cm, then what is the length of OP? **[2018-1]**



(a) 5 cm (b) 6 cm
(c) $2\sqrt{29}$ cm (d) $2\sqrt{37}$ cm

114. A circle is inscribed in an equilateral triangle of side of length *l*. The area of any square inscribed in the circle is **[2018-II]**

(a) $\frac{l^2}{2}$ (b) $\frac{\sqrt{3}l^2}{4}$

(c) $\frac{l^2}{4}$ (d) $\frac{l^2}{6}$

115. There is a path of width 5 m around a circular plot of land whose area is 144π m^2. The total area of the circular plot including the path surrounding it is **[2018-II]**

(a) $349\pi m^2$ (b) $289\pi m^2$
(c) $209\pi m^2$ (d) $149\pi m^2$

116. An equilateral triangle, a square and a circle have equal perimeter. If T, S and C denote the area of the triangle, area of the square and area of the circle respectively, then which one of the following is correct? **[2018-II]**

(a) T < S < C (b) S < T < C
(c) C < S < T (d) T < C < S

117. The chord of a circle is $\sqrt{3}$ times its radius. The angle subtended by this chord at the minor arc is k times the angle subtended at the major arc. What is the value of k? **[2018-II]**

(a) 5 (b) 2
(c) 1@2 (d) 1@5

118. The area of the region bounded externally by a square of side 2a cm and internally by the circle touching the four sides of the square is **[2018-II]**

(a) $(4 - \pi)a^2$ (b) $(\pi - 2)a^2$
(c) $(8 - \pi)a^2/2$ (d) $(\pi - 2)a^2/2$

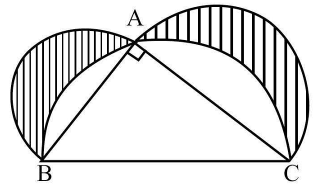
119. In the figure given below, ABC is a right-angled triangle where $\angle A = 90°$, AB = p cm and AC = q cm. On the three sides as diameters semicircles are drawn as shown in the figure. The area of the shaded portion, in square cm is **[2018-II]**

(a) pq

(b) $\frac{\pi(p^2+q^2)}{2}$

(c) $\pi(p^2 + q^2)$

(d) $\frac{pq}{2}$



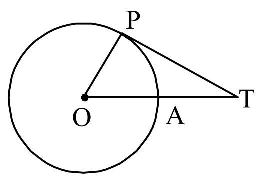
120. In the figure given below, the radius of the circle is 6 cm and AT = 4 cm. The length of tangent PT is **[2018-II]**

(a) 6 cm

(b) 8 cm

(c) 9 cm

(d) 10 cm



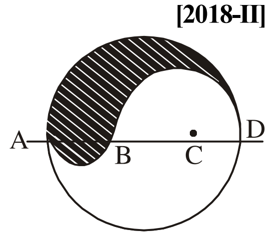
121. In the figure given below, ABCD is the diameter of a circle of radius 9 cm. The lengths AB, BC and CD are equal. Semicircles are drawn on AB and BD as diameters as shown in the figure. What is the area of the shaded region? **[2018-II]**

(a) 9π

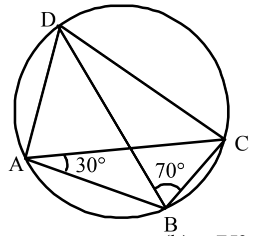
(b) 27π

(c) 36π

(d) 81π



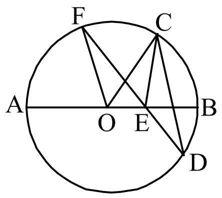
122. In the figure given below, what is $\angle BCD$ equal to? **[2018-II]**



(a) 70° (b) 75°
(c) 80° (d) 90°

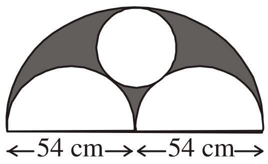
123. In the figure given below, AB is the diameter of the circle whose centre is at O. Given that $\angle ECD = \angle EDC = 32°$, then $\angle CEF$ and $\angle COF$ respectively are **[2018-II]**

(a) $32°, 64°$
(b) $64°, 64°$
(c) $32°, 32°$
(d) $64°, 32°$



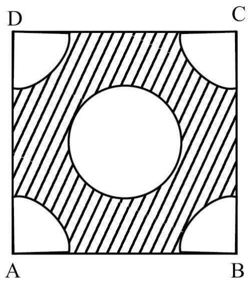
124. In the figure given below, the diameter of bigger semicircle is 108 cm. What is the area of the shaded region? **[2018-II]**

(a) $201\pi\, cm^2$
(b) $186.3\,\pi\, cm^2$
(c) $405\pi\, cm^2$
(d) $769.5\pi\, cm^2$



125. In the figure given below, ABCD is a square of side 4 cm. Quadrants of a circle of diameter 2 cm are removed from the four corners and a circle of diameter 2 cm is also removed. What is the area of the shaded region? **[2018-II]**

(a) $5\frac{7}{9}cm^2$
(b) $7\frac{7}{9}cm^2$
(c) $9\frac{5}{7}cm^2$
(d) $9\frac{5}{6}cm^2$



126. If the lengths of two parallel chords in a circle of radius 10 cm are 12 cm and 16 cm, then what is the distance between these two chords ? **[2019-I]**

(a) 1 cm or 7 cm (b) 2 cm or 14 cm
(c) 3 cm or 21 cm (d) 4 cm or 28 cm

127. Considering two opposite vertices of a square of side 'a' as centres, two circular arcs are drawn within the square joining the other two vertices, thus forming two sectors. What is the common area in these two sectors ? **[2019-I]**

(a) $a^2\left(\pi+\frac{1}{2}\right)$ (b) $a^2\left(\pi-\frac{1}{2}\right)$

(c) $a^2\left(\frac{\pi}{2}-1\right)$ (d) $a^2\left(\frac{\pi}{2}+1\right)$

128. Two circles are drawn with the same centre. The circumference of the smaller circle is 44 cm and that of the bigger circle is double the smaller one. What is the area between these two circles ? **[2019-I]**

(a) 154 square cm (b) 308 square cm
(c) 462 square cm (d) 616 square cm

129. A 12 m long wire is cut into two pieces, one of which is bent into a circle and the other into a square enclosing the circle. What is the radius of the circle ? **[2019-I]**

(a) $\frac{12}{\pi+4}$ (b) $\frac{6}{\pi+4}$

(c) $\frac{3}{\pi+4}$ (d) $\frac{6}{\pi+2\sqrt{2}}$

130. In a circle of radius 8 cm, AB and AC are two chords such that $AB = AC = 12$ cm. What is the length of chord BC ? **[2019-I]**

(a) $2\sqrt{6}$ cm (b) $3\sqrt{6}$ cm

(c) $3\sqrt{7}$ cm (d) $6\sqrt{7}$ cm

131. Two equal circles intersect such that each passes through the centre of the other. If the length of the common chord of the circles is $10\sqrt{3}$ cm, then what is the diameter of the circle ? **[2019-I]**

(a) 10 cm (b) 15 cm
(c) 20 cm (d) 30 cm

132. Consider the following statements : **[2019-I]**

1. The number of circles that can be drawn through three non-collinear points is infinity.
2. Angle formed in minor segment of a circle is acute.

Which of the above statements is/are correct ?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

133. A piece of wire is in the form of a sector of a circle of radius 20 cm, subtending an angle 150° at the centre. If it is bent in the form of a circle, then what will be its radius? **[2019-II]**

(a) $\frac{19}{3}$ cm (b) 7 cm
(c) 8 cm (d) None of these

134. A square is drawn such that its vertices are lying on a circle of radius 201 mm. What is the ratio of area of circle to that of square ? **[2019-II]**

(a) 11 : 7 (b) 7 : 11
(c) 20 : 19 (d) 19 : 20

135. What is the ratio of the area of a square inscribed in a semicircle of radius r to the area of square inscribed in a circle of radius r ? **[2019-II]**

(a) 1 : 2 (b) 2 : 5
(c) 2 : 3 (d) 3 : 5

136. The area of a sector of a circle of radius 4 cm is 25.6 cm^2. What is the radian measure of the arc of the sector ? **[2019-II]**

(a) 2.3 (b) 3.2
(c) 3.3 (d) 3.4

137. A line segment AB is the diameter of a circle with centre at O having radius 6.5 cm. Point P is in the plane of the circle such that $AP = x$ and $BP = y$. In which one of the following cases the point P does ***not*** lie on the circle ? **[2019-II]**

(a) $x = 6.5$ cm and $y = 6.5$ cm
(b) $x = 12$ cm and $y = 5$ cm
(c) $x = 5$ cm and $y = 12$ cm
(d) $x = 0$ cm and $y = 13$ cm

138. Two circles of radii 20 cm and 16 cm intersect and the length of common chord is 24 cm. If d is the distance between their centres, then which one of the following is correct ? **[2020-I]**

(a) $d < 26$ cm (b) 26 cm $< d <$ 27 cm
(c) 27 cm $< d <$ 28 cm (d) $d > 28$ cm

139. In a circle of radius 5 cm, *AB* and *AC* are two chords such that *AB* = *AC* = 8 cm. What is the length of chord *BC?* **[2020-I]**

(a) 9 cm (b) 9.2 cm
(c) 9.6 cm (d) 9.8 cm

140. Two circles touch internally. The sum of their areas is 136π cm^2 and distance between their centres is 4 cm. What are the radii of the circles ? **[2020-I]**

(a) 11 cm, 7 cm (b) 10 cm, 6 cm
(c) 9 cm, 5 cm (d) 8 cm, 4 cm

141. If area of a circle and a square are same, then what is the ratio of their perimeters ? **[2020-I]**

(a) $2\sqrt{\pi}$ (b) $\sqrt{\pi}$
(c) $\frac{\sqrt{\pi}}{2}$ (d) $\frac{\sqrt{\pi}}{4}$

142. A circle of diameter 8 cm is placed in such a manner that it touches two perpendicular lines. Then another smaller circle is placed in the gap such that it touches the lines and the circle. What is the diameter of the smaller circle ? **[2020-I]**

(a) $4(3-\sqrt{2})$ cm (b) $4(3-2\sqrt{2})$ cm
(c) $8(3-\sqrt{2})$ cm (d) $8(3-2\sqrt{2})$ cm

143. Suppose a region is formed by removing a sector of 20° from a circular region of radius 30 feet. What is the area of this new region ? **[2020-I]**

(a) 150 π square feet (b) 550 π square feet
(c) 650 π square feet (d) 850 π square feet

144. If an arc of a circle of radius 6 cm subtends a central angle measuring 30°, then which one of the following is an approximate length of the arc ? **[2020-I]**

(a) 3.14 cm (b) 2.15 cm
(c) 2.14 cm (d) 2 cm

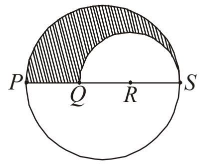
145. What is the area of the largest square plate cut from a circular disk of radius one unit ? **[2020-I]**

(a) 4 square units (b) $2\sqrt{2}$ square units
(c) π square units (d) 2 square units

146. Out of 4 identical balls of radius *r,* 3 balls are placed on a plane such that each ball touches the other two balls. The 4th ball is placed on them such that this ball touches all the three balls. What is the distance of centre of 4th ball from the plane? **[2020-I]**

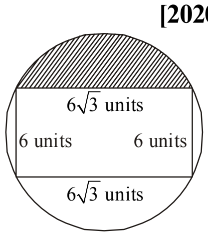
(a) $2\frac{\sqrt{2}}{3}r$ unit (b) $\frac{\sqrt{3}+2\sqrt{2}}{\sqrt{2}}r$ unit
(c) $\frac{r}{3-2\sqrt{2}}$ unit (d) $\frac{\sqrt{3}+2\sqrt{2}}{\sqrt{3}}r$ unit

147. Let *PQRS* be the diameter of a circle of radius 9 cm. The length *PQ, QR* and *RS* are equal. Semi-circle is drawn with *QS* as diameter (as shown in the given figure). What is the ratio of the shaded region to that of the unshaded region? **[2020-I]**



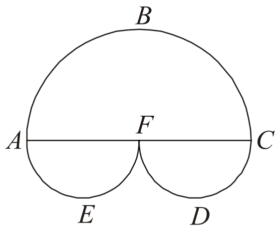
(a) 25 : 121 (b) 5 : 13 (c) 5 : 18 (d) 1 : 2

148. In the given figure, what is the area of the shaded region ? **[2020-I]**



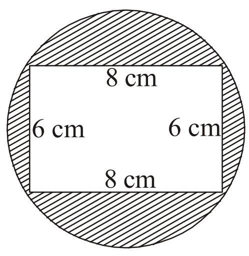
(a) $9(\pi-\sqrt{3})$ sq. units
(b) $3(4\pi-3\sqrt{3})$ sq. units
(c) $3(3\pi-4\sqrt{3})$ sq. units
(d) $9(\sqrt{3}-\pi)$ sq. units

149. In the given figure, there are three semi circles *ABC, AEF* and *CDF.* The distance between *A* and C is 28 units and *F* is the mid-point of *AC.* What is the total area of the three semi circles ? **[2020-I]**



(a) 924 square units
(b) 824 square units
(c) 624 square units
(d) 462 square units

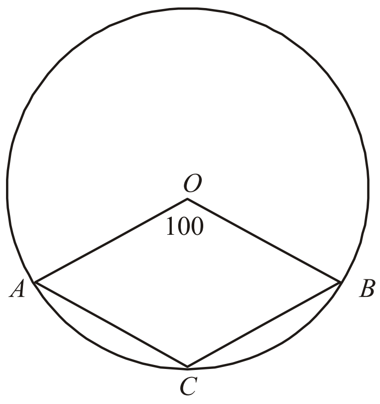
150. What is the approximate area of the shaded region in the figure given ? **[2020-I]**



(a) 15.3 cm^2 (b) 25.5 cm^2
(c) 28.4 cm^2 (d) 30.5 cm^2

HINTS & SOLUTIONS

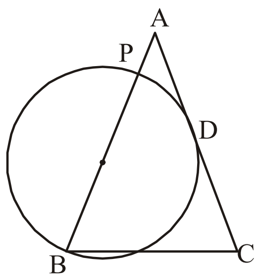
1. (d) Given, $\angle AOB = 100°$



$\therefore$ Reflex $\angle AOB = 360° - \angle AOB$
$= 360° - 100° = 260°$

$\therefore$ $\angle ACB = \dfrac{\text{Reflex } \angle AOB}{2} = \dfrac{260°}{2}$
$= 130°$

2. (c) By using theorem,



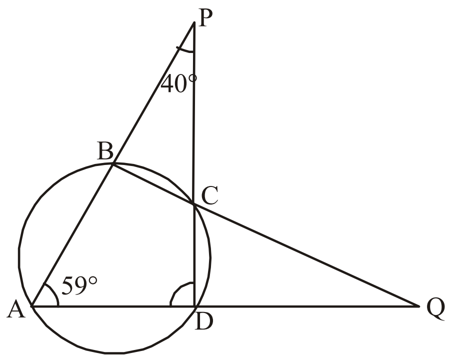
$$AB \times AP = AD^2 = \left(\frac{AC}{2}\right)^2 = \frac{1}{4}(AC)^2$$

$\Rightarrow$ $AB \times AP = \dfrac{1}{4}(AB)^2$

($\because$ $AC = AB$ given)

$\Rightarrow$ $AB = 4AP$

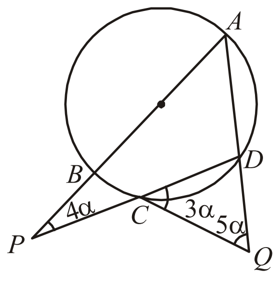
3. (c) Given $\angle PAQ = 59°$
and $\angle APD = 40°$
In ΔADP



$\angle ADP = 180° - 59° - 40° = 81°$
$\angle ADC + \angle ABC = 180°$
(cyclic quadrilateral)
$\angle ABC = 180° - 81° = 99°$
Now in ΔABQ
$\angle ABQ + \angle BAQ + \angle AQB = 180°$
$\therefore$ $\angle AQB = 180° - (99° + 59°)$
$= 180° - 158° = 22°$

4. (b) Given $\dfrac{x}{3} = \dfrac{y}{4} = \dfrac{z}{5} = \alpha$ (say)

$\therefore$ $x = 3\alpha$, $y = 4\alpha$ and $z = 5\alpha$



Since, $\angle DCQ = \angle BCP = 3\alpha$

(vertically opposite angle)

In ΔDCQ, $\angle CDQ = 180° - (3\alpha + 5\alpha) = 180° - 8\alpha$ by properation of cyclic quadrilateral,
$\angle QDC = \angle CBA = 180° - 8\alpha$ $\Rightarrow \angle PBC = 8\alpha$
In ΔPBC,
$\angle P + \angle B + \angle C = 180°$

$\therefore$ $4\alpha + 8\alpha + 3\alpha = 180° \Rightarrow \alpha = \dfrac{180°}{15} \Rightarrow \alpha = 12°$

$\therefore$ $x = 36°$, $y = 48°$, $z = 60°$

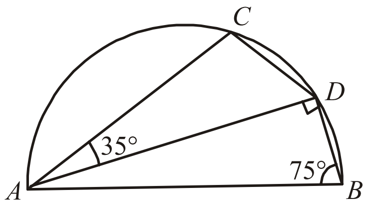
5. (b) By using theorem,

$$(PT)^2 = PA \times PB$$

$$\therefore \quad (6)^2 = 4 \times (4 + AB) \Rightarrow \frac{36}{4} = 4 + AB$$

$$\therefore \quad AB = 5 \text{ cm}$$

6. (a) Since, ΔADB is a right angled triangle at D.



$$\therefore \quad \angle DAB = 180° - (90° + 75°)$$

$$\Rightarrow \quad \angle DAB = 15°$$

Also, $ABCD$ is cyclic quadrilateral.

$$\therefore \quad \angle CAB + \angle BDC = 180°$$

$$\Rightarrow \quad \angle BDC = 180° - (35° + 15°) = 130°$$

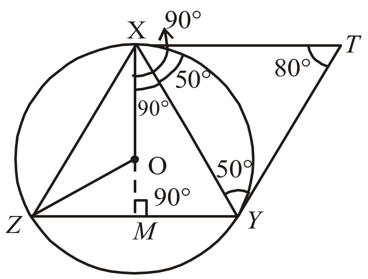
7. (d) Reflex $\angle AOC = 360° - 100° = 260°$

$$\angle ABC = \frac{1}{2} \text{ Reflex } \angle AOC$$

$$\angle ABC = \frac{1}{2} \times 260° = 130°$$

8. (b) $\angle AQP = \frac{1}{2} \times \angle AOP = \frac{75°}{2} = 37.5°$

9. (d) Since, $XT = YT$



$$\therefore \quad \angle TXY = \angle TYX = 50°$$

Also, OX is perpendicular to XT.

$$\therefore \quad \angle OXT = 90°$$

$$\therefore \quad \angle OXY = 90° - 50° = 40°$$

Also, OM is perpendicular to ZY.

In ΔXMY,

$$\angle MXY + \angle XYM + \angle XMY = 180°$$

$$\therefore \quad 40° + \angle XYM + 90° = 180°$$

$$\Rightarrow \quad \angle XYM = 50°$$

Also, by property of circle,

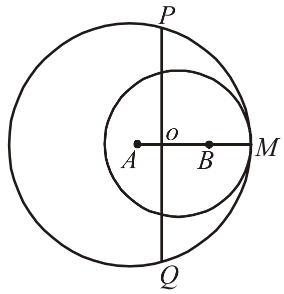
$$\angle TXY = \angle XZY = 50°$$

In ΔXYZ,

$$\angle X + \angle Y + \angle Z = 180°$$

$$\therefore \quad \angle X = 180° - 50° - 50° = 80°$$

10. (c)



$$AM = 5 \text{ cm}$$

$$BM = 3 \text{ cm}$$

$$AB = 5 - 3 = 2 \text{ cm}$$

PQ Bisect AB

$$\therefore \quad OA = OB = 1 \text{ cm}$$

In ΔAOP

$$\therefore \quad OP = \sqrt{AP^2 - OA^2} = \sqrt{5^2 - 1^2}$$

$$= \sqrt{25 - 1} = \sqrt{24} = 2\sqrt{6}$$

$$\therefore \quad PQ = 2 \times OP = 2 \times 2\sqrt{6} = 4\sqrt{6} \text{ cm}$$

11. (a) We know that, the triangle of same segment of a circle makes an equal angles.

$$\therefore \quad \angle XBY = \angle XAY = 45°$$

In ΔBXY, $\angle BXY + \angle XBY + \angle BYX = 180°$

$\Rightarrow$ $50° + 45° + \angle BYX = 180°$

($\because$ $\angle BXY = 50°$)

$$\Rightarrow \quad \angle BYX = 180° - 95° = 85°$$

12. (a) $OA = OB = AB$

(given)

So, ΔAOB is an equilateral triangle.

$$\Rightarrow \quad \angle AOB = 60°$$

We know that, the angle subtended at the centre of circle is twice the angle subtended at circumference of the circle.

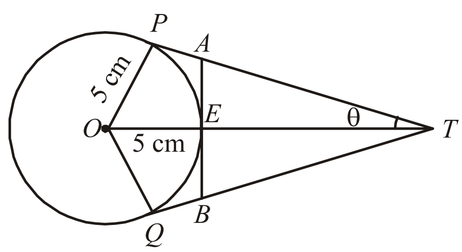
$$\therefore \quad \angle AOB = 2 \angle APB$$

$$\Rightarrow \quad \angle APB = \frac{60°}{2} = 30°$$

13. (b) $OT = 13$ cm, $OE = 5$ cm, $= OP$

(radius)

$$ET = 13 - 5 = 8 \text{ cm}$$



In ΔOPT, $13^2 - 5^2 = PT^2 \Rightarrow PT = 12$

Let $\angle ATE = \theta$

In ΔOPT, $\tan\theta = \frac{OP}{PT} = \frac{5}{12}$ ... (*i*)

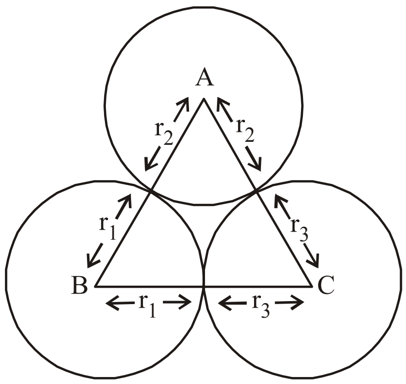
Now in ΔATE, $\tan\theta = \frac{AE}{ET} = \frac{AE}{8}$

[from Eq. (*i*)]

$\Rightarrow AE = 8\tan\theta = 8\times\frac{5}{12} = \frac{10}{3}$

$\therefore AB = 2AE = 2\times\frac{10}{3} = \frac{20}{3}$ cm

14. (a) $r_1 + r_2 = 4$
$r_2 + r_3 = 6$
$r_3 + r_1 = 8$

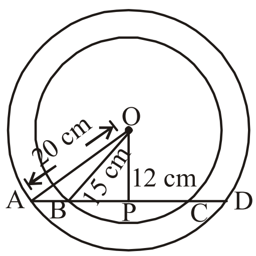


$(r_1 + r_2) + (r_2 + r_3) + (r_3 + r_1)$
$= 4 + 6 + 8$
$2(r_1 + r_2 + r_3) = 18$

$r_1 + r_2 + r_3 = \frac{18}{2} = 9$

$\therefore r_1 + r_2 + r_3 = 9$

15. (b) Two circles have three points in common only when C_1 and C_2 are same circle.
16. (b) From a point on the circle only one tangent can be drawn to a circle.
17. (a) Given $OA = 20$ cm
$OB = 15$ cm and $OP = 12$ cm

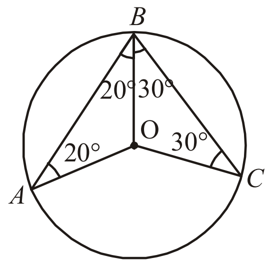


$AP = \sqrt{AO^2 - OP^2}$
$= \sqrt{20^2 - 12^2}$
$= \sqrt{400 - 144}$
$= \sqrt{256} = 16$ cm

$BP = \sqrt{15^2 - 12^2}$
$= \sqrt{225 - 144} = \sqrt{81}$
$= 9$ cm

$\therefore AB = AP - BP = 16 - 9$
$= 7$ cm

18. (d)

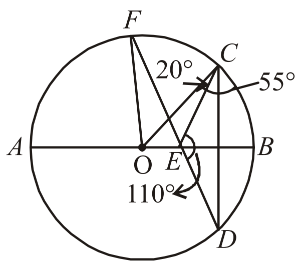


Join OB. $OA = OB = OC$
Then,
$\angle OAB = \angle OBA = 20°$
$\angle OCB = \angle OBC = 30°$
$\angle ABC = 50°$

We know that $\angle ABC = \frac{1}{2}\angle AOC$

$\therefore \angle AOC = 2\,\angle ABC = 2\times 50° = 100°$

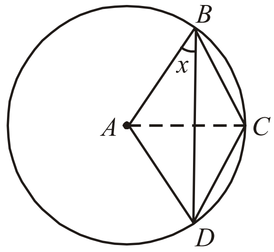
19. (b) The Given, $EC = ED$



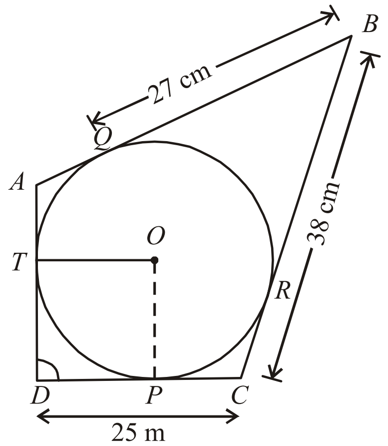
$\Rightarrow \angle EDC = \angle ECD = 35°$
Since, $\angle OCD = 55°$
Then, $\angle OCE = 20°$
By using then theorem that triangle on the same segment of a circle makes as equal angles.
Here, OE is a segment, which makes a ΔOFE and ΔOCE.
Therefore, $\angle OCE = \angle EFO = 20°$

20. (d) Given that AB = BC = CD, also since AB is the radius then AB = AC = AD = radius, so we have that: AB

= BC = CD = AC = AD, so basically we have two equilaterial triangles ABC and ACD with common base of AC (ABC and ACD are mirror images of each other). Line segment BD cuts the angle ABC in half and since all angles in equileteral traiangle equal to 60 degrees then x = 60/2 = 30 degrees.



21. (b) Given
$$BC = 38 \text{ cm}$$
$$QB = 27 \text{ cm}$$
$$DC = 25 \text{ cm}$$
$$AD \perp DC$$



We know that tangents are always equal, when they drawn to the circle from a point outside the circle.

$\therefore$ $BQ = BR = 27$ cm
$RC = BC - BR = 38 - 27 = 11$ cm
$RC = PC = 11$ cm
$DC = 25$ cm
$DP = DC - PC = 25 - 11 = 14$ cm
$DP = OT = OP$

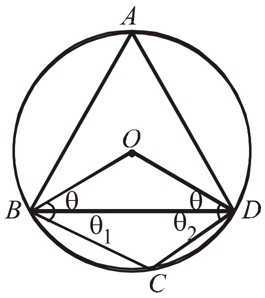
$\therefore$ Radius of the circle = 14 cm

22. (d) The sum of opposite angles in cyclic quadrilateral is always 180°.

$\therefore$ $\angle ACQ + \angle APQ = 180°$
$75° + \angle APQ = 180°$
$\therefore$ $\angle APQ = 180° - 75° = 105°$
$\angle ACQ + \angle QCR = 180°$ ($\because$ Straight line)
$75° + \angle QCR = 180°$
$\angle QCR = 180° - 75° = 105°$
$\angle CQR = 180° - 105° - 30° = 45°$

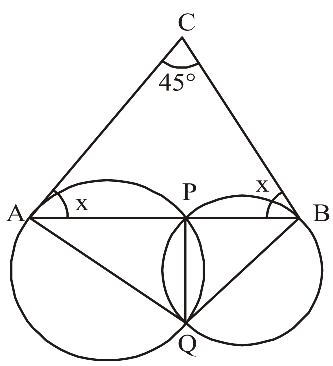
Since, $\angle APQ + \angle BPQ = 180°$ (Straight line)
$\therefore$ $105° + \angle BPQ = 180°$
$\angle BPQ = 75°$
In ΔBPQ $\angle B + \angle P + \angle Q = 180°$
$\angle B + 75° + 45° = 180°$
$\Rightarrow$ $\angle B = 60°$ $\therefore$ $\angle CBA = 60°$

23. (b) Given $\angle OBD + \angle ODB = \angle CBD + \angle CDB$



Let $\angle OBD = \angle ODB = \theta$
and $\angle DBC = \theta_1$, $\angle BDC = \theta_2$
$\therefore$ $\theta + \theta = \theta_1 + \theta_2$
... (*i*)
$\Rightarrow$ $2\theta = \theta_1 + \theta_2$
$\therefore$ $\angle BOD = 180° - 2\theta$
$\Rightarrow$ $\angle BCD = \dfrac{360° - (180° - 2\theta)}{2}$
(by properties of circle)
$\Rightarrow$ $180° - (\theta_1 + \theta_2) = 90° + \theta$
$\Rightarrow$ $180° - 2\theta = 90° + \theta$
$\Rightarrow$ $90° = 3\theta$
$\Rightarrow$ $\theta = 30°$
$\therefore$ $\angle BOD = 120°$
$\angle BAD = 60°$

24. (d) The tangents drawn from an outer point on a circle are always equal = $\angle CBA$.
Therefore, $\angle CAB = \angle CBA$



$\therefore$ $45° + x + x = 180°$
$\Rightarrow$ $2x = 180° - 45°$

$\Rightarrow \quad x = 67\frac{1^\circ}{2}$

$\angle AQP = \angle x = \angle BQP$

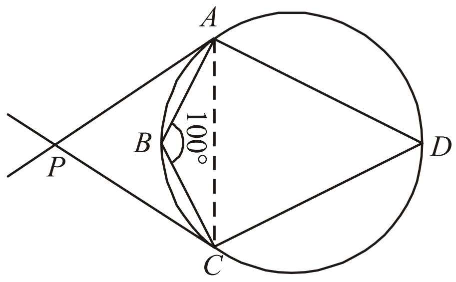
$= 67\frac{1^\circ}{2}$

(alternate interior segments properties)

$\Rightarrow \quad \angle AQB = \angle AQP + \angle BQP$

$= 67\frac{1^\circ}{2} + 67\frac{1^\circ}{2} = 135^\circ$

25. (b) We know that, the sum of opposite angles of a cyclic quadrilateral is always 180°.



$\therefore \quad \angle B + \angle D = 180^\circ$

$\Rightarrow \quad 100 + \angle D = 180^\circ$

$\Rightarrow \quad \angle D = 80^\circ$

$\therefore \quad \angle ACP = \angle PAC = 80^\circ$

(by theorem of alternate interior segment)

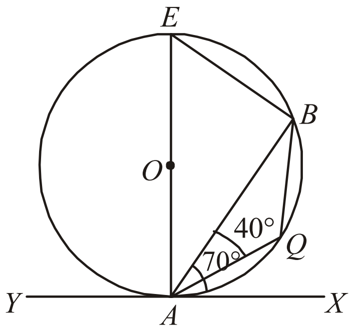
In ΔPAC,

$\angle P + \angle PAC + \angle PCA = 180^\circ$

$\Rightarrow \quad \angle P + 80^\circ + 80^\circ = 180^\circ$

$\Rightarrow \quad \angle P = 180^\circ - 160^\circ = 20^\circ$

26. (b) Given , $\angle BAX = 70^\circ$ and $\angle BAQ = 40^\circ$



$\angle QAX = 70^\circ - 40^\circ = 30^\circ$

$\therefore \quad \angle EAX = 90^\circ$

$\Rightarrow \quad \angle EAB = 90^\circ - 70^\circ = 20^\circ$

Since, $AQBE$ is a cyclic quadrilateral.

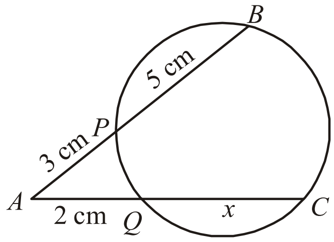
$\therefore \quad \angle EAQ + \angle EBQ = 180^\circ$

$\Rightarrow \quad \angle EBQ = 180^\circ - 60^\circ = 120^\circ$

But $\quad \angle EBA = 90^\circ$

$\therefore \quad \angle ABQ = 120^\circ - 90^\circ = 30^\circ$

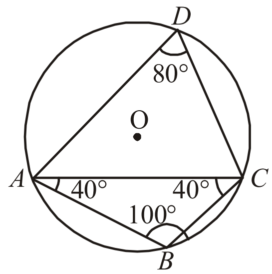
27. (c) By using theorem, we have



$AB \times AP = AC \times AQ \quad \Rightarrow 8 \times 3 = (2 + x) \times 2$

$\Rightarrow \quad \frac{8 \times 3}{2} = 2 + x \quad \Rightarrow x = 10 \text{ cm}$

28. (d) Since, $AB = CB$



$\therefore \quad \angle CAB = \angle ACB = 40^\circ$

$\Rightarrow \quad \angle ABC = 180^\circ - 2\,(40^\circ) = 100^\circ$

We know that, in cyclic quadrilateral, the sum of opposite angles is 180°.

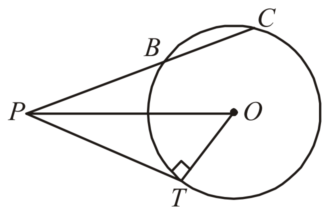
$\therefore \quad \angle B + \angle D = 180^\circ$

$\Rightarrow \quad \angle D = 180^\circ - 100^\circ = 80^\circ$

29. (a) Given, $\quad PO = 10$ cm, radius $OT = 6$ cm

and $\quad PB = 5$ cm

In ΔOTP,



$(OP)^2 = (PT)^2 + (OT)^2$

$\Rightarrow \quad (10)^2 = (PT)^2 + 6^2$

$\Rightarrow \quad PT = 8$ cm

From properties of circle,

$(PT)^2 = PB \times PC$

$\Rightarrow \quad 8^2 = 5 \times (BC + PB)$

$\Rightarrow \quad 64 = 5\,(BC + 5) \Rightarrow 5BC = 39$

$\therefore \quad BC = 7.8$ cm

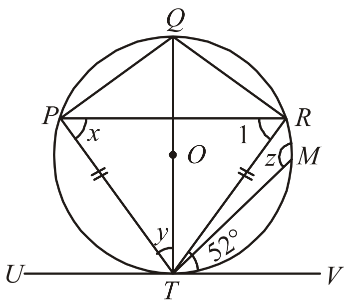
30. (c) $\quad x = \angle VTR = 52^\circ$

$x + z = 180^\circ$

(since, $PTMR$ is a cyclic quadrilateral)

$\Rightarrow \quad 52^\circ + z = 180^\circ$

$\Rightarrow \quad z = 128^\circ$

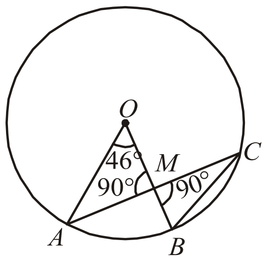


In ΔPTR, $PT = TR$
(given)

$$x = \angle 1 = 52°$$
$$\angle PTU = \angle 1 = 52° \Rightarrow \angle QTU = y + 52°$$
$$\Rightarrow 90° = y + 52° \Rightarrow y = 38°$$
$$\therefore x + y + z = 52° + 38° + 128° = 218°$$

31. (c) Since, angle subtend on the circumference is half of the angle subtend on centre.

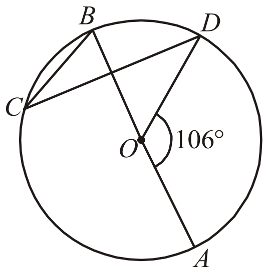
$$\therefore \angle ACB = \frac{1}{2}\angle AOB$$
$$= \frac{1}{2}\times 46° = 23°$$



In $\Delta\ MCB$,

$$\angle C + \angle B + \angle M = 180°$$
$$\Rightarrow 23° + \angle B + 90° = 180°$$
$$\therefore \angle B = 67°$$

32. (d) $\angle BOD = 180° - 106° = 74°$

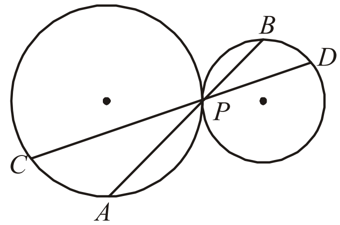


Since, $\angle BOD$ is an angle made by arc BD on centre. Here, $\angle BCD$ is an angle made by arc BD on circumference.

$$\therefore \angle BCD = \frac{1}{2}\times\angle BOD$$
$$= \frac{1}{2}\times 74° = 37°$$

33. (d) Infinite number of circles passing through a given pair of points.

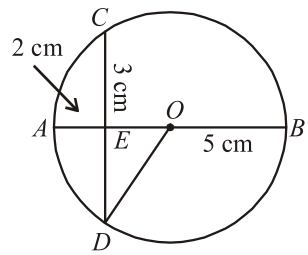
34. (d) It is clear from the figure that none of the option is correct.



35. (a) A line perpendicular to the given line, passing through the given point is the required locus.

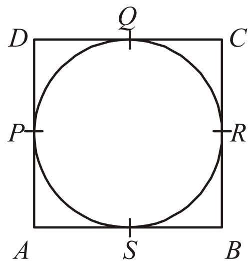
36. (b) In ΔOED,

$$(OD)^2 = (DE)^2 + (EO)^2$$



$$\Rightarrow (5)^2 = (DE)^2 + (3)^2$$
$$\Rightarrow (DE)^2 = 25 - 9 = 16$$
$$\therefore DE = 4 \text{ cm}$$

37. (c) When two tangents drawn from an external point to a circle, the length of the tangent are equal.



$$AS = AP \quad \text{... (i)}$$
$$BS = BR \quad \text{... (ii)}$$
$$CQ = CR \quad \text{... (iii)}$$
$$DP = DQ \quad \text{... (iv)}$$

Adding (i), (ii), (iii) and (iv), we get.

$$AS + BS + CQ + DQ = AP + BR + DP + CR$$
$$= AB + CD = AD + BC$$

$\therefore$ Option (c) is correct.

38. (c) Here, $r_1 = 9$ cm and $r_2 = 4$ cm

$r_1 + r_2 = 9 + 4 = 13$ cm

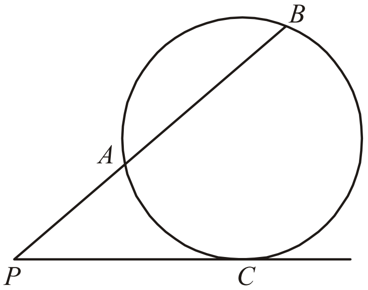
and $r_1 - r_2 = 9 - 4 = 5$cm

Also, $d = 13$ cm

Here, $d = r_1 + r_2$
$= 13$ cm

Hence, two circles touch each other externally, so three total no of common tangents are three.

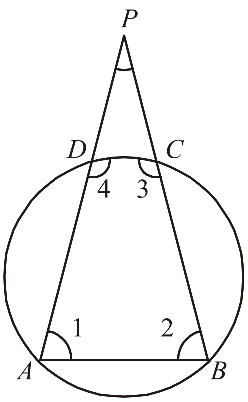
39. (a) If a secant to a circle intersect circle at points A and B PC is a tangent to circle, then



$$PC^2 = PA \times PB$$

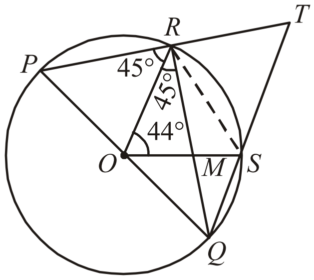
which is equivalent to area of rectangle with PA and PB as sides is equal to the area of square with PC as side.

40. (b) Given, $\angle BAD = 60°, \angle ADC = 105°$
in cycle quadrilateral $ABCD$,



$\angle 1 + \angle 3 = 180°$
$\Rightarrow \angle 3 = 180° - 60° = 120°$
Now, $\angle BCD + \angle DCP = 180°$
(straight line)
$\Rightarrow \angle DCP = 180° - 120° = 60°$
and, $\angle ADC + \angle CDP = 180°$
(straight line)
$\Rightarrow 105° + \angle CDP = 180° \Rightarrow \angle CDP = 75°$
Now in ΔCPD,
$\angle DCP + \angle CDP + \angle DPC = 180°$
$\Rightarrow 60° + 75° + \angle DPC = 180°$
$\therefore \angle DPC = 180° - 135° = 45°$

41. (d) Since, OR is a bisector of $\angle PRQ$.
$\therefore \angle PRO = \angle ORQ = 45°$
Also, $OP = OR$
$\therefore \angle OPR = 45°$

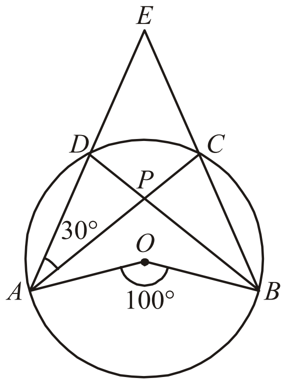


In ΔORS,

$$OR = OS \Rightarrow \angle ORS = \angle OSR = \frac{180° - 44°}{2} = 68°$$

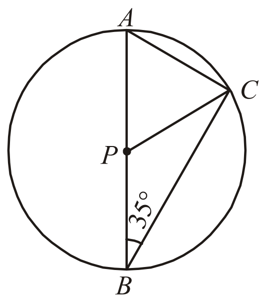
$\therefore \angle MRS = 68° - 45° = 23°$
$\Rightarrow \angle PRS = 90° + 23° = 113°$
By properties of cyclic quadrilateral.
$\angle PRS + \angle PQS = 180°$
$\Rightarrow \angle PQS = 180° - 113° = 67°$
In ΔPTQ,
$\angle QPT + \angle PQT + \angle PTQ = 180°$
$\Rightarrow \angle PTQ = 180° - 45° - 67° = 68°$

42. (b) Since, $\angle ADB = \frac{1}{2}\angle AOB = 50°$



In ΔDPA, $\angle DAP + \angle ADP + \angle DPA = 180°$
$\Rightarrow 30° + 50° + \angle DPA = 180° \Rightarrow \angle DPA = 100°$
Also, DPB be a straight line.
$\therefore \angle DPA + \angle APB = 180°$
$\Rightarrow \angle APB = 180° - 100° = 80°$

43. (d) $PC = PB$ (radii of circle)



$\Rightarrow \quad \angle PBC = \angle PCB$

(angles opposite to equal sides)

$\Rightarrow \quad \angle PCB = 35°$

and $\quad \angle ACB = 90°$

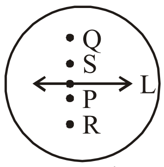
(angle in semi-circle)

$\Rightarrow \quad \angle PCA + \angle PCB = 90°$

$\Rightarrow \quad \angle PCA = 90° - 35° = 55°$

44. (a) Only one circle passing through all the vertices of a given triangle.

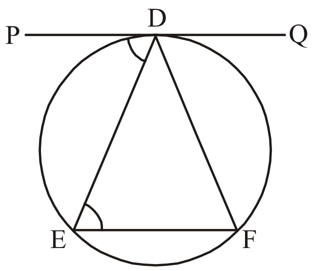
45. (c) I. It is clear from the figure that points Q, S and R in a straight line.



It is true

II. Since, PQ is parealled to EF.

$\therefore \angle PDE = \angle DEF$ (alternate angle)



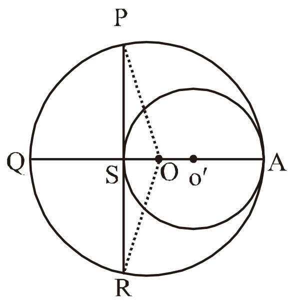
Also, $\quad \angle PDE = \angle EFD$

(angle in the alternate segments of chord ED)

$\therefore \quad \angle DEF = \angle DFE$

Therefore, ΔDEF is an isosceles triangle.

46. (d)



Let O is centre of big circle, and O' is centre of smaller circle. Both are touch internally each other.

OA = 6 cm O'A = 4cm

Here PR is longest chord of big circle

$$P.S = \frac{PR}{2}$$

OS = AS – OA

= 8 – 6 = 2 cm.

In ΔPSO

$(PS)^2 + (OS)^2 = (OP)^2$

$\Rightarrow (PS)^2 + (2)^2 = (6)^2$

$PS = \sqrt{36-4} = \sqrt{32} = 4\sqrt{2}$

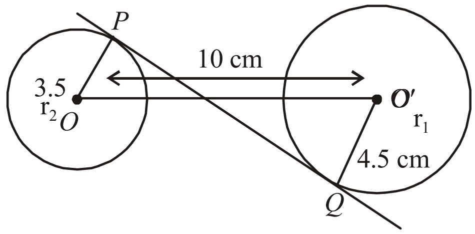
Now,

$$PS = \frac{PR}{2}$$

PR = 2 × PS

$= 2 \times 4\sqrt{2} = 8\sqrt{2}$

47. (c)



Length of common tangent,

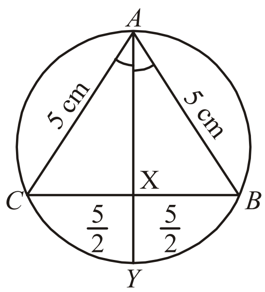
$= \sqrt{d^2 - (r_1 + r_2)^2}$

$= \sqrt{10^2 - (4.5 + 3.5)^2}$

$= \sqrt{100 - 64} = \sqrt{36}$

= 6 cm

48. (c) In ΔABC,



$$BX = \frac{5}{2}\text{cm}, \; CX = \frac{5}{2}\text{cm}$$

and $\quad AX = \frac{\sqrt{3}}{2} \times 5 = \frac{5\sqrt{3}}{2}$ cm

AY and BC are the chord of circle.

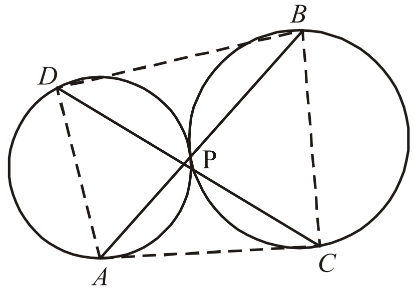
$\therefore \quad AX \cdot XY = BX \cdot XC$

$$\Rightarrow \quad \frac{5\sqrt{3}}{2} . XY = \frac{5}{2} \cdot \frac{5}{2}$$

$$\Rightarrow \quad XY = \frac{5}{2\sqrt{3}}$$

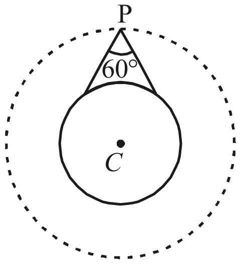
$$\therefore \quad AX \cdot AY = \left(\frac{5\sqrt{3}}{2} + \frac{5}{2\sqrt{3}}\right) \times \frac{5\sqrt{3}}{2} = 25\,\text{cm}^2$$

49. (d)

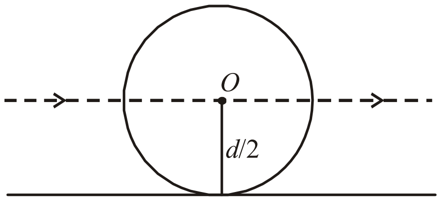


It is clear from the figure that $ACBD$ is a quadrilateral.

50. (b) The locus of P is a circle concentric with C.



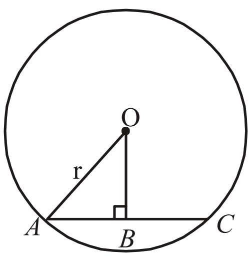
51. (b)



The locus of the centre of the front wheel of the bicycle is a line parallel to the path of the wheel of the bicycle at a height $d/2$ cm.

52. (c) Given, $AC = \sqrt{3}\,r$

$$AB = \frac{\sqrt{3}}{2}r$$



In ΔOAB,

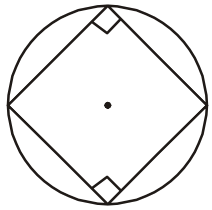
$$(OB)^2 = (OA)^2 - (AB)^2$$

$$= r^2 - \left(\frac{\sqrt{3}}{2}r\right)^2$$

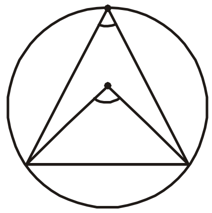
$$= r^2 - \frac{3}{4}r^2 = \frac{r^2}{4}$$

$$\Rightarrow \quad OB = \frac{r}{2}$$

53. (d) I. It is true that opposite angles of a cyclic quadrilateral are supplementary.



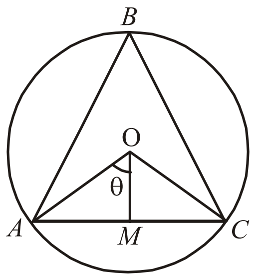
II. It is also true that the angle subtend by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.



Hence, both statements are individually true but neither statements implies of each other.

54. (c) $OA = 3$ cm and $AC = 3$ cm ... (Given)

$$AM = \frac{3}{2}\text{cm}$$



In ΔOAM,

$$\sin\theta = \frac{AM}{OA} = \frac{3/2}{3}$$

$$\Rightarrow \quad \sin\theta = \frac{1}{2} \Rightarrow \theta = 30°$$

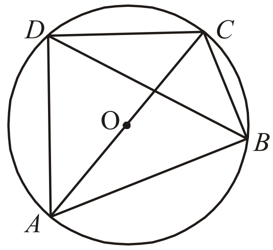
$$\therefore \quad \angle AOC = 2\theta = 60°$$

$$\angle ABC = \frac{1}{2}\angle AOC$$

(By property of circle)

$$= \frac{60°}{2} = 30°$$

55. (c) $\because \angle ADC = 90°$ (angle in semi-circle is a right angle)

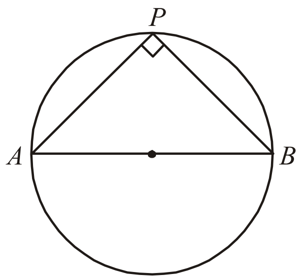


$$\angle ADB = \angle ADC - \angle BDC = 90° - 42° = 48°$$
$$\angle ADB = \angle ACB = 48°$$

56. (b) The total number of circles passes through three non-collinear points is one.

57. (d) Given,

$$AB^2 - AP^2 - BP^2 = 0$$



$$AB^2 = AP^2 + BP^2$$

Angle in a semi circle is a right angle.

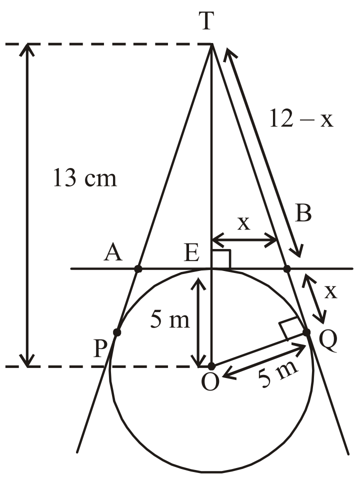
Thus, this is a equation of a circle.

Hence, locus of point P is a circle.

58. (b) In ΔOQT

$$OT^2 = OQ^2 + TQ^2$$

$$\Rightarrow (13)^2 = (5)^2 + (TQ)^2$$



$\Rightarrow TQ^2 = 169 - 25 = 144 \Rightarrow TQ = 12$ cm

Then, in ΔTEB,

$$TB^2 = EB^2 + TE^2$$

$\therefore$ $(EB = BQ)$ (Beacuse they are tangent)

$\Rightarrow$ $(12 - x)^2 = BQ^2 + TE^2$

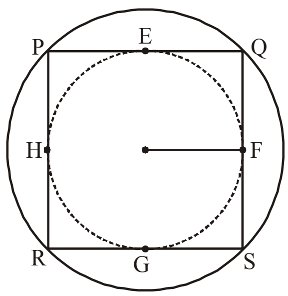
$\Rightarrow$ $144 + x^2 - 24x = x^2 + (8)^2$

$\Rightarrow$ $144 + x^2 - 24x = x^2 + 64 \Rightarrow 24x = 80$

$\Rightarrow$ $x = \frac{20}{6} = \frac{10}{3}$ cm

$\therefore$ $AB = 2EB = 2x = 2 \times \frac{10}{3} = \frac{20}{3}$ cm

59. (b)

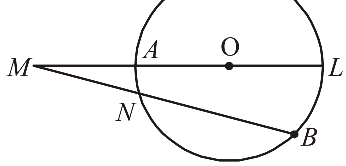


Here $PQ = QS = SR = RP$

Since, E, F, G and H are the mid-point of the chords.

The locus obtained is the circumference of the circle concentric with the given circle and having radius equal to the distance of the chords from the centre.

60. (d) Since, secants $\angle A$ and BN are intersecting at an exterior point M, then



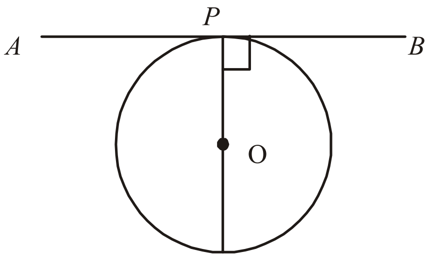
$$LM \times AM = BM \times NM$$

$$\Rightarrow \frac{MA}{MB} = \frac{MN}{LM} < 1$$

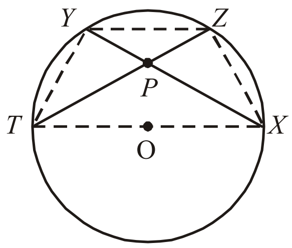
61. (c) By definition of tangent.

A tangent to a circle is a straight line that touches the circle at a single point. Also, tangent at the end points of a diameter of a circle is perpendicular to the diameter.

So, both statements are correct.



62. (c) When two chords of a circle are intersect internally, then they are divided in a proportion.



i.e., $PX \cdot PY = PZ \cdot PT$

In ΔPXZ and ΔPTY,

$\angle ZPX = \angle YPT$

(vertically opposite angles)

$\angle PZX = \angle PYT$

(angle in same segment)

$\angle PXZ = \angle PTY$

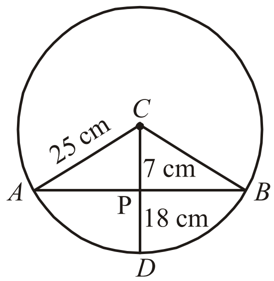
(angles in same segment)

$\therefore \quad \Delta PXZ \sim \Delta PTY$

Hence, the both statements are correct.

63. (d) $CD = 25$ cm $\qquad d = 50$ cm

$PD = 18$ cm $\qquad r = \frac{d}{2} = \frac{50}{2} = 25$ cm



$\therefore \quad CP = CD - PD$

$= 25 - 18 = 7$ cm

$AP = \sqrt{(25)^2 - (7)^2} = \sqrt{625 - 49}$

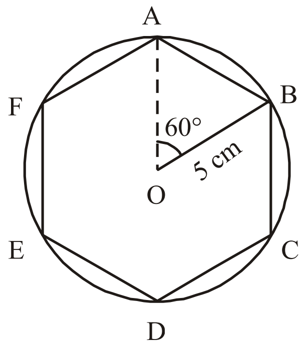
$= \sqrt{576} = 24$ cm

$\therefore \quad AB = 2AP = 24 \times 2 = 48$ cm

64. (a) $OB = OA =$ radius

Also, $\angle AOB = 60° \left(\frac{360°}{6} = 60°\right)$

and $\angle OAB = \angle OBA = 60°$



So, ΔAOB is an equilateral triangle.

Then, $AB = 5$ cm

So, Area, x = Area of circle – Area of hexagon

$= \pi r^2 - \frac{3\sqrt{3}(a)^2}{2}$

$= \frac{22}{7} \times (5)^2 - \frac{3\sqrt{3}}{2} \times (5)^2$

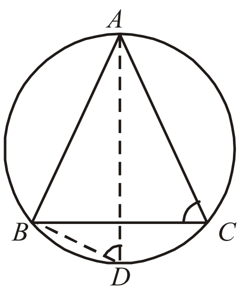
$(\because r = a = 5)$

$= 78.57 - 64.95 = 13.62 \text{ cm}^2$

65. (b) The perpendicular bisector of the chord of a circle always pass through the centre. So, statement I is wrong.

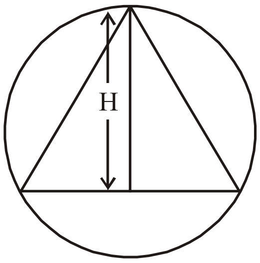
The angle in a semi-circle is a right angle. So, Statement II is correct.

66. (b)



$\angle ADB = \angle ACB = 60°$ (angles in the same segment are equal)

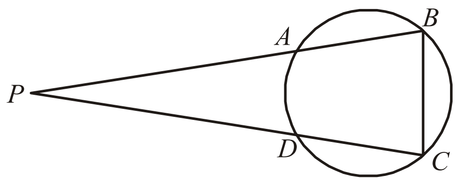
67. (d) Circumradius $= \frac{2}{3} \times$ Height



$\therefore$ Height $= \frac{10 \times 3}{2} = 15$ cm

So, length of perpendicular drawn from center = 15 – 10 = 5 cm,

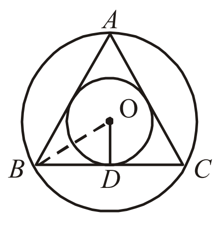
68. (d) AB and CD are chords when produced meet externally at P.



$\therefore \quad AP \times BP = CP \times DP$

Now, as $AC \nparallel BD$ and ΔPAC is not similar to ΔPDB

69. (a) In ΔABC



$AB = BC = CA$

... (given)

$\therefore$ ΔABC is an equilateral triangle.

Now radius of incircle $= \frac{\text{Side}}{2\sqrt{3}}$

$= \frac{AB}{2\sqrt{3}}$

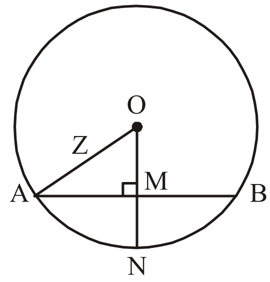
Radius of circumcircle $= \frac{\text{Side}}{\sqrt{3}} = \frac{AB}{\sqrt{3}}$

$\therefore$ Required Ratio $= \frac{AB/\sqrt{3}}{AB/2\sqrt{3}} = \frac{2}{1} = 2:1$

70. (d) Let O be the centre of circle and AB be the chord of an arc.

Given that

Length of chord $AB = x$, Radius of circle $OA = z$ and height of an arc $MN = y$



$AM = MB = \frac{x}{2} \quad \because OM \perp AB$

and $OM = ON - MN = z - y$

$[\because ON = z$ (radius) and $MN = y]$

In ΔOMA, $OA^2 = OM^2 + AM^2$

[by Pythagoras theorem]

$\Rightarrow \quad z^2 = (z-y)^2 + \left(\frac{x}{2}\right)^2$

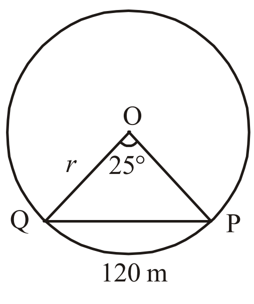
$\Rightarrow \quad z^2 = z^2 + y^2 - 2yz + \frac{x^2}{4}$

$\Rightarrow \quad 2yz - y^2 = \frac{x^2}{4}$

$\Rightarrow \quad 4(2yz - y^2) = x^2$

$\Rightarrow \quad 4y(2z - y) = x^2$

71. (c) From figure, arc length $PQ = \frac{2\pi r}{360°} \times \theta$



$\therefore \quad 120 = \frac{2 \times 3.14 \times r \times 25°}{360°}$

$\Rightarrow\ r = \dfrac{120 \times 360}{2 \times 3.14 \times 25} = \dfrac{43200}{157} = 275.15$ m

$r = 275\ m$ (approx)

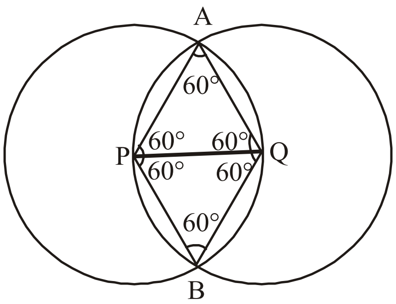
72. (c) We know that $x + y + \dfrac{xy}{100}$

Here $x = 6\%$ and $y = 6\%$

$= 6 + 6 + \dfrac{6 \times 6}{100}$

$= 12 + .36 = 12.36\%$

73. (c)

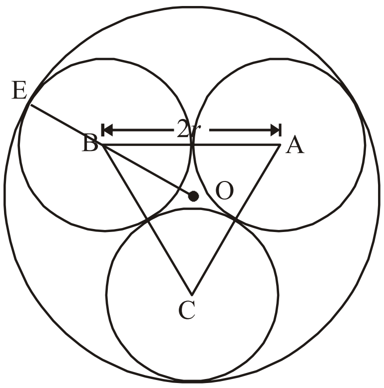


Area of common circle.

$< 2 \times$ Area of equilateral triangle APQ

$< 2 \times \dfrac{\sqrt{3}}{4} r^2 \ < \dfrac{\sqrt{3}}{2} r^2$

74. (d)



OE is the radius of big circle (R)

OE = OB + BE $\Rightarrow OB = \dfrac{2r}{\sqrt{3}}$

$\therefore$ OE $= \dfrac{2r}{\sqrt{3}} + r \ = \dfrac{2r + \sqrt{3}r}{\sqrt{3}} \ = \dfrac{r(2+\sqrt{3})}{\sqrt{3}}$

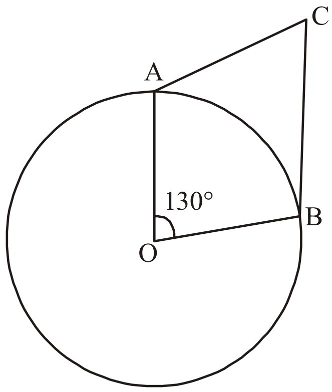
$\because$ Area of big circle $= \pi R^2$

$= \pi \left\{ \dfrac{r(2+\sqrt{3})}{\sqrt{3}} \right\}^2 = \pi r^2 \left(\dfrac{(2+\sqrt{3})}{\sqrt{3}} \right)^2 ; \ \left(r = \dfrac{d}{2} \right)$

$= \dfrac{\pi d^2 (2+\sqrt{3})^2}{4 \times 3} = \dfrac{\pi d^2 \left(2+\sqrt{3}\right)^2}{12}$

(c) 50° (d) 40°

75. (c)

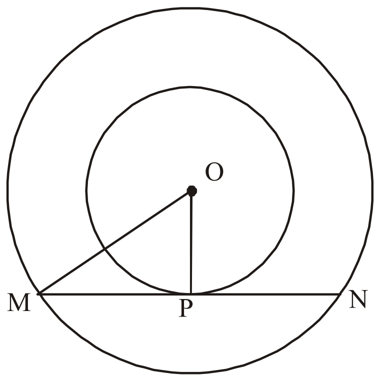


OA and OB are radii of circle, AC and BC are tangents

Now, ∠AOB + ∠ACB = 180°

∠ACB = 180° – 130° = 50°

76. (a)



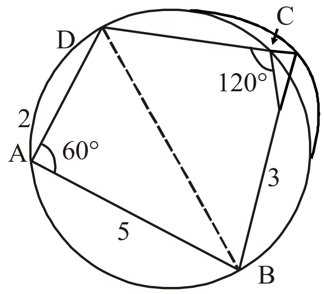
MN = 24 cm, MP = 12 cm

Radius of outer circle $= \dfrac{26}{2} = 13$ cm

OP $= \sqrt{(OM)^2 - (MP)^2}$

$= \sqrt{169 - 144} = \sqrt{25} = 5$ cm

77. (a)



In ΔABD-

$$\cos 60° = \frac{2^2 + 5^2 - BD^2}{2.2.5}$$

$$\frac{1}{2} = \frac{4 + 25 - BD^2}{20}$$

$BD^2 = 29 - 10$

$BD = \sqrt{19}$

In ΔBCD-

$$\cos 120° = \frac{3^2 + CD^2 - BD^2}{2.3.CD}$$

$$-\frac{1}{2} = \frac{9 + CD^2 - \perp 9}{6\ CD}$$

$CD^2 - 10 = -3\ CD$

$CD^2 + 3\ CD = x$

Let $CD = x$

$x^2 + 3x - 10 = 0$

$(x + 5)(x - 2) = 0$

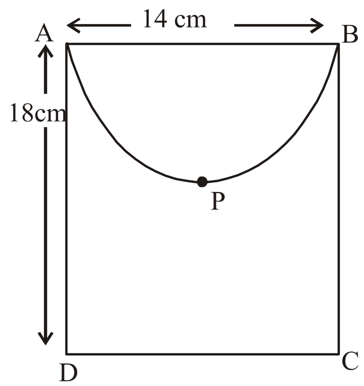
$x \neq -5$

$x = 2$

$CD = 2$ cm

So, option (a) is correct.

78. (d)



Remaining Area

$= $ Area of ABCD – Area of APB

$$= 18 \times 14 - \frac{1}{4}\pi\left(\frac{14}{2}\right)^2$$

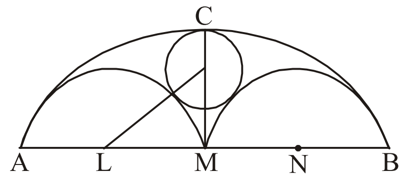
$$= 252 - \frac{1}{2} \times \frac{22}{7} \times \frac{14}{2} \times \frac{14}{2} = 252 - 77 = 175 \text{ cm}^2$$

So, option (d) is correct.

79. (c) $AB = 2a$ i.e., $AL = LM = MN = NB = \frac{a}{2}$

Let $OC = r$

In ΔOML, $OM = a - r$ and $OL = OP + PL = r + \frac{a}{2}$



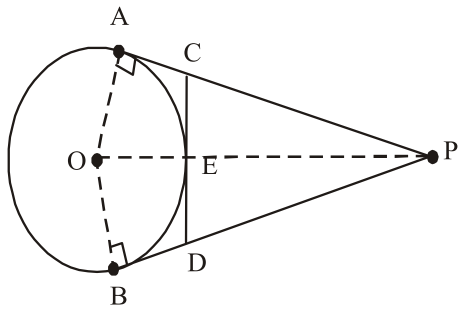
$$\therefore \left(\frac{a}{2}\right)^2 + (a - r)^2 = \left(r + \frac{a}{2}\right)^2$$

$$\Rightarrow \frac{a^2}{4} + a^2 - 2ar + r^2 = r^2 + \frac{a^2}{4} + ar$$

$$\Rightarrow a^2 = 3ar \Rightarrow r = \frac{a}{3}$$

So, option is correct.

80. (d)



$PA = PC + CA$

$\Rightarrow PC + CA = 16$

$\Rightarrow PC + CE = 16$ ------------(i)

Again,

$\Rightarrow PB = PD + DB$

$\Rightarrow PD + DB = 16$

$\Rightarrow$ PD + ED = 16

PD + ED = 16 ------------(ii)

eq (i) + eq (ii) -

PC + CE + PD + ED = 16 + 16

PC + PD + CE + ED = 32

PC + PD + CD = 32

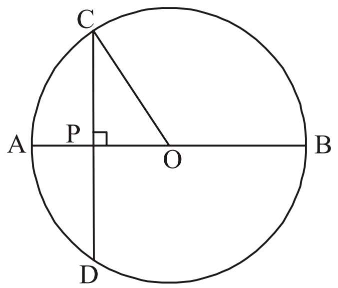
Perimeter of Δ PCD = 32 cm

So, option (d) is correct.

81. (c) Let OA = r and OP = x

$\therefore$ BP = r + x and PA = r – x

$$\frac{r+x}{r-x} = \frac{2}{1} \Rightarrow 3x = r$$



$OP^2 + PC^2 = OC^2$

($\because$ CP $\perp$ AB)

$$\frac{r^2}{9} + PC^2 = r^2$$

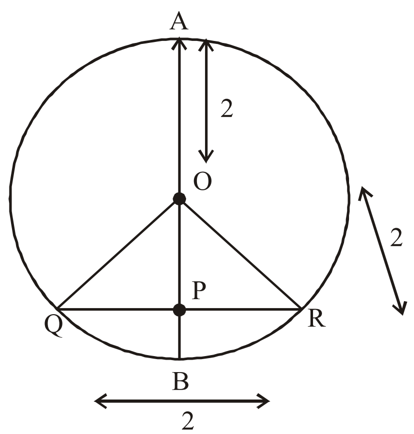
$$PC^2 = \frac{8r^2}{9}$$

$CP = \frac{2\sqrt{2}}{3}r$ and $CD = \frac{4\sqrt{2}}{3}r = \frac{2\sqrt{2}}{3}d$ (where d = 2r)

So, option (c) is correct.

82. (b) Let us consider a circle of radius 2 units.

Diameter = AB = 2 × 2 = 4 units



QR be a chord of circle

then QR = 2 units

Let O be the centre of the circle

Given AP > BP

In right angled triangle POR

By applying pythagorus theorem, we get

$$OP = \sqrt{(RO)^2 - (PR)^2}$$

$$OP = \sqrt{(2)^2 - (1)^2} = \sqrt{4-1} = \sqrt{3}$$

$OP = \sqrt{3}$

AP = AO + OP

$= 2 + \sqrt{3}$ (where OA is the radius of the circle)

$\therefore$ Option (b) is correct.

83. (c) Let radius = r

Length of an arc = 100 m

But length of arc = $2\pi r\left(\frac{\theta}{360}\right)$

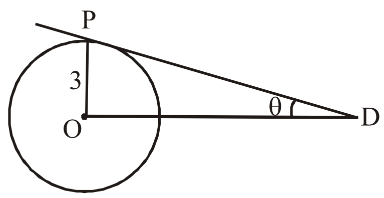
$$\Rightarrow \quad 100 = 2\pi \times r \times \frac{36}{360}$$

$$\Rightarrow \quad r = \frac{100 \times 360}{2\pi \times 36} = \frac{500}{\pi} m$$

$\therefore$ Option (c) is correct.

84. (c) Given D is an external point DP is tangent to circle.

Then DP = 4



OP = 3 (radius of circle)

$$\Rightarrow \quad \tan\theta = \frac{OP}{DP} = \frac{3}{4}$$

$$\Rightarrow \quad OD = \sqrt{OP^2 + DP^2} = \sqrt{(3)^2 + (4)^2}$$

$$OD = \sqrt{9+16} = 5$$

$$\Rightarrow \quad \sin\theta = \frac{3}{5}$$

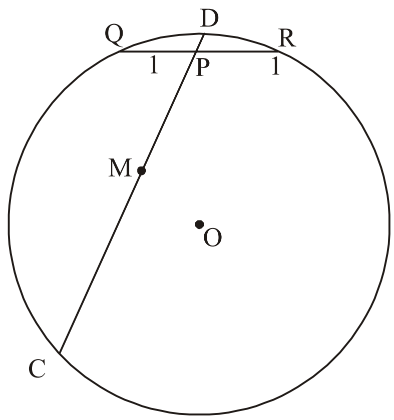
$\therefore$ Option(c) is correct.

85. (c) $CP \times PD = QP \times PR$

$\Rightarrow$ $m \times n = 1 \times 1$

$\Rightarrow$ $m \times n = 1$ ... (i)

Option (a) sol. ?



Now $x^2 - 10x + 1 = 0$

$\Rightarrow$ $x = \dfrac{10 \pm \sqrt{100-4}}{2}$

$\Rightarrow$ $x = 5 \pm 4\sqrt{3}$

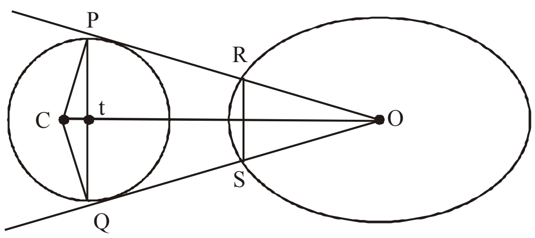
$\therefore$ $m \times n = (5+4\sqrt{3})(5-4\sqrt{3})$

$\Rightarrow$ $m \times n = 25 - 16 \times 3$

$\Rightarrow$ $m \times n = -23$ ...(ii)

From eq. (i) and (ii), we can say that statement 2 is not correct.

86. (c)



Given a circle with centre c.

OP and OQ are tangents to the circle from Q + O point O outside the circle.

Given $OR \times SQ = OS \times RP$

$\Rightarrow$ $\dfrac{OR}{RP} = \dfrac{OS}{SQ}$

$\Rightarrow$ $RS \parallel PQ$ (By Basic proportionality theorem)

(1) Also $CP = CQ$ = radius of the circle.

A perpendicular drawn from P to Q,

Draw circle X and Y with centre O and radius OR and DS respectively.

Since $RS \parallel PQ$

Here O is the center of circle X and Y both Radius OR and OS lies in the same circle.

$\Rightarrow$ $OR = OS \Rightarrow$ Area of Circle X = Area of circle Y

$\Rightarrow$ $X = Y$

Statement (1) is true.

(2) Also we know that if two tangents are drawn to the circle then $\angle POC = \angle QOC$ and $\angle PCO = \angle QCO$

Also we know that $CP = CQ$ = radius

So ΔPtc and ΔQtc are similar by AA similarly.

i.e., $\angle P = \angle Q = 45°$ ...(1)

and $\angle t = \angle t = 90°$

Also $\angle PCO = \angle QOC$ (Alternate angles)

$\angle POC = \angle QCO$ (Alternate angles)

from (1) if $\angle P = \angle Q = 45°$

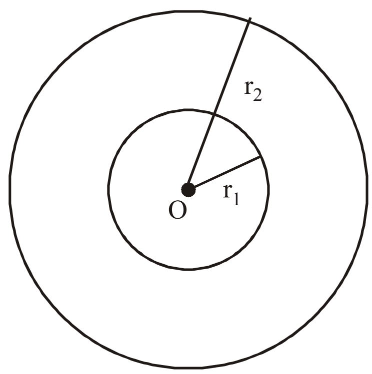
$\Rightarrow$ $\angle QCO = \angle PLO = 45°$

$\Rightarrow$ $\angle POC + \angle QCO = 45° + 45° = 90°$

Statement (2) is true.

$\therefore$ Option (c) is correct.

87. (c) Let two cocentric circular rings with centre O and



Radius of large ring = r_2

Radius of smaller ring = r_1

Area of circular both

= Area of larger ring – Area of smaller ring ...(1)

Given circumference of larger ring = 3 × circumference of smaller ring

$\Rightarrow$ $2\pi r_2 = 3 \times 2\pi r_1$

$\Rightarrow$ $r_2 = 3r_1$

Also given

Area of circular both = n (area of smaller ring) ...(2)

Comparing (1) & (2) we have

Area of larger ring – Area of smaller ring

= n(Area of smaller ring)

$\Rightarrow \quad \pi r_2^2 - \pi l_1^2 = n\left(\pi r_1^2\right)$

$\Rightarrow \quad \pi r_2^2 \,(n+1)\pi r_1^2$

$\Rightarrow \quad (n+1)r_1^2 = r_2^2$

$\Rightarrow \quad (n+1)r_1^2 = (3r_1)^2$

$(\because r_2 = 3l_1)$

$\Rightarrow \quad n+1=9$

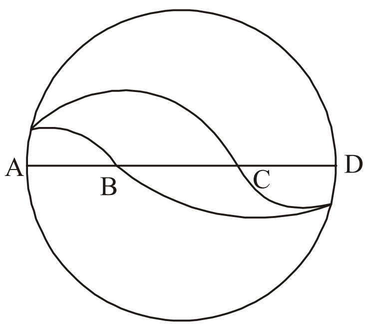
$\Rightarrow \quad n=8$

$\therefore$ Option (c) is correct.

88. (d)

89. (c) Given AD be the diameter of circle.

This circle consist of three semi-circles.



So Area of circle = Area of all 3 semi circles ...(i)

AD is the diameter of all circles.

Let AD = d

Then radius = d/2

Given are of circle = 707 m^2

From (i) we get

$$707 = \frac{\pi}{2}\left(\frac{d}{2}\right)^2 + \frac{\pi}{2}\left(\frac{d}{2}\right)^2 + \frac{\pi}{2}\left(\frac{d}{2}\right)^2$$

$$707 = \frac{3\pi}{2}\left(\frac{d}{2}\right)^2$$

$$\Rightarrow \left(\frac{d}{2}\right)^2 = \frac{707 \times 2}{3\pi}$$

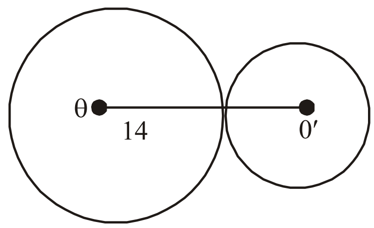
Area of shaded region $= \frac{\pi}{2}\left(\frac{d}{2}\right)^2$

$$= \frac{\pi}{2} \times \frac{707 \times 2}{3\pi} = \frac{707}{3} m^2$$

Lost of levelling the shaded region = 63 rs/m^2

Lost $= 63 \times \frac{707}{3} = 21 \times 707 = 14847$ Rs.

90. (d)



Let radius of both circles be R and r respectively

$\Pi R^2 + \Pi r^2 = 130\Pi$

$R^2 + r^2 = 130$ and $R + r = 14$ given

$(R+r) = 14$

Squaring Both side

$R^2 + r^2 + 2rR = 196$

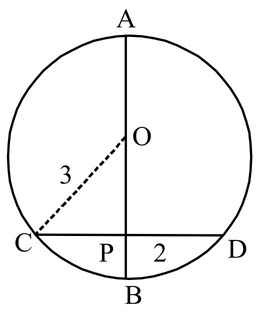
$2rR = 196 - 130 = 66$

$(R+r)^2 - 4rR = 196 - 132$

$(R-r)^2 = 64$

$R - r = 8$

91. (b) in Δ OCP OC = 3 cm, CP = 1 cm



$OP^2 = OC^2 - CP^2$

$OP^2 = 3^2 - 1^2 = 8$

$OP = \sqrt{8}$

$AP = 3 + \sqrt{8}$, $BP = 3 - \sqrt{8}$

$$\frac{AP}{BP} = \frac{3+\sqrt{8}}{3-\sqrt{8}}$$

92. (d) Radius of circle = $\frac{5}{22}$ m

distance covered = 7 km or 7000 m

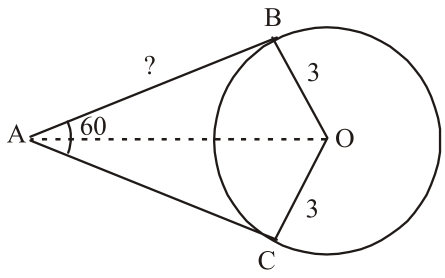
distance covered in 1 revolution = $2\Pi r = 2 \times \frac{22}{7} \times \frac{5}{22}$

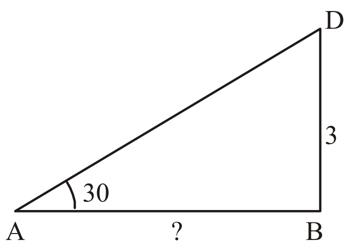
$$= \frac{10}{7} \text{ m}$$

Number of revolutions = $\frac{7000}{10} \times 7 = 4900$

93. (a) in Δ OAB $\angle A = 30°$, OB = 3cm

$\frac{OB}{AB} = \tan 30°$





$\frac{3}{AB} = \frac{1}{\sqrt{3}}$

AB = $3\sqrt{3}$

94. (c) Radius of both the wheel is x cm and y cm

Distance covered by front wheel in n rotation be $2\pi xn$

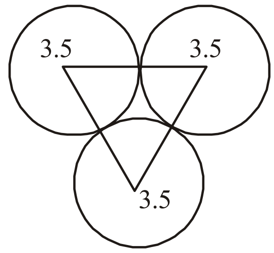
Than by rear wheel in rT

revolution will be = $T 2\pi y$

$2\pi xn = 2\pi y T$

$T = \frac{2\pi nx}{2\pi y} = \frac{nx}{y}$

95. (c)



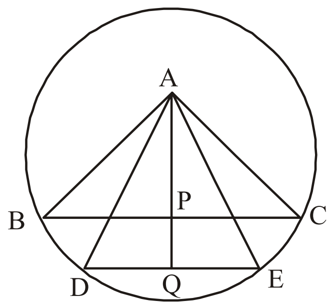
Shaded region area

= area Δ – area 3 sector

$= \frac{\sqrt{3}}{4}(7)^2 - 3 \times \pi(3.5)^2 \frac{60}{360}$

$= \frac{\sqrt{3}}{4} 49 - \frac{22}{7} \times \frac{3.5 \times 3.5}{2} = \frac{49}{8}(2\sqrt{3} - \pi)$

96. (c)



in Δ APB

$AB = \frac{13}{2}$ cm, $BP = \frac{12}{2} = 6$ cm

AP = ?

$AP^2 = AB^2 - BP^2$

$AP^2 = \left(\frac{13}{2}\right)^2 - \left(\frac{12}{2}\right)^2 = \left(\frac{5}{2}\right)^2$

$AP = \frac{5}{2}$

in Δ AQD

$AQ^2 = AD^2 - DQ^2$

$= \left(\frac{13}{2}\right)^2 - \left(\frac{5}{2}\right)^2 = \left(\frac{12}{2}\right)^2$

$AQ = \frac{12}{2} = 6$

$PQ = 6 - \frac{5}{2} \Rightarrow \frac{7}{2} = 3.5$

97. (c) area of square = 121 cm^2

side = $\sqrt{121} = 11$

perimeter = 4 × 11 = 44 curcumference of circle

$2\pi r = 44$

$$r = \frac{44 \times 7}{22 \times 2} = 7$$

Area = π(7)2

$$= \frac{22}{7} \times 7 \times 7 = 154 \text{ cm}^2$$

98. (b) When radius = r

$C = 2\pi r$

area = πr^2

$$2\pi(r + x) = \frac{115}{100} 2\pi r$$

When Radius = x

$C = 2\pi(r + x)$

Area = $\pi(r + x)^2$

$$r + x = \frac{115}{100} r$$

New area

$$\pi(r + x)^2 = \pi\left(\frac{115}{100} r\right)^2$$

$$\text{increase in area} = \pi\left(\frac{115}{100}\right)^2 - \pi r^2$$

$$\% \text{ increase} = \frac{\pi r^2\left[\frac{115^2}{100^2} - 1\right]}{\pi r^2} \times 100$$

$$= \frac{(115+100)(115-100)}{100}$$

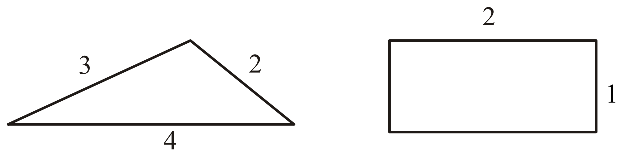
32.25 %

or

% increase in C = % increase in R
increase in area = $15 + 15 + \dfrac{15 \times 15}{100}$

= 32.25%

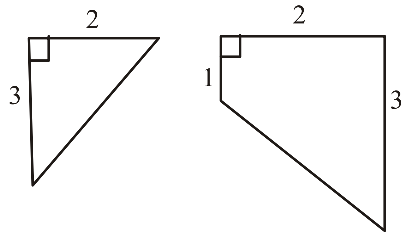
99. (c)



$$S = \frac{2+3+4}{2} = \frac{9}{2} \qquad \text{as } r = 2 \times 1 = 2m^2$$

$$\text{Area} = \frac{\sqrt{\frac{9}{2}\left(\frac{9}{2}-2\right)\left(\frac{9}{2}-3\right)\left(\frac{9}{2}-4\right)}}{\sqrt{\frac{9}{2}\left[\frac{5}{2}\right]\left(\frac{3}{2}\right)\left(\frac{1}{2}\right)}}$$

$$= \frac{3}{4}\sqrt{15}\ m^2$$



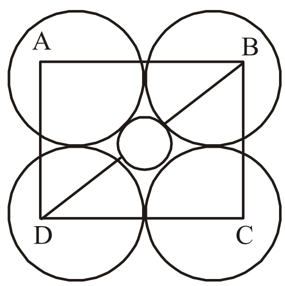
$$\text{Area} = \frac{1}{2} \times 2 \times 3 = 3\ m^2 \qquad \text{Area} = \frac{1}{2}[1+3] \times 2 = 4$$

$$\text{Total area} = 2 + 3 + 4 + \frac{3}{4}\sqrt{5}$$

$$9 + \frac{3}{4}\sqrt{5}$$

100. (a) D is diameter of each circle

∴ Side of square = D



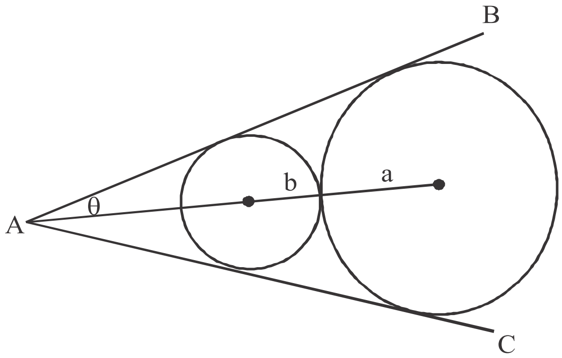
Diagonal of square = $\sqrt{D^2 + D^2} = D\sqrt{2}$

Diameter of shaded circle = $D\sqrt{2} - D = D(\sqrt{2} - 1)$

Diameter = $\sqrt{2}\,D - D = D(\sqrt{2} - 1)$

101. (b) We are given that a rectangular cello tape of length 4 cm and breadth 0.5 cm is used for joining each pair of edges. Therefore, area of the cello tape used for each face of the cube is a^2. Thus, total area of the cello tape used is $6a^2$. Now, we have $6a^2 = 6(4 \times 0.4)(4 \times 0.5) = 6.2.2 = 24$ sq. cm.

102. (b)



Let the radius of the bigger circle be a and radius of the smaller circle be b.

Then the angle made by direct common tangents when two circles of radius a and b touch externally is given by $\theta = 2\sin^{-1}\left(\frac{a-b}{a+b}\right)$.

We are given that area of the bigger circle = 9 area of the smaller circle

$\Rightarrow \pi a^2 = 9\pi b^2 \Rightarrow a^2 = 9b^2 \Rightarrow a = 3b$

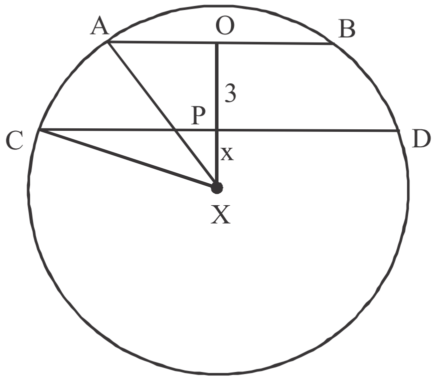
Let us consider $\angle BAC = \theta$

Thus,

$$\theta = 2\sin^{-1}\left(\frac{a-b}{a+b}\right) = 2\sin^{-1}\left(\frac{3b-b}{3b+b}\right) = 2\sin^{-1}\left(\frac{2b}{4b}\right)$$

$$= 2\sin^{-1}\left(\frac{1}{2}\right) = 2\sin^{-1}(\sin 30°) = 2 \times 30° = 60°$$

103. (c)



Given that $AB = 4$ cm and $CD = 10$ cm, let the radius of the circle be r cm.

Since the perpendicular from the center of a circle to a chord bisects the chord, therefore $AO = OB = 2$ cm and $CP = PD = 5$ cm.

In ΔAOX, by Pythagoras theorem, we have

$$2^2 + (3+x)^2 = r^2 \Rightarrow 4 + 9 + 6x + x^2 = r^2 \quad ...(1)$$

Similarly, in ΔCPX, we have

$$5^2 + x^2 = r^2 \Rightarrow 25 + x^2 = r^2 \quad ...(2)$$

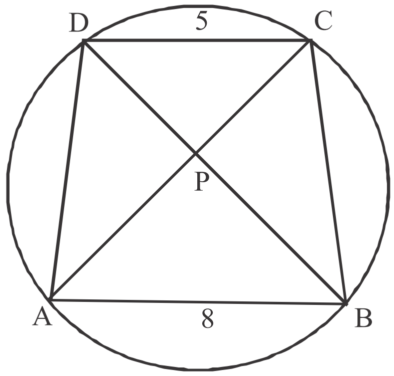
Equation (1) and (2) gives

$$4 + 9 + 6x + x^2 = 25 + x^2 \Rightarrow 13 + 6x = 25 \Rightarrow 6x = 12 \Rightarrow x = 2$$

Now, equation (2) gives

$$25 + 4 = r^2 \Rightarrow r^2 = 29 \Rightarrow r = \sqrt{29} \text{ cm}$$

104. (d)



Since $ABCD$ is a cyclic quadrilateral and a trapezium, therefore $AB \,\square\, CD$.

Now, in APB and CPD, we have

$\angle CDP = \angle ABP$

$\angle DCP = \angle PAB$

Therefore, by AA similarity criteria, $\Delta APB \sim \Delta CPD$

Now, the ratio of areas of similar triangles is equal to ratio of the squares of one of its proportional sides, therefore,

$$\frac{ar.(\Delta APB)}{ar.(\Delta CPD)} = \frac{AB^2}{CD^2} \Rightarrow \frac{24}{ar.(\Delta CPD)} = \frac{8^2}{5^2}$$

$$\Rightarrow \frac{24}{ar.(\Delta CPD)} = \frac{64}{25}$$

$$\Rightarrow ar.(\Delta CPD) = \frac{24 \times 25}{64} = \frac{75}{8} = 9.375 \text{ sq. cm}$$

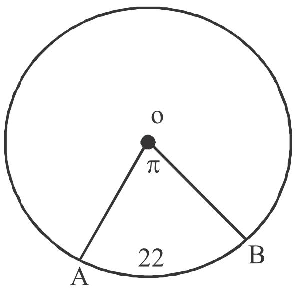
105. (a) We are given that $r_1 = 9$ cm, $r_2 = 4$ cm and the distance between the centers of two circles is 13 cm.

Therefore, the length of the direct common tangent of these circles is given by $\sqrt{d^2 - (r_1 - r_2)^2}$ where d is the distance between the centers of the two circles.

$$\sqrt{d^2 - (r_1 - r_2)^2} = \sqrt{13^2 - (9-4)^2} = \sqrt{169 - 25}$$

$$= \sqrt{144} = 12 \text{ cm}$$

106. (b)



Length of the arc

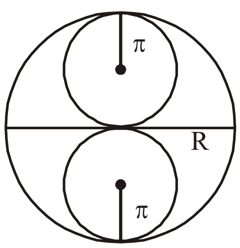
$$= \frac{\theta}{360} \times 2\pi r = 22 \Rightarrow \frac{\pi}{360} \times 2\pi r = 22$$

This is in radians. To convert in degrees, we will multiply $\frac{180}{\pi}$.

$$\frac{\theta}{360} \times 2\pi r = 22 \Rightarrow \frac{\pi}{360} \times 2\pi r = 22$$

$$\Rightarrow \frac{180}{\pi} \times \frac{\pi}{360} \times 2\pi r = 22 \Rightarrow r = 7 \text{ cm.}$$

107. (a)



Let radius of a smaller circular region be r area of a smaller region = πr^2

radius of bigger circle = 2r

Area of bigger circle = $\pi \times (2r)^2 = 4\pi r^2$

Total area of two smaller circles = $2 \times \pi r^2 = 2\pi r^2$

Area of remaining part = $4\pi r^2 - 2\pi r^2 = 2\pi r^2$

$$\text{Required value} = \frac{2\pi r^2}{4\pi r^2} \times A = \frac{A}{2}$$

108. (c) Tangent $= \sqrt{10^2 - (4.5 + 3.5)^2} = 6$ cm

109. (a)

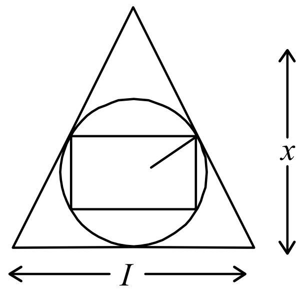
110. (d)

111. (b) $\angle ACB = 90 - \frac{\angle A \times B}{2} = 90 - \frac{50}{2} = 65$

112. (b)

113. (d) $OP = \sqrt{AB^2 + PB^2 + OC \times PB}$

114. (d)



Side of equilateral $\Delta = \ell$

Then,

$$\text{Inradius of circle} = \frac{\ell}{2\sqrt{3}}$$

which is diagonal of the square inscribe in the circle

Let, side of square = x

Then,

$$x^2 + x^2 = \left(\frac{\ell}{\sqrt{3}}\right)^2$$

$$2x^2 = \frac{\ell^2}{3}$$

$$\text{Area of square, } x^2 = \frac{\ell^2}{6}$$

115. (b) Area of circular plot

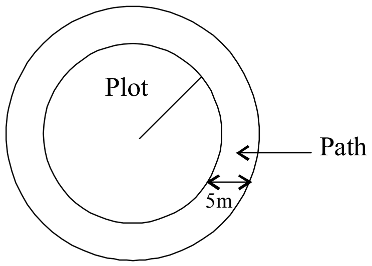
$= \pi r^2 = \pi(144)$

$\therefore$ r = 12 cm

Radius of circular plot including the path

=12+5= 17m

Area = $\pi(17)^2 = 289\pi$ m^2



116. (a) Let perimeter of equilateral triangle = Perimeter of square
= Perimeter of circle = 12 unit

Then, side length of equilateral triangle $= \frac{12}{3}$ unit = 4 unit

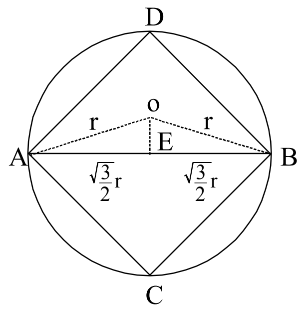
Side length of Square $= \frac{12}{4} = 3$ unit.

Side length of Square Circle $= \frac{12}{2 \times 3.14} = 1.91$ unit

Area of equilateral traingle T = $\frac{\sqrt{3}}{4}(4)^2 = 4\sqrt{3}$

= 6.928 unit.
Area of square S = $(3)^2 = 9$ unit
Area of circle C = $\pi\,(1.91)^2 = 11.46$ unit
Hence, C > S > T.

117. (b)



From ΔAOE, $\sin(\angle AOE) = \frac{AE}{AO} = \frac{\frac{\sqrt{3}}{2}r}{r} = \frac{\sqrt{3}}{2}$

$\therefore \sin(\angle AOE) = \sin\left(\frac{\pi}{3}\right)$

$\therefore \angle AOE = \frac{\pi}{3}$

From property of cyclic quadrilateral,

$\angle AOB = \frac{1}{2} \times (2 \times \angle AOB) = \frac{\pi}{3}$

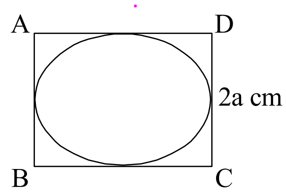
and $\angle ACB = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$\therefore K = 2.$

118. (a) Side length of square = Diameter of circle = 2a cm.
Now area bounded between two fig. = Area of square – Area of circle

$= (2a)^2 - \pi\left(\frac{2a}{2}\right)^2$

$= (4-\pi).a^2.$



119. (d) In ΔABC, by phythagoras

$BC = \sqrt{p^2+q^2}$

Area of the semicircle with diameter BC

$= \frac{\pi}{2}\left(\frac{\sqrt{p^2+q^2}}{2}\right)^2$

$= \frac{\pi}{8}(p^2+q^2)$

Area of the semicircle with diameter AB

$\frac{\pi}{2}\left(\frac{p}{2}\right)^2 = \frac{\pi p^2}{8}$

Area of the semicircle with diameter AC

$= \frac{\pi}{2}\left(\frac{q}{2}\right)^2 = \frac{\pi q^2}{8}$

Area of the triangle ABC = $\frac{1}{2}$ pq.

Area of shaded region =

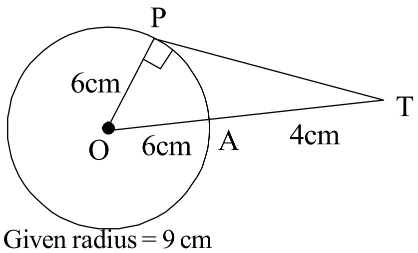
$\frac{\pi p^2}{8} + \frac{\pi q^2}{8} + \frac{1}{2}pq - \frac{\pi}{8}(p^2+q^2)$

$= \frac{\pi(P^2+q^2)}{8} + \frac{1}{2}pq - \frac{\pi}{8}(p^2+q^2)$

$= \frac{1}{2}pq.$

120. (b) As PT is a tangent to the circle at point P.
Then OP $\perp$ PT

Now, From ΔOPT, $PT = \sqrt{(OT)^2 - (OP)^2}$
$= \sqrt{(10)^2 - (6)^2}$
$= 8\,cm$



121. (b) Given radius = 9 cm

$\therefore$ diameter ABCD = 18 cm

Given that AB = BC = CD = 6cm

Area of the semicircle with diameter AD

$$=\frac{\pi}{2}(9)^2=\pi\frac{81}{2}$$

Area of the semicircle with diameter

$$AB=\frac{\pi}{2}(3)^2=\pi\times\frac{9}{2}$$

Area of the semicircle with diameter

$$BD=\frac{\pi}{2}(6)^2=\frac{\pi}{2}36$$

Area of the shaded portion

$$=\frac{81\pi}{2}+\frac{9\pi}{2}-\frac{36\pi}{2}=\frac{54\pi}{2}=27\pi$$

122. (c) $\angle BAC = \angle BDC$ {Angle made by same chord BC in the same side}

Now, from ΔBCD, sum of angles = 180°

$\angle CBD + \angle BDC + \angle BCD = 180°$

$\therefore$ $\angle BCD = 180° - 100° = 80°$

123. (b) In ΔCDE, $\angle CDE = \angle EDC = 32°$

$\therefore$ $\angle DEC = 180° - \angle ECD - \angle EDC$

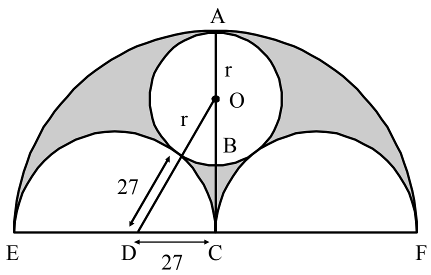
$= 180° - 32° - 32° = 116°$

Again, $\angle CED + \angle CEF = \angle DEF = 180°$

$116° + \angle CEF = 180°$

$\therefore$ $\angle CEF = 180° - 116° = 64° = \angle COF$

124. (c)



Draw a perpendicular OC on the line CD.

Let OA = r

$\Rightarrow$ OC = 54 – r

OD = 27 + r

In ΔOCD,

$(27 + r)^2 = (54 - r)^2 + (27)^2$

$\Rightarrow r = 18$

Area of the semi circle with diameter EF $=\frac{\pi}{2}(54)^2$

Area of the semi circle with diameter EC = Area of the semicircle with diameter $CF=\frac{\pi}{2}(27)^2$

Area of the circle with diameter AB = $\pi(18)^2$

Area of the shaded portion

$$=\frac{\pi}{2}(54)^2-\left[\frac{\pi}{2}(27)^2+\frac{\pi}{2}(27)^2+\pi(18)^2\right]$$

$= 405\pi\ cm^2$

125. (c) Area of the square = $(4)^2$ = 16 cm^2

Area of circle of the center

$$4\times\frac{\pi}{4}\left(\frac{2}{2}\right)^2=\pi\ cm^2$$

Total area removed = $\pi + \pi = 2\pi$ cm^2

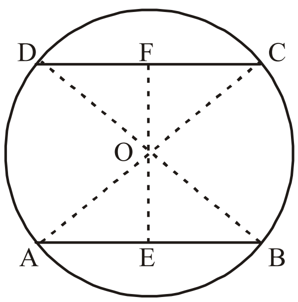
$$2\times\frac{22}{7}=\frac{44}{7}cm^2$$

$\therefore$ Area of shaded region

$$=16-\frac{44}{7}=9\frac{5}{7}cm^2$$

126. (b) **Case-I :** When chords are on opposite site of Center

Let *AB* and *CD* are two parallel chords of length 16 cm and 12 cm



OE and *OF* are $\perp$ on chord *AB* and *CD* from center '*O*'.

Then $AE = EB = \frac{AB}{2} = 8$cm

and $CF = DF = \frac{CD}{2} = 6$cm

OB = *OC* = radius of the circle = 10 cm

From $\Delta BOE, OE=\sqrt{(OB)^2-(BE)^2}$

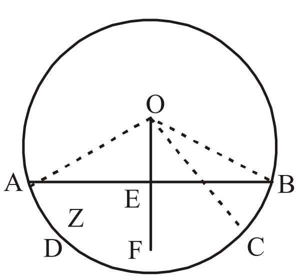
$=\sqrt{(10)^2-(8)^2}=6$

$OF=\sqrt{(OC)^2-(CF)^2}=\sqrt{(10)^2-(6)^2}=8$

Distance between two parallel chord

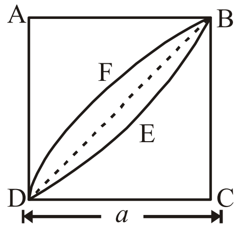
$EF = OE + OF = 6 + 8 = 14$ cm.

Case-II : When two chord are on same side of center.



Based on calculation in case I.
Distance between two chords $EF = OF - OE = 8 - 6 = 2$ cm.

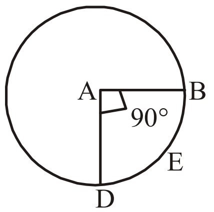
127. (c) Let arc BED of a circle with center A and arc BFD is of a circle of centre C.



Side length of the square $CD = a$
BD is a diagonal of square $ABCD$ of side length $= a$.

Then, Area of $\Delta ABD = \frac{1}{2} \times a \times a = \frac{a^2}{2}$

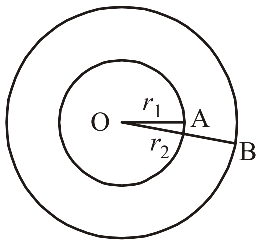
Now, Area of sector $ABED = \frac{\pi}{4} \times (AB)^2 = \frac{\pi}{4}(a)^2$



Area of BDEB $= \frac{\pi}{4}a^2 - \frac{a^2}{2} = \frac{(\pi - 2)a^2}{4}$

$\therefore$ Area of $BEDFB = 2 \times \frac{(\pi-2)a^2}{4} = a^2\left(\frac{\pi}{2} - 1\right)$

128. (c) Let radius of smaller and bigger circles are r_1 and r_2 cm



Perimeter of bigger circle $= 2 \times 44 = 88$ cm

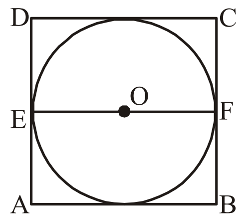
Now, $44 = 2 \times \frac{22}{7} \times r_1 \Rightarrow r_1 = 7$ cm

$88 = 2 \times \frac{22}{7} \times r_2 \Rightarrow r_2 = 14$ cm

Area between two circle $= \pi\left(r_2^2 - r_1^2\right)$

$= \frac{22}{7}(14^2 - 7^2) = 462$ sq.cm

129. (b) Let $ABCD$ is a square that inclose a circle with center 'O'.
Diameter of the circle (EF)
= Side of the square (AB) = a (let)



According to the question,
Perimeter of square + perimeter of circle = 12 m
$4 \times a + \pi . a = 12$
$\Rightarrow (4 + \pi) . a = 12$

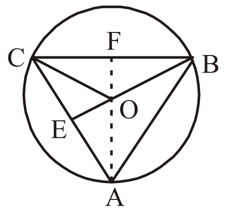
$a = \frac{12}{(4+\pi)}$

Radius of the circle $= \frac{a}{2} = \frac{6}{(4+\pi)}$

130. (d) OB = OC = 8 cm radius

From ΔAOE, $OA = AE = \frac{12}{2} = 6$ cm

$OE = \sqrt{(OA)^2 - (AE)^2} = \sqrt{(8)^2 - (6)^2} = \sqrt{28}$ cm



Now, $\Delta AEO \sim \Delta AFC$,

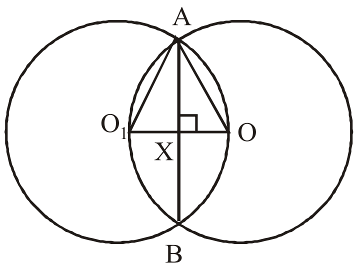
$\therefore \frac{AO}{AC} = \frac{OE}{CF} \Rightarrow CF = \frac{OE \times AC}{AO} = \frac{\sqrt{28} \times 12}{8}$

$= 3\sqrt{7}$ cm

$\therefore BC = 2 \times CF = 6\sqrt{7}$ cm

131. (c) Let two circles with center O and O' intersect each other at point A and B, such that AB is a common chord.

Then $AB = 10\sqrt{3}$ cm (Given)



Since both passes through the center of each other as shown in figure

So, O_1O is the radius of both circle

Let $O_1O = r = AO_1 = AO$

$AX = AB/2 = 5\sqrt{3}$ cm

(Since OX is perpendicular to chord AB, so bisects it)

AOO_1 forms an equilateral triangle with side = radius = r

$$\sin 60 = \frac{\sqrt{3}}{2} = \frac{AX}{AO} = \frac{5\sqrt{3}}{r}$$

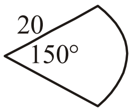
$\Rightarrow$ So $r = 10$ cm

$\Rightarrow$ diameter = 20 cm

132. (d) I. Number of circle that can be drawn from three non-collinear poitns is only one and if three points are collinear, then number of circle that can be drawn from these three points is zero.

II. Angle formed in a minor segment is obtuse and angle formed in major segment is always acute.

Hence, both statements are false.

133. (d) 

From $\ell = r.\theta$

here $\theta = 150° = \dfrac{5\pi}{6}$

$$\ell = \frac{5\pi}{6} \times 20 = \frac{50}{3} \times \frac{22}{7}$$

$$\text{Perimeter} = 2 \times 20 + \frac{50 \times 22}{3 \times 7} = 40 + \frac{50 \times 22}{3 \times 7}$$

r = radius of required circle

then, $2\pi r = 40 + \dfrac{50 \times 22}{3 \times 7}$

$$\Rightarrow 2 \times \frac{22}{7} r = 40 + \frac{50 \times 22}{3 \times 7}$$

$$\Rightarrow \frac{44}{7} r = \frac{44}{7} \times \frac{25}{3} = 40$$

$$\Rightarrow \left(r - \frac{25}{3}\right)\frac{44}{7} = 40 \Rightarrow r - \frac{25}{3} = 40 \times \frac{7}{44} = \frac{70}{11}$$

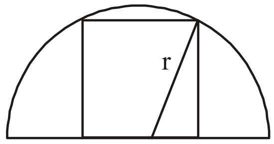
$$\Rightarrow r = \frac{25}{3} + \frac{70}{11} = \frac{485}{33}$$

134. (a) $D_S = 2R$ $\quad D_S^2 = (2R)^2$

$$\frac{\text{Area of circle}}{\text{Area of square}} = \frac{\pi R^2}{\frac{1}{2} D_S^2} = \frac{\pi}{2} = \frac{22}{14} = \frac{11}{7}$$

135. (b) Side of the square inscribed in a semicircle of radius r is s, then $(r)^2 = (s)^2 + \left(\dfrac{s}{2}\right)^2 \Rightarrow r^2 = \dfrac{5s^2}{4}$

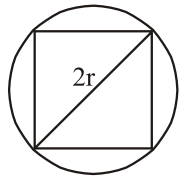
Area $(s)^2 = \dfrac{4r^2}{5}$



When square is drawn inside a full circle, then its

Side $= \dfrac{2r}{\sqrt{2}}$

$\therefore$ Area $= \dfrac{4r^2}{2} = 2r^2$



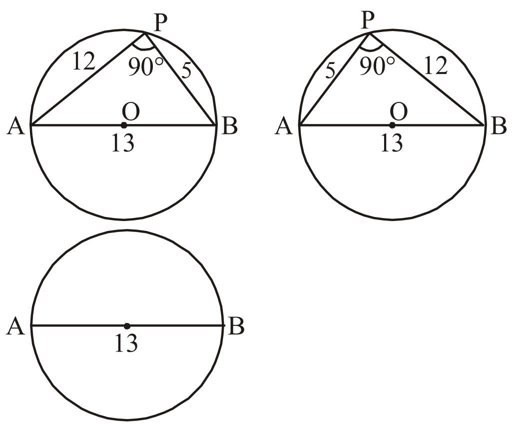
$$\therefore \text{Ratio} = \frac{\frac{4r^2}{5}}{2r^2} = \frac{2}{5} = 2:5$$

136. (b) $\pi(16) = 360°$

$$\therefore \text{Sector} = \frac{25.6}{\pi \times 16} \times 360$$

$$= \frac{25.6}{180 \times 16} 360 = 3.2$$

137. (a)



P does not lie on the circle at x = 6.5cm and y = 6.5 cm because, as <APB = 90°
then, $AB^2 = AP^2 + BP^2$

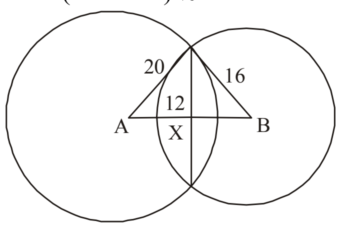
$(13)^2 \neq (6.5)^2 + (6.5)^2$

Hence, point P does not lies on the circle

138. (b) $AX = \sqrt{400 - 144} = 16$

$BX = \sqrt{256 - 144} = \sqrt{112} = 10.6$

$AB = (16 + 10.6) \approx 26.6$



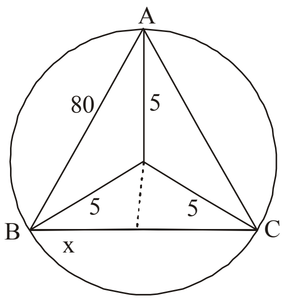
139. (c) $R = \dfrac{abc}{4\Delta}$

$$R = \frac{8 \times 8 \times c}{4 \times \frac{1}{2} \times c\sqrt{64 - \frac{c^2}{4}}}$$

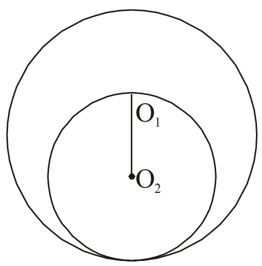
$$5 = \frac{32}{\sqrt{64 - \frac{c^2}{4}}}$$

$$\Rightarrow \quad 64 - \frac{c^2}{4} = \frac{1024}{25} \Rightarrow \frac{c^2}{4} = \frac{576}{25}$$

$$\Rightarrow \quad \frac{c}{2} = \frac{24}{5} \Rightarrow c = 9.6 \text{ cm.}$$



140. (b)



$\pi r_1^2 + \pi r_2^2 = 136\pi$

$r_1^2 + r_2^2 = 136$ ...(i)

$r_1 - r_2 = 4$ ...(ii)

from equation (i) and (ii)

$(r_1 - r_2)^2 = r_1^2 + r_2^2 - 2r_1 . r_2$

$\Rightarrow 16 = 136 - 2r_1.r_2$

$r_1.r_2 = 60$

Again from equation (ii)

$(r_1 + r_2)^2 = r_1^2 + r_2^2 + 2r_1.r_2$

$(r_1 + r_2)^2 = 136 + 120$

$(r_1 + r_2)^2 = 256$

$\therefore \quad r_1 + r_2 = 16$ ...(iii)

From equation (ii) and (iii) we get

$$\begin{array}{r} r_1 + r_2 = 16 \\ r_1 - r_2 = 4 \\ \hline r_1 = 10 \\ r_2 = 6 \end{array}$$

141. (c) Let radius of cirlce is 'r' and side of the square is a then, from question

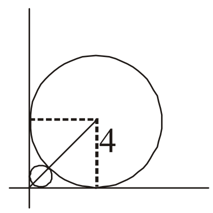
$\pi r^2 = a^2 \Rightarrow \dfrac{r}{a} = \dfrac{1}{\sqrt{\pi}}$

Now $\dfrac{2\pi r}{4a} = \dfrac{2\pi}{4\sqrt{\pi}} = \dfrac{\sqrt{\pi}}{2}$

142. (d) $(\sqrt{2} + 1)d = (\sqrt{2} - 1)8$

$\Rightarrow \quad d = (\sqrt{2} - 1)^2 8$

$= (3 - 2\sqrt{2})8\text{cm}$



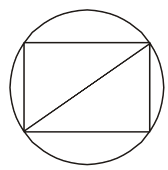
143. (d) $\text{Area} = \dfrac{340}{360} \times \pi \times (30)^2 = 850\pi$

144. (a) $30° = \dfrac{\pi}{6} \Rightarrow \text{Arc length} = r\theta = \dfrac{6 \times 3.14}{6} = 3.14 \text{ cm}$

145. (d) Area of Largest square = $\dfrac{d^2}{2}$

Where 'd' is the diameter of the circular disk and d = 2r = 2

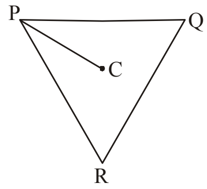
Hence, Area = $\dfrac{(2)^2}{2} = 2sq.4$ units.

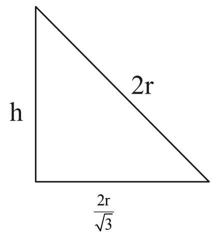


146. (d) The centre of three lower balles form an equilateral triangle. Let C be the centroid of this triangle. C is at the distance of $\frac{2r}{\sqrt{3}}$ form each vertex of lower ball.

Let h = height of centre of 4th ball from C.

then, $h = \sqrt{(2r)^2 - \left(\frac{2r}{\sqrt{3}}\right)^2} = 2r\sqrt{\frac{2}{3}}$





$\therefore$ height of centre of 4^{th} ball from plane

$= h + r = 2r\sqrt{\frac{2}{3}} + r = r\left(\frac{2\sqrt{2}+\sqrt{3}}{\sqrt{3}}\right)$ units

147. (b) $\frac{\text{Shaded Area}}{\text{Total Area}} = \frac{\frac{1}{2}\left(\pi\times 9^2 - \pi\times 6^2\right)}{\pi\times 9^2} = \frac{5}{18}$

$\therefore \frac{\text{Shaded area}}{\text{Unshaded area}} = \frac{5}{18-5} = \frac{5}{13}$

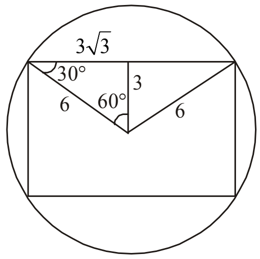
148. (b) Radius of circle $= \sqrt{(3)^2 + \left(3\sqrt{3}\right)^2} = \sqrt{9+27}$

Area of circle $= \pi r^2 = 36\pi$

Reqd. Area $= \frac{\pi(6)^2}{3} - 3\times 3\sqrt{3}$

$= 12\pi - 9\sqrt{3}$

$= 3(4\pi - 3\sqrt{3})$ Sq. units



149. (d) Area of three semicircles.

= Area of semicirlce ABC + Area of Semicircle AEF + Area of semicircle CDF.

As AC = 28 unit

Then radius of semicirlce ABC $= \frac{28}{2} = 14$ unit

Radius of semicircle AEF = CDF $= \frac{14}{2}$

= 7 unit.

Required Area $= \frac{\pi}{2}(14^2 + 7^2 + 7^2)$

= 462 sq. unit

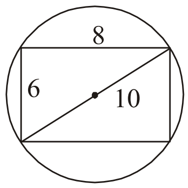
150. (d) Area of circle $= \pi r^2$

$= \pi\left(\sqrt{3^2+4^2}\right)^2 = \pi\left(\sqrt{25}\right)^2 = 25\pi$

Area of rectangle = a × b = 6 × 8

$\therefore$ Reqd. Area $= \pi \times 5^2 - 6 \times 8$

$= 25\pi - 48 = 30.5$ cm^2



CHAPTER

25 Statistics

1. What is the geometrical mean of the variate which takes values 210, 201, 102, 20, 12, 10, 2, 1 and 0? **[2007-I]**
(a) 3 (b) 2
(c) 1 (d) 0

2. What type of classification is needed to enumerate the female population of India? **[2007-I]**
(a) Geographical (b) Chronological
(c) Qualitative (d) Quantitative

3. If the mean of the variable X is M, what is the mean of variable $Y = \frac{(X-4)}{5}$? **[2007-I]**
(a) $\frac{(M-4)}{5}$ (b) M
(c) M – 5 (d) 5M – 4

4. If mean of y and $\frac{1}{y}$ is M, then what is the mean of y^3 and $\frac{1}{y^3}$? **[2007-II]**
(a) $\frac{M(M^2-3)}{3}$ (b) M^3
(c) $M^3 - 3$ (d) $M(4M^2 - 3)$

5. The following observation have been arranged in ascending order: **[2007-II]**
29, 32, 48, 52, x, x + 3, 71, 75, 80, 92
If the median of data is 61.5, then what is the value of x?
(a) 54 (b) 60
(c) 62 (d) 56

6. For less than ogive, the cumulative frequencies are plotted against which of the following? **[2007-II]**
(a) Upper limits of class intervals
(b) Lower limits of the class intervals
(c) Mid-points of the class intervals
(d) Either (b) or (c)

7. For the following frequency distribution:

Class interval	0-5	5-10	10-15	15-20	20-25	25-30
Frequency	10	15	30	80	40	20

If m is the value of mode, then which one of the following is correct? **[2007-II]**
(a) 5 < m < 10 (b) 10 < m < 15
(c) 15 < m < 20 (d) 20 < m < 25

8. The cumulative frequency curve of a frequency distribution with 6 classes and total frequency 60 is a straight line. Consider the following statements:
I. The first and the last classes have a frequency of 10 each.
II. Both the middle classes have a total frequency of 30.
III. The frequency distribution does not have a mode.
Which of the above statements are correct? **[2007-II]**
(a) I and II (b) I and III
(c) II and III (d) I, II and III

9. Square diagrams are drawn to represent the following data:

Country	Pakistan	India	Myanmar	China
Labour	36	81	25	100
Production (in ₹)				

Using the scale 1 cm^2 = ₹ 25, what is the length of the representative square for India? **[2007-II]**
(a) 1.8 cm (b) 1.2 cm
(c) 1 cm (d) 2 cm

10. An average Indian family allocates its monthly income under different heads as follows:

Items	Percentage Share
Food	40
House Rent	15
Saving	x
Transport	12
Miscellaneous	23

A pie diagram of this data is to be drawn. What is the value of x, if the angle which the sector representing saving makes at the centre is 36°? **[2008-I]**
(a) 13 (b) 11
(c) 10 (d) 8

11. From a series of 50 observations, an observation with value 45 is dropped but the mean remains the same. What was the mean of 50 observations? **[2008-I]**
(a) 50 (b) 49
(c) 45 (d) 40

12. The yield of paddy per plot of one acre were obtained from a number of plots from two different districts in a state and are summarized in the following table:

Yield of paddy per plot in quintals	District A Number of plots	District B Number of plots
38.0 – 41.0	25	14
41.0 – 44.0	36	29
44.0 – 47.0	59	35
47.0 – 50.0	30	54
50.0 – 53.0	25	41

Which of the following statements is correct? **[2008-I]**
(a) The mode for district A is higher than the mode for district B
(b) The mode for district B is higher than the mode for district A
(c) Both the distributions are symmetric
(d) Both the distributions have the same mean

13. If every number of a finite set is increased by any number k, the measure of central tendency should also increase by k. Which one of the following measures of central tendency does not have this property? **[2008-I]**
(a) Arithmetic mean
(b) Median
(c) Mid-range, i.e. the arithmetic mean of the largest and smallest numbers
(d) Geometric mean

14. If the median of the distribution (arranged in ascending order) 1, 3, 5, 7, 9, x, 15, 17 is 8, what is the value of x? **[2008-I]**
(a) 11 (b) 13
(c) $9 < x < 15$ (d) $9 \leq x \leq 15$

15. In Statistics, a suitable graph for representing the partitioning of total into subparts is **[2008-I]**
(a) An ogive (b) A pictograph
(c) A histogram (d) A pie chart

16. Suppose X is some statistical variable with mean μ. Let $x_1, x_2, \ldots x_n$ be its deviations from mean with the respective frequencies $f_1, f_2, \ldots, f_n$. What is the value of the sum $x_1 f_1 + x_2 f_2 + \ldots + x_n f_n$? **[2008-II]**
(a) 0 (b) 1
(c) μ (d) $\mu + 1$

17. Sets A, B and C contain 5 numbers each. The medians of the numbers in these sets are 3, 8 and 11, respectively. What is the median of the combined 15 numbers of the three sets? **[2008-II]**
(a) 8 (b) 7
(c) $\frac{22}{3}$ (d) Cannot be determined

18. Which one of the following statements is not correct with reference to a histogram? **[2008-II]**
(a) Frequency curve is obtained by joining the mid-points of the top of the adjacent rectangles with smooth curves
(b) Histogram is drawn for continuous data
(c) The height of the bar is proportional to the frequency of that class
(d) Mode of the distribution can be obtained from the histogram

19.

X	0	1	2	3	4
Frequency	4	f	9	g	4

The table above gives the frequency distribution of a discrete variable X with two missing frequencies. If the total frequency is 25 and the arithmetic mean of X is 2, then what is the value of the missing frequency f? **[2008-II]**
(a) 4 (b) 5
(c) 6 (d) 7

20. Assume that population densities of 5 major states of India are given. Which one of the following diagrams is suitable to represent the data? **[2008-II]**
(a) Single bar diagram
(b) Percentage bar diagram
(c) Pie diagram
(d) Since population density is a ratio, it cannot be represented by any diagram

21. Which one of the following represents statistical data? **[2009-I]**
(a) The names of all owners of shops located in a shopping complex
(b) A list giving the names of all states of India
(c) A list of all European countries and their respective capital cities
(d) The volume of a rainfall in certain geographical area, recorded every month for 24 consecutive months

22. The arithmetic mean of a set of 10 numbers is 20. If each number is first multiplied by 2 and then increased by 5, then what is the mean of new numbers? **[2009-I]**
(a) 20 (b) 25
(c) 40 (d) 45

23. Consider the following types of data:
I. Marks of students who appeared for a test of 100 marks.
II. Collar sizes of 200 shirts sold in a week.
III. Monthly incomes of 250 employees of a factory.

For which of the above data, mode is a suitable measure of central tendency? **[2009-I]**
(a) Both I and II (b) Only II
(c) Both I and III (d) I, II and III

24. The mean of 25 observations is 36. The mean of first 13 observations is 32 and that of last 13 observations is 39. What is the value of 13th observation? **[2009-I]**
(a) 20 (b) 23
(c) 32 (d) 40

25. Data on percentage distribution of area of land in acres owned by households in two districts of a particular state are as follows:

Land Holding	District A	District B
0.01 – 0.99	5.62	13.53
1.0 – 2.49	18.35	21.84
2.5 – 7.49	47.12	39.32
7.5 – 12.49	19.34	12.15
12.5 – 19.99	7.21	7.43
20.0 – 29.99	2.36	5.73

What is the appropriate diagram to represent the above data? **[2009-I]**
(a) Pie diagram (b) Histogram
(c) Bar chart (d) None of the above

26. The following table shows the percentage of male and female coffee drinkers and non-coffee drinkers in two towns A and B.

Attributes	Town A		Town B	
	Male	**Female**	**Male**	**Female**
Coffee drinkers	40%	5%	25%	15%
Non-coffee drinkers	20%	35%	30%	30%

If the total population of the towns A and B are 10000 and 20000 respectively, then what is the total number of female coffee drinkers in both towns? **[2009-II]**

(a) 8000 (b) 6000
(c) 3500 (d) 2500

27. The median of three positive integers, two of which are equal, is 5. What is the least possible value of the arithmetic means of these integers? **[2009-II]**

(a) 2
(b) 3
(c) 4
(d) No such least possible value exists

28. Which one of the following can be obtained from a histogram? **[2009-II]**

(a) Mean (b) Median
(c) Mode (d) None of these

29. Examples of data are given below:

I. Information on households collected by an investigator by door to door visits.
II. Data on the percentage of literates, sexwise, for the different districts of a state collected from records of the census of India.
III. General information about families, collected by telephonic interviews.

Which one of the following in respect of the above is correct? **[2009-II]**

(a) I and II are primary data
(b) I and III are primary data
(c) II and III are primary data
(d) I, II and III are primary data

30. What is the geometric means of the observations 125, 729, 1331? **[2009-II]**

(a) 495 (b) 1485
(c) 2221 (d) None of these

31. Consider the following pairs of numbers: **[2009-II]**

I. (8, 12)
II. (9, 11)
III. (6, 24)

Which pairs of number have the same harmonic means?

(a) I and II (b) II and III
(c) I and III (d) I, II and III

32. The marks of the students of a class who appeared for a test in English are represented in the following frequency table.

Class interval	Frequency
1 – 10	9
11 – 20	22
21 – 30	—
31 – 40	20
41 – 50	12
51 – 60	8
	100 (total frequency)

What is/are the modal class(es)? **[2010-I]**

(a) Only 10.5 – 20.5
(b) Only 20.5 – 30.5
(c) 10.5 – 20.5 and 20.5 – 30.5
(d) There is no modal class

33. What is the weighted mean of first 10 natural numbers whose weights are equal to the corresponding number? **[2010-I]**

(a) 7 (b) 5.5
(c) 5 (d) 4.5

DIRECTIONS (Qs. 34-35): *Read the following information carefully to answer the questions that follow.*

The average age of 6 persons living in a house is 23.5 years. Three of them are majors and their average age is 42 years. The difference in ages of the three minor children is same.

34. What is the mean of the ages of minor children? **[2010-I]**

(a) 3 years (b) 4 years
(c) 5 years (d) 6 years

35. What is the median of the ages of minor children? **[2010-I]**

(a) 3 years (b) 5 years
(c) 7 years (d) Cannot be determined

36. A new frequency distribution is constructed by doubling each frequency of the original distribution keeping the other entries intact. The following measures are computed for both the tables:

I. Arithmetic mean
II. Median
III. Harmonic mean

Which of the following statements with reference to above is correct? **[2010-I]**

(a) Corresponding values of I and II only are equal in both the distributions
(b) Corresponding values of I and III only are equal in both the distributions
(c) Corresponding values of II and III only are equal in both the distributions
(d) Corresponding values of I, II and III are equal in both the distributions

37. Consider the following data:

Year	Birth rate	Death rate
1911 – 21	48.1	38.5
1921 – 31	46.4	36.3
1931 – 41	45.2	31.2
1941 – 51	39.9	27.4
1951 – 61	41.7	22.8
1961 – 71	41.1	35.9
1971 – 81	37.1	14.8

For which period is the natural growth rate minimum? **[2010-I]**

(a) 1911-21 (b) 1921-31
(c) 1951-61 (d) 1961-71

38. With the help of histogram one can prepare **[2010-I]**

(a) frequency polygon (b) frequency curve
(c) frequency distribution (d) All of the above

39. The table below gives the number of members of a club classified by sex and nativity:

Sex \ Nativity	Locals	Migrants	Total
Male	85	45	130
Female	35	35	70
Total	120	80	200

The above data are represented by a pie diagram. What is the sectorial angle of the area representing male-migrant category? **[2010-I]**

(a) 45° (b) 22.5°
(c) 81° (d) 67.5°

DIRECTIONS (Qs. 40-42): *Read the following information carefully to answer the questions that follow.*

The arithmetic mean, geometric mean and median of 6 positive numbers a, a, b, b, c, c, where a < b < c are $\frac{7}{3}$, 2, 2, respectively.

40. What is the sum of the squares of all the six numbers? **[2010-I]**

(a) 40 (b) 42
(c) 45 (d) 48

41. What is the value of c? **[2010-I]**

(a) 1 (b) 2
(c) 3 (d) 4

42. What is the mode? **[2010-I]**

(a) 1 (b) 2
(c) 1, 2 and 4 (d) None of these

43. Consider the following data:

x	1	2	3	4	5
f	3	5	9	–	2

If the arithmetic mean of the above distribution is 2.96, then what is the missing frequency? **[2010-I]**

(a) 4 (b) 6
(c) 7 (d) 8

44. For a set of positive numbers, consider the following statements:

I. If each number is reduced by 2, then the geometric mean of the set may not always exist.
II. If each number is increased by 2, then the geometric mean of the set is increased by 2.

Which of the above statements is/are correct? **[2010-II]**

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

45. Consider the following frequency distribution:

Class	0 – 10	0 – 20	0 – 30	0 – 40	0 – 50
Frequency	3	8	14	14	20

What is the above frequency distribution known as? **[2010-II]**

(a) Cumulative distribution in more than type
(b) Cumulative distribution in less than type
(c) Continuous frequency distribution
(d) None of the above

46. Consider the following statements in respect of histogram.

I. Histogram is an equivalent graphical representation of the frequency distribution.
II. Histogram is suitable for continuous random variables, where the total frequency of an interval is evenly distributed over the interval.

Which of the statements given above is/are correct? **[2010-II]**

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

47. What is the median of the values 11, 7, 6, 9, 12, 15 and 19? **[2010-II]**

(a) 9 (b) 11
(c) 12 (d) 15

48. Let $(x_1, y_1), (x_2, y_2), ..., (x_n, y_n)$ are n pairs of positive numbers. The arithmetic mean and geometric mean of any set of positive numbers $(c_1, c_2, ..., c_n)$ are denoted by $M(c_i)$ and $G(c_i)$, respectively.

Consider the following:

I. $M(x_i + y_i) = M(x_i) + M(y_i)$
II. $G(x_i y_i) = G(x_i)\, G(y_i)$

Which of the above is/are correct? **[2010-II]**

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

49. The arithmetic mean of 100 numbers was computed as 89.05. It was later found that two numbers 92 and 83 have been misread as 192 and 33, respectively. What is the correct arithmetic mean of the numbers? **[2010-II]**

(a) 88.55 (b) 87.55
(c) 89.55 (d) Cannot be determined

DIRECTIONS (Qs. 50-51): *The item-wise expenditure of a Non-Government Organisation for the year 2008-09 is given below:*

Item	Expenditure (in Rs. lakh)
Salary of employees	6
Social welfare activities	7
Office contingency	3
Vehicle maintenance	4
Rent and hire charges	2.5
Miscellaneous expenses	1.5

The above data are represented by a pie diagram.

50. What is the central angle of the largest component? **[2011-I]**
(a) 120° (b) 105°
(c) 90° (d) 85°

51. What is the difference between central the angles of the largest and the smallest component? **[2011-I]**
(a) 90° (b) 85°
(c) 82.5° (d) 77.5°

DIRECTIONS (Qs. 52-53): *The following table gives the frequency distribution of life length in hours of 100 electric bulbs having median life 20 h.*

Life of bulbs (in hours)	Number of bulbs
8 – 13	7
13 – 18	x
18 – 23	40
23 – 28	y
28 – 33	10
33 – 38	2

52. What is the missing frequency 'x'? **[2011-I]**
(a) 31 (b) 27
(c) 24 (d) 14

53. What is the missing frequency 'y'? **[2011-I]**
(a) 27 (b) 24
(c) 14 (d) 11

54. Consider the following statements in respect of a histogram:
I. The histogram consists of vertical rectangular bars with a common base such that there is no gap between consecutive bars.
II. The height of the rectangle is determined by the frequency of the class it represents.
Which of the statements given above is/are correct? **[2011-I]**
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

55. The arithmetic mean of 10 numbers was computed as 7.6. It was later discovered that a number 8 was wrongly read as 3 during the computation. What should be the correct mean? **[2011-I]**
(a) 7.1 (b) 7.6
(c) 8.1 (d) 8.6

56. Which one among the following statements is correct? **[2012-I]**
(a) Simple bar diagrams are those diagrams which show two characteristics of the data
(b) In pie diagrams all the items are converted into angles
(c) A bar diagram is one in which data are shown in terms of bars
(d) Bar diagrams present data through length and breadth

57. Two following characteristics relate to the persons participating in athletic events:
I. height of the person.
II. colour of the eye of the person.
III. number of times a person correctly hits a target in ten attempts.
Which of the following in respect of the above is correct? **[2012-I]**
(a) I is a continuous variable, II is not a variable and III is a discrete variable
(b) I is a continuous variable, II and III are discrete variables
(c) I and III are discrete variables and II is not a variable
(d) I, II and III all are discrete variables

58. Consider the following statements:
I. The data collected by the investigator to be used by himself are called primary data.
II. The data obtained from government agencies are called secondary data.
Which of the above statements is/are correct? **[2012-I]**
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

59. Which one among the following statements is not correct? **[2012-I]**
(a) For size of readymade garments, mode is the best suitable measure
(b) For average rate of increase when the rate of population growth is given, geometric mean is best suitable
(c) For average rate of speed when different distances are covered by different rates of speed, harmonic mean is best suitable
(d) For average level of intelligence of students in a class, arithmetic mean is the best suitable

60. Let the observations at hand be arranged in increasing order. Which one of the following measures will not be affected when the smallest and the largest observations are removed? **[2012-I]**
(a) Mean (b) Median
(c) Mode (d) Standard deviation

61. If the population figures are given for each state of India, then the data can be classified as **[2012-I]**
(a) qualitative (b) quantitative
(c) chronological (d) geographical

62. The frequency distribution of a variate x is as given below:

x	2	3	4	5	6
Frequency	2	7	14	22	30

What is the cumulative frequency corresponding to variate $x = 5$? **[2012-I]**
(a) 36 (b) 44
(c) 45 (d) 52

63. Consider the following statements in respect of the set $S = \{1, 2, 3, ..., n\}$.

I. $\frac{(n+1)}{2}$ is the median of the numbers in S.

II. n is the mode of the numbers in S.

Which of the above statements is/are correct? **[2012-I]**

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

64. The arithmetic mean and geometric mean of two numbers are 14 and 12, respectively. What is the harmonic mean of the numbers? **[2012-I]**

(a) 10 (b) 18
(c) $\frac{32}{3}$ (d) $\frac{72}{7}$

65. Consider the following table:

x	0	1	2	3	4
y	100	90	70	40	0

How are x and y related? **[2012-II]**

(a) $y = 100 - 10x$ (b) $y = 100 - 5x^2$
(c) $y = 20 - x - x^2$ (d) $y = 100 - 5x - 5x^2$

66. In histogram, the width of the bars is proportional to **[2012-II]**

(a) Frequency (b) Number of classes
(c) Class interval (d) None of these

67. Which one of the following relations for the numbers 10, 7, 8, 5, 6, 8, 5, 8 and 6 is correct? **[2012-II]**

(a) Mean = Median (b) Mean = Mode
(c) Mean > Median (d) Mean > Mode

68. The mean of 100 values is 45. If 15 is added to each of the first forty values and 5 is subtracted from each of the remaining sixty values, the new mean becomes **[2012-II]**

(a) 45 (b) 48
(c) 51 (d) 55

69. Which of the following is/are correctly matched?

I. Weight of a person : Continuous variable.

II. Educational qualification of the person : Attribute

Select the correct answer using the codes given below. **[2012-II]**

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

70. Consider the following distribution:

Value of the variable	1	2	3	4	5
Frequency	3	f	6	5	3

For what value of f, is the arithmetic mean of the above distribution 3.1? **[2012-II]**

(a) 2 (b) 3
(c) 4 (d) 5

71. The average of u, v, w, x, y and z is 10. What is the average of u + 10, v + 20, w + 30, x + 40, y + 50 and z + 60? **[2013-II]**

(a) 30 (b) 35
(c) 40 (d) 45

72. If m is the mean of p, q, r, s, t, u and v, then what is $(p - m) + (q - m) + (r - m) + (s - m) + (t - m) + (u - m) + (v - m)$ equal to? **[2013-II]**

(a) 0 (b) s
(c) $\frac{(p+v)}{2}$ (d) None of these

DIRECTIONS (Qs. 73-74): *Read the following information carefully and answer the questions given below.*

The median of the following distribution is 14.4 and the total frequency is 20.

Class Interval	0 – 6	6 – 12	12 – 18	18 – 24	24 – 30
Frequency	4	x	5	y	1

73. What is x equal to? **[2013-II]**

(a) 4 (b) 5
(c) 6 (d) 7

74. What is the relation between x and y? **[2013-II]**

(a) $2x = 3y$ (b) $3x = 2y$
(c) $x = y$ (d) $2x = y$

75. There are 45 male and 15 female employees in an office. If the mean salary of the 60 employees is ₹ 4800 and the mean salary of the male employees is ₹ 5000, then the mean salary of the female employees is **[2013-II]**

(a) ₹ 4200 (b) ₹ 4500
(c) ₹ 5600 (d) ₹ 6000

76. The mean of 7 observations is 7. If each observation is increased by 2, then the new mean is: **[2013-II]**

(a) 12 (b) 10
(c) 9 (d) 8

77. Which of the following are the examples of discrete variables?

I. Number of errors per page in a book.

II. Height of individuals measured in centimetre.

III. Waiting time to failure of electric bulbs.

IV. Number of leaves on branches of a tree.

Select the correct answer using the codes given below. **[2013-II]**

(a) Only I (b) I and IV
(c) III and IV (d) II and IV

DIRECTIONS (Qs. 78-80): *Read the following information carefully to answer the questions that follow.*

In a frequency distribution having class intervals 0-10, 10-20, 20-30 and 30-40 the respective frequencies are x, x + 8, x – 2 and x – 4 and the arithmetic mean of the distribution is 17.8.

78. The value of x is **[2013-II]**

(a) 3 (b) 6
(c) 8 (d) 12

79. The median lies in which one of the following class intervals? **[2013-II]**

(a) 0-10 (b) 10-20
(c) 20-30 (d) 30-40

80. The modal class is: **[2013-II]**

(a) 0-10 (b) 10-20
(c) 20-30 (d) 30-40

81. Consider the following statements:

I. A frequency distribution condenses the data and reveals its important features.

II. A frequency distribution is an equivalent representation of original data.

Which of the above statements is/are correct? **[2013-II]**

(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

82. Which one of the following statements is correct? **[2013-II]**
(a) A frequency polygon is obtained by connecting the corner points of the rectangles in a histogram
(b) A frequency polygon is obtained by connecting the mid-points of the tops of the rectangles in a histrogram
(c) A frequency polygon is obtained by connecting the corner points of the class intervals in a histogram
(d) None of the above

83. Consider the following:
I. The arithmetic mean of two unequal positive numbers is always greater than their geometric mean.
II. The geometric mean of two unequal positive numbers is always greater than their harmonic mean.
Which of the above statements is/are correct? **[2014-I]**
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

84. Consider the following statements in respect of a discrete set of numbers.
I. The arithmetic mean uses all the data is always uniquely defined.
II. The median uses only one or two numbers from the data and may not be unique.
Which of the above statements is/are correct? **[2014-I]**
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

85. The geometric mean of $(x_1, x_2, x_3, ..., x_n)$ is x and the geometric mean of $(y_1, y_2, y_3, ..., y_n)$ is y. Which of the following is/are correct?
I. The geometric mean of $(x_1 y_1, x_2 y_2, x_3 y_3, ..., x_n y_n)$ is XY.
II. The geometric mean of $\left(\frac{x_1}{y_1}, \frac{x_2}{y_2}, \frac{x_3}{y_3}, ..., \frac{x_n}{y_n}\right)$ is $\frac{X}{Y}$.
Select the correct answer using the code given below. **[2014-I]**
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

86. The following table gives 'less than' type frequency distribution of income per day.

Income (in ₹) less than	Number of persons
1500	100
1250	80
1000	70
750	55
500	32
250	12

What is the modal class? **[2014-I]**
(a) 250-500 (b) 500-750
(c) 750-1000 (d) None of these

87. Which of the following items of information is a good example of statistical data? **[2014-I]**
(a) A table of logarithms of numbers
(b) A list of names of 120 students of a class
(c) A list of annual incomes of the members of a club
(d) Holiday list of the offices of Government of India in the year 2013

88. Consider the following in respect of variate which takes values 2, 2, 2, 2, 7, 7, 7 and 7.
I. The median is equal to mean.
II. The mode is both 2 and 7.
Which of the above statements is/are correct? **[2014-I]**
(a) Only I
(b) Only II
(c) Both I and II
(d) Neither I nor II

89. Consider the following statements pertaining to a frequency polygon of a frequency distribution of a continuous variable having seven class intervals of equal width.
I. The original frequency distribution can be reconstructed from the frequency polygon.
II. The frequency polygon touches the X-axis in its extreme right and extreme left.
Which of the above statements is/are correct? **[2014-I]**
(a) Only I (b) Only II
(c) Both I and II (d) Neither I nor II

90. The mean of the following distribution is 18.

Class interval	Frequency
11 – 13	3
13 – 15	6
15 – 17	9
17 – 19	13
19 – 21	f
21 – 23	5
23 – 25	4

What is the value of f? **[2014-I]**
(a) 8 (b) 9
(c) 10 (d) 11

91. The class which has maximum frequency is known as **[2014-II]**
(a) median class (b) mean class
(c) modal class (d) None of these

92. Consider the following statements related to cumulative frequency polygon of a frequency distribution, the frequencies being cumulated from the lower end of the range : **[2014-II]**
1. The cumulative frequency polygon gives an equivalent representation of frequency distribution table.
2. The cumulative frequency polygon is a closed polygon with one horizontal and one vertical side. The other sides have non–negative slope.
Which of the above statements is / are correct ?
(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

93. Consider the following data : **[2014-II]**
1. Number of complaints lodged due to road accidents in a state within a year for 5 consecutive years.
2. Budgetary allocation of the total available funds to the various items of expenditure.

Which of the above data is / are suitable for repsentation of a pie diagram ?

(a) Only 1 (b) Only 2
(c) Both 1 and 2 (d) Neither 1 nor 2

94. When we take class intervals on the X–axis and corresponding frequencies on the Y-axis and draw rectangles with the areas proprotional to the frequencies of the respective class intervals, the graph so obtained is called **[2014-II]**

(a) bar diagram
(b) frequency curve
(c) ogive
(d) None of the above

95. If x_i's are the mid–points of the class intervals of grouped data, f_i's are the corresponding frequencies and $\bar{x}$ is the mean, then what is $\sum f_i(x_i - \bar{x})$ equal to? **[2014-II]**

(a) 0 (b) –1
(c) 1 (d) 2

96. Ten observations 6, 14, 15, 17, $x + 1$, $2x - 13$, 30, 32, 34 and 43 are written in ascending order. The median of the data is 24. What is the value of x? **[2014-II]**

(a) 15 (b) 18
(c) 20 (d) 24

97. If A, G and H are the arithmetic, geometric and harmonic means between a and b respectively, then which one of the following relations is correct? **[2015-I]**

(a) G is the geometric mean between A and H
(b) A is the arithmetic mean between G and H
(c) H is the harmonic mean between A and G
(d) None of the above

98. The geometric mean of three positive numbers a, b, c is 3 and the geometric mean of another three positive numbers d, e, f, is 4. Also, at least three elelments in the set $\{a, b, c, d, e, f\}$ are distinct. Which one of the following inequalities gives the best information about M, the arithmetic mean of the six numbers ? **[2015-I]**

(a) $M > 2\sqrt{3}$
(b) $M > 3.5$
(c) $M \geq 3.5$
(d) It is not possible to set any precise lower limit for M

99. There are five parties A, B, C, D and E in an election. Out of total 100000 votes cast, 36000 were cast to party A, 24000 to party B, 18000 to party C, 7000 to party D and rest to party E. What angle will be allocated for party E in the pie chart? **[2015-I]**

(a) 15° (b) 54°
(c) 60° (d) 72°

For the next four (4) items that follow:

Consider the following frequency distribution :

Class	Frequency
0-10	4
10-20	5
20-30	7
30-40	10
40-50	12
50-60	8
60-70	4

100. What is the mean of the distribution? **[2015-I]**

(a) 37.2 (b) 38.1
(c) 39.2 (d) 40.1

101. What is the median class? **[2015-I]**

(a) 20–30 (b) 30–40
(c) 40–50 (d) 50–60

102. What is the median of the distribution? **[2015-I]**

(a) 37 (b) 38
(c) 39 (d) 40

103. What is the mode of the distribution? **[2015-I]**

(a) 38.33 (b) 40.66
(c) 42.66 (d) 43.33

104. If a variable takes discrete values a + 4, a – 3.5, a – 2.5, a –3, a–2, a + 0.5, a + 5 and a – 0.5 where a > 0, then the median of the data set is **[2015-II]**

(a) a – 2.5 (b) a – 1.25
(c) a – 1.5 (d) a – 0.75

105. If each of n numbers $x_i = i$ (i = 1, 2, 3,..... n) is replaced by $(i + 1)\,x_i$, then the new mean is **[2015-II]**

(a) $\frac{n+3}{2}$ (b) $\frac{n(n+1)}{2}$

(c) $\frac{(n+1)(n+2)}{3n}$ (d) $\frac{(n+1)(n+2)}{3}$

106. The weighted arithmetic mean of first 10 natural numbers whose weights are equal to the corresponding numbers is equal to **[2015-II]**

(a) 7 (b) 14
(c) 35 (d) 38.5

107. The election result in which six parties contested was depicted by a pic chart. Party A had an angle 135° on this pic chart. If it secured 21960 votes, how many valid votes in total were cast? **[2016-I]**

(a) 51240 (b) 58560
(c) 78320 (d) 87840

108. The mean and median of 5 observations are 9 and 8 respectively. If 1 is substracted from each observation, then the new mean and the new median will respectively be **[2016-I]**

(a) 8 and 7
(b) 9 and 7
(c) 8 and 9
(d) Cannot be determined due to insufficient data

109. The age distribution of 40 children is as follows:

Age (in years)	5–6	6–7	7–8	8–9	9–10	10–11
No. of children	4	7	9	12	6	2

Consider the following statements in respect of the above frequency distribution: **[2016-I]**

1. The median of the age distribution is 7 years.
2. 70% of the children are in the age group 6–9 years.
3. The modal age of the children is 8 years.

Which of the above statements are correct? **[2016-I]**

(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

110. Suppose $x_1 = \lambda^8$ for $0 < 10$, where $\lambda > 1$. **[2016-I]**
Which one of the following is correct?

(a) AM – Median (b) GM – Median
(c) GM – Median (d) AM – Median

111. Suppose $x_1 = \frac{1}{1}$ for i = 1, 2, 3, ...11. **[2016-I]**

Which one of the following is not correct?

(a) AM > 1/6 (b) GM > 1/6
(c) HM > 1/6 (d) Median – HM

112. The average score of class X is 83. **[2016-II]**
The average score of class Y is 76.
The average score of class Z is 85.
The average score of class X and Y is 79
and average score of class Y and Z is 81.
What is the average score of X, Y and Z ?

(a) 81.5 (b) 80.5
(c) 79.0 (d) 78.0

113. For x > 0, if a variable takes discrete values x + 4, x – 3.5, x – 2.5, x – 3, x – 2, x + 0.5, x – 0.5, x + 5, then what is the median ? **[2016-II]**

(a) x – 1.25 (b) x – 0.5
(c) x + 0.5 (d) x + 1.25

114. The median of a set of 9 distinct observations is 20.5. If each of the largest 4 observations of the set is increased by 2, then the median of the new set **[2016-II]**

(a) is increased by 2
(b) is decreased by 2
(c) is two times the original median
(d) remains the same as that of original set

115. Number of credit cards held by an individual can be treated as **[2016-II]**

(a) Qualitative data
(b) Discrete data
(c) Categorical data
(d) None of the above

116. A pie chart depicts the classification of total funds of an organization according to different sources of funds. A particular sector of pie chart for corporate tax has 108° angle at' the centre. What is the percentage of income from corporate tax to total funds ? **[2016-II]**

(a) 20% (b) 25%
(c) 30% (d) 35%

117. Consider the following statements : **[2016-II]**

1. The classes of type 15–19, 20–24, 25–29 etc. are exclusive classes.
2. The classes of type 15–20, 20–25, 25–30 etc. are inclusive classes.

Which of the above statements is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

118. Suppose the class interval 10–15 has frequency 30, then what is the frequency density of this class interval? **[2016-II]**

(a) 2 (b) 3
(c) 5 (d) 6

119. Consider the following distribution : **[2017-I]**

Class	Frequency
0 – 20	17
20 – 40	28
40 – 60	32
60 – 80	f
80 – 100	19

If the mean of the above distribution is 50, what is the value of f ?

(a) 24 (b) 34
(c) 56 (d) 96

120. In a pie diagram, there are four slices with angles 150°, 90°, 60° and 60°. A new pie diagram is formed by deleting one of the slices having angle 60° in the given pie diagram. In the new pie diagram **[2017-I]**

(a) The largest slice has angle 150°
(b) The smallest slice has angle 70°
(c) The largest slice has angle 180°
(d) The smallest slice has angle 90°

121. In an asymmetrical distribution, if the mean and median of the distribution are 270 and 220 respectively, then the mode of the data is **[2017-I]**

(a) 120 (b) 220
(c) 280 (d) 370

122. An individual purchases three qualities of pencils. The relevant data is given below : **[2017-I]**

Quality	Price per Pencil (in ₹)	Money spent (in ₹)
A	1.00	50
B	1.50	x
C	2.00	20

It is known that the average price per pencil is ₹ 1.25. What is the value of x ?

(a) ₹10 (b) ₹ 30
(c) ₹ 40 (d) ₹ 60

123. Consider the following frequency distribution :

x	Frequency	Cumulative frequency
1	8	8
2	10	18
3	f_1	29
4	f_2	45

What are the values of f_1 and f_2 respectively ?

(a) 10 and 17 (b) 17 and 10
(c) 11 and 16 (d) 16 and 11

124. What is the total number of boys studying Statistics and Physics? **[2017-II]**

(a) 180 (b) 240
(c) 250 (d) 310

125. The number of girls studying Statistics is what percent (approximate) of the total number of students studying Chemistry? **[2017-II]**

(a) 58.8 (b) 73.5
(c) 78.7 (d) 80.6

126. In which subjects is the difference between the number of boys and girls equal? **[2017-II]**

(a) Mathematics and Chemistry
(b) Statistics and Chemistry
(c) Mathematics and Physics
(d) Mathematics and Statistics

127. What is the difference between the number of boys studying Mathematics and the number of girls studying Physics? **[2017-II]**

(a) 20 (b) 30
(c) 60 (d) 80

128. What is the ratio of the total number of boys to the total number of girls? **[2017-II]**

(a) 67 : 83 (b) 17 : 26
(c) 27 : 19 (d) 189 : 179

129. Frequency density of a class is computed by the ratio **[2017-II]**

(a) Class frequency to the class width
(b) Class frequency to total frequency
(c) Class frequency to total number of classes
(d) Cumulative frequency up to that class to total frequency

130. A small company pays each of its 5 category 'C' workers ₹ 20,000, each of its 3 category 'B' workers ₹ 25,000 and a category 'A' worker ₹ 65,000. The number of workers earning less than the mean salary is **[2017-II]**

(a) 8 (b) 5
(c) 4 (d) 3

131. The pie diagrams on the monthly expenditure of two families A and B are drawn with radii of two circles taken in the ratio 16 : 9 to compare their expenditures.

Which one of the following is the appropriate data used for the above mentioned pie diagrams? **[2017-II]**

(a) ₹ 16,000 and ₹ 9,000 (b) ₹ 8,000 and ₹ 4,500
(c) ₹ 25,600 and ₹ 8,100 (d) ₹ 4,000 and ₹ 3,000

132. Consider the following statements :

Statement I : The value of a random variable having the highest frequency is mode.

Statement II : Mode is unique.

Which one of the following is correct in respect of the above statements? **[2017-II]**

(a) Both Statement I and Statement II are true and Statement II is the correct explanation of Statement I
(b) Bothe Statement I and Statement II are true but Statement II is not the correct explanation of Statement I
(c) Statement I is true but Statement II is false
(d) Statement I is false but Statement II is true

133. Which one of the following is not correct?

The proportion of various items in a pie diagram is proportional to the **[2017-II]**

(a) Areas of slices
(b) Angles of slices
(c) Lengths of the curved arcs of the slices
(d) Perimeters of the slices

134. The geometric mean of x and y is 6 and the geometric mean of x, y and z is also 6. Then the value of z is **[2017-II]**

(a) 12 (b) $\sqrt{6}$

(c) 6 (d) $\sqrt[3]{6}$

135. The total number of live births in a specific locality during different months of a specific year was obtained from the office of the Birth Registrar. This set of data may be called **[2017-II]**

(a) Primary data (b) Secondary data

(c) Recorded data (d) Countable data

136. The heights (in cm) of 5 students are 150, 165, 161, 144 and 155. What are the values of mean and median (in cm) respectively? **[2017-II]**

(a) 165 and 161 (b) 155 and 155

(c) 160 and 155 (d) 155 and 161

137. In the following table of inverse variation, what are the values of A, B and C respectively? **[2018-I]**

M	15	− 6	2	C
N	− 4	A	B	60

(a) 10, –30, –1 (b) 10, –1, 30

(c) –30, 10, –1 (d) –1, –30, 10

138. A Pie Chart is drawn for the following data : **[2018-I]**

Sector	Percentage
Agriculture and Rural Development	12.9
Irrigation	12.5
Energy	27.2
Industry and Minerals	15.4
Transport and Communication	15.9
Social Services	16.1

What is the angle (approximately) subtended by the Social Services Sector at the centre of the circle? **[2018-I]**

(a) 45° (b) 46°

(c) 58° (d) 98°

139. The arithmetic mean of two numbers is 10 and their geometric mean is 8. What are the two numbers? **[2018-I]**

(a) 15, 5 (b) 12, 8

(c) 16, 4 (d) 18, 2

140. The arithmetic mean of 11 observations is 11. The arithmetic mean of the first 6 observations is 10.5 and the arithmetic mean of the last 6 observations is 11.5. What is the sixth observation? **[2018-I]**

(a) 10.0 (b) 10.5

(c) 11.0 (d) 11.5

Consider the following for the next two (02) questions.

In a triangle ABC, a, b and c are the lengths of the sides and p, q and r are the lengths of its medians.

141. Which one of the following is correct? **[2018-I]**

(a) $(a + b + c) < (p + q + r)$

(b) $3 (a + b + c) < 4 (p + q + r)$

(c) $2 (a + b + c) > 3 (p + q + r)$

(d) $3 (a + b + c) > 4 (p + q + r)$

142. Data on ratings of hotels in a city is measured on **[2018-II]**

(a) Nominal scale (b) Ordinal scale

(c) Interval scale (d) Ratio scale

143. The median of 19 observations is 30. Two more observations are made and the values of these are 8 and 32. What is the median of the 21 observations? **[2018-II]**

(a) 32

(b) 30

(c) 20

(d) Cannot be determined due to insufficient data

144. As the number of observations and classes increases, the shape of a frequency polygon **[2018-II]**

(a) Tends to become jagged

(b) Tends to become increasingly smooth

(c) Stay the same

(d) Varies only if data become more reliable

145. Let $\overline{x}_1$ and $\overline{x}_2$ (where $\overline{x}_2 > \overline{x}_1$) be the means of two sets comprising n_1 and n_2 (where $n_2 < n_1$) observations respectively. If $\overline{x}$ is the mean when they are pooled, then which one of the following is correct? **[2018-II]**

(a) $\overline{x}_1 < \overline{x} < \overline{x}_2$ (b) $\overline{x} > \overline{x}_2$

(c) $\overline{x} < \overline{x}_1$ (d) $(\overline{x}_1 - \overline{x}) + (\overline{x}_2 - \overline{x}) = 0$

146. Consider the following statements: **[2018-II]**

Statement I : Median can be computed even when the end intervals of a frequency distribution are open.

Statement II : Median is a positional average.

Which one of the following is correct in respect of the above statements?

(a) Both Statement I and Statement II are true and Statement II is the correct explanation of Statement I.

(b) Both Statement I and Statement II are true and Statement II is not the correct explanation of Statement I.

(c) Statement I is true but Statement II is false

(d) Statement I is false but Statement II is true

147. Consider the following grouped frequency distribution : **[2019-I]**

x	f
0 - 10	8
10-20	12
20 - 30	10
30 - 40	p
40 - 50	9

If the mean of the above data is 25. 2, then what is the value of p ?

(a) 9 (b) 10

(c) 11 (d) 12

148. Consider the following frequency distribution : **[2019-I]**

x	f
8	6
5	4
6	5
10	8
9	9
4	6
7	4

What is the median for the distribution ?

(a) 6 (b) 7

(c) 8 (d) 9

149. Diagrammatic representation of data includes which of the following ? **[2019-I]**

1. Bar diagram
2. Pie-diagram
3. Pictogram

Select the correct answer using the code given below :

(a) 1 and 2 only (b) 2 and 3 only

(c) 1 and 3 only (d) 1, 2 and 3

150. The data collected from which one of the following methods is ***not*** a primary data ? **[2019-I]**

(a) By direct personal interviews

(b) By indirect personal interviews

(c) By schedules sent through enumerators

(d) From published thesis

151. The monthly expenditure of a person is ₹ 6, 000. The distribution of expenditure on various items is as follows : **[2019-I]**

	Item of expenditure	*Amount (in ₹)*
1.	Food	2,000
2.	Clothing	660
3.	Fuel and rent	1,200
4.	Education	480
5.	Miscellaneous	1,660

If the above data is represented by a percentage bar diagram of height 15 cm, then what are the lengths of the two segments of the bar diagram corresponding to education and miscellaneous respectively ?

(a) 1.25 cm and 5 cm (b) 1.2 cm and 4. 15 cm

(c) 1.2 cm and 3.5 cm (d) 4.15 cm and 6 cm

152. If the mean of m observations out of n observations is n and the mean of remaining observations is m, then what is the mean of all n observations ? **[2019-I]**

(a) $2m-\frac{m^2}{n}$ (b) $2m+\frac{m^2}{n}$

(c) $m-\frac{m^2}{n}$ (d) $m+\frac{m^2}{n}$

153. Which one of the following pairs is correctly matched ?

(a) Median – Graphical location

(b) Mean – Graphical location

(c) Geometric mean – Ogive

(d) Mode – Ogive

154. The following pairs relate to frequency distribution of a discrete variable and its frequency polygon. Which one of the following pairs is ***not*** correctly matched ? **[2019-I]**

(a) Base line of the polygon – X-axis

(b) Ordinates of the vertices of the polygon – Class frequencies

(c) Abscissa of the vertices of the polygon – Class marks of the frequency distribution

(d) Area of the polygon – Total frequency of the distribution

DIRECTIONS: *Read the following frequency distribution for two series of observations and answer the* ***two*** *items that follow* **[2019-II]**

Class interval	Frequency	
	Series-I	Series-II
10-20	20	4
20-30	15	8
30-40	10	4
40-50	x	2x
50-60	y	y
Total	100	100

155. What is the mean of frequency distribution of Series-I ?

(a) 33.6 (b) 35.6

(c) 37.6 (d) 39.6

156. What is the mode of the frequency distribution of Series-II?

(a) 26 (b) 36

(c) 46 (d) 56

DIRECTIONS: *Read the following information and answer the* ***four*** *items that follow .* **[2019-II]**

Let the distribution of number of scooters of companies X and Y sold by 5 showrooms (A, B, C, D and E) in a certain year be denoted by $S1$ and the distribution of number of scooters of only company X sold by the five showrooms in the same year be denoted by $S2$.

Showroom	A	B	C	D	E	Total number of scooters sold
S1(in%)	19	21	15	33	12	6400
S2 (in %)	24	18	20	30	8	3000

157. Number of scooters of company *Y* sold by showroom *E* is what per cent of the number of scooters of both companies sold by showroom *C* ?

(a) 52 (b) 54
(c) 55 (d) 56

158. Number of scooters of both the companies sold by showroom *B* is what per cent more than the number of scooters of company *X* sold by showroom *A* ?

(a) $78\frac{2}{3}$ (b) $83\frac{1}{3}$
(c) $86\frac{2}{3}$ (d) $88\frac{1}{3}$

159. What is the average number of scooters of company *Y* sold by the showrooms *A*, *C* and *E* ?

(a) $461\frac{1}{3}$ (b) $431\frac{1}{3}$
(c) $426\frac{1}{3}$ (d) $416\frac{1}{3}$

160. What is the difference between the number of scooters of both companies sold by showroom *A* and total number of scooters of company *X* sold by showrooms *B* and *E* together?

(a) 416 (b) 426
(c) 432 (d) 436

DIRECTIONS: *Read the following information and answer the **four** items that follow :* **[2019-II]**

The data shows that Indian roads are turning deadlier over the years.

Year	2014	2015	2016	2017
Number of bikers killed	40957	46070	52750	48746
Number of pedestrians killed	12330	13894	15746	20457
Number of cyclists killed	4037	3125	2585	3559

161. What was the average number of pedestrians killed per day in the year 2017?

(a) 51 (b) 53
(c) 54 (d) 56

162. What is the approximate percentage change in the pedestrians fatalities during the period 2014−17 ?

(a) 66% (b) 68%
(c) 71% (d) 76%

163. What is the average number of bikers killed daily in road accidents in the year 2017 ?

(a) 163 (b) 152
(c) 147 (d) 134

164. What is the average number of cyclists killed daily in road accidents in 2017 ?

(a) 10 (b) 12
(c) 19 (d) 21

165. What is the median of the data 3, 5, 9, 4, 6, 11, 18? **[2020-I]**

(a) 6 (b) 6.5
(c) 7 (d) 7.5

166. In a pi-diagram there are three sectors. If the ratio of the angles of the sectors is 1 : 2 : 3, then what is the angle of the largest sector? **[2020-I]**

(a) 200° (b) 180°
(c) 150° (d) 120°

DIRECTIONS: *for the following three (03) items: Read the following information and answer the three items that follow:*

[2020-I]

The following data presents count of released convicts who have served prison terms (*X*), those who have received some educational or technical training during their term (*Y*) and those who were offered Company placement (Z) respectively, from six different jails A, B, C, D, E and F, in the year 2010.

	X	Y	Z
A	86	45	25
B	1305	903	461
C	2019	940	474
D	1166	869	416
E	954	544	254
F	1198	464	174

167. Jails with highest and smallest percentage of trained convicts are respectively.

(a) F and D (b) D and F
(c) C and A (d) D and A

168. Jail with highest placement rate of trained convicts is

(a) F (b) D
(c) B (d) A

169. Jails from which more than half of the trained convicts are offered jobs, are

(a) A, B and C (b) A, B and D
(c) A, D and E (d) A, E and F

HINTS & SOLUTIONS

1. (d) The variables are: 210, 201, 102, 20, 12, 10, 2, 1 and 0.
∴ Geometric mean of variates
$= \sqrt[9]{210 \times 201 \times 102 \times 20 \times 12 \times 10 \times 2 \times 1 \times 0}$
$= \sqrt[9]{0} = 0$

2. (d) Quantitative is needed to enumerate the female population of India.

3. (a) Since, the mean of the variable X is M, then mean of the variable $Y = \frac{(X-4)}{5}$ is $\frac{M-4}{5}$.

4. (d) Mean of y and $\frac{1}{y} = M$

$\Rightarrow \frac{y + \frac{1}{y}}{2} = M \Rightarrow y + \frac{1}{y} = 2M$ ... (i)

Now, mean of y^3 and $\frac{1}{y^3}$ is

$$\frac{y^3 + \frac{1}{y^3}}{2} = \frac{\left(y + \frac{1}{y}\right)^3 - 3\left(y + \frac{1}{y}\right)}{2}$$

$$\Rightarrow \frac{y^3 + \frac{1}{y^3}}{2} = \frac{(2M)^3 - 6M}{2}$$

$$= \frac{(2M)\left[(2M)^2 - 3\right]}{2} = M(4M^2 - 3)$$

5. (b) n = 10, which is even ...[Given]
∴ Median (M) =

$$\frac{\text{Value of } \left[\left(\frac{n}{2}\right)\text{th} + \left(\frac{n}{2}+1\right)\text{th}\right] \text{ term}}{2}$$

$\Rightarrow 61.5 = \frac{\text{5th term} + \text{6th term}}{2}$

$\Rightarrow 61.5 = \frac{x + x + 3}{2} = \frac{2x+3}{2}$

$\Rightarrow 2x + 3 = 123$
$\Rightarrow 2x = 123 - 3 = 120$
$\therefore x = 60$

6. (a) For an ogive, the cumulative frequencies are plotted as a upper limit of class intervals.

7. (c): Here, maximum frequency is 80, hence mode will be between 15-20.

8. (a) Since, the frequency in a straight line, so we take all classes have equal frequency, i.e. 10
I. It is true that first and last class have 10 frequency.
II. Both the middle classes have 10 + 10 = 20.
III. Since, all have equal frequency, so we cannot determined the mode.

9. (a) ₹ 25 = 1 cm^2

∴ ₹ 1 = $\frac{1}{25}$ cm^2

∴ ₹ 81 = $\frac{81}{25}$ cm^2 = Area of square

Side of square = $\sqrt{\frac{81}{25}} = \frac{9}{5} = 1.8$ cm

10. (c) We know that:

$$\text{Central angle} = \frac{\text{Value of item}}{\underline{\text{Sum of values of items}}} \times 360°$$
x

$\therefore 36° = \frac{x}{40 + 15 + x + 12 + 23} \times 360°$

$\Rightarrow \frac{36°}{360°} = \frac{x}{90 + x}$

$\Rightarrow 90 + x = 10x$
$\Rightarrow 9x = 90$
$\therefore x = 10$

11. (c) Let the observation mean = x
∴ Sum of 50 observations = 50x
According to question,

$\therefore \frac{50x - 45}{49} = x$

$\Rightarrow 50x - 45 = 49x$
$\therefore x = 45$

12. (b) **For District A:**
Maximum frequency = 59
∴ Modal class = 44-47
∴ $l = 44, f_1 = 59, f_0 = 36, f_2 = 30, h = 3$

$\therefore \text{Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$

$= 44 + \frac{59 - 36}{2 \times 59 - 36 - 30} \times 3$

$= 44 + \frac{23 \times 3}{52} = 44 + 1.33 = 45.33$

For District B:
Maximum frequency = 54
∴ Modal class = 47-50
∴ $l = 47, f_1 = 54, f_0 = 35, f_2 = 41, h = 3$

$\therefore \text{Mode} = 47 + \frac{54 - 35}{2 \times 54 - 35 - 41} \times 3$

$= 47 + \frac{19 \times 3}{32} = 47 + 1.78 = 48.78$

Mode of District B > mode of District A.

13. (d) If every number of a finite set is increased by any number k, the measure of central tendency should also increase by k. Geometric mean does not have this property.
14. (d) Distributions are 1, 3, 5, 7, 9, x, 15, 17.
Total number of terms = 8
[even]
$\therefore$ Median

$$= \frac{\text{Value of } \frac{8}{2}\text{th term} + \text{Value of } \left(\frac{8}{2}+1\right)\text{th term}}{2}$$

$$= \frac{\text{Value of } 4^{th} \text{ term} + \text{Value of } 5^{th} \text{ term}}{2}$$

$$= \frac{7+9}{2} = \frac{16}{2} = 8$$

Now, the distribution is arranged in ascending order.
So, $9 \le x \le 15$.
15. (d) In statistics, a suitable graph for representing the partitioning of total into sub parts is a pie chart.
16. (a) According to property of arithmetic mean zero,
$\therefore x_1 f_1 + x_2 f_2 + ... + x_n f_n = 0$
17. (d) Median cannot be determined from the given data.
18. (c) The height of the bar is not proportional to the frequency of the class.
19. (a)

X	f	fx
0	4	0
1	f	f
2	9	18
3	g	3g
4	4	16

$\Sigma fx = f + 3g + 34$

$N = 25, \ \overline{X} = 2$
$f + g + 17 = 25$
$f + g = 8$
... (i)

$$\overline{X} = \frac{\Sigma fx}{N}$$

$$2 = \frac{f + 3g + 34}{25}$$

$25 \times 2 = f + 3g + 34$
$\Rightarrow f + 3g = 16$
... (ii)
From equation (i) and (ii),

$$\begin{array}{r} f + 3g = 16 \\ \underline{\underset{-}{f} + \underset{-}{\ \ g} = \underset{-}{8}} \\ 2g = 8 \end{array}$$

$$g = \frac{8}{2} = 4$$

f = 4, g = 4.

20. (c) To determine the population of 5 major states of India, the best suitable data is pie diagram.
21. (d) The volume of rainfall in certain geographical area, recorded every month for 24 consecutive months.
22. (d) Given that:
Let $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9$ and a_{10} be 10 numbers.

Then, $\frac{a_1 + a_2 + a_3 + ... + a_{10}}{10} = 20$

According to question,

$$= \frac{(2a_1 + 5) + (2a_2 + 5) + (2a_3 + 5) + ... + (2a_{10} + 5)}{10}$$

$$= \frac{(2a_1 + 2a_2 + 2a_3 + ... + 2a_{10}) + 5 \times 10}{10}$$

$$= \frac{2(a_1 + a_2 + a_3 + ... + a_{10})}{10} + \frac{50}{10}$$

$= 2 \times 20 + 5 = 45$
23. (c) Only I and III, mode is a suitable measure of central tendency.
24. (b) Total number of observations = 25
Sum of 25 observations = 25 × 36 = 900
Sum of first 13 observations = 13 × 32 = 416
Sum of last 13 observations = 13 × 39 = 507
$\because$ 13^{th} observation = 416 + 507 – 900
= 923 – 900 = 23
25. (c) Because there is a gap between two adjacent bars, so both the districts can be represented by bar chart.
26. (c) Total number of female coffee drinkers
= 5% of 10000 + 15% of 20000 = 500 + 3000 = 3500
27. (c) Let three positive integers whose median is 5 are 1, 5, 6

$$\therefore \text{ A.M.} = \frac{1+5+6}{3} = \frac{12}{3} = 4$$

28. (c) Mode can be obtained from a histogram.
29. (b) Statements I and III are primary data.
30. (a) G.M. = $\sqrt[3]{125 \times 729 \times 1331} = \sqrt[3]{5^3 \times 9^3 \times 11^3}$
= 5 × 9 × 11 = 495
31. (c) I. H.M. = $\frac{2 \times 8 \times 12}{8 + 12} = \frac{2 \times 8 \times 12}{20}$

$$\left(\because \text{harmonic mean (HM) of a and b} = \frac{2ab}{a+b}\right)$$

$$= \frac{48}{5} = 9.6$$

II. H.M. = $\frac{2 \times 9 \times 11}{9 + 11} = \frac{2 \times 9 \times 11}{20} = 9.9$

III. H.M. = $\frac{2 \times 6 \times 24}{6 + 24} = \frac{2 \times 6 \times 24}{30} = \frac{48}{5} = 9.6$

Thus, I^{st} and III^{rd} pairs have same harmonic means.

32. (b) Let the frequency of class interval 21 – 30 be f.
$9 + 22 + f + 20 + 12 + 8 = 100$
$\Rightarrow 71 + f = 100$
$\Rightarrow f = 100 - 71 = 29$
$\because$ Highest frequency = 29
$\therefore$ Modal class = Class corresponding to 29
$= 20.5 - 30.5$

33. (a) $\because$ Weighted mean

$$= \left(\frac{w_1 x_1 + w_2 x_2 + ... + w_{10} x_{10}}{w_1 + w_2 + ... + w_{10}}\right)$$

$$\bar{x}_n = \frac{1\times1+2\times2+...+10\times10}{1+2+3+...+10} = \frac{1^2+2^2+...+10^2}{1+2+...+10}$$

$$= \frac{385}{55} = 7$$

By using the formula,

$$\boxed{\Sigma n^2 = \frac{n(n+1)(2n+1)}{6} \text{ and } \Sigma n = \frac{n(n+1)}{2}}$$

$$= \frac{\frac{10(10+1)(20+1)}{6}}{10\left(\frac{10+1}{2}\right)} = \frac{10\times11\times21}{6\times55} = 7$$

Solution (34-35):
Total age of six persons = $23.5 \times 6 = 141$ years
Total age of three major persons = $42 \times 3 = 126$ years
$\therefore$ Total age of three minor children = $141 - 126 = 15$ years
The difference in ages of the three minor children is same.
Therefore, we take ages may be:
5, 5, 5; 3, 5, 7; 2, 5, 8 and 1, 5, 9
In all the cases, median will be 5 years.

34. (c) Mean age of minor children = $\frac{15}{3}$ = 5 years.

35. (b) Median age of minor children = 5 years.

36. (d) If we double each value of original frequency distribution, then mean, median and harmonic mean remain same. Hence, option (d) is correct.
Since, in case observation since arithmetic mean, median and harmonic mean is dependent of change of origin but if we multiply the frequency of same quantity, then, these are independent.

37. (d)

Year	Birth rate	Death rate	Growth Rate = B.R. – D.R.
1911 – 21	48.1	38.5	9.6
1921 – 31	46.4	36.3	10.1
1931 – 41	45.2	31.2	14.0
1941 – 51	39.9	27.4	12.5
1951 – 61	41.7	22.8	18.9
1961 – 71	41.1	35.9	5.2
1971 – 81	37.1	14.8	22.3

It is clear from the above table that minimum growth rate is 5.2 in the year 1961-1971.

38. (d) With the help of histogram, we can prepare frequency polygon, frequency curve and frequency distribution.

39. (c) Male migrants category in pie diagram

$$= \frac{45°}{200} \times 360° = 81°$$

Solutions (40-42):
$a < b < c$
Total numbers = 6
Increasing order a, a, b, b, c, c

$$\therefore \text{ Median} = \frac{\left(\frac{6}{2}\right)\text{th term} + \left(\frac{6}{2}+1\right)\text{th term}}{2}$$

$$= \frac{\text{3rd term} + \text{4th term}}{2}$$

$$2 = \frac{b+b}{2} = b$$

$$\text{Arithmetic mean} = \frac{a+a+b+b+c+c}{6}$$

$$\Rightarrow \frac{7}{3} = \frac{a+b+c}{3}$$

$\Rightarrow a + b + c = 7$
$\Rightarrow a + c = 7 - 2 = 5$
... (i)

$$\text{Geometric mean} = \left(a^2 \times b^2 \times c^2\right)^{\frac{1}{6}}$$

$$\Rightarrow 2 = (abc)^{\frac{1}{3}}$$

$\Rightarrow abc = 8$

$$\Rightarrow ac = \frac{8}{2} = 4$$

... (ii)

$$\Rightarrow c = \frac{4}{a}$$

From equation (i),

$$a + \frac{4}{a} = 5$$

$$\Rightarrow \frac{a^2+4}{a} = 5$$

$\Rightarrow a^2 - 5a + 4 = 0$
$\Rightarrow a^2 - 4a - a + 4 = 0$
$\Rightarrow a(a - 4) - 1(a - 4) = 0$
$\Rightarrow (a - 4)(a - 1) = 0$
if a = 1 then c = 4
a = 4 then c = 1
a = 1, c = 4 and b = 2

40. (b) Required sum = $2(a)^2 + 2(b)^2 + 2(c)^2$
$= 2(1)^2 + 2(2)^2 + 2(4)^2$
$= 2 + 8 + 32 = 42$

41. (d) The value of c is 4.

42. (d) Mode = 3 (Median) – 2 (Mean)

$= 3(2) - 2\left(\frac{7}{3}\right) = \frac{18-14}{3} = \frac{4}{3}$

43. (b)

x	f	xf
1	3	3
2	5	10
3	9	27
4	f_1	$4f_1$
5	2	10
Total	$19 + f_1$	$50 + 4f_1$

$\therefore$ Mean $= \frac{\Sigma x_i f_i}{\Sigma f_i}$

$\Rightarrow 2.96 = \frac{50 + 4f_1}{19 + f_1}$ [given]

$\Rightarrow 56.24 + 2.96 f_1 = 50 + 4f_1$

$\Rightarrow 6.24 = 1.04 f_1$

$\Rightarrow f_1 = 6$

44. (a) I. Let the numbers be 1, 3, 5. If each number is reduced by 2, then new number will be –1, 1, 3.

$\therefore$ G $= 3\sqrt{-1 \times 1 \times 3} = \sqrt{-3}$ does not exist

II. If we increased each number by 2 then the geometric mean of set is not increased by 2.

45. (b) **Cumulative Frequency:** The number of values less than the upper class boundary for the currect class. So, given class is less than type class.

46. (c) We know that histogram is an equivalent graphical representation of the frequency distribution and is suitable for continuous random variables, where, the total frequency of an interval is evenly distributed over the interval.
Hence, both the given statements are correct.

47. (b) Arrange in ascending order:

6, 7, 9, 11, 12, 15, 19

Now, n = 7 which is odd

$\therefore$ Median = Value of $\left(\frac{7+1}{2}\right)$th term

= Value of 4^{th} term = 11

48. (c) Given $(x_1, y_1), (x_2, y_2), ..., (x_n, y_n)$ are n pairs of positive numbers.

I. $M(x_n + y_n)$

$= \frac{y(x_1 + y_1) + (x_2 + y_2) + ... + (x_n + y_n)}{n}$

$= \frac{(x_1 + x_2 + ... + x_n) + (y_1 + y_2 + ... + y_n)}{n}$

$= M(x_n) + M(y_n)$

$\Rightarrow M(x_n + y_n) = M(x_n) + M(y_n)$

II. $G(x_n\, y_n) = \{(x_1 y_1)(x_2 y_2)(x_3 y_3)...(x_n y_n)\}^{\frac{1}{n}}$

$= (x_1 \cdot x_2 ... x_n)^{\frac{1}{n}} \cdot (y_1 \cdot y_2 ... y_n)^{\frac{1}{n}}$

$= G(x_n) \cdot G(y_n)$

$\Rightarrow G(x_n, y_n) = G(x_n) \cdot G(y_n)$

Thus, both statements are true.

49. (a) Arithmetic Mean of 100 numbers = 89.05
So, sum of the 100 numbers = 89.05 × 100 = 8905
Now,
Two numbers 92 and 83 has been read as 192 and 33.
Comparatively increased = (192 + 33) – (92 + 83)
= 225 – 175 = 50
Sum is decreased by 50.
So, sum of 100 numbers = 8905 – 50 = 8855
Again,

Average of 100 numbers $= \frac{8855}{100} = 88.55$

50. (b) Total expenditure = 6 + 7 + 3 + 4 + 2.5 + 1.5
= ₹ 24 lakh

$\therefore$ Central angle of largest component $= \frac{360°}{24} \times 7$
$= 105°$

51. (c) Difference between the expenditure of the largest and smallest component = 7 – 1.5 = 5.5

$\therefore$ Required angle $= \frac{5.5}{24} \times 360° = 82.5°$

Solutions (52-53):

Number of total bulbs = 100

$\therefore$ 7 + x + 40 + y + 10 + 2 = 100

$\Rightarrow$ x + y = 41 ... (i)

Life of bulbs (in hours)	Number of bulbs	Cumulative Frequency
8 – 13	7	7
13 – 18	x	7 + x
18 – 23	40	47 + x
23 – 28	y	47 + x + y
28 – 33	10	57 + x + y
33 – 38	2	59 + x + y
	N = 100	

The median life is 20 h, so median interval will be (18-23).

Here, $l = 18$, $\frac{N}{2} = 50$

c = 7 + x, f = 40, h = 5

$\therefore$ Median $= l + \frac{\left(\frac{N}{2} - C\right)}{f} \times h$

$\Rightarrow\ 20 = 18 + \dfrac{(50-7-x)}{40} \times 5$

$\Rightarrow\ 2 = \dfrac{50-7-x}{8}$

$\Rightarrow\ 16 = 50 - 7 - x$

$\Rightarrow\ x = 43 - 16$

$\Rightarrow\ x = 27$

52. (d) Missing frequency 'x' is 27
Put the value of x in equation (i), we get
$y = 41 - 27 = 14$

53. (c) Missing frequency 'y' is 14.

54. (c) **Statement I :**
A graph which displays the data by using vertical bars of various heights in rectangular shapes to represent frequencies. Such that there is no gap between consecutive bars and also the height of the rectangle.
Statement II :
The height of the rectangle is determined by the frequency of the class it represents.
So, both the statements are correct.

55. (c) Correct A.M. = $\dfrac{n\bar{x} - (\text{Sum of wrong observations}) + (\text{Sum of correct observations})}{n}$

$= \dfrac{10 \times 7.6 - 3 + 8}{10} = \dfrac{81}{10} = 8.1$

56. (a) Simple Bar Diagram represents only one variable. For examples: Sales, Production, Population, Figures, etc. For various years may be shown by simple bar charts. Since these are of the same width and vary only in heights (or lengths). It becomes very easy for readers to study the relationship. Simple bar diagrams are very popular in practice. A bar can be either vertical or horizontal; vertical bars are more popular.

57. (a)

58. (c) **Primary Data:**
(i) Primary data are always original as it is collected by the investigator.
(ii) Suitability of the primary data will be positive because it has been systematically collected.
(iii) Primary data are expensive and time consuming.
(iv) Extra precautions are not required.
(v) Primary data are in the shape of raw material.
(vi) Possibility of personal prejudice.
Secondary Data:
(i) Secondary data lacks originality. The investigator makes use of the data collected by other agencies.
(ii) Secondary data may or may not suit the objects of enquiry.
(iii) Secondary data are relatively cheaper.
(iv) It is used with great care and caution.
(v) Secondary data are usually in the shape of readymade products.
(vi) Possibility of lesser degree of personal prejudice.
So, both I and II are correct.

59. (d) Since, intelligence of students is an attribute, arithmetic mean is not suitable method.

60. (b) In an increasing order arrangements of observations, the median will not be affected when the smallest and the largest observations are removed.

61. (d) Geographical Data it is simply a collection of information that can describes objects and things with relation to space. So, population figure of a state classified into geographical data.

62. (c)

x	2	3	4	5	6
f	2	7	14	22	30
cf	2	9	23	45	75

Cumulative frequency corresponding to variate x = 5 is 45.

63. (d) $S = \{1, 2, 3, ..., n\}$
Statement I
Median : It is the middle term. So, $\dfrac{n+1}{2}$, here it is not define n is even or odd.
Hence, we cannot say that $\dfrac{n+1}{2}$ is median. It is not correct.
Mode : It is the value that appears most of ten in a set of data.
$S = \{1, 2, 3, ..., n\}$
Here, all elements in S have same frequency.
So, both the statements are not correct.

64. (d) Arithmetic mean, A = 14
Geometric mean, G = 12
We know that,
$G^2 = AH$
$\therefore\ (12)^2 = 14 \times H$
$\Rightarrow\ H = \dfrac{12 \times 12}{14} = \dfrac{72}{7}$

65. (d) Given data,

x	0	1	2	3	4
y	100	90	70	40	0

Here is only relation which satisfy the above data, i.e. $y = 100 - 5x - 5x^2$
(i) $x = 0,\ y = 100 - 0 - 0 = 100$
(ii) $x = 1,\ y = 100 - 5 - 5 = 100 - 10 = 90$
(iii) $x = 2,\ y = 100 - 10 - 20 = 100 - 30 = 70$
(iv) $x = 3,\ y = 100 - 15 - 45 = 100 - 60 = 40$
(v) $x = 4,\ y = 100 - 20 - 80 = 100 - 100 = 0$

66. (c) A diagram consisting of rectangles, whose area is proportional to the frequency of a variance and whose width is equal to the class interval.

67. (a) Given numbers are 10, 7, 8, 5, 6, 8, 5, 8 and 6
Arrange in ascending order
5, 5, 6, 6, 7, 8, 8, 8, 10
Total term, n = 9 (odd)
Now,

(i) Mean $= \dfrac{5+5+6+6+7+8+8+8+10}{9}$

$= \dfrac{63}{9} = 7$

(ii) Median $= \left(\dfrac{n+1}{2}\right)$th term $= \left(\dfrac{9+1}{2}\right)$th term

= 5th term = 7

(iii) Mode = 8 because of higher frequency term

∴ Mean = Median

68. (b) Given that, mean of 100 values is 45

∴ Sum of 100 values, i.e. $\sum_{i=1}^{100} x = 45 \times 100 = 4500$

According to condition,

$$\sum_{i=1}^{40}(x_i + 15) + \sum_{i=41}^{100}(x_i - 5)$$

$$= \sum_{i=1}^{40} x_i + 15 \times 40 + \sum_{i=41}^{100} x_i - 5 \times 60$$

$$= \left(\sum_{i=1}^{40} x_i + \sum_{i=41}^{100} x_i\right) + 600 - 300 = \sum_{i=1}^{100} x_i + 300$$

= 4500 + 300 = 4800

[from equation (i)]

∴ New mean $= \dfrac{4800}{100} = 48$

69. (a) **Continuous Variable :**

It is available whose value is obtained by measuring.

Examples:

(i) Height of students in class.

(ii) Weight of students in class.

(iii) Weight of a person.

Here, weight of a person is continuous variable while educational qualification of the person is Discrete Variables.

70. (b) Given the arithmetic mean of the above distribution = 3.1

x	f	fx
1	3	3
2	f	2f
3	6	18
4	5	20
5	3	15
	N = 17 + f	Σ fx = 56 + 2f

Arithmetic Mean $= \dfrac{\Sigma fx}{N}$

$3.1 = \dfrac{56 + 2f}{17 + f}$

$52.7 + 3.1f = 56 + 2f$

$1.1f = 3.3$

$f = \dfrac{3.3}{1.1} = 3$

71. (d) Given average of 6 letters = 10

$\Rightarrow \dfrac{u+v+w+x+y+z}{6} = 10$

According to the question,

$$\frac{z+60+u+10+v+20+w+30+x+40+y+50}{6}$$

$$= \frac{(u+v+w+x+y+z)+(60+10+20+30+40+50)}{6}$$

$$= \frac{u+v+w+x+y+z}{6} + \frac{210}{6} = 10 + \frac{210}{6} = 45$$

72. (a) $\dfrac{p+q+r+s+t+u+v}{7} = m$

$\Rightarrow p + q + r + s + t + u + v = 7m$

$\therefore (p - m) + (q - m) + (r - m) + (s - m) + (t - m) + (u - m) + (v - m)$

$= (p + q + r + s + t + u + v) - 7m$

$= 7m - 7m = 0$

Solutions (73-74):

Class interval	Frequency	Cumulative frequency
0 – 6	4	4
6 – 12	x	4 + x
12 – 18	5	9 + x
18 – 24	y	9 + x + y
24 – 30	1	10 + x + y
	20	

According to question,

Here, 10 + x + y = 20

$\Rightarrow x + y = 20 - 10$

$\Rightarrow x + y = 10$ ... (i)

73. (a) Median $= l + \left(\dfrac{\frac{n}{2} - cf}{f}\right) \times h$

$\Rightarrow 14.4 = 12 + \left[\dfrac{\frac{20}{2} - (4+x)}{5}\right] \times 6$

$\Rightarrow 14.4 = 12 + \dfrac{10 - 4 - x}{5} \times 6$

$\Rightarrow 14.4 - 12 = \dfrac{6 - x}{5} \times 6$

$\Rightarrow 2.4 = \dfrac{36 - 6x}{5}$

$\Rightarrow$ $12 = 36 - 6x$
$\Rightarrow$ $6x = 24$
$\therefore$ $x = 4$

74. (b) Now, putting the value of x in equation (i), then,
$4 + y = 10$
$\Rightarrow$ $y = 6$

$$\frac{x}{y} = \frac{4}{6} = \frac{2}{3}$$

$\Rightarrow$ $3x = 2y$

75. (a) Given that,
Number of male employees (M) = 45
Number of female employees (F) = 15
Mean salary of male employee $(\bar{x}_M)$ = ₹ 5000
Total number of employees = (M + F) = 45 + 15 = 60
Mean salary of employees $(\bar{x}_{MF})$ = ₹ 4800
Let mean salary of female employee is $\bar{x}_F$
By formula,

$$\bar{x}_{MF} = \frac{M\,\bar{x}_M + F\,\bar{x}_F}{(M+F)}$$

$$\Rightarrow 4800 = \frac{45 \times 5000 + 15 \times \bar{x}_F}{60}$$

$\Rightarrow$ $4800 \times 60 - 45 \times 5000 = 15 \times \bar{x}_F$
$\therefore$ $\bar{x}_F = 4800 \times 4 - 3 \times 5000$
$= 300(16 \times 4 - 50) = 300 \times 14 = 4200.$

76. (c) Given that, mean of 7 observations = 7

$$\Rightarrow \frac{1}{7}\sum_{i=1}^{7} x_i = 7 \Rightarrow \sum_{i=1}^{7} x_i = 49$$

... (i)
According to question,
Each observation is increased by 2. Then the new mean,

$$= \frac{1}{7}\sum_{i=1}^{7}(x_i + 2) = \frac{1}{7}\left(\sum_{i=1}^{7} x_i + 2 \times 7\right)$$

$$= \frac{1}{7}(49 + 14) = \frac{1}{7} \times 63 = 9$$

77. (b) **Discrete Variable:**
It is a variable whose value is obtained by counting.
Examples:
(i) Number of students present.
(ii) Number of red marbles in a jar.
(iii) Students' grade level.
Continuous Variable:
It is a variable whose value is obtained by measuring.
Examples:
(i) Height of students in class.
(ii) Weight of students in class.
(iii) Time it takes to get to school.
(iv) Distance travelled between class.
So, statement I and IV are examples of discrete variables.

Solutions (78-80):

78. (d)

C.I.	x midvalues (xi)	f	fxi
0 – 10	5	x	5x
10 – 20	15	x + 8	15x + 120
20 – 30	25	x – 2	25x – 50
30 – 40	35	x – 4	35x – 140
		N = 4x + 2	Σ fxi = 80x – 70

Arithmetic Mean = 17.8

$$17.8 = \frac{\Sigma fxi}{N} = \frac{80x - 70}{4x + 2}$$

$17.8(4x + 2) = 80x - 70$
$71.2x + 35.6 = 80x - 70$
$8.8x = 105.6$
$x = 12$

79. (b)

C.I.	f	C.F.
0 – 10	x = 12	12
10 – 20	x + 8 = 20	32
20 – 30	x – 2 = 10	42
30 – 40	x – 4 = 8	50
	N = 50	

$$\therefore \frac{N}{2} = \frac{50}{2} = 25$$

Hence, median class is 10-20.

80. (b) New, modal class is 10-20, because it has maximum frequency, i.e. 20.

81. (c) A frequency distribution is a summary of the data set in which the interval of possible values is divided into sub-intervals known as class.

82. (b) In a frequency polygon, a line graph is drawn by joining all the mid-points of the top of the bars of a histogram. It gives the idea about the shape of the data distribution.
The two end-points of a frequency polygon always lie on the x-axis.

83. (c) The decreasing order of mean are:
Arithmetic mean > Geometric mean > Harmonic mean

84. (a) **Statement I:**
The Arithmetic Mean is obtained by sum of all the elements of the data set and dividing by the number of elements and it is always uniquely defined.
Statement II:
The median is the middle element when the data set is arranged in order of magnitude.
Mean, Median and mid-range always exist and are unique.

85. (c) Geometric mean of $(x_1, x_2, x_3, ..., x_n)$

$= (x_1 \cdot x_2 \cdots x_n)^{\frac{1}{n}} = X$

Geometric mean of $(y_1, y_2, y_3, ..., y_n)$

$= (y_1 \cdot y_2 \cdots y_n)^{\frac{1}{n}} = Y$

$\therefore$ Geometric mean of $(x_1 y_1, x_2 y_2, ..., x_n y_n)$

$= (x_1 \cdot x_2 \cdots x_n)^{\frac{1}{n}} \times (y_1 \cdot y_2 \cdots y_n)^{\frac{1}{n}}$

$= (x_1 y_1 \cdot x_2 y_2 \cdots x_n y_n)^{\frac{1}{n}} = XY$

Geometric mean of $\left(\frac{x_1}{y_1}, \frac{x_2}{y_2}, \cdots \frac{x_n}{y_n}\right)$

$$= \frac{(x_1 \cdot x_2 \cdots x_n)^{\frac{1}{n}}}{(y_1 \cdot y_2 \cdots y_n)^{\frac{1}{n}}}$$

$$= \left(\frac{x_1}{y_1} \cdot \frac{x_2}{y_2} \cdots \frac{x_n}{y_n}\right)^{\frac{1}{n}} = \frac{X}{Y}$$

86. (b)

Income less than	Class interval	Number of persons	Frequency
1500	1250 – 1500	100	20
1250	1000 – 1250	80	10
1000	750 – 1000	70	15
750	500 – 750	55	23
500	250 – 500	32	20
250	0 – 250	12	12

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 250 + \left(\frac{20 - 12}{40 - 12 - 23}\right) \times 250$$

$$= 250 + \frac{8}{5} \times 250$$

$= 250 + 400 = 650$

So, the modal class is 500-750.

87. (c) **Statistical data:**
In statistics and quantitative research methodology, a data sample is a set of data collected and or selected from a different sources and good example of statistical data – A list of annual incomes of the members of a club.

88. (c) I. Mean of all observations $= \frac{2 \times 4 + 7 \times 4}{8} = 4.5$

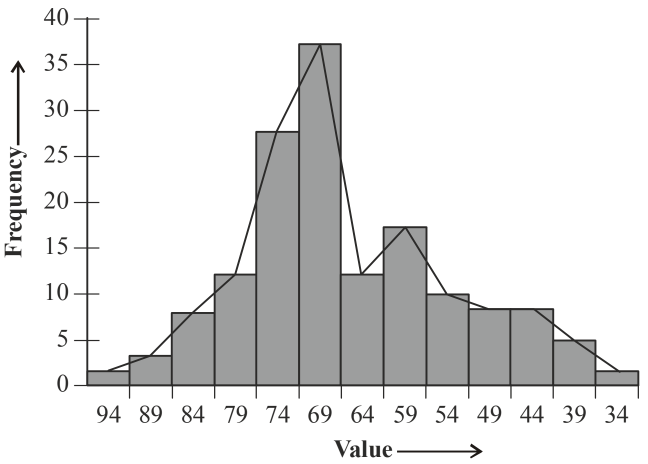
For median, first we arrange in ascending order = 2, 2, 2, 2, 7, 7, 7, 7

$\therefore$ Median $= \frac{4\text{th} + 5\text{th}}{2} = \frac{2+7}{2} = 4.5$

II. Mode is both 2 and 7, since frequency of occurrence is same, i.e. maximum frequency.

89. (c) **Statement I :**
Frequency polygons are a graphical device for understanding the shapes of distribution. They serve the same purpose as histograms. It is formed by joining the mid-points of histogram.
Statement II :
Frequency polygon touch the x-axis in its extreme left and extreme right of graph. See graph below:

Histogram / Frequency Polygon



90. (a)

Classes	Mid-values (x_i)	Frequency (f_i)	$d = x_i - 18$	$u_i = \frac{x_i - 18}{2}$	$f_i u_i$
11 – 13	12	3	–6	–3	–9
13 – 15	14	6	–4	–2	–12
15 – 17	16	9	–2	–1	–9
17 – 19	18	13	0	0	0
19 – 21	20	f	2	1	f
21 – 23	22	5	4	2	10
23 – 25	24	4	6	3	12
		40 + f			f – 8

Mean (x) = A + $\frac{\sum f_i u_i}{\sum f} \times h = 18 + \frac{f-8}{40+f} \times 2$

Given mean = 18

$18 = 18 + \frac{f-8}{40+f} \times 2$

$\Rightarrow$ 2f – 16 = 0 $\Rightarrow$ 2f = 16

$f = \frac{16}{2} = 8$

f = 8

91. (c) The modal class means that the class which has maximum frequency.
92. (a) Here, Statement 1 is correct but Statement 2 is not correct.
93. (c) Both Statements 1 and 2 are suitable for represention of a pie diagram.
94. (d) In bar diagram, the frequency is shown by the height of the bar whereas in histogram the frequency is shown by the area of the bar. So obtained graph is histogram.
95. (a) $\sum f_i(x_i - \bar{x}) = 0$ because sum of product of deviations and frequencies from mean value will be 0.
96. (c) Observations can by arranged in ascending order. 6, 14, 15, 17, x + 1, 2x – 13, 30, 32, 34 and 43.
Here, n = 10
[even]
$\therefore$ Median

$$= \frac{\text{Value of}\left(\frac{n}{2}\right)\text{th term} + \text{Value of}\left(\frac{n}{2}+1\right)\text{th term}}{2}$$

$$= \frac{\text{Value of}\left(\frac{10}{2}\right)\text{th term} + \text{Value of}\left(\frac{10}{2}+1\right)\text{th term}}{2}$$

$$= \frac{\text{Value of 5th term} + \text{Value of 6th term}}{2}$$

$$= \frac{x+1+2x-13}{2} = \frac{3x-12}{2}$$

But given, median = 24

$\therefore \frac{3x-12}{2} = 24$

$\Rightarrow$ 3x – 12 = 24 × 2 = 48

$\therefore$ 3x = 48 + 12

$\Rightarrow$ 3x = 60

$\therefore \Rightarrow x = 20$

Hence, the value of x is 20.

97. (a) $A \geq G \geq H$ between a and b
So, G is the Geometric mean between A and H.
98. (b) G.M. of a,b,c = 3

$\Rightarrow (abc)^{\frac{1}{3}} = 3 \Rightarrow abc = 27$

Also, a, b, c are in Geometric progression

So a = 1, b = 3, c = 9

Geometric mean of d,e,f = 4

$\Rightarrow (def)^{\frac{1}{3}} = 4$

$\Rightarrow def = 64$

Also, d, e, f are in Geometric progression

So, d = 2, e = 4, f = 8

Set = {1, 3, 9, 2, 4, 8}

Arithmetic mean

$= \frac{1+3+9+2+4+8}{6}$

$= \frac{27}{6} = \frac{9}{2} = 4.5 > 3.5$

So option (b) is correct

99. (b) Votes cast in favour of E
= 100000 – (36000 + 24000 + 18000 + 7000)
= 15,000
Angle allocated for party E in

Pie chart = $\frac{360°}{100000} \times 15000 = 54°$

Solutions (Qs. 100-103):

Class	Mid Values	Frequency F_i	$d_i = xi - 35$	$U_i = \frac{x_i - 35}{10}$	$f_i U_i$	Cumulative Frequency
0 – 10	5	4	–30	–3	–12	4
10–20	15	5	–20	–2	–10	9
20–30	25	7	–10	–1	–7	16
30–40	35	10	0	0	0	26
40–50	45	12	10	1	12	38
50–60	55	8	20	2	16	46
60–70	65	4	30	3	12	50

$$\sum F_i U_i = 11$$

$$N = \sum f_i = 50$$

$$\frac{N}{2} = \frac{50}{2} = 25$$

100. (a) Mean $= A + h\left[\frac{\sum fiUi}{N}\right]$

$= 35 + 10 \times \frac{11}{50} = 37.2$

101. (b) Median Class = 30 – 40.
l =30, F = 16, f =10, h = 10
102. (c) Median = $l + \frac{\frac{N}{2} - F}{f} \times h$

$= 30 + \frac{25-16}{10} \times 10 = 30 + 9 = 39.$

103. (d) Mode = $l+\frac{f-f_1}{2f-f_1-f_2}\times h$

Here, the maximum frequency is 12 their class is 40 – 50, then $l = 40$ $f = 12$, $f_1 = 10$, $f_2 = 8$

mode = $40+\frac{12-10}{2\times12-10-8}\times10$

$= 40+\frac{2}{6}\times10$ = 40 + 3.33 = 43.33

104. (b) Arranging the data in ascending order.

(a – 3.5), (a – 3), (a – 2.5), (a – 2), (a – 0.5), (a + 0.5), (a + 4), (a + 5).

Total terms = 8

$$\text{Medium} = \frac{\text{4th term} + \text{5th term}}{2}$$

$$= \frac{a-2+a-0.5}{2}$$

$$= \frac{2a-2.5}{2}$$

= a – 1.25

So, option (b) is correct.

105. (d) $(i+1)x^i = (i+1)x^i$ where i = 1, 2, 3, ____n

$$\sum_{i=1}^{n} i(i+1) = 1.2+2.3+3.4+4.5+\text{-------meters}$$

$$= \sum_{n=1}^{n} T_n$$

$$= \sum n(n+1)$$

$$= \sum n^2 + \sum n$$

$$= \frac{(n+1)n(2n+1)}{6}+\frac{n(n+1)}{2}$$

$$\text{Mean} = \frac{1}{n}\left[\frac{(n+1)n(2n+1)}{6}+\frac{n(n+1)}{2}\right]$$

$$= \frac{(n+1)}{2}\left[\frac{2n+1}{3}+1\right]$$

$$= \frac{(n+1)}{2}\frac{(2n+4)}{3}$$

$$= \frac{(n+1)(n+2)}{3}$$

So, option (d) is correct.

106. (a) Weighted arithmetic mean

$$= \frac{1\times1+2\times2+3\times3+4\times4+----+10\times10}{1+2+3+----10}$$

$$= \frac{(1)^2+(2)^2+(3)^2+----+(10)^2}{1+2+3+----10}$$

$$= \frac{(10)(10+1)(2\times10+1)/6}{(10\times11)/2}$$

$$= \frac{10\times11\times21\times2}{6\times10\times11}$$

= 7

So, option (a) is correct.

107. (b) In a pie chart

$$\text{Central angle} = \frac{\text{Frequency}}{\text{Total frequency}}\times360$$

Here central angle = 135°

Frequency i.e., secured votes = 21960

Find valid votes in total.

$$\Rightarrow\quad 135 = \frac{21960}{\text{total valid votes}}\times360$$

$$\Rightarrow\quad \text{total valid votes} = \frac{21960}{135}\times360$$

= 58560

∴ Option (b) is correct.

108. (a) Let 5 observations be x, y, z, p and q.

$$\text{Mean} = \frac{x+y+z+p+q}{5}$$

But mean = 9

$\Rightarrow$ x + y + z + p + q = 45

Also 1 is subtracted from each observation, then

$$\text{Mean} = \frac{(x-1)+(y-1)+(z-1)+(p-1)+(q-1)}{5}$$

$$= \frac{x+y+z+p+q-5}{5} = \frac{45-5}{5}$$

= 8

New mean = 8

Since n = 5 is odd

$$\text{Median} = \frac{n+1}{2} = \frac{5+1}{2} = 3$$

i.e., 3rd observation.

$\Rightarrow$ z is median

But median = p

$\Rightarrow$ z = p

New median = 8 – 1 = 7

Mean = 8

Median = 7

∴ Option (a) is correct.

109. (c) $\frac{N}{2}=\frac{40}{2}=20$

Age (in years)	No. of children	Cumulative frequency
5–6	4	4
6–7	7	11
7–8	9	20
8 9	12	32
9–10	6	38
10–11	2	40
Total	40	

Median class i.e, (7 – 8)

$$\text{Median} = \ell + \left(\frac{\frac{N}{2} - f_{pc}}{fm}\right) \times h$$

$$= 7 + \left(\frac{20-11}{9}\right) \times 1$$

$$= 7 + 1 = 8$$

Median age = 8 years

Hence, statement-1 is not correct.

Total number of childrens in the age group 6 – 9 years = 7 + 9 + 12 = 28 percentage of children in the age group 6 – 9 years.

$$= \frac{28}{40} \times 100 = 70$$

Hence statement-2 is correct.

Mode ℓ class is (8 – 9)

$$\text{Mode} = \ell + \left(\frac{f_m - f_{mp}}{2f_m - f_{mp} - f_{ms}}\right) \times h$$

$$= 8 + \left(\frac{12-9}{2 \times 12 - 9 - 6}\right) \times 1$$

$$= 8 + \frac{1}{3} = 8$$

(Nearest integer)

Hence model age = 8 years

Therefore statement-3 is correct.

110. (c)

111. (c) $x_i = \frac{1}{i}$ $i = 1, 2, 3 .... 11$

Then $x_1, x_2, x_3 .... x_{11}$

$$= 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, ... \frac{1}{11}$$

$\text{Median} = \frac{n+1}{2}$ Here n is odd

$\Rightarrow$ Median $\frac{11+1}{2}$ = 6th observation

$$\text{Median} = \frac{1}{6}$$

A.M. between four numbers

$$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}$$

$$\text{A.M} = \frac{1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}}{4} = \frac{\frac{12+6+4+3}{12}}{4} = \frac{25}{36} = 0.6$$

$\Rightarrow$ If A.M. of four numbers $7\frac{1}{6}$

$\Rightarrow$ If A.M. of Eleven numbers $7\frac{1}{6}$

$$\text{G.M. of two numbers} = \left(1 \times \frac{1}{2}\right)^{\frac{1}{2}} = \frac{1}{\sqrt{2}} = \frac{1}{1.414}$$

$\Rightarrow$ G.M. of 11 Numbers also $7\frac{1}{6}$

H.M. of Eleven numbers

$$\frac{11}{1+2+3+4+5+6+7+8+9+10+11}$$

$$= \frac{11}{66} = \frac{1}{6} = \text{Median}$$

$\therefore$ Option (c) is correct.

112. (a) Total score class x = 83x

class y = 76y

class z = 85 z

$$\text{Given} = \frac{83x + 76y}{x+y} = 79$$

$$83x + 76y = 79x + 79y$$

$$4x = 3y$$

$$\frac{x}{3} = \frac{y}{4} \quad ...(1)$$

$$\text{and } \frac{76y + 85z}{y+z} = 81$$

$$4z = 5y$$

$$\frac{y}{4} = \frac{z}{5} \quad ...(ii)$$

From (i) and (ii)

$$\frac{x}{3}=\frac{y}{4}=\frac{z}{5}$$

Let x, y and z be 3t, 4t and 5t

then

$$\frac{83x+76y+85z}{x+y+z}=\frac{83(3t)+76(4t)+85(5t)}{3t+4t+5t}$$

$=81.50$

113. (a) After an anging data in ascending order

$x-3.5, x-3, x-2.5. x-2\ x-0.5, x+0.5$

$x+4, x+5$

$\because$ Then number of term is even

$$\text{then average} = \frac{\frac{n}{2}^{th}\text{ term}+\left(\frac{n}{2}^{th}+1\right)\text{term}}{2}$$

$$=\frac{x+2+x-0.5}{2}=x-1.25$$

114. (d) let data be

a, b, c, d, 20.5, e, F, G, H

after increasing largest 4 no. by 2

a, b, c, d, 20.5, e + 2, F + 2, G + 2, H + 2

$\therefore$ Median remains the same

115. (c) Categorical data

116. (c) Required percentage $=\frac{108}{360}\times 100=30\%$

117. (c) Both the statements are correct.

118. (d) Density of CI $=\frac{30}{15-10}$

119. (a)

LI	X_1	$d_i = A - x_1$	$u_1=\frac{d_i}{20}$	F_1	F_1u_i
0 – 20	10	40	2	17	34
20 – 40	30	20	1	28	28
40 – 60	50	0	0	32	0
60 – 80	70	–20	1	F	F
80 – 100	90	–40	2	19	–38
				$\Sigma F_1 = 96 + F$	$24 - F$

$A = 50$

$$\bar{X} = A+\frac{\Sigma F_i u_i}{\Sigma F_i}\times h$$

$$50=50+\frac{24-f\times 20}{96+F}$$

$F = 24$

120. (c) Four slices 150°, 90°, 60°, 60°

when 60° is deleted

Remaining 150°, 90°, 60°

Total = 300°

While making Pie chart where 300° is taken as 100%

$$\frac{150}{300}\times 100=50\%$$

$$\frac{90°}{300}\times 100=30\%$$

$$\frac{60°}{300}\times 100=20\%$$

50% of 360° will be 180°

$\therefore$ largest slice as angle – 180°

121. (a) Mode = 3 median – 2 mean

mode = 3 (220) – 2 (270)

= 660 – 540 = 120

122. (b) Number of Type A pencil $=\frac{50}{1}=50$

Number of Type B pencil $=\frac{x}{1.50}$

Number of Type C pencil $=\frac{20}{2}=10$

$$\text{Average}=\frac{\text{Total money spent}}{\text{total no. of pencil}}=1.25$$

$$=\frac{x+50+20}{50+10+\frac{x}{1.50}}=1.25$$

$$=70+x=1.25\left(60+\frac{x}{1.50}\right)$$

$$70+x=75.00+\frac{1.25}{1.50}x$$

$$x-\frac{125}{150}x=5$$

$$\frac{25}{150}x=5$$

$x=30$

123. (c)

X	F	CF
1	8	8
2	10	18
3	F_1	29
4	F_2	45

$F_1 = 29 - 18 = 11$

$F_2 = 45 - 29 = 16$

124. (c) because $180 + 70 = 250$

125. (b) According to the question,

$$250 = x\% \text{ of } 340 \Rightarrow x = \frac{250 \times 100}{340} = 73.5\%$$

126. (c) Difference in the number of boys and girls studying Mathematics $= 150 - 90 = 60$

Difference in the number of boys and girls studying Physics $= 180 - 120 = 60$

127. (b) Difference in the number of boys studying Mathematics and Physics $= 180 - 150 = 30$

128. (a) Total number of boys $= 150 + 180 + 70 + 136 = 536$

Total number of girls $= 90 + 120 + 250 + 204 = 664$

Ratio of number of boys to number of girls

$$= \frac{536}{664} = \frac{67}{83}$$

129. (a)

130. (a) Mean salary $= \dfrac{20000 \times 5 + 25000 \times 3 + 65000}{9}$

$$= \frac{100000 + 75000 + 65000}{9}$$

$$= \frac{240000}{9} = 26666.6$$

The number of workers earning salaries less than the mean salary are category 'B' and 'C' workers

i.e. $5 + 3 = 8$.

131. (a) $16000 : 9000 = 16 : 9$

132. (c) as mode can be more than one.

133. (d) as perimeter of the complete pie diagram is equal to the circumference which gives the length of the arcs of each slice.

134. (c) We know that geometric mean of two numbers $= \sqrt{xy}$, where x and y are two numbers.

This means, $\sqrt{xy} = 6$ (given)

$\sqrt{xy} = 6 \Rightarrow xy = 36$

Now, we have geometric mean of three numbers $= \sqrt[3]{xyz}$, where x, y and z are three numbers.

This means, $\sqrt[3]{xyz} = 6$ (given)

$\sqrt[3]{xyz} = 6 \Rightarrow \sqrt[3]{36z} = 6 \Rightarrow 36z = 6^3 = 216$

$$\Rightarrow z = \frac{216}{36} = 6$$

135. (b) as secondary data is the data collected from sources other than the user itself.

136. (b) Mean = sum of all the observations ÷ total number of observations

$$= \frac{150 + 165 + 161 + 144 + 155}{5} = \frac{775}{5} = 155$$

Median = the middle value after arranging in ascending or descending order in case of odd number of observations

Arranging in ascending order, we get

144, 150, 155, 161, 165

Therefore, median = 155

137. (a)

M	15	$15 \div (-2.5) = -6$	$-6 \div (-3) = 2$	C = $2 \div (-2) = -1$
N	–4	A = $-4 \times (-2.5) = 10$	B = $10 \times (-3) = -30$	$-30 \times (-2) = 60$

Hence, values of A, B and C are 10, –30 and –1 respectively.

138. (c) Required angle $= \dfrac{16.1}{100} \times 360 = 57.96 \approx 58°$

139. (c)

140. (c) Sum of 11 observations $= 11 \times 11 = 121$

Sum of first 6 observations $= 10.5 \times 6 = 63$

Sum of last 6 observations $= 11.5 \times 6 = 69$

Value of 6th observations $= 69 + 63 - 121 = 132 - 121 = 11$

141. (b)

142. (b) Data on ratings in a city is measured on ordinal scale.

143. (b) Added values are 8 and 32 median is 30.
Here, $8 < 30 < 32$.

So, Median of 21 observation = $(11)^{th}$ tern = 10^{th} term of 19 observations of given date.
Hence, median will be unchenged.

144. (b) As the mumber of observations and classes increases, the shape of frequency polygon tends to become increasingly smooth.

145. (a) Let if possible $\overline{x}_1 < \overline{x} < \overline{x}_2$
Consider
$\overline{x}_1 < \overline{x}$
$$\overline{x}_1 < \frac{n_1\overline{x}_1 + n_2\overline{x}_2}{n_1 + n_2}$$
$n_1\overline{x}_1 + n_2\overline{x}_1 < n_1\overline{x}_1 + n_2\overline{x}_2$
$\Rightarrow \quad \overline{x}_1 < \overline{x}_2$
[it is true as given in Question]
Hence $\overline{x}_1 < \overline{x}$
...(1)
Consider $\overline{x} < \overline{x}_2$
$$\frac{n_1\overline{x}_1 + n_2\overline{x}_2}{n_1 + n_2} < \overline{x}_2$$
$n_1\overline{x}_1 + n_2\overline{x}_2 < n_1\overline{x}_2 + n_2\overline{x}_2$
$\Rightarrow \overline{x}_1 < \overline{x}_2$ [True as given in Question]
Hence $\overline{x} < \overline{x}_2$
From (1) & (2)
$\overline{x}_1 < \overline{x} < \overline{x}_2$

146. (a) Median is a positional averages so it can be computed even when the end intervals of a frequency distribution are open.
Here both statements are correct and statement II is correct explanation of statement I.

147. (c) Mean = (sum of ∞) / (sum of f) = (5*5 + 12*15 + 10*25 + p*35 + 9*45) / (8 + 12 + 10 + P + 9) = 25.2 (875 + 35P) / (39 + P) = 25.2 $\Rightarrow$ P = 11

148. (c) Summation of frequencies = 6 + 4 + 5 + 8 + 9 + 6 + 4 = 42 Median = mid value = average of 21^{st} and 22^{nd} value Arranging data in increasing order we get

x	***f***
4	6
5	4
6	5
7	4
8	6
9	9
10	8

So mid value i.e 21^{st} and 22^{nd} value = 8

149. (d) All three are types of data representation
Pictogram uses pictures so show different identities with different numbers

150. (d) Primary data is information that you collect specifically for the **purpose** of your research project. An advantage of primary data is that it is specifically tailored to your research needs. A disadvantage is that it is expensive to obtain.

151. (b) 15 cm corresponds to ₹6000
Education = 480/6000 * 15 cm = 1.2 cm
Miscellaneous = 1660/6000 * 15cm = 4.15 cm

152. (a) Mean of m observations out of n observation is n
$\therefore$ Sum of m observation = $m.n$
Number of remaining observation = $(n - m)$
Mean of remaining observation = m
$\therefore$ Sum of remaining observation $= (n - m).m = mn - m^2$
Sum of all n observations
$= mn + mn - m^2 = 2mn - m^2$
$\therefore$ Mean of all n observation
$$= \frac{2mn - m^2}{n} = 2m - \frac{m^2}{n}$$

153. (a) Median can be traced using frequency polygon curve. It has a graphical location on the curve. Ogive is a graph showing a situation such as the number of hour students study. It is a cumulative frequency curve. Data values are shown on the x-axis while cumulative frequency are shown on y-axis. Thus, Ogive does not show mean or mode. Hence option (a) is correct.

154 (d) Area of the polygon gives sum of $f_i x_i$ not sum of frequency distribution (f_i).

155. (c) x + y = 100 – (20 + 15 + 10) = 55
2x + y = 100 – (4 + 8 + 4) = 84
$\Rightarrow$ x = 29 and y = 26
Max. of freg. distribution of series I
$$= \frac{(15\times20)+(25\times15)+(35\times10)+(45\times29)+(55\times26)}{100}$$
$$= \frac{(300+375+350+1305+1430)}{100} = \frac{3760}{100} = 37.6$$

156. (c) Mode of frequency distribution of series II is 46.

157. (c) No. of scooters of Y sold by E
= 12% × 6400 – 8% × 3600
= 528
No. of total scooters by showroom
c = 15% × 6400 = 960
$$\text{Reqd. \%} = \frac{528}{960}\times100 = 55\%$$

158. (c) Number of scooters of both company sold by showroom B
$$= 6400 \times \frac{21}{100} = 1344.$$
Number of scooter of company x sold by showroom A
$$= 3000\times\frac{24}{100} = 720$$
$$\text{Required percentage} = \left(\frac{1344 - 720}{720}\right)\times100 = 86\frac{2}{3}\%$$

159. (a) Reqd. Avg.
$$= \frac{(19+15+12)\%\times6400 - (24+20+8)\%\times3000}{3}$$
$$= \frac{46\times64 - 52\times30}{3} = \frac{1384}{3} = 461\frac{1}{3}$$

160. (d) Number of scooters of both company x and y sold by showroom A
$$6400\times\frac{19}{100} = 1216$$

Number of scooter of company X sold by B and E together $= 3000 \times \left(\frac{18+8}{100}\right) = 780$

Difference in number $= 1216 - 780 = 436$

161. (d) Average number of pedestrians killed per day in year 2017 $= \frac{20457}{365} \approx 56.$

162. (a) Pedestrians fatalities in 2014 = 12330
Pedestrians fatalities in 2017 = 20457

Percentage change $= \left(\frac{20457-12330}{12330}\right) \times 100$
$= 66\%$

163. (d) Average number of bikers killed daily in road accident in the year 2017

$= \frac{48746}{365} \approx 134.$

164. (a) Average number of cyclists killed daily in ground accident in 2017 $= = \frac{3559}{365} \approx 10.$

165. (a) Arrange the number assending order
3, 4, 5, 6, 9, 11, 18
Therefore 4^{th} term out of 7 is Median.
$\therefore$ Median = 6

166. (c) Sum of angles of a pie diagram = 360°
$\therefore$ $\theta + 2\theta + 3\theta = 360°$
$6\theta = 360°$
$\theta = 60°$
Hence, angle of Largest sector
$6\theta = 3 \times 60°$
$= 180°$

167. (b) Jail A $= \frac{45}{86} = 52.3\%$

Jail B $= \frac{903}{1305} = 69.19\%$

Jail C $= \frac{940}{2019} = 46.55\%$

Jail D $= \frac{869}{1166} = 74.45\%$

Jail E $= \frac{544}{954} = 57.02\%$

Jail F $= \frac{464}{1198} = 38.73\%$

Hence, Jail with highest and lowest percentage of frained convicts are from jail D and jail F respectively.

168. (d) Highest placement rate of trained convicts are from jail A

$= \frac{25}{45} = 55.56\%$

169. (a) Jail A $= \frac{25}{45} \times 100$
$= 55.56\%$

Jail B $= \frac{461}{903} = 51.50\%$

Jail C $= \frac{474}{940} = 50.04\%$

Jail D $= \frac{416}{869} = 47.87\%$

Jail E $= \frac{254}{544} = 46.69\%$

Jail F $= \frac{174}{464} = 37.5\%$

Hence, Jail A, B and C are the required answer.